CS112 Data Structures - Final - Spring 2022

Name: _____ NetID: _____

- **WRITE your name and NetID on EVERY page.**
- **DO NOT REMOVE** the staple/crimp in your exam.
- **DO NOT BEGIN** until instructed to do so.
- WRITE NEATLY AND CLEARLY. If we cannot read your handwriting, you will not receive credit. Please plan your space usage. No additional paper will be given.
- This exam is worth 150 points.

Problem 1 – Miscellaneous (40 points)

1. (10 points) For this problem write the fastest algorithm (measured by worst-case big O). Describe an algorithm (give concise steps, do not write code) that given an unsorted array of length *m* and a sorted array of length *n*, finds the common items. Assume there are no duplicates in either array and, *m* is much larger than *n*. What is the worst-case big O running time?

   5 points. Algorithm
   - For each item in m array, do a binary search in n array
   5 points. O(m log n)

   [If they sort the first array with an O(m log m) algo, it's not fastest,
   since m is much bigger than n
   And since this is a simple problem, no partial credit.]

2. **(6 points)** In the worst case, how many compares, in big O notation, to insert in a BST? Explain.
   3 points. O(n)
   3 points. In the worst case each node is to the left (or each node to the right) of its parent forming a list of n nodes.

3. **(6 points)** In the worst case, how many compares, in big O notation, to insert in a d-way heap? Explain.
   3 points. O ($\log_d$ n)
   3 points. Each node has d children therefore the tree height is $\log_d$ n. During insert a new key is compared against the parent (1 compare) and can swim from the leaf level to the root.

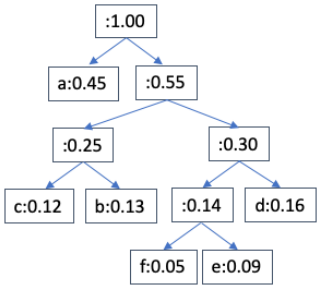
4. (18 points) On the Huffman Coding assignment the `TreeNode` class houses a `CharFreq` object "data" representing a certain character and its frequency, and `TreeNodes` "left" and "right" representing the left and right subtrees of the binary tree.

Name: _____ NetID: _____

```java
public class TreeNode {
    private CharFreq data;
    private TreeNode left;
    private TreeNode right;

    // We can create with data and both children
    public TreeNode(CharFreq d, TreeNode l, TreeNode r) {
        data = d;
        left = l;
        right = r;
    }

    // We can create with only data, children are null
    public TreeNode(CharFreq d) { this(d, l: null, r: null); }

    // No arguments sets everything to null
    public TreeNode() { this(d: null, l: null, r: null); }

    // Getters and setters
    public CharFreq getData() { return data; }
    public TreeNode getLeft() { return left; }
    public TreeNode getRight() { return right; }

    public void setData(CharFreq d) { data = d; }
    public void setLeft(TreeNode l) { left = l; }
    public void setRight(TreeNode r) { right = r; }
}
```

```java
public class CharFreq implements Comparable<CharFreq> {
    private Character character;
    private double probOcc;

    // We can set both the Character and double at once
    public CharFreq(Character c, double p) {
        character = c;
        probOcc = p;
    }

    // No arguments makes a null character and prob 0
    public CharFreq() { this(c: null, p: 0); }

    // Allows us to use Collections.sort() to sort by probOcc
    public int compareTo(CharFreq cf) {
        Double d1 = probOcc, d2 = cf.probOcc;
        if (d1.compareTo(d2) != 0) return d1.compareTo(d2);
        return character.compareTo(cf.character);
    }

    // Getters and setters
    public Character getCharacter() { return character; }
    public double getProbOcc() { return probOcc; }

    public void setCharacter(Character c) { character = c; }
    public void setProbOcc(double p) { probOcc = p; }
}
```

Write the following **RECURSIVE** `print` method that prints in preorder the probabilities in the Huffman Tree.



The `print` method prints the probabilities in this tree as follows:
1.0 0.45 0.55 0.25 0.12 0.13 0.30 0.14 0.05 0.09 0.16

```java
// Prints the probabilities according to an preorder
// traversal of the tree

private static void print (TreeNode root) {
      // COMPLETE THIS RECURSIVE METHOD
}
```

```java
private static void print(TreeNode root) {
    if ( root == null ) return; // 2 point for base case

    // 4 points for printing before left and right calls
    System.out.println(root.getData().getProbOcc() + " ");

    // 4 points for going left before going right
    print (root.getLeft());  // 4 points
    print (root.getRight()); // 4 points
}
```

Name: _____ NetID: _____

Problem 2 - Priority Queue (16 points)

Suppose that a MAX heap is implemented using an array. The heap is used to keep track of the frequency of the words in a book, the word that appears most often in the book has the highest frequency and is located at the root (array index 1).

