Week 5 Discussion 1A

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Learning Assistants

November 1, 2024

Midway Review

K-Nearest Neighbors

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K-Nearest Neighbors

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c. We can tune for the most optimal value of k for a given dataset.
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b. Linear Regression uses MSE, Logistic Regression uses hinge loss, and Softmax Regression uses CE.
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Gradient Descent

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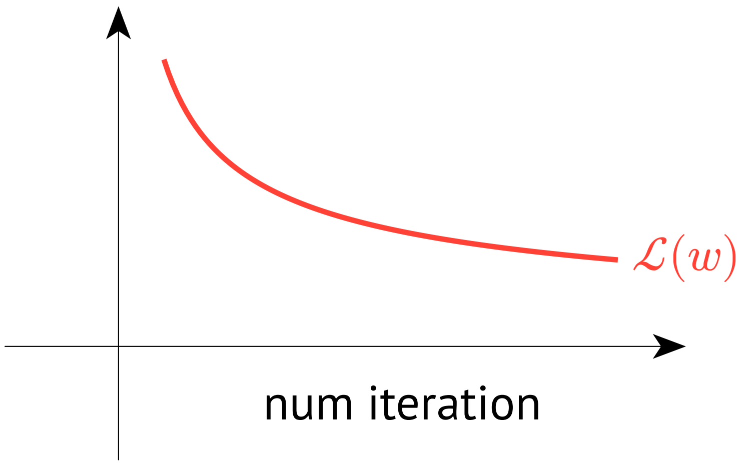
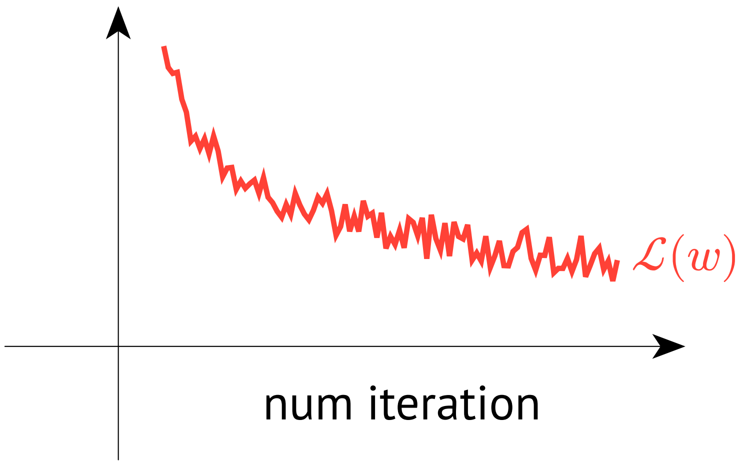
Solution: Use a running mean of gradients to build up **momentum** in a general direction.

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

Gradient Descent

Which of the following are true about the training loss curves below?





Gradient Descent

a. Graph A is **Mini-Batch Gradient Descent** – gradient estimates using batch and minibatch are equivalent.
b. Graph A is **Batch Gradient Descent** – accurate gradient estimates, resulting in a decrease in loss at every iteration.
c. Graph B is **Mini-Batch Gradient Descent** – noisy gradient estimates, resulting in oscillations in the loss trajectory.
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Regularization

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Universal Approximation Theorem

What does this claim?

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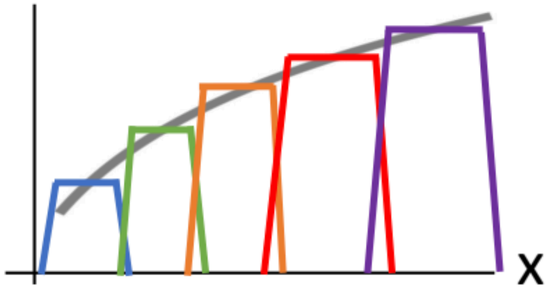
What does this claim? With enough neurons, a neural network can approximate any function f.

However, this does not mean a neural network can efficiently learn such an approximation.



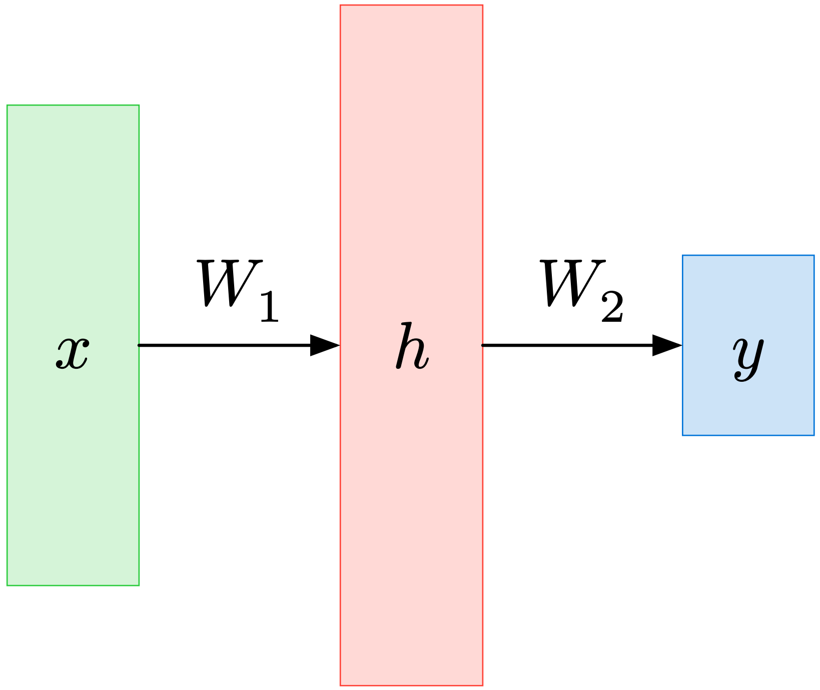
Approximate functions with bumps!

Neural Networks

A **Neural Network** is a computational model that makes decisions and predictions in a way inspired by biological systems.

Generally consists of:

- Input x
- Hidden Layer(s) h_i
- Output Layer y
- Activation Functions $\varphi(x)$



Neural Networks

Suppose we have a neural network $F : \mathbb{R}^{N\times(10\cdot 10)} \to \mathbb{R}^{N\times 1}$ that predicts a class based on a grayscale image.

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F = nn.Sequential(
  nn.Linear(100, 10, bias=False),
  nn.ReLU(),
  nn.BatchNorm(10),
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How many **trainable parameters** does F have? $100 \cdot 10 + 2 \cdot 10 + 10 \cdot 1 + 1 = 1031$

Activation Functions

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Why do we need non-linearity anyways?

Activation Functions

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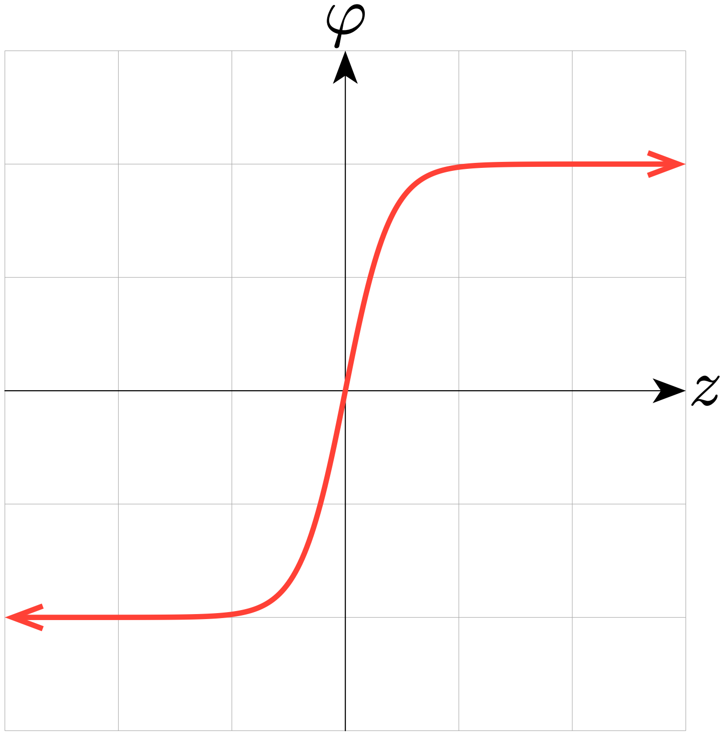
Why do we need non-linearity anyways? Suppose we had a linear activation φ. You can think of this as another matrix.

$$
\begin{aligned}
y &= W_2\varphi(W_1x + b_1) + b_2 \\
&= W_2\varphi W_1x + W_2\varphi b_1 + b_2 \\
&= (\varphi W_2W_1)x + (\varphi W_2b_1 + b_2)
\end{aligned}
$$

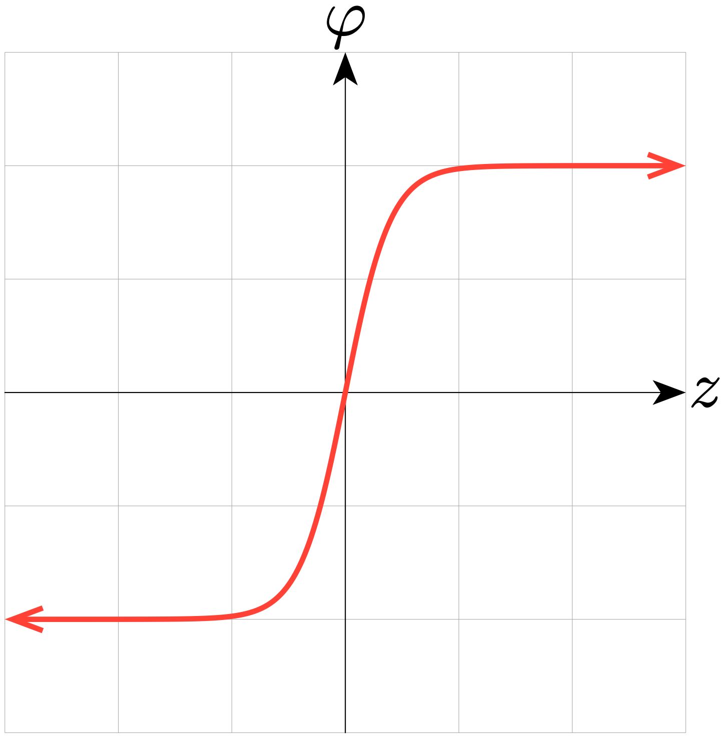
Same as $y = W_3x + b_3$, where $W_3 = \varphi W_2W_1$ and $b_3 = \varphi W_2b_1 + b_2$.

