Neutron Update

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Neutron-Nucleus Cross Section

- Contributions from nuclear, electromagnetic (electrostatic, magnetic moment, polarisation), new interactions
- Collect into coherent scattering length b_c, electromagnetic contribution χ_{em} and new contribution χ_{new}.
- $\chi_{em} = Z(b_F + b_I)/b_c$, Foldy scattering length and intrinsic neutron-electron scattering length.
- f(q) is the form factor of the atom, modeled to 10^{-4} accuracy by $f \approx [1 + 3(q/q_0)^2]^{-0.5}$, $q_0 = 6.86$ Å^{-1}
- b_S Schwinger scat. length, b_i incoherent scat. length, both assumed negligible.

$$V_{\text{new}}(r) = -\frac{1}{4\pi} g^2 Q_1 Q_2 \frac{e^{-\mu r}}{r},$$

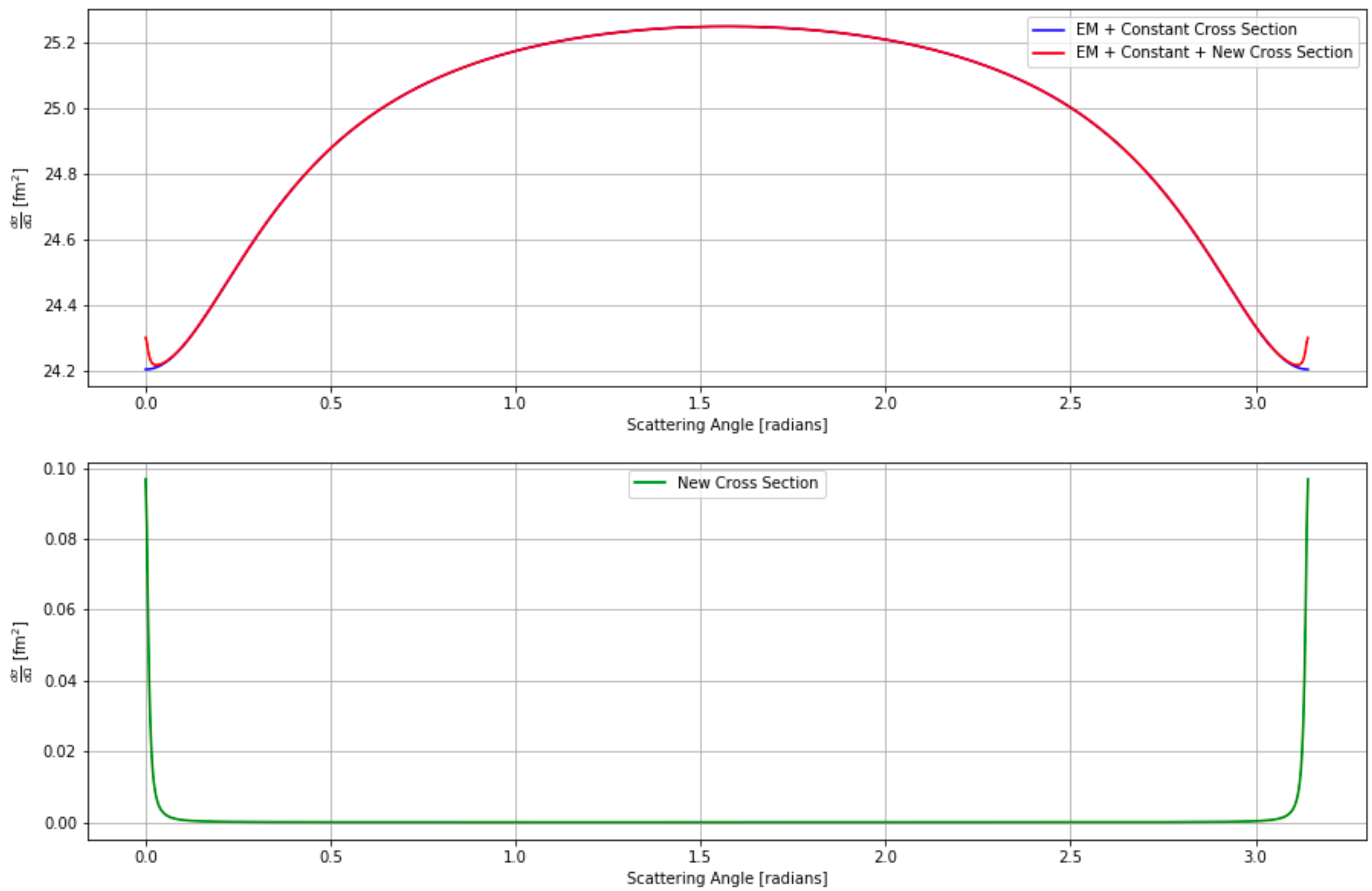
$$\frac{d\sigma}{d\Omega}(q) = b_c^2 \left\{ 1 + \chi_{\text{em}}[1 - f(q)] + \chi_{\text{new}} \frac{\mu^2}{q^2 + \mu^2} \right\}^2 + b_s^2(q) + b_i^2 + O(b_F^2)$$

$$\simeq b_c^2 \left\{ 1 + 2\chi_{\text{em}}[1 - f(q)] + 2\chi_{\text{new}} \frac{\mu^2}{q^2 + \mu^2} \right\},$$