During input every word is inserted into the heap as follows: a word is read from the book and then the word is searched in the heap:

(i) if the word is not found it is then inserted into the heap with frequency 1.
(ii) if the word is found its frequency is increased by 1.

Answer the following questions.

a. **(4 points)** What is the worst-case running time, in big O notation, of searching for a word in the heap? Explain.

2 points. O(n)
2 points. Heaps have no symmetric order (inorder traversal yields words in ascending order) so linear search is performed.

b. **(6 points)** Assume that a word has been searched and determined that it is not in the heap. What is the worst-case running time, in big O notation, of inserting a new word into the heap (after heap invariants are restored)? Explain.

3 points. O (1)
3 points. A word is inserted with frequency 1 at the first empty array slot from left to right. The MAX-heap order invariant is not violated because the parent word will not have a smaller frequency than 1.

c. **(6 points)** Assume that a word has been searched and determined that it is in the heap. What is the worst-case running time, in big O notation, to update a word's frequency (heap invariants are expected to be restored after insert is complete)? Explain.

3 points. O (log n)
3 points. Suppose that there are n words in the heap. All words have the same frequency. When a word at the leaf level has its frequency increased it can swim all the way to the root.

Name: _____ NetID: _____

Problem 3 - Hash table (20 points)

The following keys will be inserted in sequence to a hash table. For simplicity, we omit the "values" associated with the keys.

14 8 27 10 15 90 11 7 12 17

1. **(16 points)** Assume the Separate-Chaining Symbol Table API discussed in class is used. The table size is denoted by `m` and the hash function is `hash(key) = key % m`. The initial table size is 3. *Note that keys are inserted at the front of the list*. The threshold of the load factor is 2. So, when the load factor is larger than 2, rehashing should be performed. Suppose we would double the table size when we do rehashing. Show the contents of the two hash tables before rehashing and after rehashing.

   Before rehashing
   [0] ---> 90 ---> 15 ---> 27
   [1] ---> 10
   [2] ---> 8 ---> 14
   // 1 point for each key

   After rehashing
   [0] ---> 12 ---> 90
   [1] ---> 7
   [2] ---> 14 ---> 8
   [3] ---> 27 ---> 15
   [4] ---> 10
   [5] ---> 17 ---> 11
   // 1 points for each key

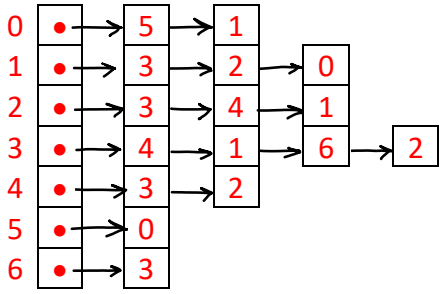
2. **(4 points)** What is the running time (big O) for rehashing given the input size n? Give the reasoning.

   1 point. O(n).
   3 points. Each of the n keys will need to be remapped to the new table

Name: _____ NetID: _____

Problem 4 - Graph (40 points)

1. **(6 points)** Assume an **<u>undirected</u>** graph G with 7 vertices and 8 edges (*v*,*w*) (an edge between vertices *v* and *w*). Based on the adjacency list Java implementation discussed in class where a *new edge is added to front of list*. If the list of 8 edges (0, 1) (1, 2) (2, 4) (2, 3) (3, 6) (5, 0) (3, 1) (3, 4) were added in sequence to construct the graph G, show the vertex-indexed array of lists.



// 1 point for each index 0-4
// 0.5 point for each 5 and 6

2. Answer the following questions based on the adjacency list in problem 4.1.
   a. (2 points) If a single for loop is used to iterate over the list of vertices adjacent to a given vertex v, what is the number of iterations? degree(v) OR the number of adjacent vertices to v OR the number of neighbors to the vertex v

   b. (3 points) Given the code below, what is the number of times StdOut.println statement is executed, assuming G is the **<u>undirected graph</u>** constructed in problem 4.1?
   2*8 times OR 2E // G.E() = 8;

```
for ( int v = 0; v < G.V(); v++ )
    for ( int w : G.adj(v) )
        StdOut.println( v + "-" + w );
```

3. Answer the following questions based on the Java code below and the adjacency list representation of graph G constructed in problem 4.1.

```
private void dfs(Graph G, int v) {
    marked[v] = true;
    for (int w : G.adj(v)) {
        if (!marked[w]) {
            edgeTo[w] = v;
            dfs(G, w);
        }
    }
}
```

Name: _____ NetID: _____

a. (5 points) Write the sequence of vertices visited of a method call to dfs(G, 0).