Activation Functions



- Name:
- Formula:
- Properties:

Activation Functions



- Name: tanh
- Formula:
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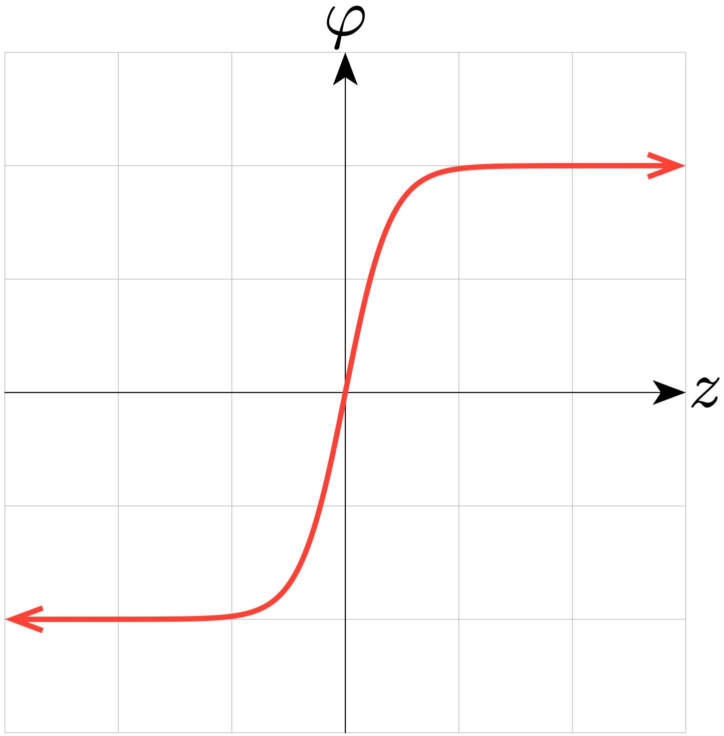

- Name: tanh
- Formula: $\varphi(z) = \frac{e^{2x}-1}{e^{2x}+1}$
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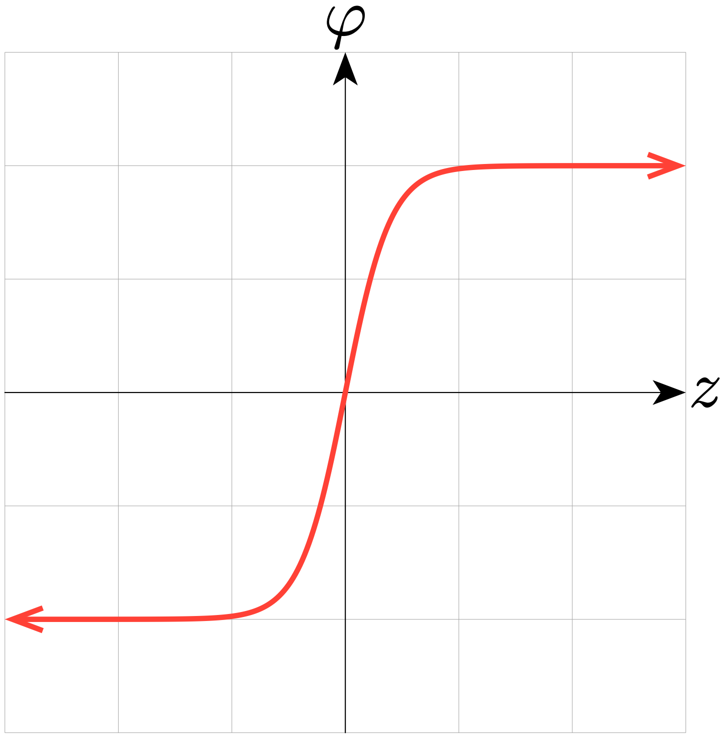

- Name: tanh
- Formula: $\varphi(z) = \frac{e^{2x}-1}{e^{2x}+1}$
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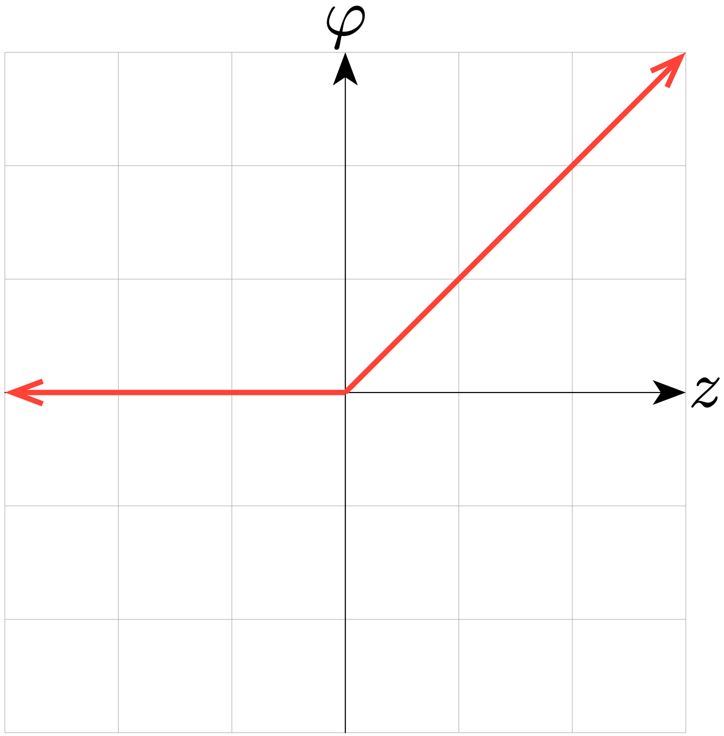

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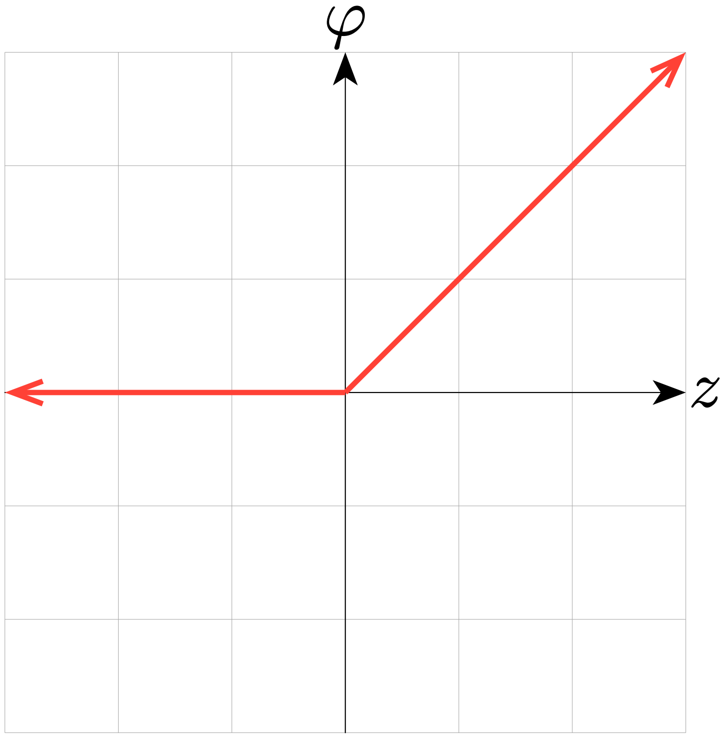

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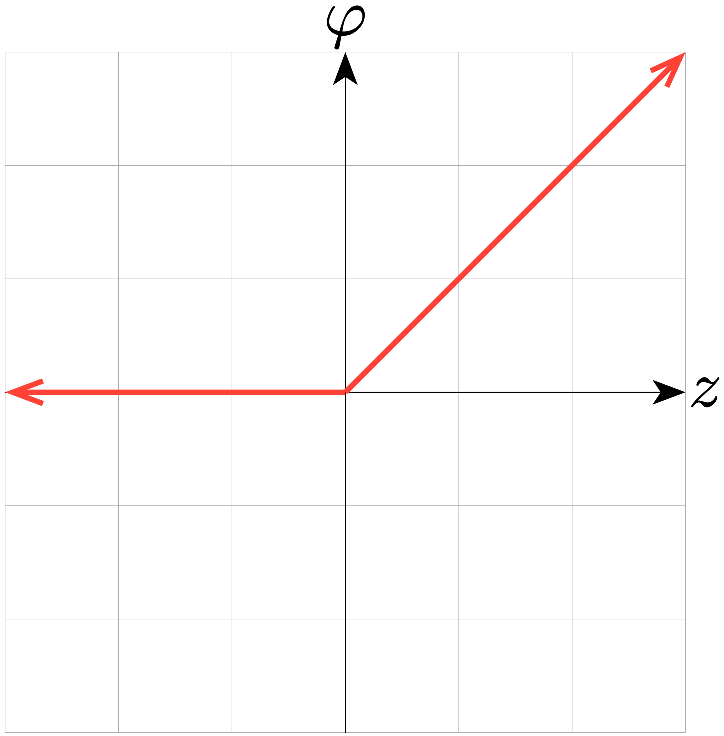

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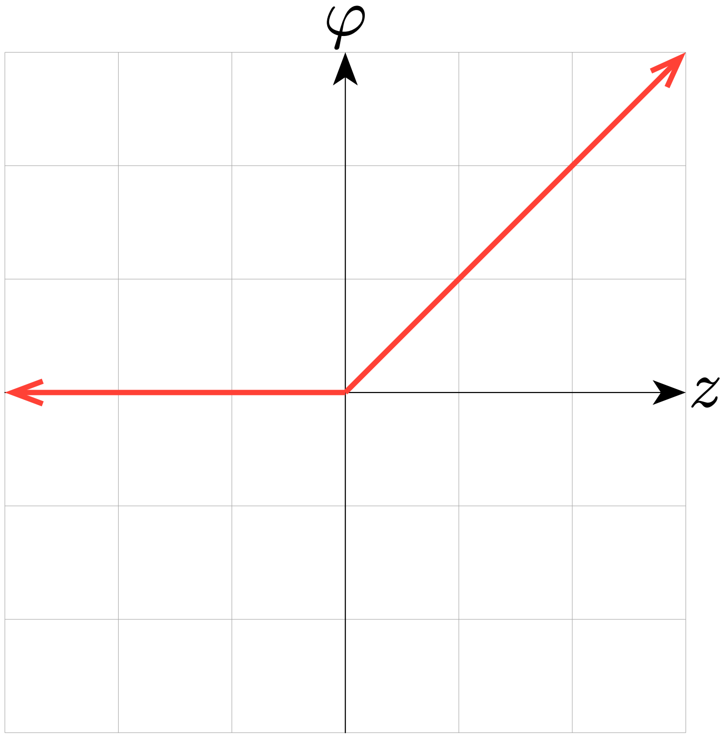


- Name: Rectified Linear Unit (ReLU)
- Formula: $\varphi(z) = \max(0, z)$
- Properties: Non-linear, differentiable everywhere except $x = 0$, $\nabla > 0$ when $z > 0$

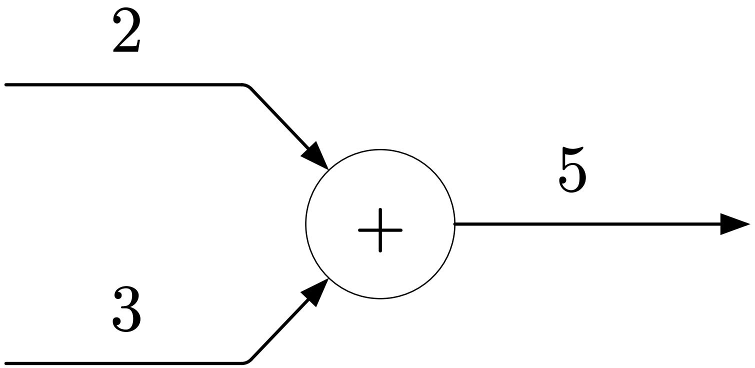
Backpropagation

Helpful gradient gates to remember!

Backprropagation

Helpful gradient gates to remember!

