The Mamba Mentality as a Basketball Manifestation of the Memoryless Property

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Introduction

Kobe Bryant capped off his Hall of Fame career by scoring 60 points against the Utah Jazz at home. He did so in exactly 50 field goal attempts. In this respect, that last game serves as a perfect microcosm of Kobe's 20 seasons in the league; an elite scorer unmatched by anyone at his prime, and an unabashed gunslinger never hesitant to let it fly from almost anywhere on the court. It was no coincidence that Kobe was the highest volume perimeter shooter in the history of the game with 26200 attempts, surpassed only by elite inside players Karl Malone (26210) and Kareem Abdul-Jabbar (28307). It is, therefore, instructive to take a closer look at Kobe's shot selection patterns and their results. In particular we will address the question of the hot hand, the idea that field goal percentage increases following a hot streak of made shots. Via a Markov Chain Model, we demonstrate that past shots had little notable effect on Kobe's percentages. In this way, we argue that Kobe Bryant's confidence was a manifestation of the memoryless property of his shooting.

What Hot Hand?

Kobe was renowned for going on streaks where he looked unstoppable. This hot hand theory is wildly popular, applied to virtually any NBA player who makes a few shots in a row, and no doubt the casual player has experienced that sensation of confidence at the local gym. But do these assumptions hold weight when we analyse Kobe's numbers?

Let's examine how different aspects of Kobe's game change with his attempts. If the criticisms are true, and Kobe just jacked up bad shots even when he was in a funk, we could expect a higher number of attempts to correlate with a lower percentage. Alternatively, if the field percentage increases with attempts, then some credence could be given to the theory. Here is Kobe's FG percentage vs FG attempted – in the following charts the 2013-14 season has been dropped since he only played 6 games:

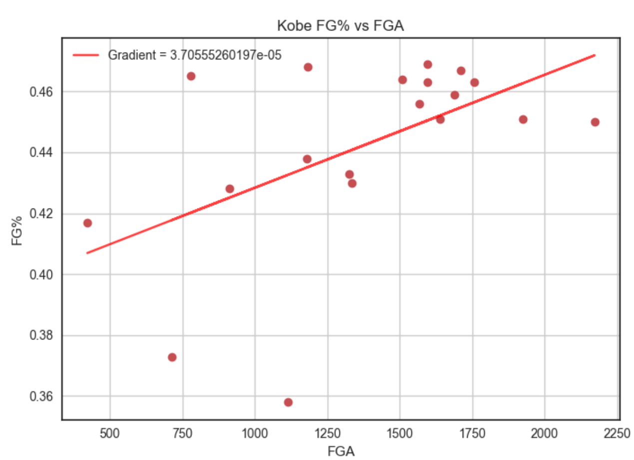

Figure 1: Kobe Bryant FGA vs FGP

There certainly some positive correlation, but this itself is not completely conclusive. It could be that more made shots simply give more confidence and result in a better shooting percentage. Correlation, however does not imply causation so this itself is not conclusive. What it does say, however is that we should pursue a further exploration of the numbers to look for the hot hand effect.

Up till now we have not made much distinction between 2 point and 3 point field goals, so let's compare FGA vs True Shooting Percentage. We use TS% This accounts for the fact that a 3 pointer is worth more and accordingly contributes more to the percentage:

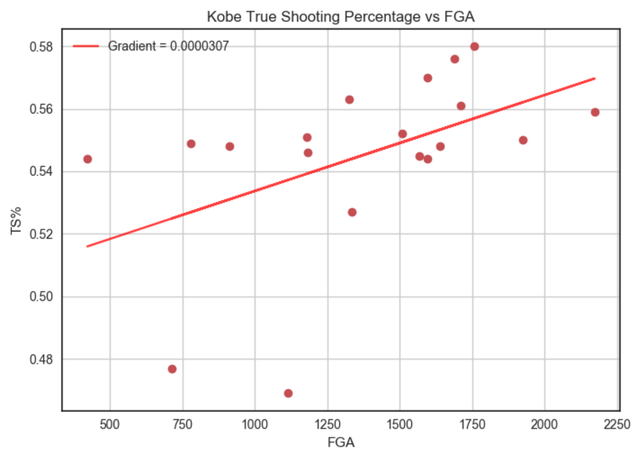

Figure 2: Kobe Bryant True Shooting Percentage vs FGA

The points here are much more clustered together, and ignoring the two points at the bottom, which were from the last few seasons when Kobe was in terminal decline, we see that the data hovers consistently around a TS% of .55 to .56. Accounting for the difference between 2 and 3 point field goals appears to make the change in percentage less pronounced. At the very least, the numbers suggest that shooting a 3 did not give Kobe proportionally more of a hot hand. That is to say, shooting a 3 did not appear to enhance Kobe's shooting much more than shooting a 2 did, suggesting that if the hot hand exists, it had a uniform impact over 2 point and 3 point field goals.