Table 1
Typical values of the various contributions δb to the total neutron scattering length b of a heavy atom

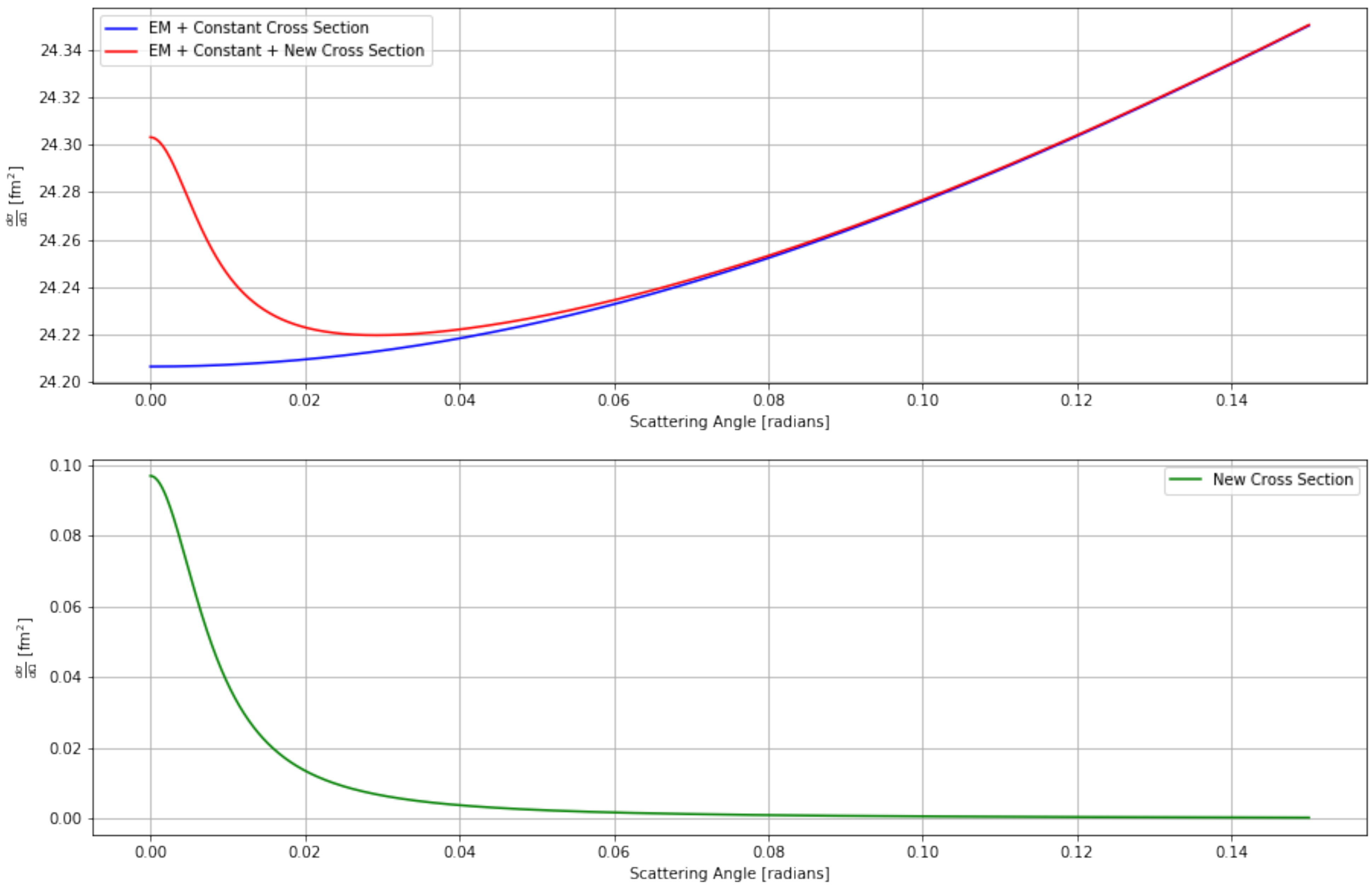
Class	Interaction	δb (fm)
I	Strong interaction	10.0
	Atomic magnetic dipole moment*	10.0
II	Spin–orbit (Schwinger)	0.1
	Foldy	0.1
	Neutron electric polarizability	0.05
	Intrinsic electrostatic	0.01
	Nuclear magnetic dipole moment*	0.005
III	Neutron electric dipole moment*	$\lesssim 10^{-8}$
	Neutron electric charge*	$\lesssim 10^{-10}$
	Weak interaction	$\sim 10^{-34}$

* If any.