Add Gate



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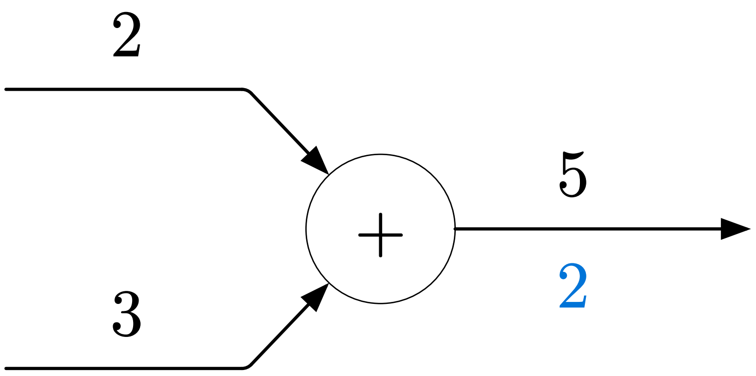

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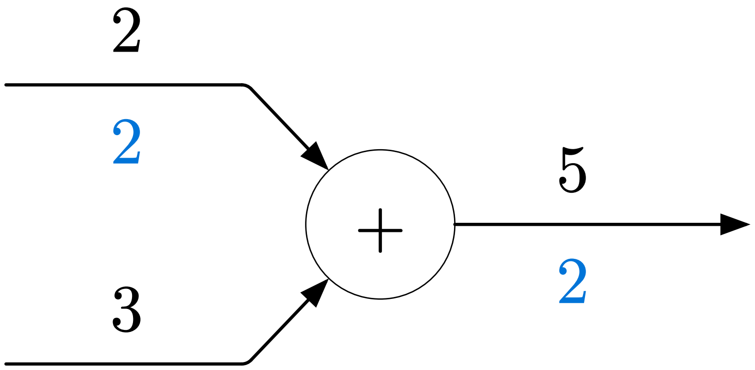

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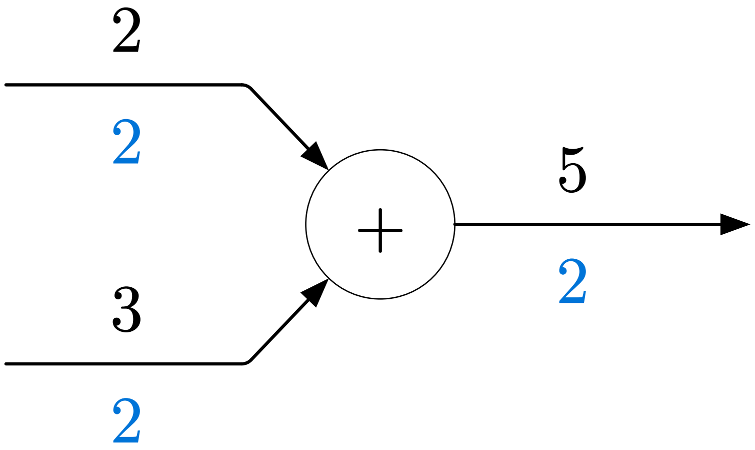

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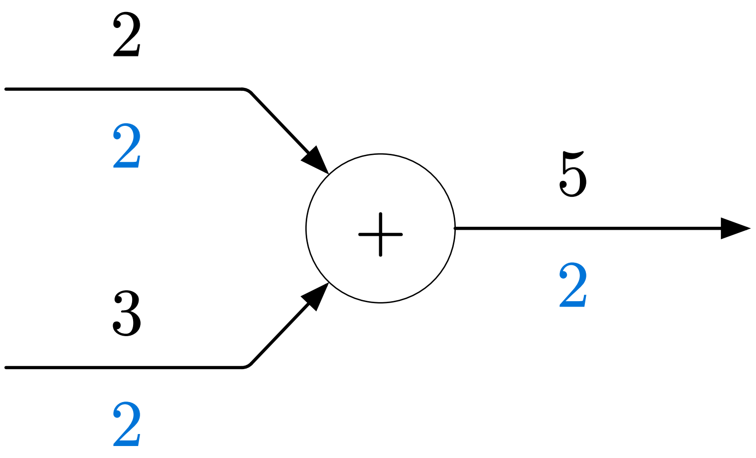

Gradient Distributor

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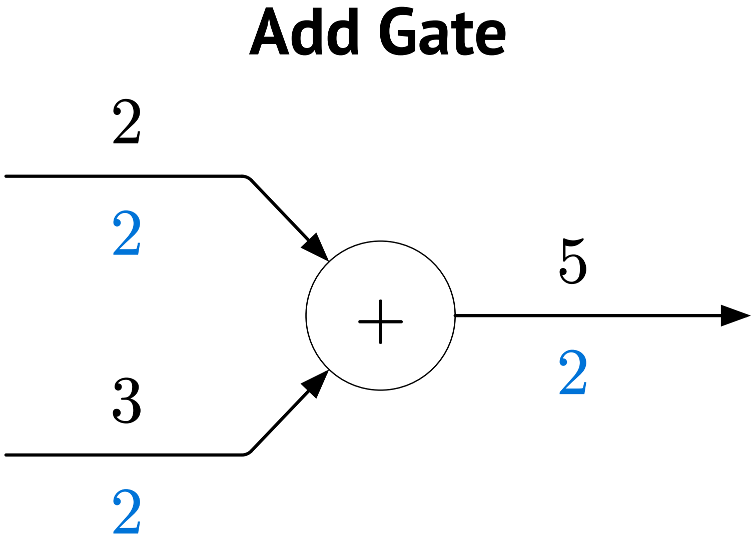

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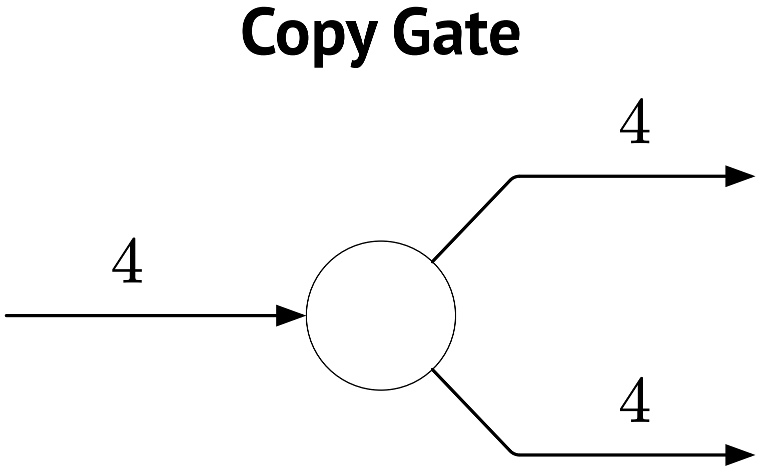

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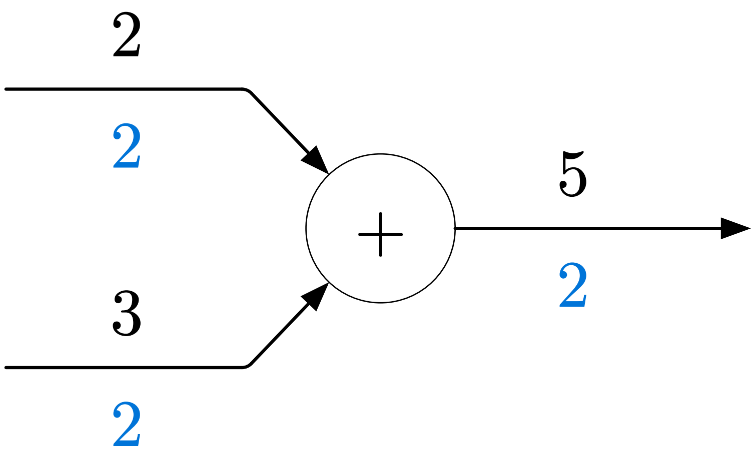

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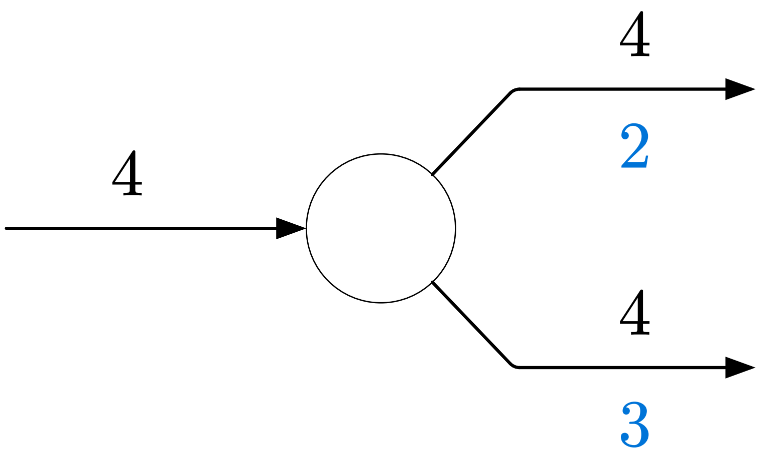

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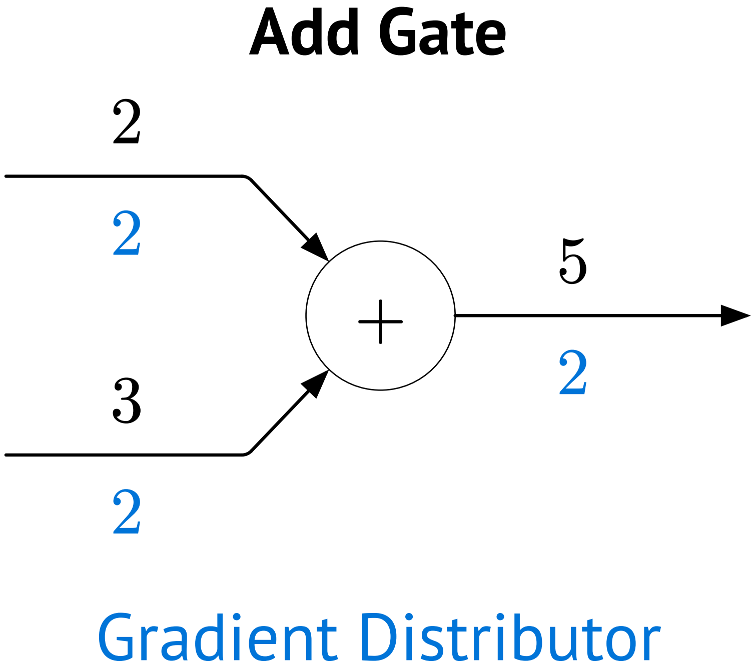

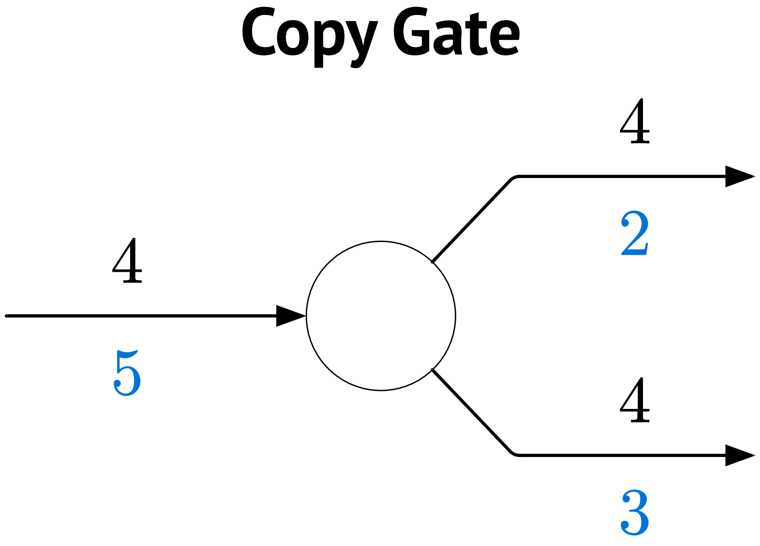

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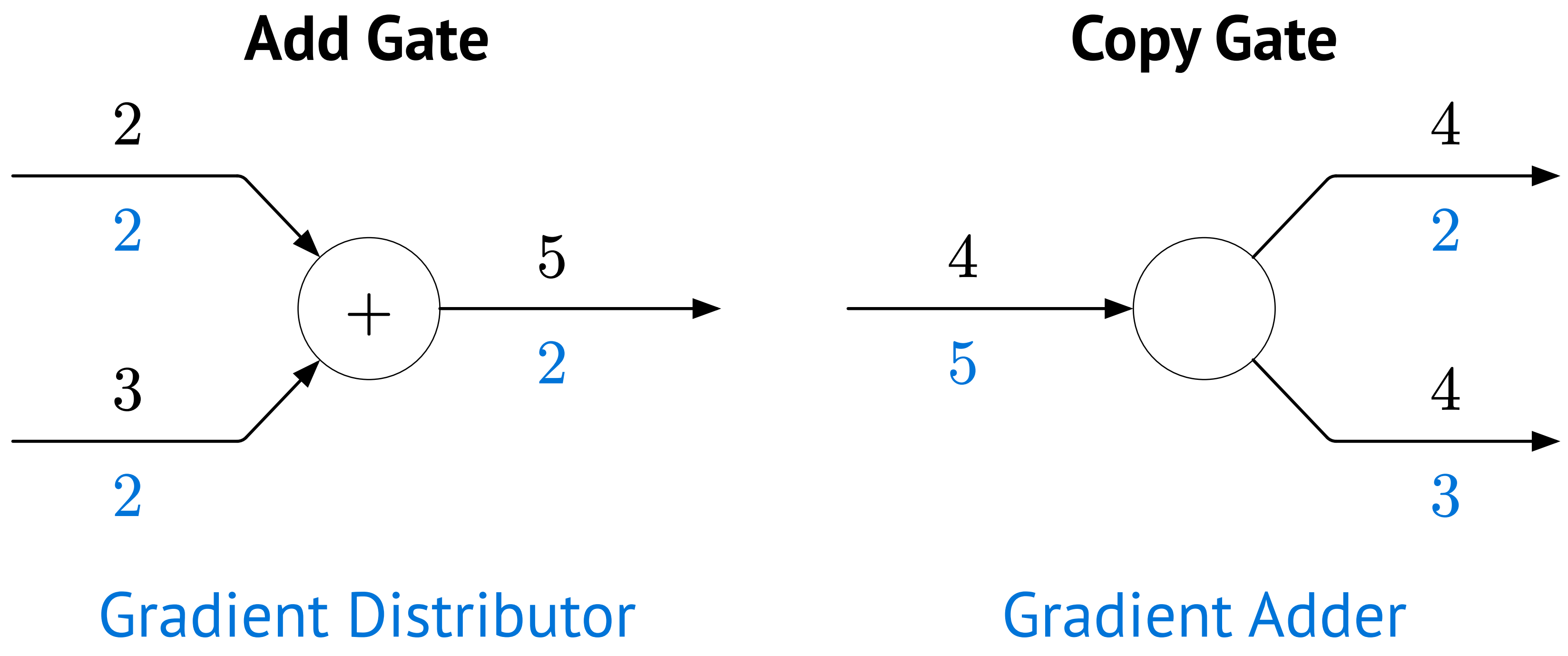