A deeper look at the Hot Hand

The data doesn't present a firm conclusion on the hot hand so far. To get more concrete results, we need to get a more concrete statistical model going. We will make one simplifying assumption, namely that if the hot hand exists, its impact does not extend forever. This means that if past shots have an impact on present shots, then the most recent shots have the most important impact. In order to make this problem somewhat tractable, we further strengthen this assumption by arguing that Kobe's shot making obeys the *Markov Property*, viz. the only shot with discernible impact on the present is the immediate shot before.

Let X_i be the random variable that is 1 if the i^{th} shot is a make and 0 otherwise. The Markov Property allows us to claim that $X_{i+1}|X_i$ is independent of $X_i|X_{i-1}$. Now we aim to estimate the probabilities of makes and misses given the previous shot, p_{make} and p_{miss}.

It is difficult to find shot tracking data for NBA players since the NBA clamped down on data scraping (the absence of which the NBA attributes to a SportVU glitch which has apparently lasted for over a year). Nevertheless, there is a Kaggle Dataset on Kobe Bryant's Shot Selection that provides information on a large amount of his field goals (not all since some data is omitted). In particular, the dataset contains 11466 attempts which come after a make, and 14232 which come after a miss. A simple manipulation via Python yields the following probabilities:

	After Make	After Miss
p_{make}	0.4396	0.4514
p_{miss}	0.5603	0.5486

The key use of our assumption is that now we can bound the errors on these probabilities. Using the Central Limit Theorem (those concerned with technicalities can find a calculation at the end), we get that there is a 95% chance that our measured $p_{make} \in [p - 0.0092, p + 0.0092]$ for the after make case and a 95% change that $p_{make} \in [p - 0.0082, p + 0.0082]$ for the after miss case (the discrepancy comes from a larger sample size). The p_{miss} values are just determined by the p_{make} values so no separate analysis for those is necessary. So these estimated values are pretty solid, and comparing them to Kobe's overall mark of $p_{make} = .447$ we see that they don't differ significantly enough to suggest any major difference after makes or misses. They are both within the 95% threshold meaning that they are sufficiently close to the overall percentages to be classed as good estimates for those percentages. Yet again, the gambler's fallacy - recast as the hot (or cold) hand- doesn't seem to survive closer scrutiny. Of course, while this analysis was very limited due to the Markov Property, others have conducted more thorough analyses which also dispute the existence of a hot hand.

A Markov Chain Analysis

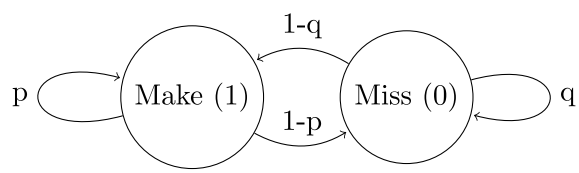

Figure 3: A simple Markov Chain model for Kobe's shooting

Just for fun, since we have modeled Kobe's shots with with a Markov model, let's consider some Markov Chain type questions and see if they give us any new insights. One obvious question we may ask is that given that Kobe missed his first shot, how many shots will he take to make his first one. The stopping time equation here is exceedingly simple:

$$\beta(0) = 1 + q\beta(0) + (1-q)\beta(0) = 1 + q\beta(0) \implies \beta(0) = \frac{1}{1-q}$$

Where q is the probability of a miss given a miss, 0.5486. Thus under this model, we would expect Kobe to take 2.22 shots before his first make, if we know his first shot was a miss. Alternatively if his shot was a make, by the same analysis as above, just replacing q with p, we estimate that Kobe takes 1.78 shots before he misses.

Are these results correct? We can employ a simple but fairly inelegant procedure to calculate these averages from the raw data. Simply take the array of shot data, and order the shots by game. We can then map game IDs to arrays containing the shot attempts in those games, and in these arrays we can simply start at every miss and see how many missed shots till the next make and then average this value over the game. Finally we average the averages over all the data. It must be remembered that our dataset isn't complete, but since it uses 25699 out of the 26200 attempts of Kobe's career, these actual numbers are very good estimates of the actual values. For brevity, let's call the average number of attempts till a make starting from a miss the cold time and the average number of attempts till a miss starting from a make the hot time. Doing the procedure above gives the following results:

	Cold Time	Hot Time
Actual	2.21	1.83
Expected	2.22	1.78

The Markov Chain model results are again very close to the actual results.

This Markov Chain is also special in that it is *irreducible* (we can get from any state in the graph to any other state) and *aperiodic* since it has self loops. So we can ask about the long term probability distribution of being in states Make and Miss. Let's construct the balance equations

$$\begin{bmatrix}\pi_0 & \pi_1\end{bmatrix}\begin{bmatrix} q & 1-q \\ 1-p & p\end{bmatrix} = \begin{bmatrix}\pi_0 & \pi_1\end{bmatrix}$$

Where $p = 0.4396$, $q = 0.5486$ from the table above. π_0 and π_1 are the long term probabilities of ending up in the miss and make states respectively. These, together with the normalisation condition ($\pi_0 + \pi_1 = 1$), yield the following:

$$\pi_0 = \frac{1-p}{2-p-q} = 0.555$$
$$\pi_1 = \frac{1-q}{2-p-q} = 0.445$$

These numbers don't appear too special, but compare 0.445, the long term probability of ending up in state 1, vs Kobe's overall career percentage, 0.447, and we see that these two numbers are remarkably close. In fact, the difference is well within the margin of error from the above CLT calculations, and so there is a strong reason to believe these numbers are supposed to be very similar. This supports the validity of the Markov Property assumption, since it produces almost exactly the long term result that we expect.