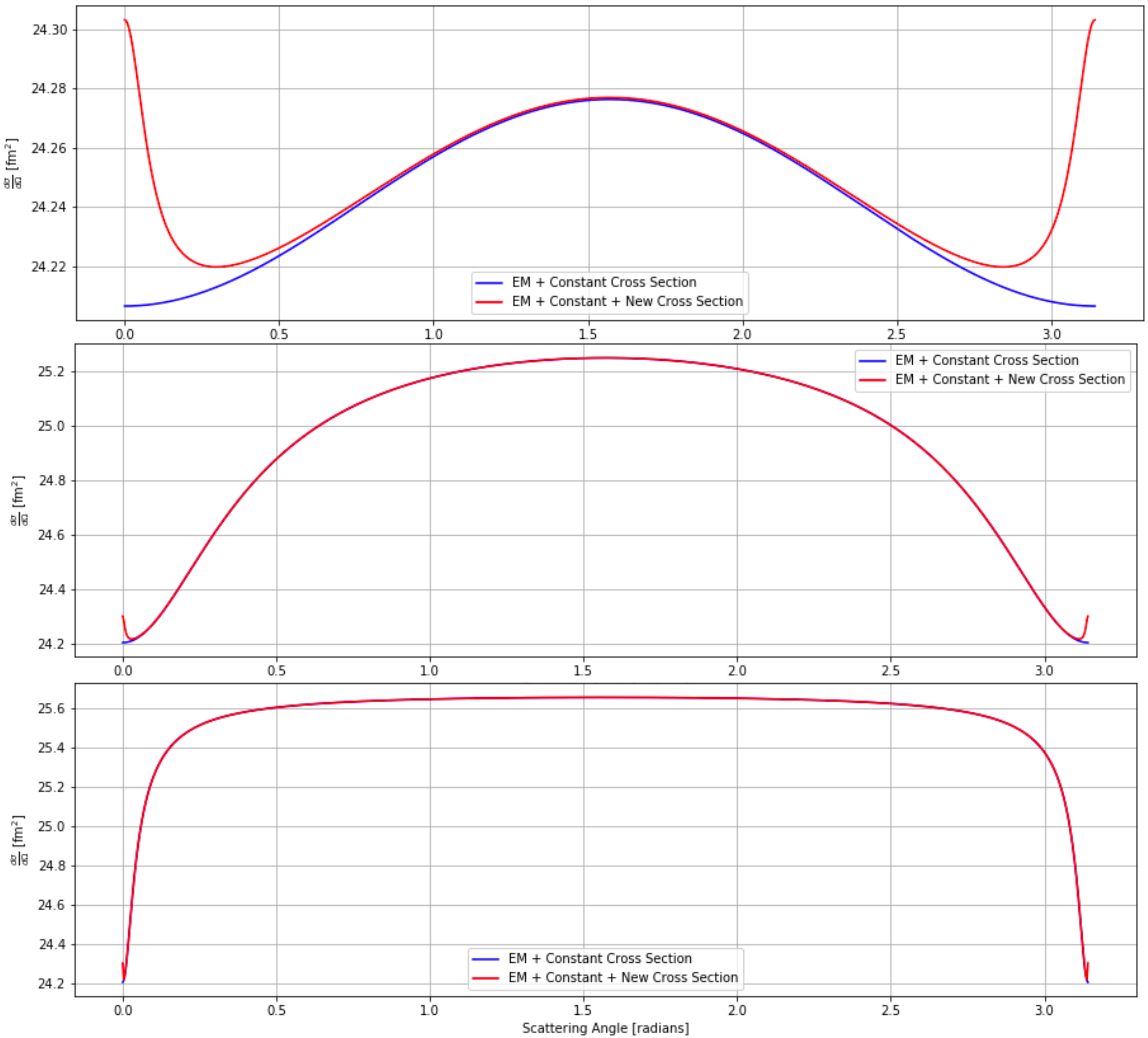


Cross Section with and without new force

λ = 1 Å (~ 0.1eV)



Cross Section with and without new force zoom in



λ = 10 Å

λ = 1 Å

λ = 0.1 Å

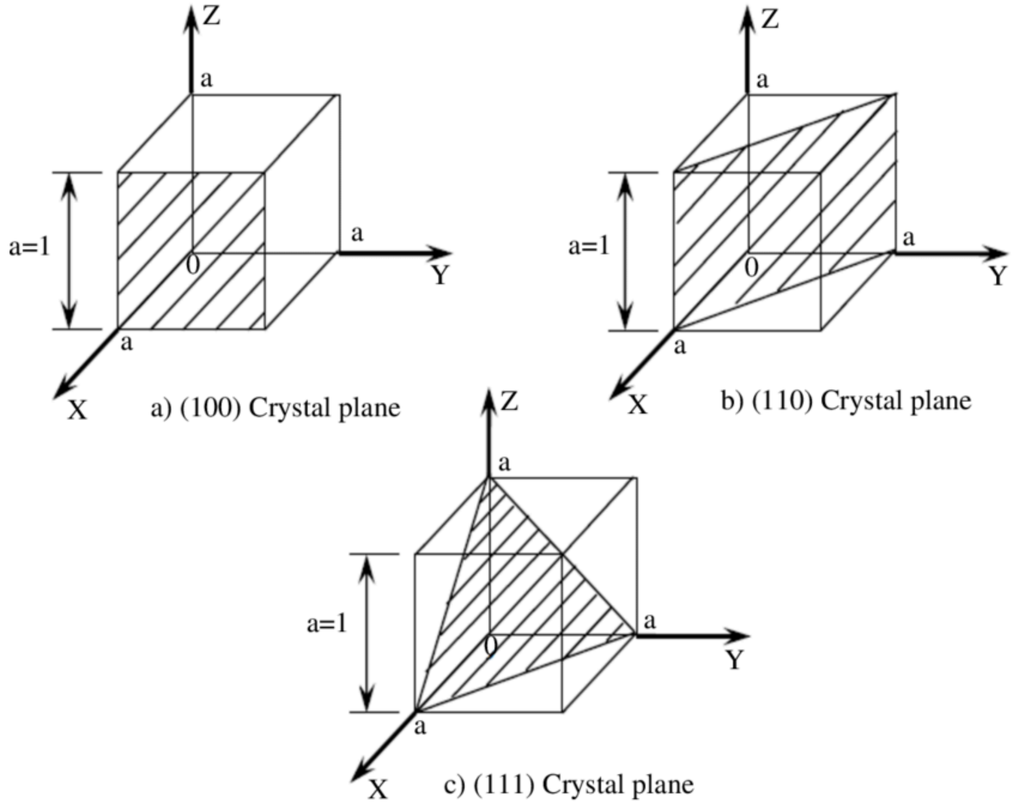
Lower energy neutrons seem easier to use

