Assignment 2b

This assignment must be submitted to the STA Reception Desk in the Cantor Building ***by 3:00pm on or before Monday 23 March 2020****.*

Your attention is drawn to the University's policy on cheating. Answers to this assignment must be individual work – any evidence suggesting the possibility of either plagiarism or collusion will be investigated. More details can be found on the module site on MyHallam.

Instructions

You should attempt all of the questions. This task is worth 12.5% of the overall module and 50% of Coursework 2. Your work will be evaluated by *grading descriptor* which can be found under *Assessment* on the module site on MyHallam.

Your submitted work should be presented legibly and neatly, with the official front sheet stapled at the front.

Please use A4 paper. To make it easier to mark, please start a new page for each question. Your complete set of pages should be stapled in the top-left hand corner without obscuring your work. **Please do not use plastic wallets, or other covers**.

- Two functions $f\colon A \to B$ and $g\colon A \to B$ are equal if $f(a) = g(a)$ for all $a \in A$. We write this as $f = g$.
- After part b), use the formula for f given in part b).

Task

The function $f\colon \mathbb{N} \to \mathbb{N}$ is defined recursively by the formula

$$f(n) = \begin{cases} 1 & n = 0, \\ 3f(n-1) - 1 & \text{otherwise.} \end{cases}$$

(a) Use the above definition to calculate $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$.

(b) Prove by induction that

$$f(n) = \frac{3^n + 1}{2}$$

for all $n \in \mathbb{N}$.

(c) Another function $g\colon \mathbb{N} \to \mathbb{Z}$ is defined by $g(n) = 2n - 1$. State $(g \circ f)(n)$ for $n \in \mathbb{N}$.

(d) A further function $h\colon \mathbb{Z} \to \mathbb{R}$ is defined by $h(n) = n^2$. State $(h \circ g)(n)$ for $n \in \mathbb{N}$.

(e) Hence, show that $((h \circ g) \circ f) = (h \circ (g \circ f))$.