Problem 1

Problem a
Newton's Method: 0.7148059123627707
Secant Method: 0.7148059123554098
Bisection Method: 0.7148056030273438

Problem b
Newton's Method: 0.2
Secant Method: 0.19999999999114973
Bisection Method: 0.19999961853027343

Problem c
Newton's Method: 2.0945514815423265
Secant Method: 2.094551481569724
Bisection Method: 2.0945506572723387

Problem d
Newton's Method: 0.6931471805600257
Secant Method: 0.6931471805757405
Bisection Method: 0.6931467056274416

Problem e
Newton's Method: 0.567143290409784
Secant Method: 0.5671432904420043
Bisection Method: 0.5671438217163085

Problem f
Newton's Method: 1.134724138401536
Secant Method: 1.134724138401522
Bisection Method: 1.134725570678711

Problem g
Newton's Method: 0.8767262153950625
Secant Method: 0.8767262153942947
Bisection Method: 0.8767266273498535

Problem h
Newton's Method: 1.2599210498948732
Secant Method: 1.2599210498948596
Bisection Method: 1.2599201202392578

Problem i
Newton's Method: 2.028757838110434
Secant Method: -2.0287578380661646
Bisection Method: 1.5707950592041016

Problem j
Bisection Method: 2.999998728434245

Problem 2

2.)

Base Case ($n=0$)

$$|g(\alpha) - g(x_n)| = |\alpha - x_{n+1}|$$

$$|g(\alpha) - g(x_0)| = |\alpha - x_1| \leq L|\alpha - x_0|$$

Induction Step

IH $|\alpha - x_n| \leq L^n |\alpha - x_0|$

$n \to n+1$

$$|\alpha - x_{n+1}| \leq L^{n+1} |\alpha - x_0| = L L^n |\alpha - x_0|$$

$n \to \infty$ $\quad L^n |\alpha - x_0| \to 0$

Geometric sum $= \frac{1}{1-L}$

sum of error $= \frac{L^n}{1-L} |x_1 - x_0| \geq |\alpha - x_n|$

Problem 3

3. $x_{n+1} = (1+e^{-x_n}) = g(x)$

$g(x) = 1 + e^{-1}$

$g'(x) = -e^{-x}$

$|g'(1)| = |-e^{-1}| = .367879441 \leq 1$

$|g'(2)| = |-e^{-2}| = .135335232 \leq 1$

- Since $g'(x) < 1$, $x \in [1,2]$ function $g(x)$ is a contraction in the interval $(0,1)$ and converges

$g'(1) = .367879441$

$g'(2) = .135335232$

$L = .367879441$

$$\frac{L^n}{1-L}|x_1 - x_0| \leq 10^{-5}$$

$$\frac{(.36788)^n}{(1-.36788)}|1-0| \leq 10^{-5}$$

$$.36788^n \leq .6321 \times 10^{-5}$$

$$n \ln .36788 \leq \ln .6321 \times 10^{-5}$$

$$n = \frac{\ln .6321 \times 10^{-5}}{\ln .36788}$$

$n = 11.97$

$= 12$ iterate

$L = .135335232$

$$\frac{(.13534)^n}{(1-.13534)}|1-0| \leq 10^{-5}$$

$$.13534^n \leq .865 \times 10^{-5}$$

$$n \ln .13534 \leq \ln 8.65 \times 10^{-6}$$

$$n = \frac{\ln 8.65 \times 10^{-5}}{\ln .13534}$$

$n = 5.87$