Scattering off Lattice

- Key feature of lattice - only allows certain momentum transfers.

- For a crystal, output intensity is given by $I(\theta) \sim F_{hkl}^2$, h,k,l Miller Indices related to lattice FT and indicates where diffractive peaks occur.

- Use Bragg Law to convert h,k,l into an angular position. For a cubic lattice:



$$F_{hkl} = \sum_{j=1}^{N} f_j e^{[-2\pi i(hx_j + ky_j + lz_j)]},$$

Sum over all atoms of unit cell. I think f_j is the single atom neutron cross section for the j^{th} atom.

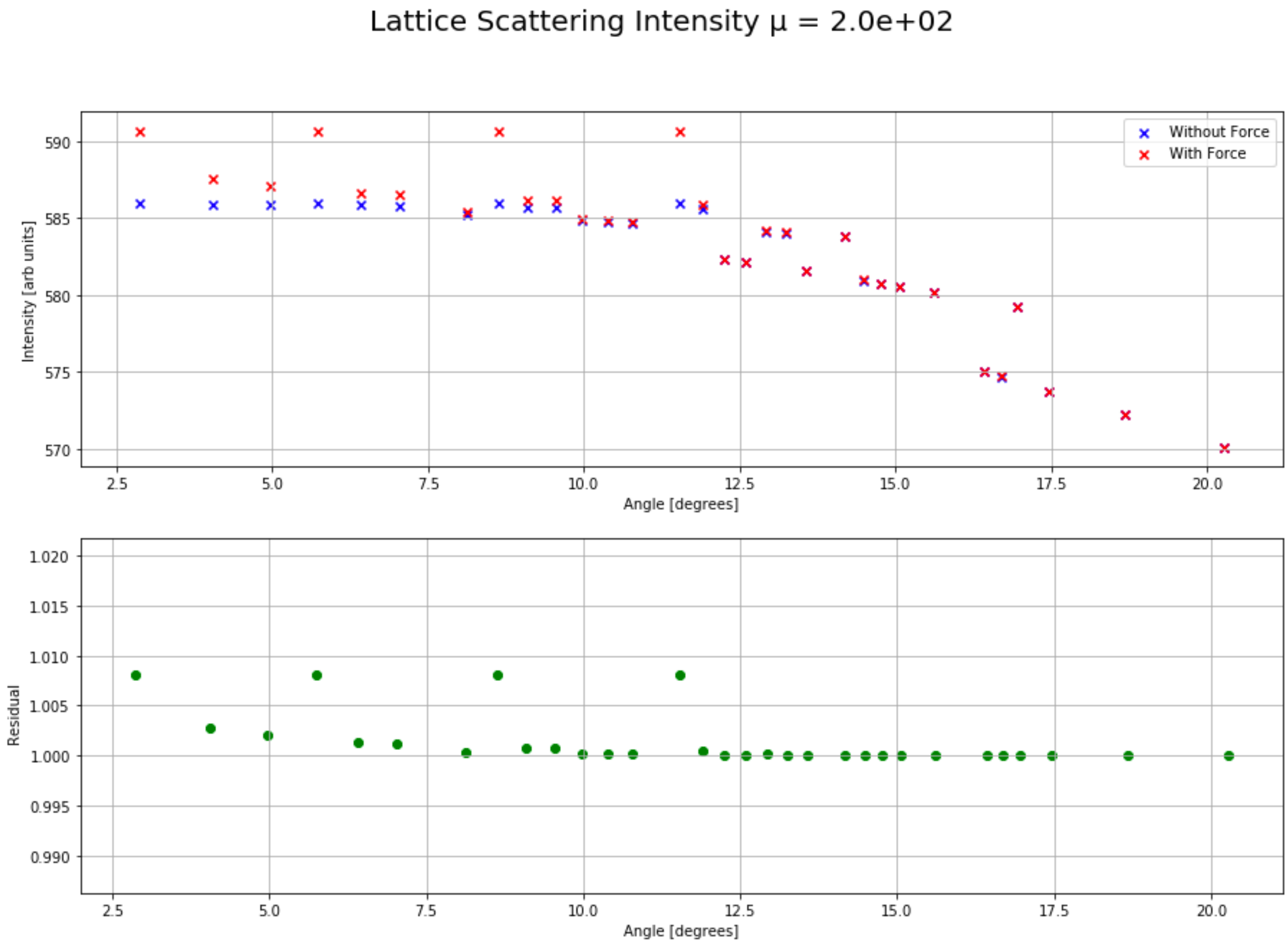
Bragg's Law

$$\lambda = 2d\sin(\theta)$$

For a cubic system

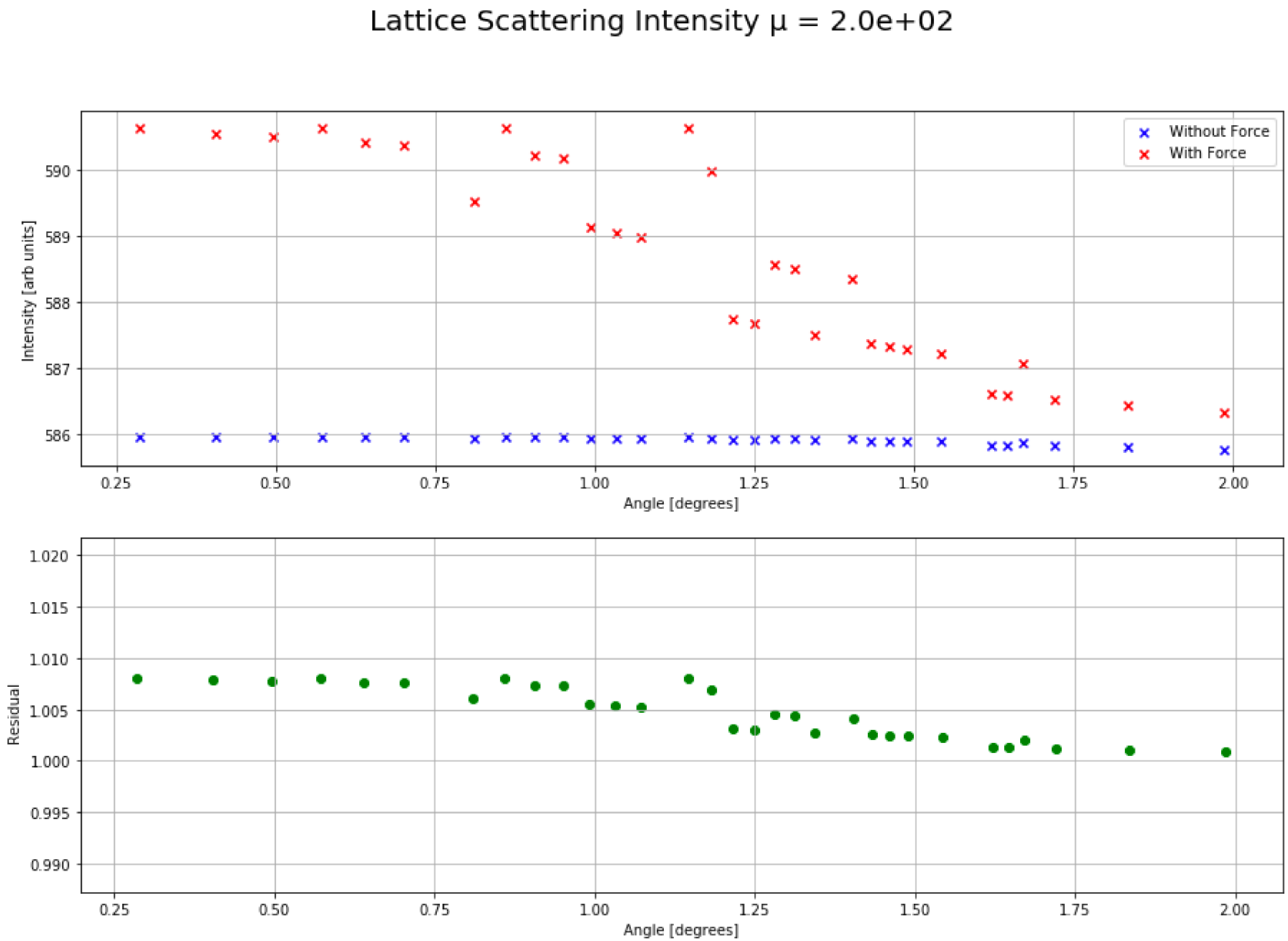
$$\frac{1}{d^2} = \frac{h^2+k^2+l^2}{a^2}$$

$$d_{hkl} = \frac{a}{\sqrt{(h^2+k^2+l^2)}} \Longrightarrow \lambda = \frac{2a\sin(\theta)}{\sqrt{(h^2+k^2+l^2)}} \Longrightarrow \sin^2(\theta) = \frac{\lambda^2}{4a^2}(h^2+k^2+l^2)$$