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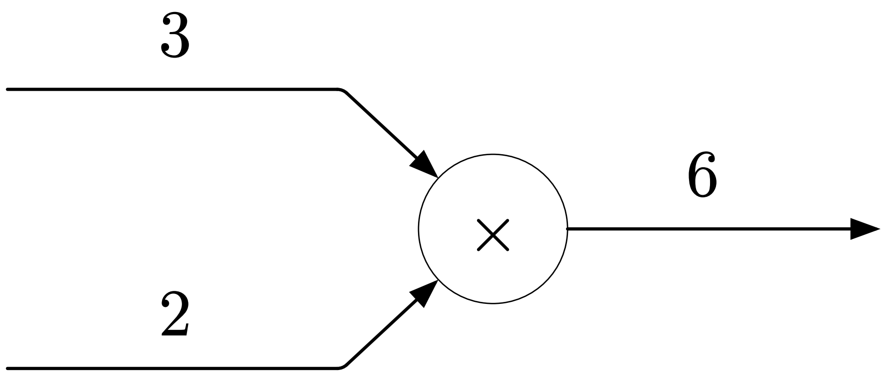

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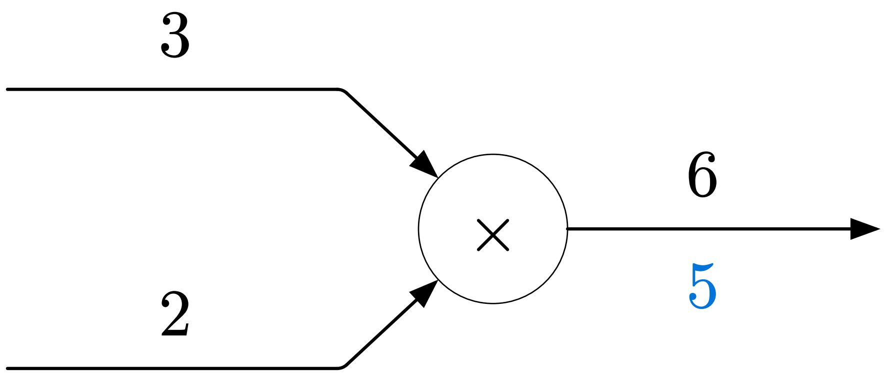

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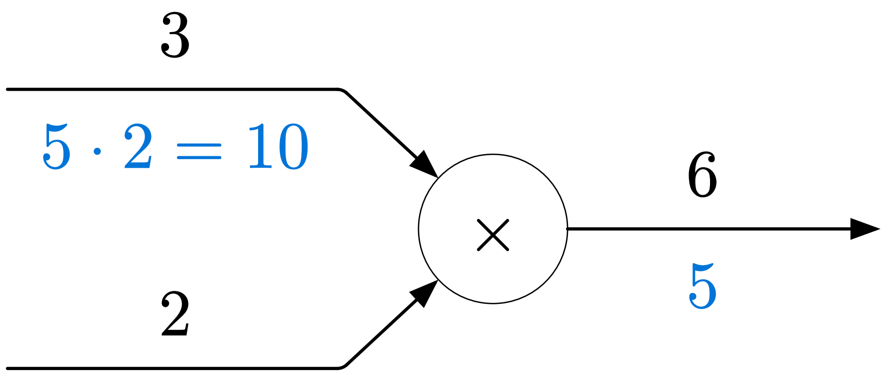

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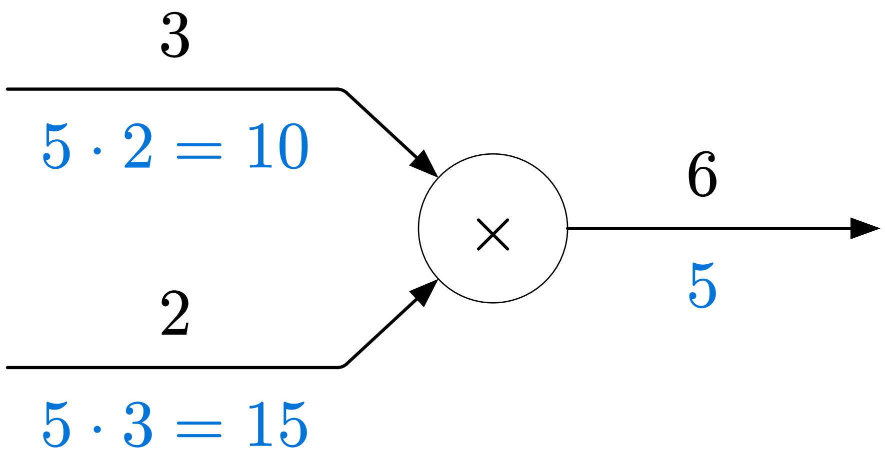

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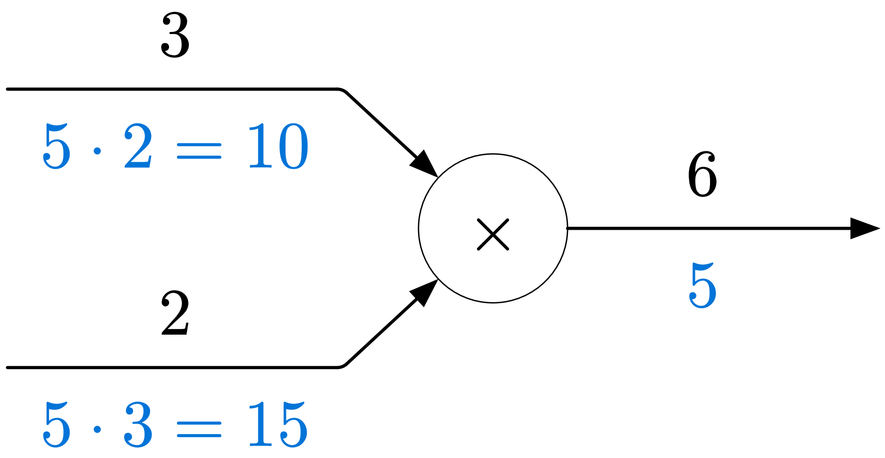

Swap Multiplier

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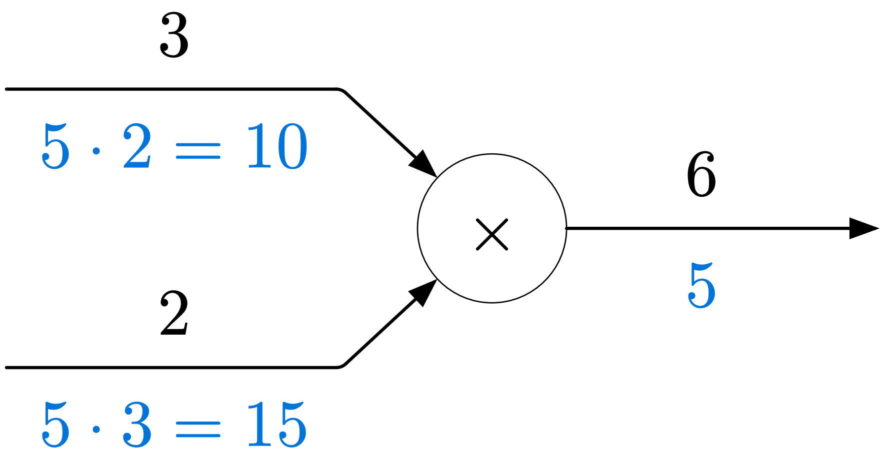

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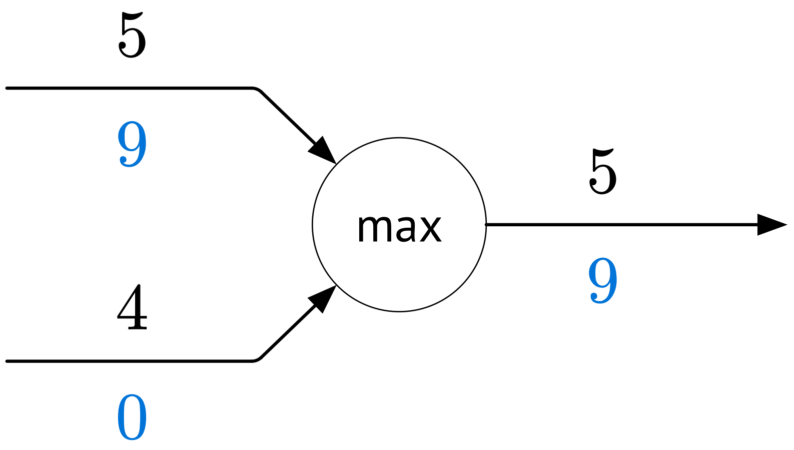

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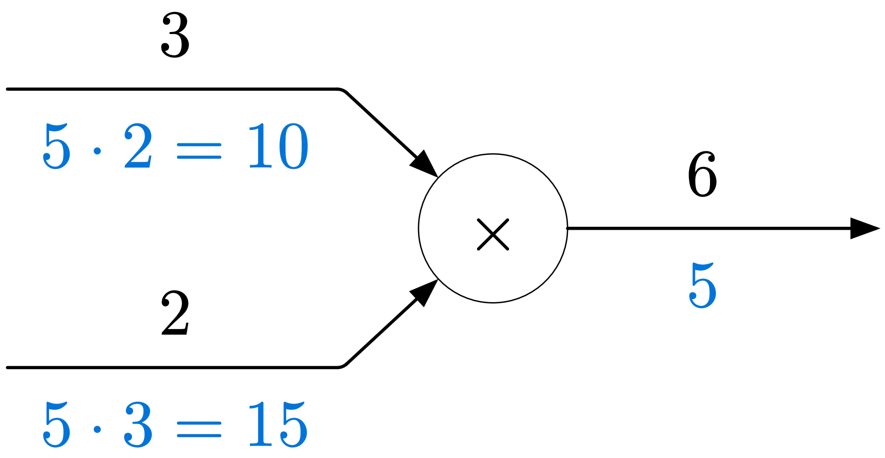

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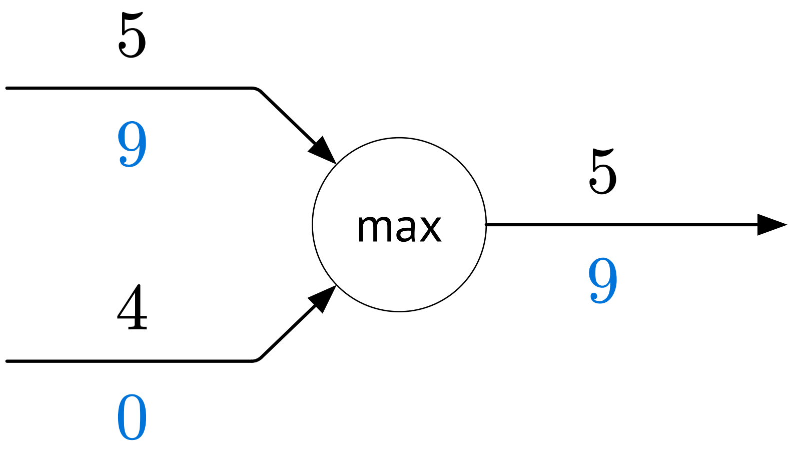

