Sheffield Hallam University

Assignment Submission Cover Sheet

Please complete all sections and attach to your assignment. You can find these details on your Assessment Statement

Student Name:

Student Number:

If this is a group submission the details of the student responsible for submitting the work is to be entered above and the name and student number of other group members below.

Course Title:

Module Title:

Module Code:

Tutor:

Assignment Title:

Deadline:

Declaration

I confirm that this assessment is my own work and that I have duly acknowledged and correctly referenced the work of others. I am aware of and understand that any breaches to the Code of Academic Conduct will be investigated and sanctioned in accordance with the Academic Conduct Regulation, found on shuspace| Rules and Regulations| Conduct and Discipline

Signature:

Date Submitted:

Learning contracts

If you have a learning contract recommendation for adjusted marking you will need to use blue stickers to alert the tutor to this.

*You **must** attach a blue sticker on all your assignments and exam scripts. If you submit work electronically you must type the wording of the sticker in blue on the front of your assignment where tutors can easily see it.*

Maths for CS – Assignment 1a

Module Code 55-402612

1. A new connective is defined by

(x * y) = (x => y) ^ (¬x => y).

a) The current definition of *x * y* is not easy to manipulate. By simplifying the right-hand side of Equation (1) above, show that there is a simpler way to write the expression.

If we take the original equation and replace the implication signs *(=>)* with their formal definition *(p => q = ¬p v q)* then we get an equation we can work on simplifying:

= (¬x v y) ^ (¬¬x v y)

= (¬x v y) ^ (x v y)

Using the law of distribution, we can take the common element *y* out of the brackets:

= y v (¬x ^ x)

Now using the law of cancellation we see that this equation simplifies down a lot:

= y v (T)

Finally using the law of identity we can discard the leftover value…

= y

…And see that the equation simplifies down to just *y*. A truth table providing proof is below:

x	**y**	¬x	x => y	¬x => y	**(x => y) ^ (¬x => y)**
F	**F**	T	T	F	**F**
F	**T**	T	T	T	**T**
T	**F**	F	F	T	**F**
T	**T**	F	T	T	**T**

b) Are there any cases where *x * y = y * x*?

Since we can now see that *x * y* simply equals *y*, for both sides of the equation to be equal they must be the same number or evaluate to the same number. Therefore, the cases where *x * y = y * x* are all the cases where *x = y*.

We can prove this by simplifying the given scenario using the simplification discovered above in question 1a:

*x * y = y*

*y * x = x*

It now becomes apparent that for these equations to be equal, *x* must equal *y*.

c) Are there any cases where *(x * y) * z = x * (y * z)*?

We can make it easier to answer this question by simplifying what we are being asked a bit:

*(x * y) * z = (y) * z = z*

*x * (y * z) = x * (z) = z*

Instantly we can see by resolving the terms inside the brackets using the simplification discovered in question 1a. that both equations are equal to *z*, meaning that they will always be equal.

2. Construct a proposition *E* involving p, q and r that yields the following truth table.

p	q	r	E
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Your proposition must include at least one conjunction (AND), one disjunction (OR), and one negation (NOT). Each symbol (p, q and r) must appear at least once. Write a brief explanation of the process you used to find your answer and prove, using a truth table, that your answer is correct.

If we look at the truth table given above, we can see that in the desired output column there are only two false values; if we negate the output we can express the desired output using only 2 minterms, and simplify the equation from there if necessary:

p	q	r	E	¬E	**¬E: Minterm**
F	F	F	T	F	
F	F	T	T	F	
F	T	F	F	T	**(¬p ^ q ^ ¬r)**
F	T	T	T	F	
T	F	F	T	F	
T	F	T	T	F	
T	T	F	F	T	**(p ^ q ^ ¬r)**
T	T	T	T	F	

The combination of these two minterms, accounting for the negation of E, allows us to express the desired output (E) using the following expression:

= ¬((¬p ^ q ^ ¬r) v (p ^ q ^ ¬r))

The first step in simplifying this is resolving the "not" acting on the whole expression:

= (p v ¬q v r) ^ (¬p v ¬q v r)

The expression can be further simplified using the law of distribution: the two minterms share a common term of (¬q v r), which can be taken out of the brackets as follows:

= (¬q v r) ^ (p v ¬p)

p	q	r	¬p	¬q	p v ¬p	¬q v r	**E**
F	F	F	T	T	T	T	**T**
F	F	T	T	T	T	T	**T**
F	T	F	T	F	T	F	**F**
F	T	T	T	F	T	T	**T**
T	F	F	F	T	T	T	**T**
T	F	T	F	T	T	T	**T**
T	T	F	F	F	T	F	**F**
T	T	T	F	F	T	T	**T**

While it seems apparent that *(p v ¬p)* should be resolved to simply *T* using the law of cancellation *(p v ¬p = T)* and then removed using the law of identity *(p ^ T = p)*, removing the term would mean the simplified expression no longer gives the desired output, E, as the truth table would only have four values if there were two inputs. Additionally, the problem requires that *p* be present somewhere in the final expression.