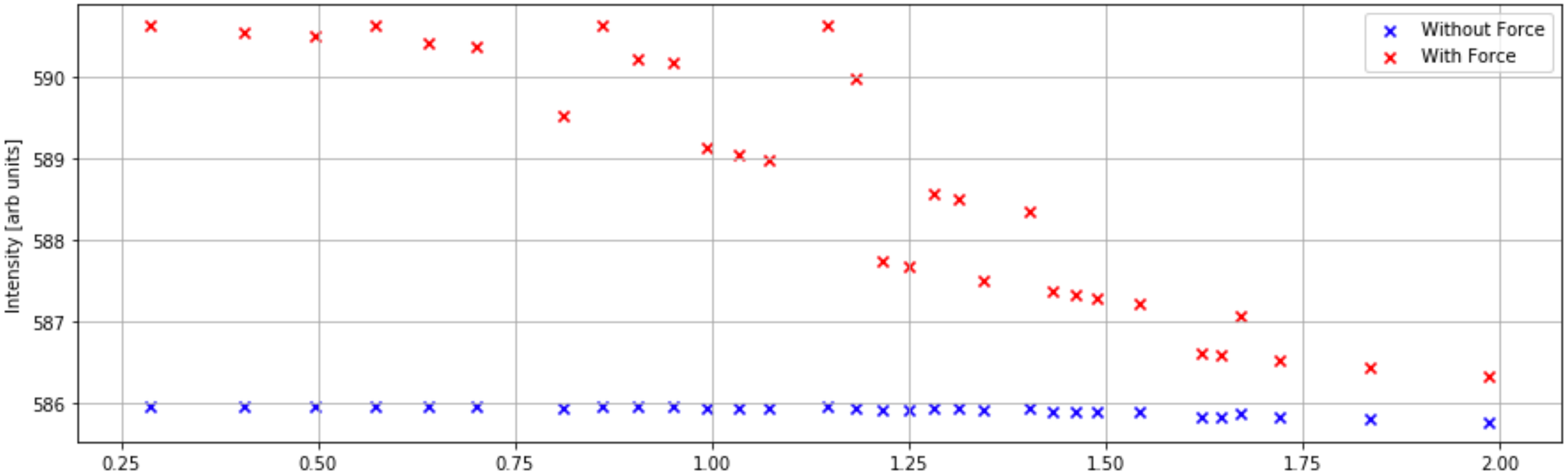
Neutrons off Cubic Xenon Lattice

$a = 10$ Å, $\mu = 200$ eV, $\lambda = 1$ Å



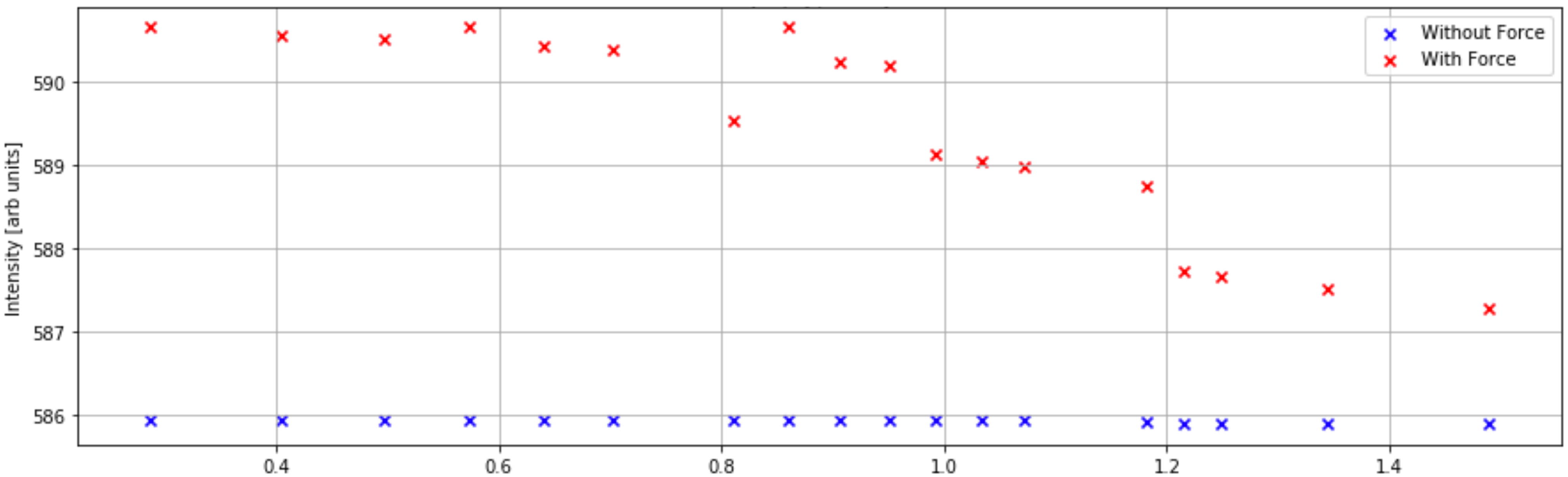
Neutrons off Cubic Xenon Lattice

$a = 100$ Å, $\mu = 200$ eV, $\lambda = 1$ Å

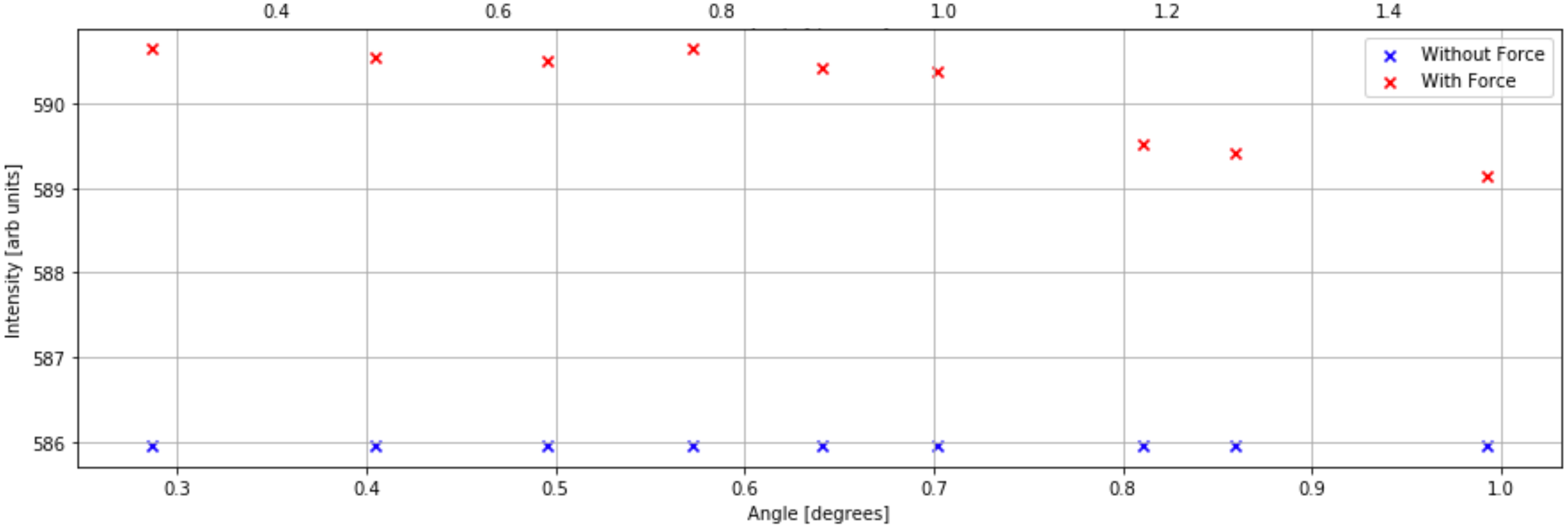


5 Miller Planes

All plots with
a = 100 Å, μ = 200 eV, λ = 1 Å

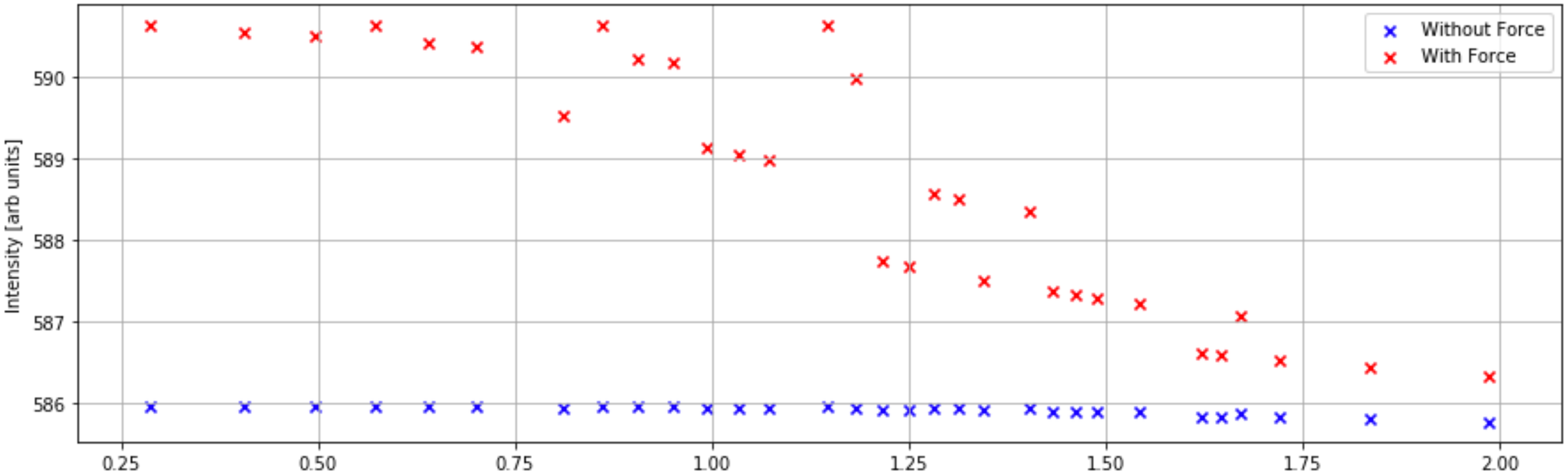


4 Miller Planes