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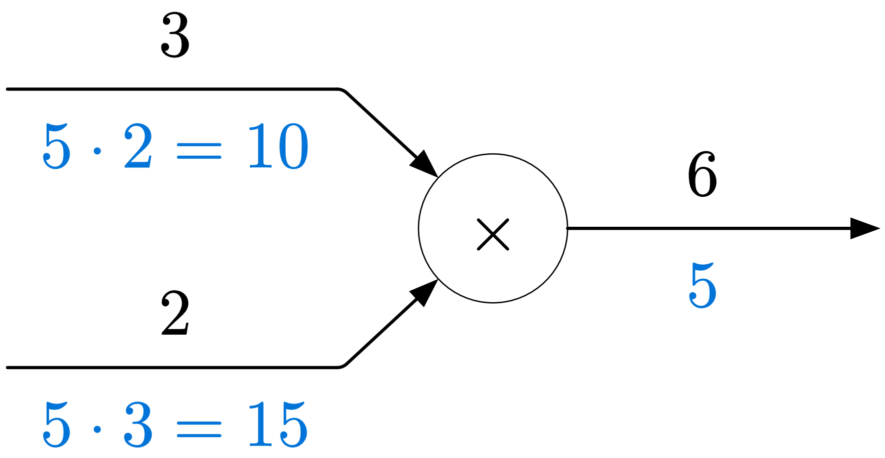

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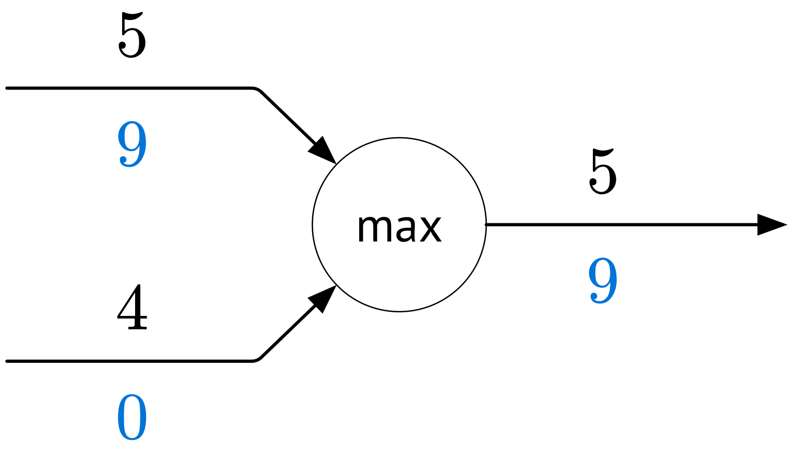

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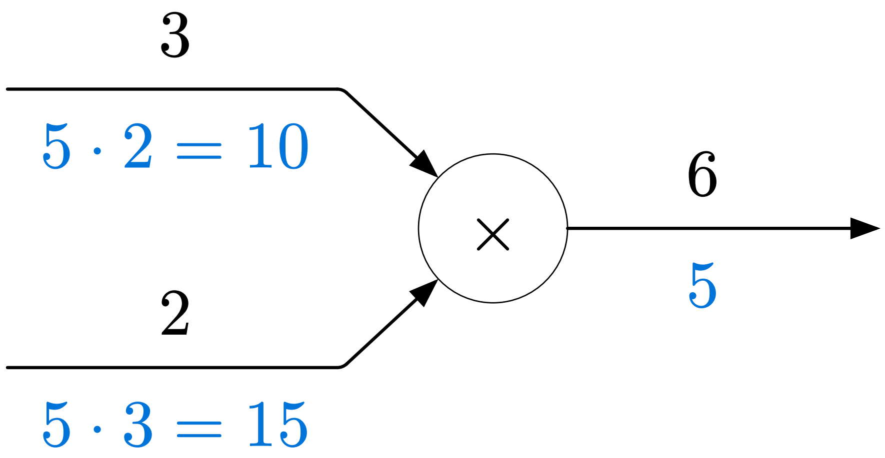

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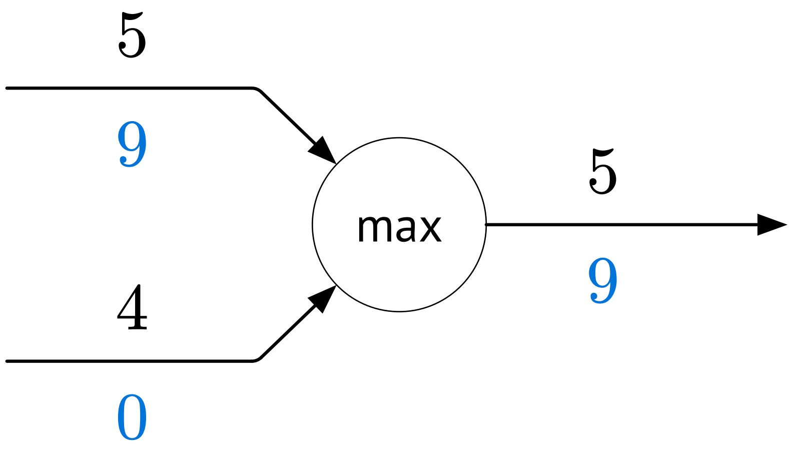
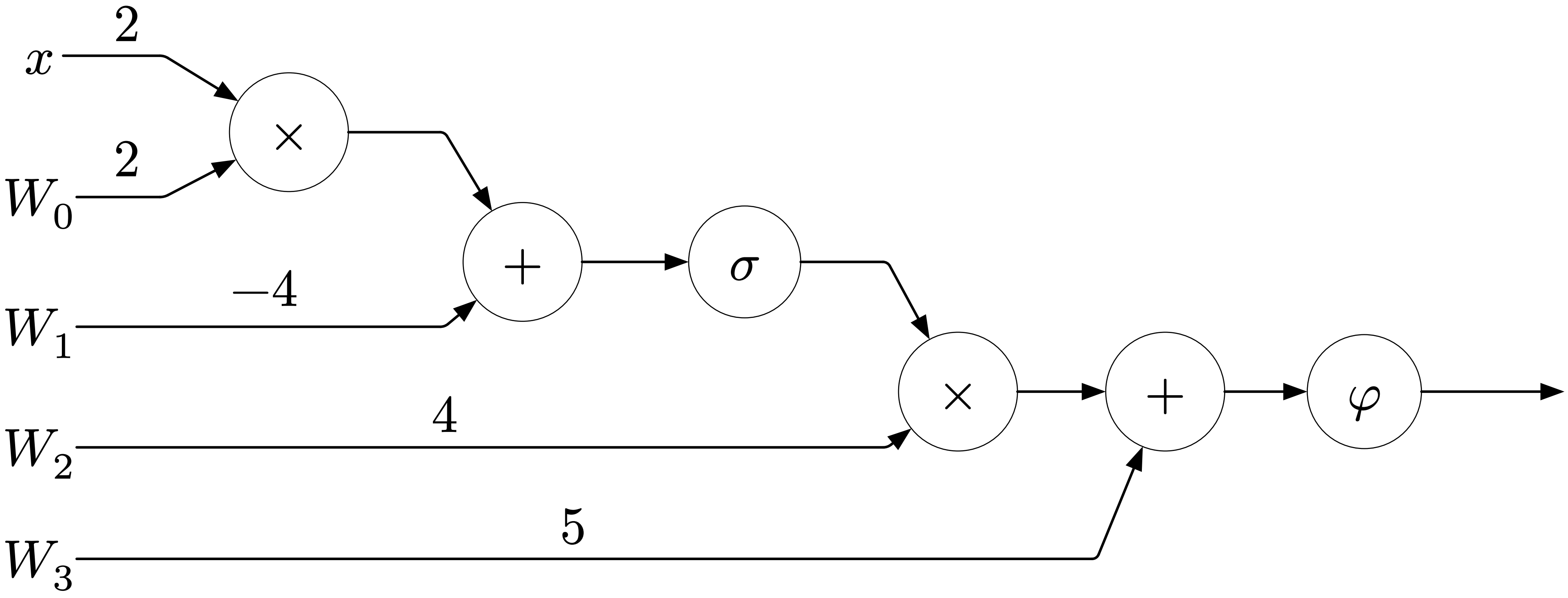


Gradient Router

Backpropagation

Compute the forward and backward pass. φ is a ReLU activation.



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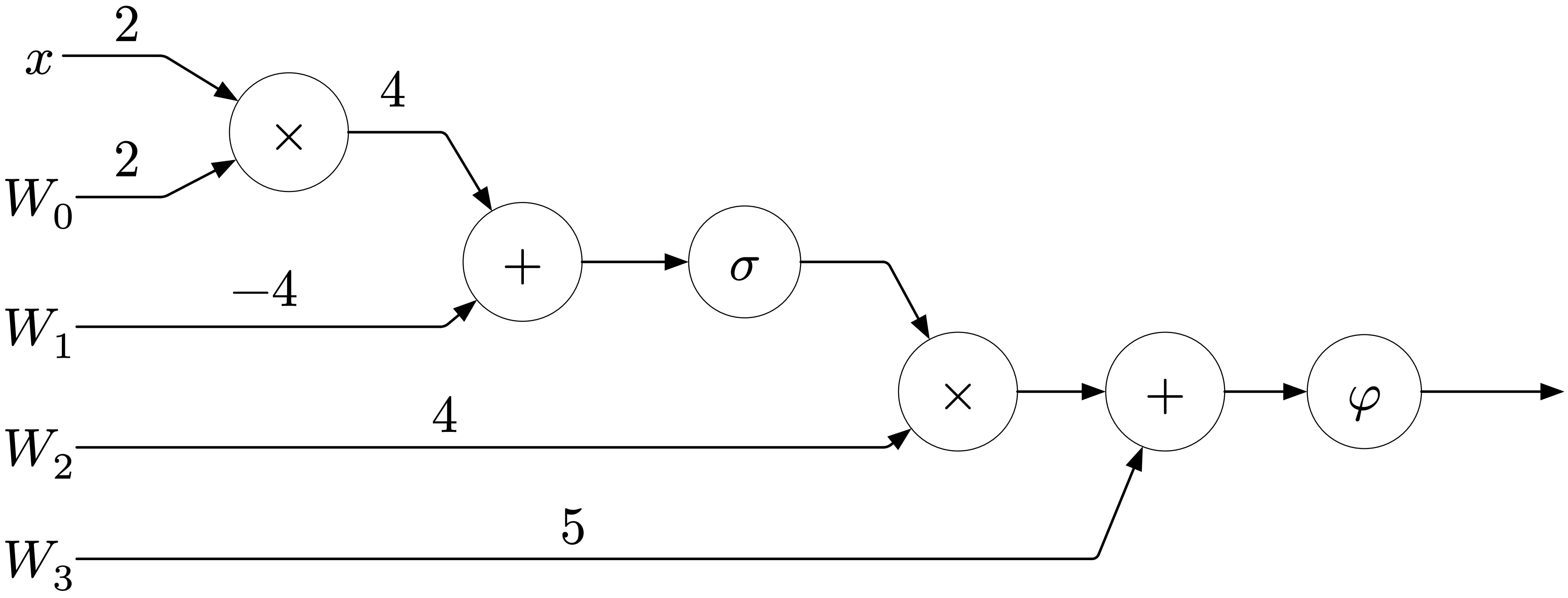