Conclusions: The benefits of a short term memory

I wouldn't say I'm a ballhog. I'm a shooter.

- Kobe Bryant

The league will need to wait for a while till it sees another gunslinger totally unperturbed by the prospect of jacking up 50 shots in his last game. However, as the above analysis of his shooting demonstrates, no matter what he felt mentally, there is concrete evidence to suggest that Kobe's shot making followed the Markov property very closely - there was little to none hot or cold hand overall. The fact that the probabilities of a make given a miss or a make are so close to his overall field goal percentage strongly suggests that each shot result overall is independent of each shot result before. The success of the Markov Chain model at predicting the length of Kobe's average cold and hot times is also a strong indication of the validity of applying the Markov Property. Of course, this does not mean you should launch full court heaves regularly. The key lies in the right balance of confidence and good shot selection with shots taken in rhythm.

The above is of course a strong statement, and most will immediately argue that anybody playing pickup can feel the effects of a hot hand. Of course, this work is not conclusive, and an immediate next step is to apply the same techniques to other prolific scorers such as Kareem Abdul- Jabbar, Michael Jordan, LeBron James, and Kevin Durant among others. Nevertheless, the sheer volume of data analysed means that our conclusions can't be disregarded just because of 'feelings'. We may attempt to reconcile this observation by noting that an elite scorer like Kobe would have a consistency between shots. That is to say his shot form is unlikely to change too much between shots. It is common basketball wisdom that the best shooters have extremely consistent shot forms and Kobe was no exception. Such factors that vary much more for the typical casual player than for a Hall of Fame scorer could have an impact on the independence of shots.

Finally, however, these numbers could provide motivation to those who seem to always lack confidence on the court during games despite having a decent level of skill. The message appears to be that if one is confident and does not let past misses affect the psyche, they could potentially develop the same independence of shot attempts that is apparent in Kobe's career. The technical term for this in statistics is being *memoryless*, that is the outcome at some time is completely independent of what happened before. Kobe's numbers back up his unabashed confidence and lack of hesitation in jacking up shots. Whether it contributed to or detracted from winning of course remains another matter - comparing stats like Win Shares vs FGA might be a good way to start thinking about that. Nevertheless, when it comes to sheer offensive production, any coach will say that there's hardly any substitute to plenty of practice, rhythm, and confidence. Don't be a ballhog on the hardwood, be a shooter. Ideally, a forgetful, memoryless shooter.

Appendix - CLT Error Calculation

Let X_j be the indicator variable that is 1 if Kobe made the j^{th} shot (with probability p) and 0 otherwise. Let our indicator, $\hat{X}$ be $\frac{1}{n-1}\sum_{j=1}^{n-1} X_{j+1}|X_j$, so that $\mathbb{E}[\hat{X}] = p$. We now wish to find some δ such that

$$\begin{aligned} \Pr(|\hat{X} - \mathbb{E}[\hat{X}]| \geq \delta) &\leq 0.05 \\ \Pr\left(\left|\frac{\hat{X} - \mathbb{E}[\hat{X}]}{\sigma}\right| \geq \frac{\delta}{\sigma}\right) &\leq 0.05 \\ \Pr\left(Z \geq \frac{\delta}{\sigma}\right) &\leq 0.05 \end{aligned}$$

To find σ, we find the variance first:

$$\begin{aligned} \mathrm{Var}(\hat{X}) &= \mathrm{Var}\left(\frac{1}{n-1}\sum_{j=1}^{n-1} X_{j+1}|X_j\right) \\ &= \frac{1}{(n-1)^2}\mathrm{Var}\left(\sum_{j=1}^{n-1} X_{j+1}|X_j\right) \\ &= \frac{1}{n-1}\mathrm{Var}(X_{j+1}|X_j) \\ &\leq \frac{1}{4(n-1)} \\ \therefore \sigma &\leq \frac{1}{2\sqrt{n-1}} \end{aligned}$$

This means that $\frac{\delta}{\sigma} \geq 2\delta\sqrt{n-1}$, so if we set the latter quantity to be the 95% threshold of the standard normal ($z = 1.96$) our resulting δ will guarantee our initial desired bound.

$$2\delta\sqrt{n-1} = 1.96 \implies n = \frac{1.96}{2\sqrt{n-1}}$$

And plugging the appropriate values of n into this gives the δ bounds quoted above.