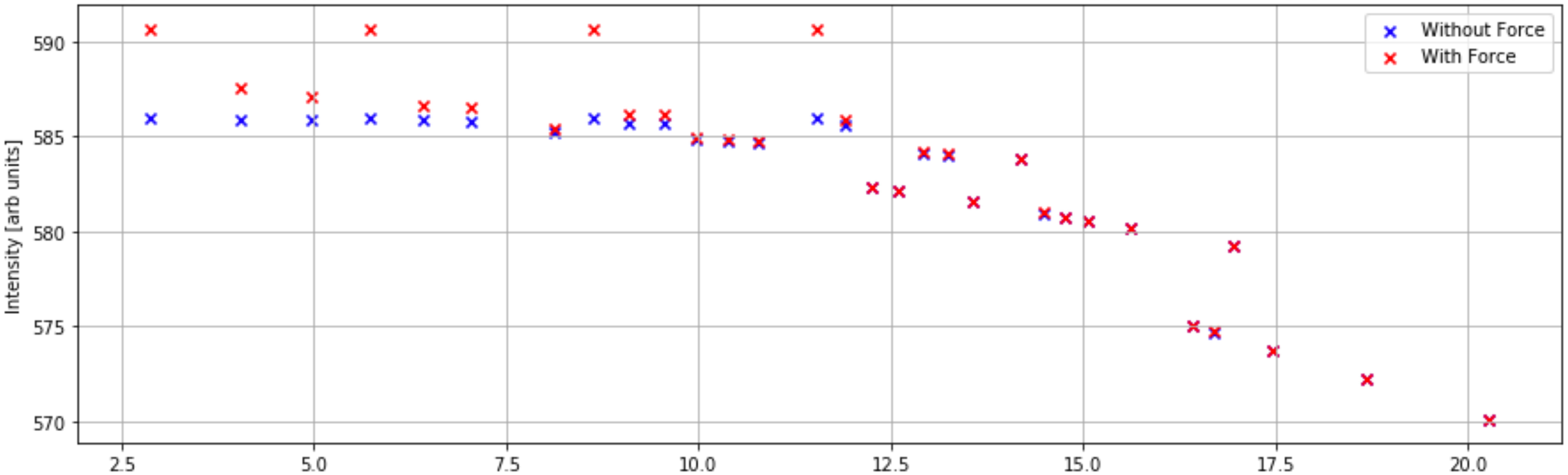


3 Miller Planes

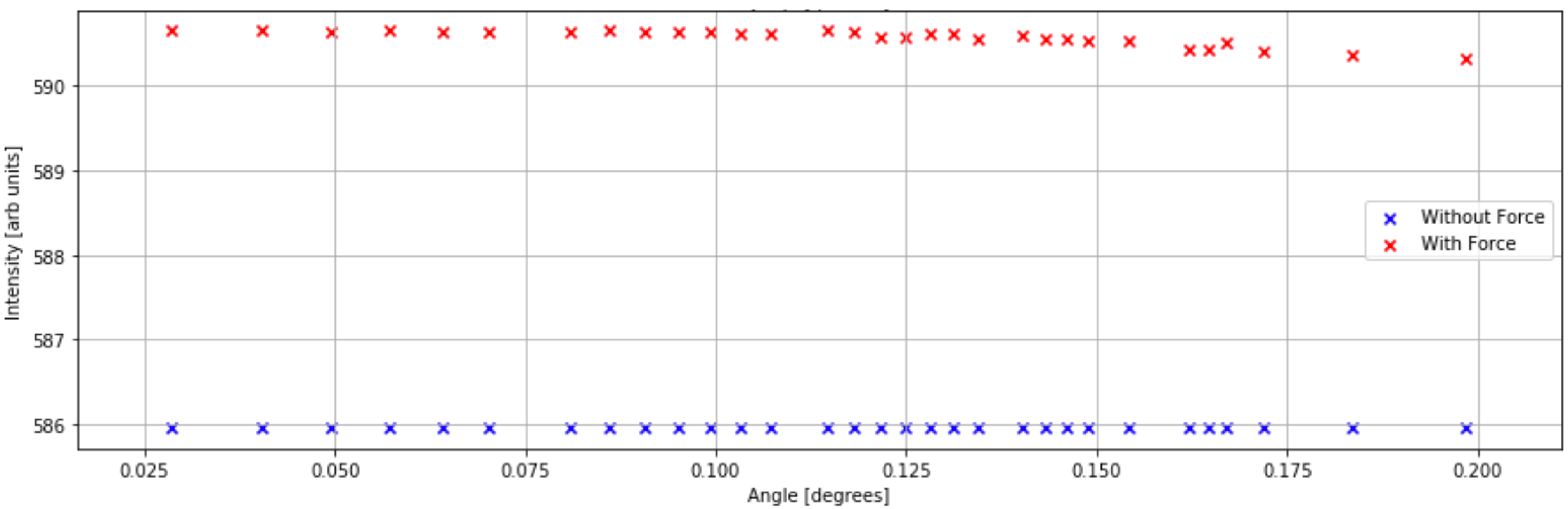
Question: How many of these peaks do we care about?



λ = 1 Å, a = 100 Å
$\lambda/a = 0.01$



λ = 10 Å, a = 100 Å
$\lambda/a = 0.1$



λ = 0.1 Å, a = 100 Å
$\lambda/a = 0.001$

For lattice λ/a is important.
Lower λ/a gives sharper differentiation in a narrower angle range.

Summary/Next Steps

- Neutron cross section dominated by strong force and electromagnetic effects, but can see new force via scattering.
- For simple atomic scattering higher λ gives better signal.
- For (primitive cubic) lattice scattering, important parameter is λ/a. Lower λ/a gives sharper contrast in smaller angular range.
- Next step is to introduce irregular lattice, i.e. unit cell axes with different lengths a,b,c and then put different atoms in the unit cell.