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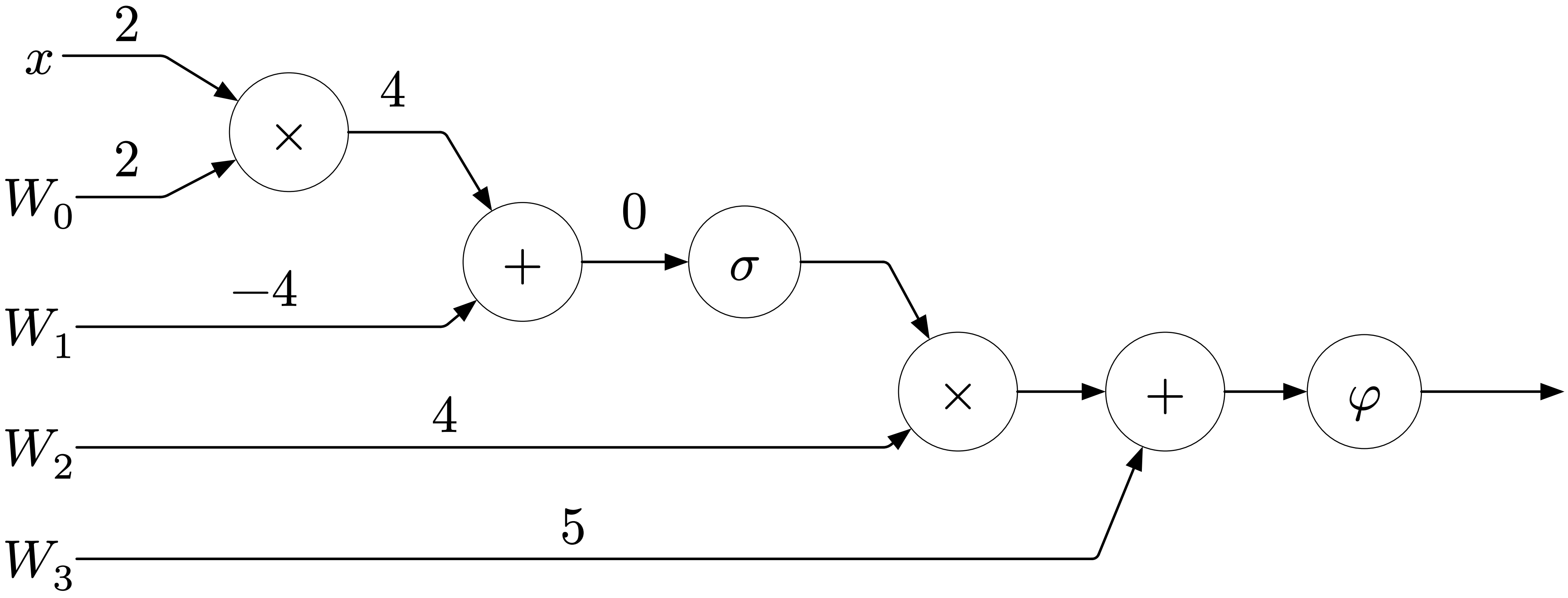

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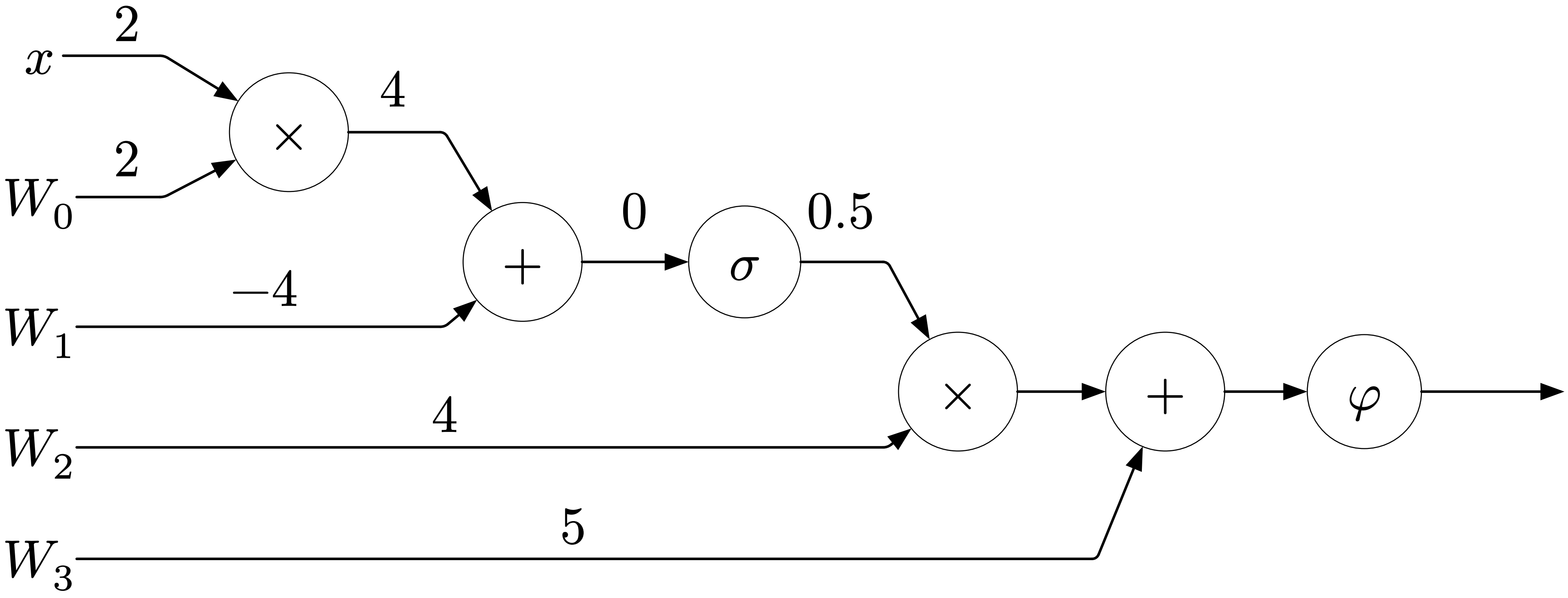

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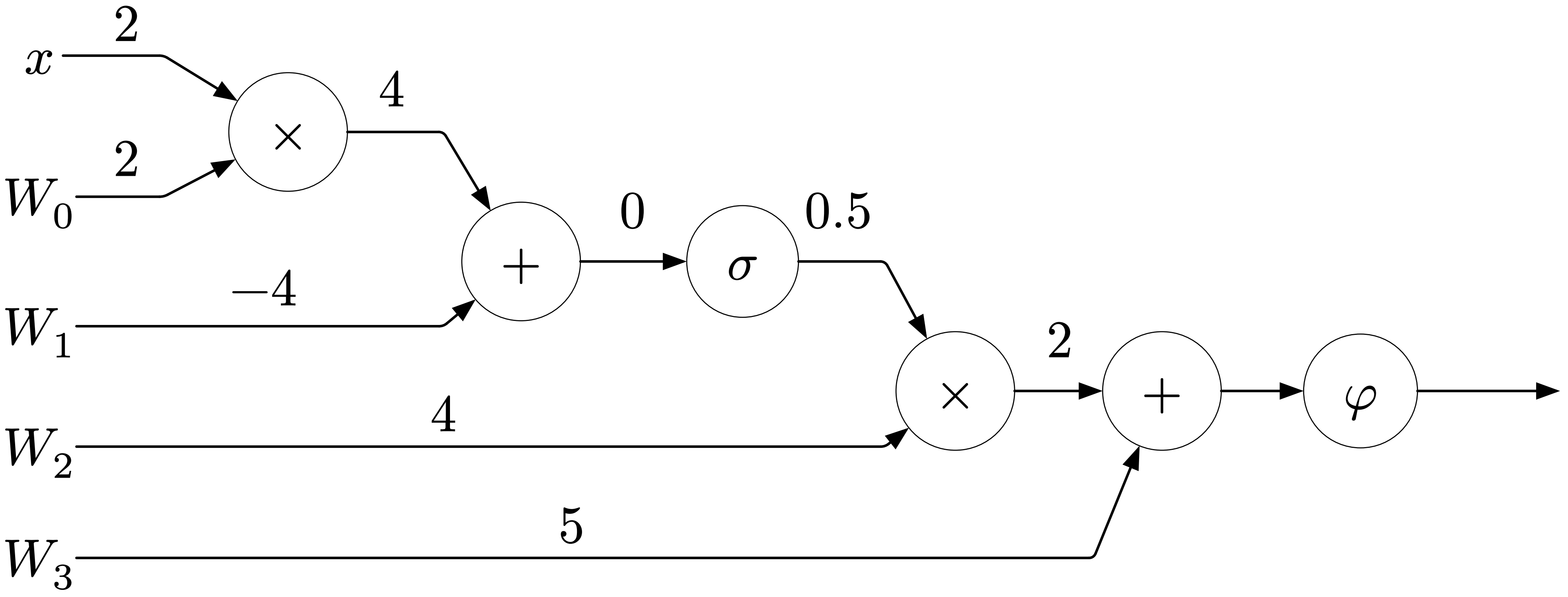

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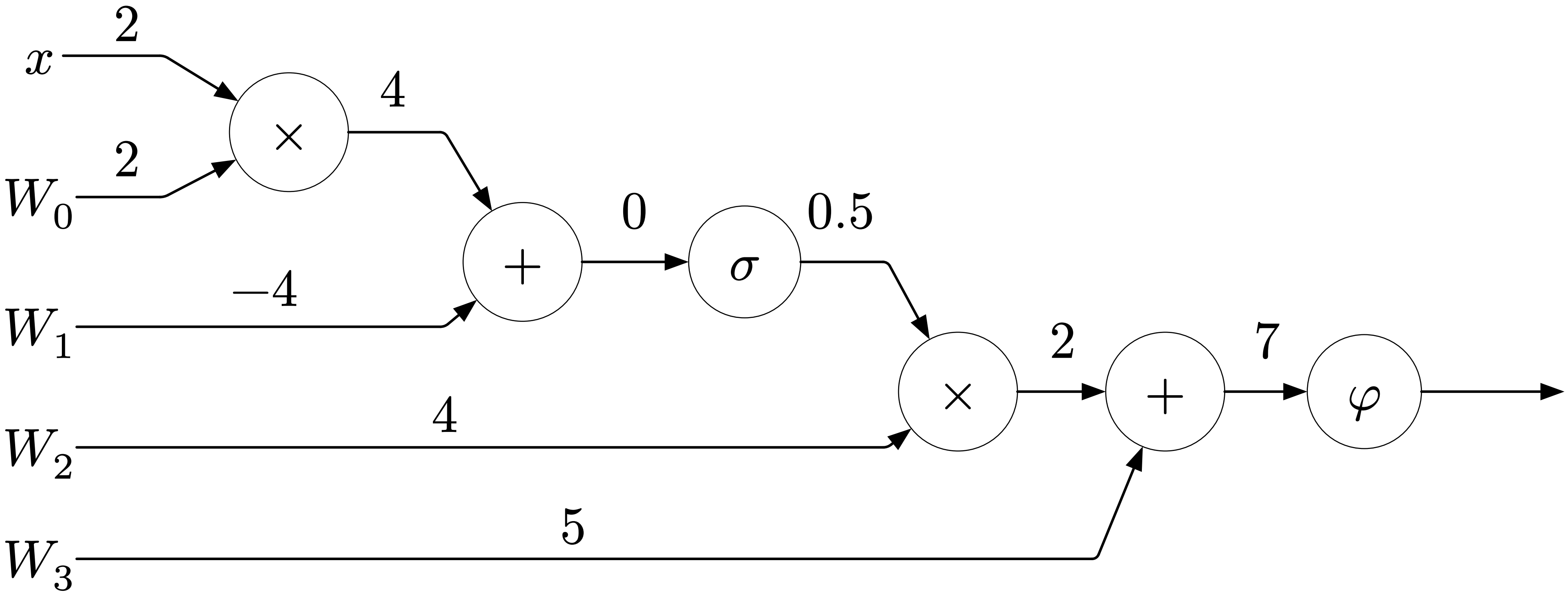