0 -> 5 -> 1 -> 3 -> 4 -> 2 -> 6

G.adj(v) returns the vertices adjacent to vertex v in the same order of the linked list.

No partial credit.

b. (5 points) write the contents of the array edgeTo[] as a consequence of the method call in 3.a.

0	1	2	3	4	5	6
-	0	4	1	3	0	3

// if all correct 5 points

// otherwise 0.8 point for each index 0 – 6

4. Answer the following questions based on the Java code below and the adjacency list representation of graph G constructed in problem 4.1.

a. (5 points) Write the sequence of vertices visited of a method call to bfs(G, 0).

0 -> 5 -> 1 -> 3 -> 2 -> 4 -> 6 No partial credit.

```
private void bfs(Graph G, int s) {
        Queue<Integer> q = new Queue<Integer>();
        distTo[s] = 0;
        marked[s] = true;
        q.enqueue(s);

        while (!q.isEmpty()) {
            int v = q.dequeue();
            for (int w : G.adj(v)) {
                if (!marked[w]) {
                    edgeTo[w] = v;
                    distTo[w] = distTo[v] + 1;
                    marked[w] = true;
                    q.enqueue(w);
                }
            }
        }
    }
```

b. (5 points) Write the contents of the array edgeTo[] as a consequence of the method call in 4.1.

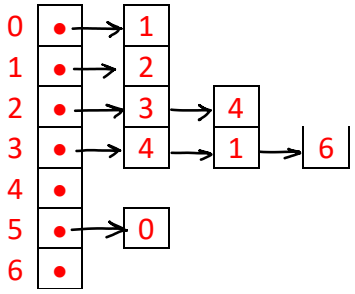
0	1	2	3	4	5	6
-	0	1	1	2	0	3

// 1 point for each index 1 – 6

Name: _____ NetID: _____

5. Assume a **directed** graph G with 7 vertices and 8 edges (*v*,*w*) (an edge between vertices *v* and *w*). Based on the adjacency list Java implementation discussed in class where a *new edge is added to front of list*.

a. (6 points) If a list of 8 edges (0, 1) (1, 2) (2, 4) (2, 3) (3, 6) (5, 0) (3, 1) (3, 4) with 7 vertices were added in sequence to construct the graph G, show the vertex-indexed array of list.



// 1 point for indexes 0, 1, 2, 3, 5
// 0.5 points indexes 4, 6

b. (3 points) What is the maximum number of edges one can add to the directed graph in 5.1?

7 * (7 -1) = 42

The maximum number of edges in a directed graph is v * (v-1), there are 7 edges in the graph.

Name: _____ NetID: _____

Problem 5 - Sorts (34 points)

1. **(5 points)** Which sorting algorithm would you use, insertion sort, selection sort, merge sort or quick sort, to sort a large array that is known to be almost sorted? Justify your answer.

   2 points. Insertion sort
   3 points. Because the best-case scenario for insertion sort is O(n) when the array is already sorted.

2. **(2 points)** Which sorting algorithm sorts an array by cutting the array in half, recursively sorting each half, and then merging the sorted halves?

   merge sort.

3. **(3 points)** Explain how does merge sort compare to heap sort, in big O notation, with respect of storage consumption?
   Merge sort uses O(n) extra storage to sort an array while heap sort does not use extra storage.
   OR
   Merge sort uses 2n storage to sort an array while heap sort uses n.
   OR
   Merge sort uses n+n/2 storage to sort an array while heap sort uses n.

4. **(24 points)** Trace the quicksort algorithm on the following array. Use the first item as the pivot when doing a split.

   25, 8, 89, 28, 15, 9, 2

   a. **(18 points)** Show the series of item swaps that are performed in the FIRST split process, and the array after each of these swaps, up to and including the step of placing the pivot at its correct location. You only need to show the FIRST split of the original array.

```
                  25   8   89   28    15    9    2
Swap 89,2  ->    25   8   2    28    15    9    89 (4 pts)
Swap 28,9  ->    25   8   2    9     15    28   89 (4 pts)
Swap 25,15 ->    15   8   2    9     25    28   89 (4 pts)
```

   b. **(12 points)** Show the full recursion tree, i.e. all splits, on original array and all subarrays.

```
25   8   89    28    15    9    2

                |
                V

                ----
```

Name: _____ NetID: _____