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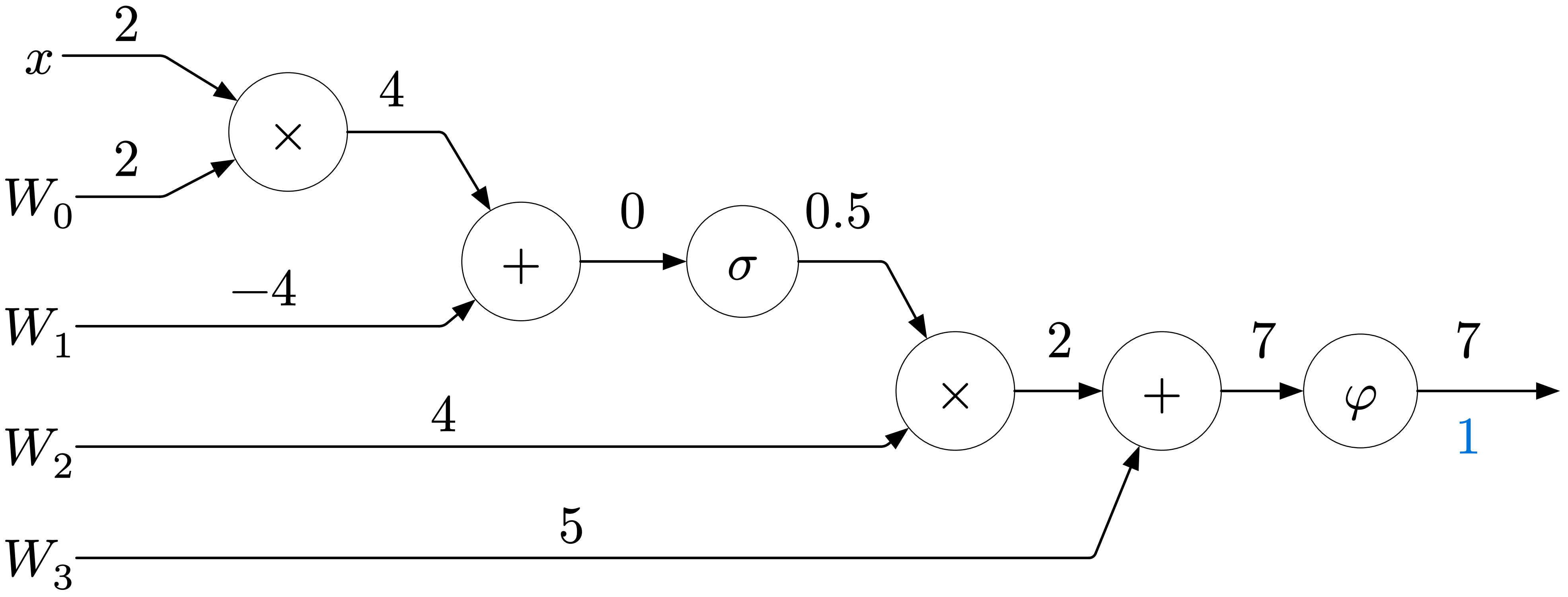

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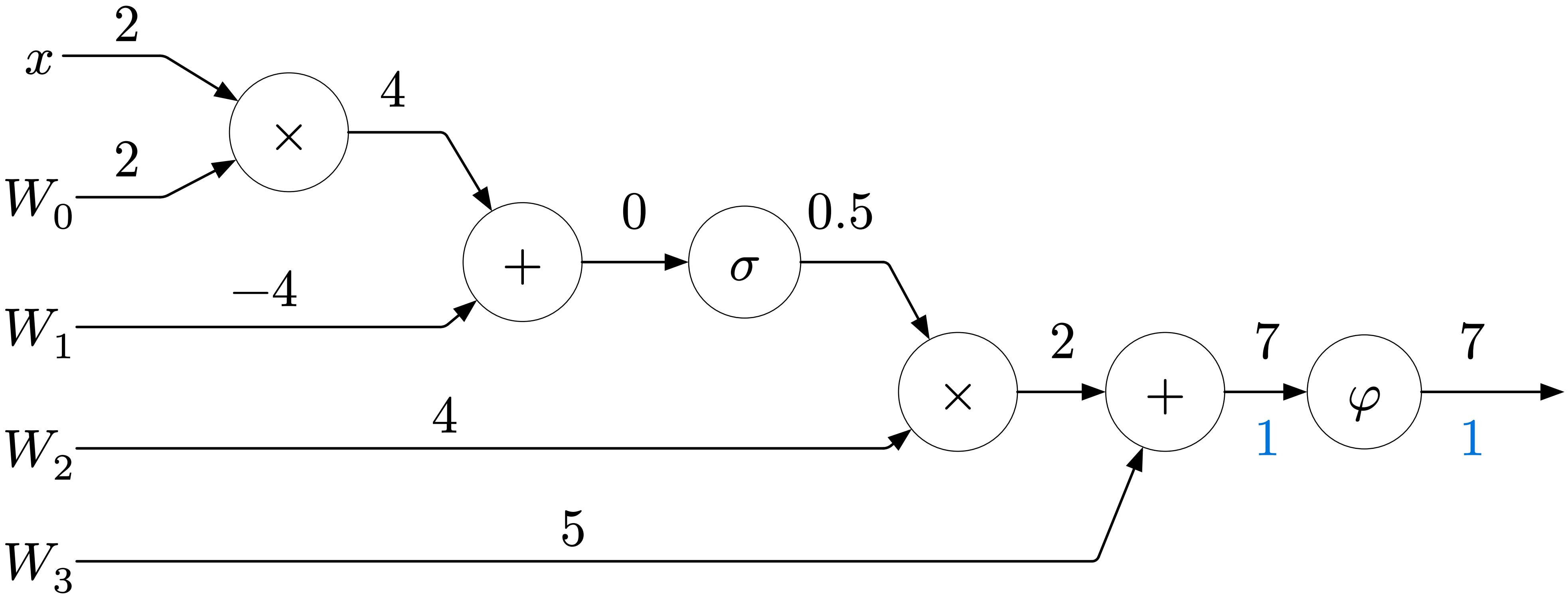

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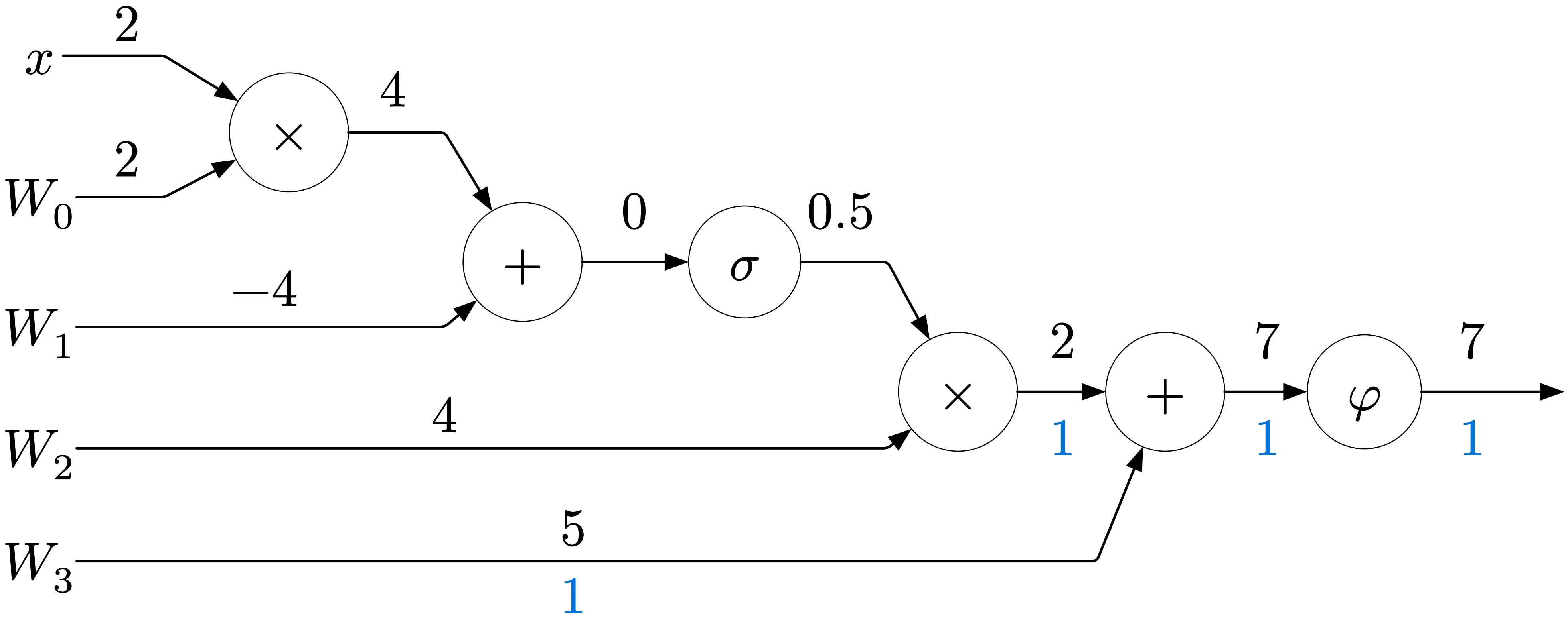

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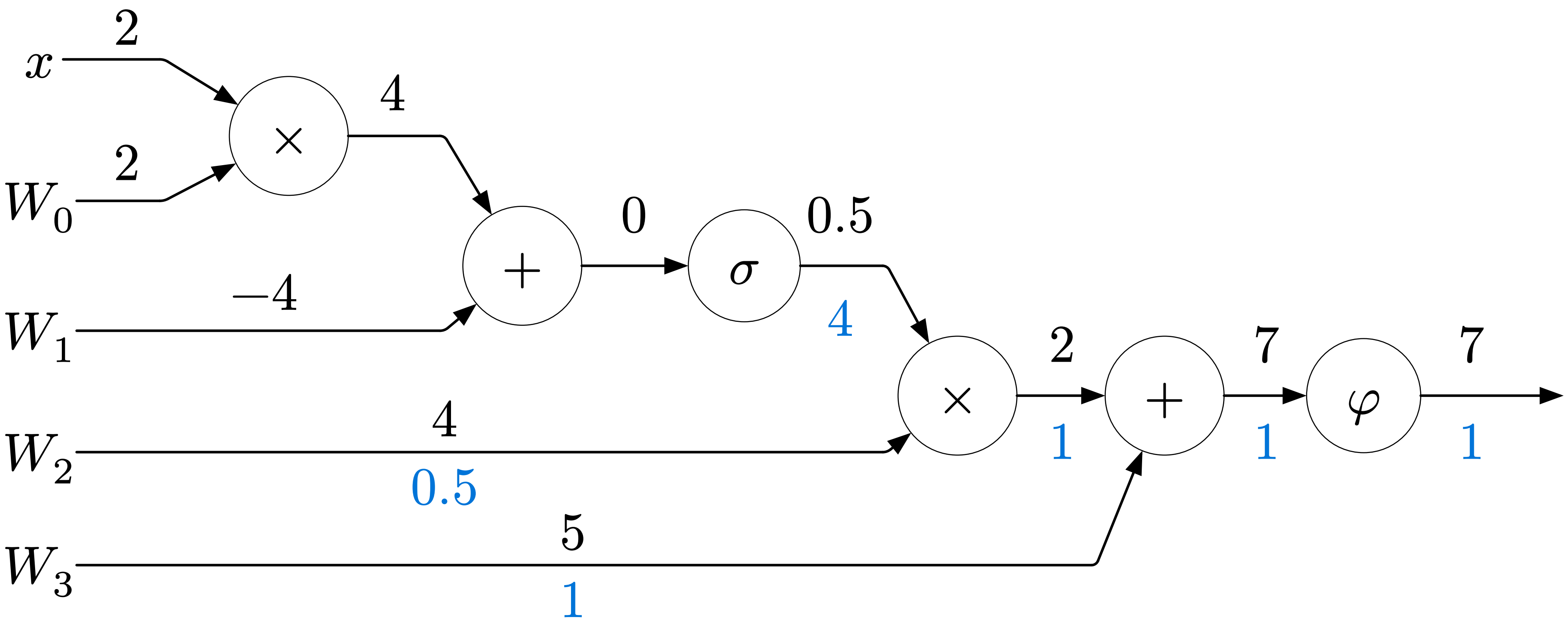

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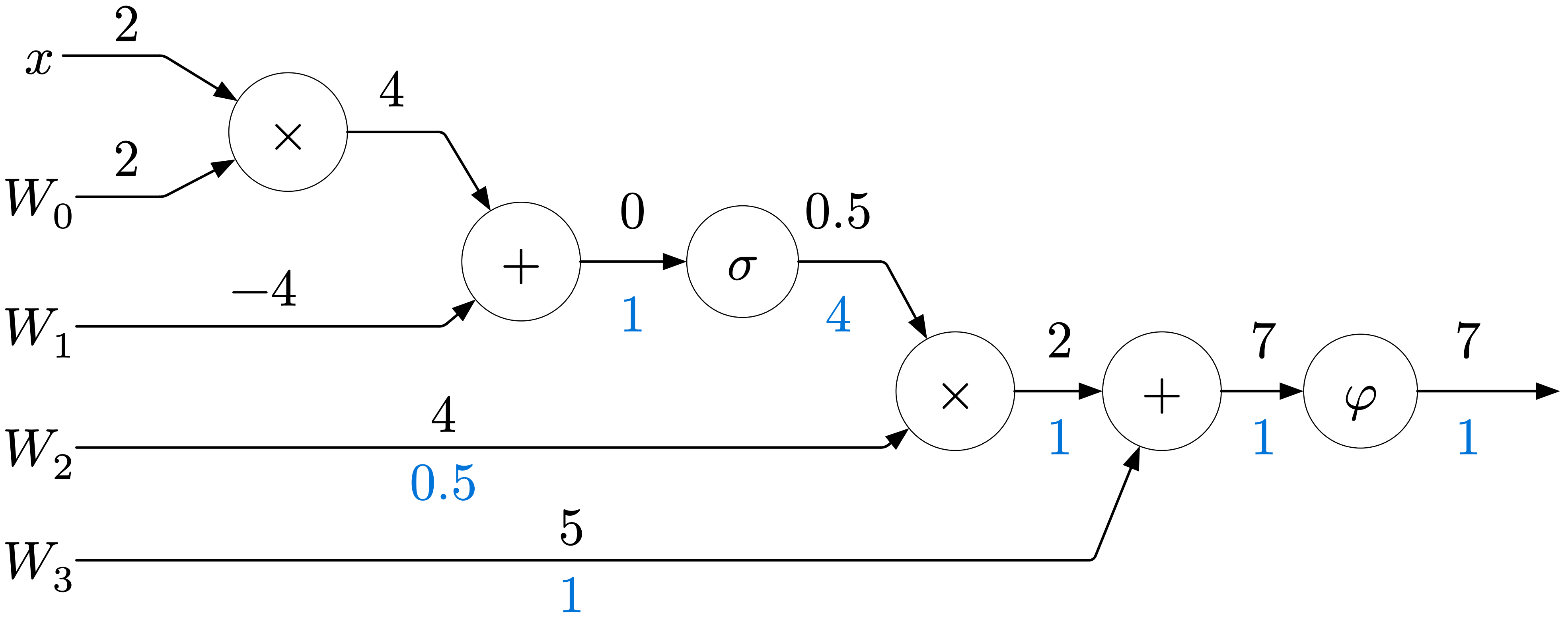

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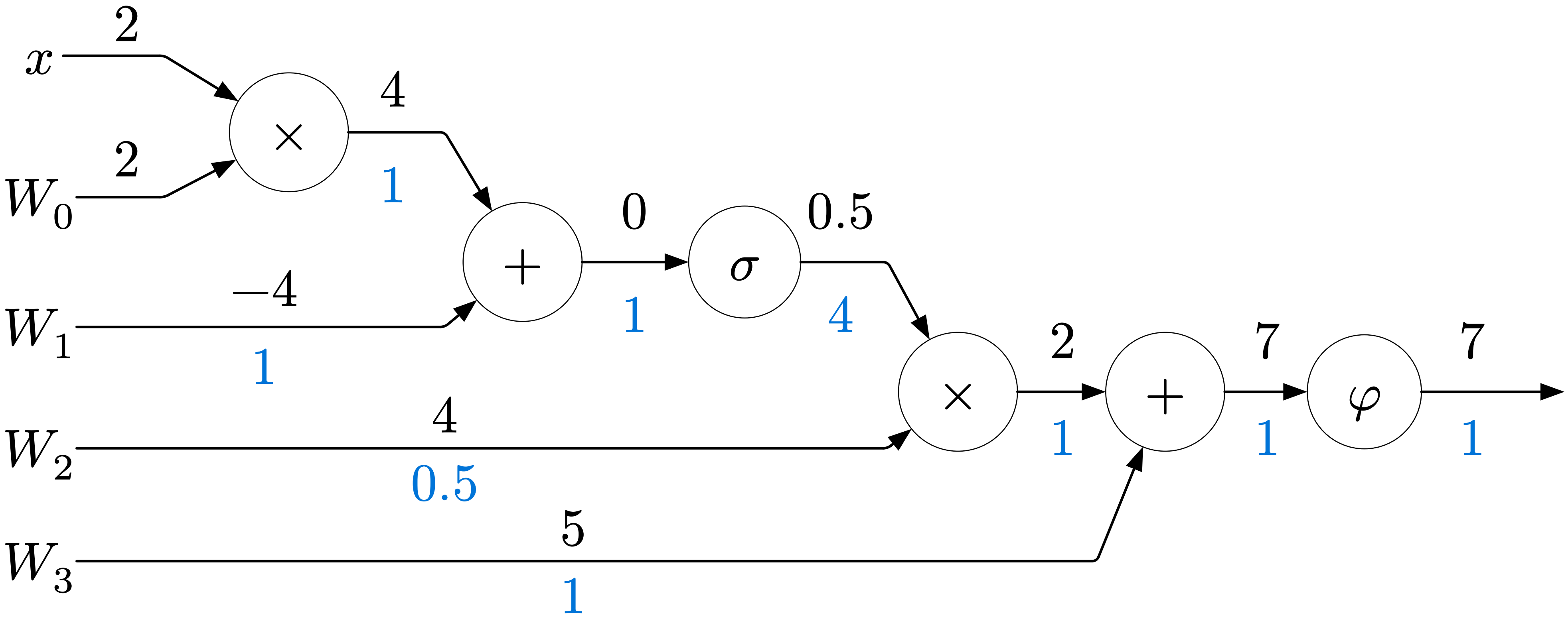
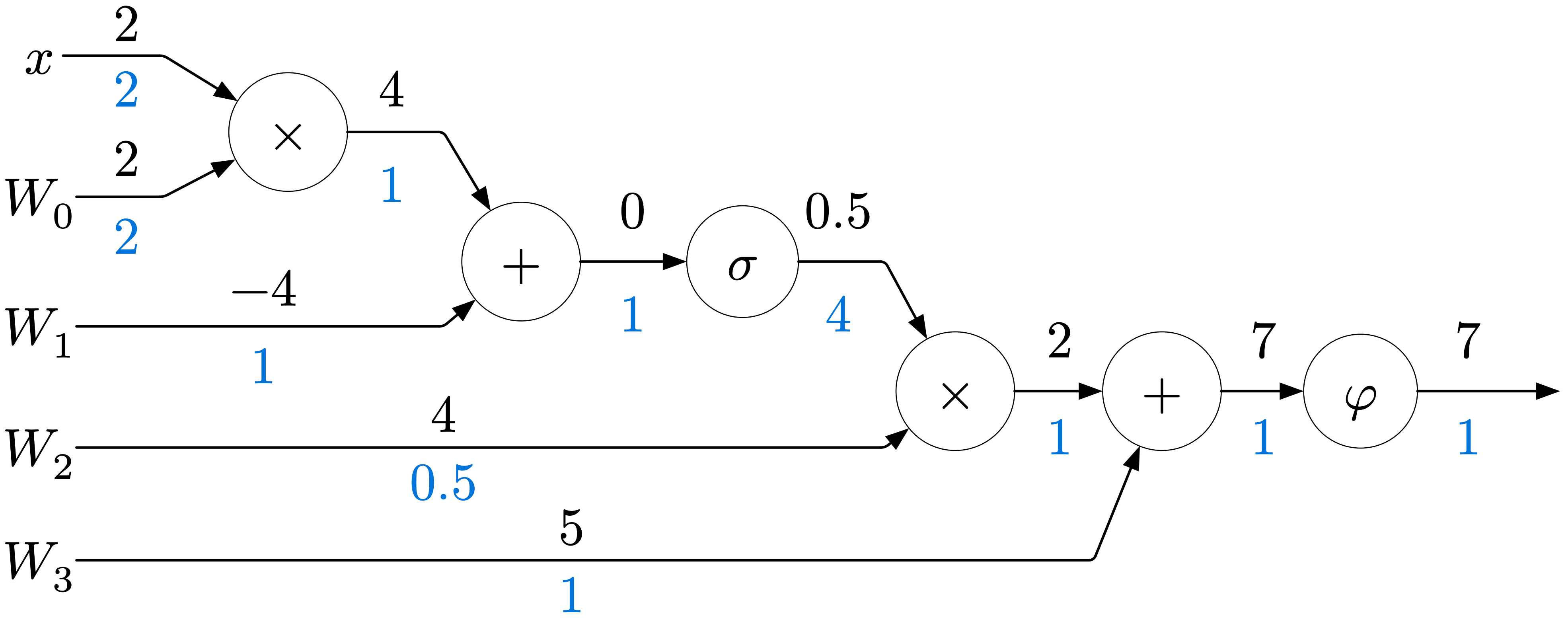


Backpropagation

Compute the forward and backward pass. φ is a ReLU activation.



Normalization

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$$\hat{\mu}_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_{i,j} - \hat{\mu}_j\right)^2$$

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μ_j, σ_j^2 are running avg

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

At test time, use aggregated statistics from training.

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a. It increases and depends on the kernel size and stride of each layer.
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c. It only depends on the kernel size of the first layer.
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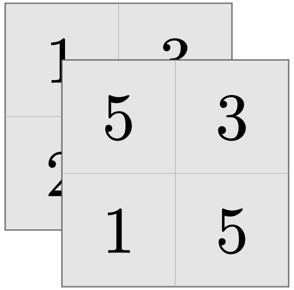
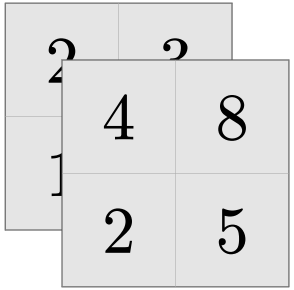
Convolutions

Given an input $x \in \mathbb{R}^{H \times W \times C}$, where $H = W = C = 2$, compute the output of a convolution with a **kernel** $K \in \mathbb{R}^{2 \times 2 \times 2}$, **padding** of 1, and **stride** of 2.



Input



Kernel

Convolutions

	Input		Kernel	
Channel 1	1	3	2	3
	2	4	1	6
Channel 2	5	3	4	8
	1	5	2	5

Convolutions

$$\begin{bmatrix}0&0&0&0\\0&1&3&0\\0&2&4&0\\0&0&0&0\end{bmatrix} \quad \begin{bmatrix}2&3\\1&6\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}6&\ \\ \ &\ \end{bmatrix}$$

Convolutions

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Convolutions

$$\begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 3 & 0 \\ \hline 0 & 2 & 4 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|}\hline 2 & 3 \\ \hline 1 & 6 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{|c|c|}\hline 6 & 3 \\ \hline 6 & \\ \hline \end{array}$$

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$$\begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 5 & 3 & 0 \\ \hline 0 & 1 & 5 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|}\hline 4 & 8 \\ \hline 2 & 5 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{|c|c|}\hline 25 & \\ \hline & \\ \hline \end{array}$$

Convolutions

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A convolution layer has 16 **filters** $K \in \mathbb{R}^{7\times 7\times 8}$ (note: $\mathbb{R}^{H\times W\times C}$), padding of 2, and stride of 3. Compute the number of parameters, including bias.

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State the dimension of output y for an input $x \in \mathbb{R}^{32\times32\times8}$?

$$H_{\text{out}} = \left\lfloor \frac{H_{\text{in}} - k + 2p}{s} \right\rfloor + 1 = 10, W_{\text{out}} = \left\lfloor \frac{W_{\text{in}} - k + 2p}{s} \right\rfloor + 1 = 10$$

$$C_{\text{out}} = 16$$

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Which of the following is true about convolutions?

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a. The number of biases is equal to the number of filters.
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c. The number of output channels is equal to the number of filters.
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