Quantum Eye I:
Bounded Cross-Basis Prediction from Single-Basis Counts via Frequency Signatures

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Abstract

We present a counts-only method for extracting *bounded, falsifiable* information from single-basis measurements and using it to *predict* cross-basis statistics in tested regimes. Four parameters—Purity (P), Structure (S), Entropy balance (E), and Quantum coherence proxy (Q)—are computed directly from raw measurement bitstrings via histogram and frequency-domain features.

Scope/limits: Single-basis outcome frequencies are not informationally complete. In general, there exist distinct density matrices with identical Z-basis counts (and identical counts-derived invariants), so this paper does *not* claim exact state recovery or preservation of "complete quantum information." Instead, we recover a bounded set of counts-derivable invariants and use them as an empirical representation for prediction and validation.

All empirical evidence in this paper is **simulator-only** (adversarially validated, counts-only mechanisms in tested regimes); no hardware runs are included.

The QSV criterion $\text{QSV} = P \cdot S \cdot E \cdot Q > \tau$ is presented as an *empirical heuristic filter* (with tunable threshold τ) that separates well-behaved runs from noise-dominated runs in tested regimes. From single-basis (Z) data we study bounded cross-basis inference targets under simulator-only validation.

The (P, S, E, Q) parameters admit an optional mapping to UCP constraint modes $(\mathsf{L}, \mathsf{C}, \mathsf{I}, \mathsf{U})$, though all results stand independently of UCP.

Contents

1 Introduction

Quantum state tomography typically requires measurements in multiple bases to reconstruct the full density matrix [3, 4]. Recent compressed-sensing approaches reduce measurement settings [2]. This paper explores an alternative: extracting maximal information from single-basis measurements using frequency-domain analysis.

UCP/REAL recap. The constraint vector $\mathbf{X} = (\mathsf{L}, \mathsf{C}, \mathsf{I}, \mathsf{U}) \in \Delta^3$ obeys the additive invariant $\mathsf{L} + \mathsf{C} + \mathsf{I} + \mathsf{U} = 1$ (UCP) and, on physical configurations, the multiplicative invariant $\mathsf{LCIU} = \kappa$ (REAL). All definitions and proofs are fixed canon from UCT Final and Papers 1–2; this paper only applies that background and remains operationally readable without it.

1.1 Contributions

1. **Raw-Data Definitions:** Four parameters (P, S, E, Q) defined directly from measurement bitstrings, not from reconstructed density matrices (Section 2).
2. **Empirical QSV Criterion:** $P \cdot S \cdot E \cdot Q > 0$ as a validated filter for state quality (Section 5).
3. **Adversarial Simulator Validation:** Causal frequency ablations, bounded identifiability, and explicit null-space failures under a simulator-only protocol (Section 4).
4. **Theoretical Framework (heuristic):** Frequency signature analysis explaining how single-basis counts can support bounded cross-basis inference under constraints (Section 6).
5. **UCP Interpretation:** Optional mapping to constraint modes (Section 7—may be skipped).

2 QSV Parameter Definitions from Raw Data

This section defines the four QSV parameters directly from single-basis measurement records. These definitions are **canonical** and are reused verbatim in Papers 8 (QE-II) and 9 (Decoherence).

2.1 Measurement Record

Definition 2.1 (Measurement Record)**.** A single-basis measurement record for an n-qubit system consists of N shots, each yielding a time-stamped, ordered bitstring

$$(\mathbf{b}^{(k)}, t^{(k)}) = \big((b_1^{(k)}, \ldots, b_n^{(k)}),\, t^{(k)}\big) \in \{0,1\}^n \times \mathbb{R}$$

for $k = 1, \ldots, N$, recorded in acquisition order. The *full transcript* $\{(\mathbf{b}^{(k)}, t^{(k)})\}_{k=1}^N$ is required for reproducibility and is stored/published alongside derived statistics.

Definition 2.2 (Outcome Frequencies)**.** For each bitstring outcome $\mathbf{s} \in \{0,1\}^n$, define the observed frequency:

$$f_{\mathbf{s}} = \frac{1}{N} \sum_{k=1}^{N} \mathbf{1}[\mathbf{b}^{(k)} = \mathbf{s}] \tag{1}$$

where $\mathbf{1}[\cdot]$ is the indicator function. The frequency vector $\mathbf{f} = (f_{\mathbf{s}})_{\mathbf{s} \in \{0,1\}^n}$ satisfies $\sum_{\mathbf{s}} f_{\mathbf{s}} = 1$.

2.2 The Four QSV Parameters

Definition 2.3 (Purity Parameter P)**.** The purity parameter measures concentration of the frequency distribution:

$$P = \sum_{\mathbf{s} \in \{0,1\}^n} f_{\mathbf{s}}^2 \tag{2}$$

Range: $P \in [2^{-n}, 1]$. $P = 1$ when all shots yield the same outcome; $P = 2^{-n}$ for uniform distribution.

Definition 2.4 (Structure Parameter S)**.** The structure parameter measures deviation from maximum entropy:

$$S = 1 - \frac{H(\mathbf{f})}{n \ln 2} \tag{3}$$

where $H(\mathbf{f}) = -\sum_{\mathbf{s}} f_{\mathbf{s}} \ln f_{\mathbf{s}}$ is the Shannon entropy (with $0 \ln 0 = 0$).

Range: $S \in [0, 1]$. $S = 1$ for deterministic outcomes; $S = 0$ for maximum entropy (uniform) [5, 1].

Definition 2.5 (Entropy Balance Parameter E)**.** The entropy balance measures autocorrelation structure in the shot sequence:

$$E = 1 - \frac{1}{n} \sum_{j=1}^{n} \left| R_j - R_j^{(\mathrm{ind})} \right| \tag{4}$$

where $R_j = \frac{1}{N-1} \sum_{k=1}^{N-1} \mathbf{1}[b_j^{(k)} = b_j^{(k+1)}]$ is the single-lag autocorrelation for qubit j, and $R_j^{(\mathrm{ind})} = p_j^2 + (1-p_j)^2$ is the expected value for independent shots with marginal probability $p_j = \frac{1}{N} \sum_k b_j^{(k)}$.

Range: $E \in [0, 1]$. $E \approx 1$ when shot-to-shot correlations match i.i.d. expectations; $E \ll 1$ indicates drift or memory effects.

Definition 2.6 (Spectral Parity Index (Q))**.** The spectral parity index is a *counts-derived proxy* measuring high-frequency spectral content on an ordered shot record:

$$Q = \frac{\sum_{|\omega| > \omega_c} |F_\omega|^2}{\sum_\omega |F_\omega|^2} \tag{5}$$

where $F_\omega = \frac{1}{N} \sum_{k=1}^{N} m^{(k)} e^{-2\pi i \omega k / N}$ is the Fourier transform of the parity sequence $m^{(k)} = (-1)^{\sum_j b_j^{(k)}}$, and $\omega_c = N/4$ is the cutoff frequency.

Order dependence and reporting requirement: Q depends on shot order. All computations in this paper use the acquisition-order transcript from Definition 2.1 (no shuffling). Reproducible reporting requires publishing the ordered transcript $\{(\mathbf{b}^{(k)}, t^{(k)})\}$; if any permutation averaging is used, report M and the averaging protocol explicitly.

Range: $Q \in [0, 1]$. $Q \approx 0$ for classical (low-frequency) patterns; higher Q indicates relatively more high-frequency parity content in the measured transcript (not a direct, basis-independent measure of coherence).

2.3 Parameter Summary

Terminology note. The quantities E and Q are **counts-derived proxies** (temporal stability and high-frequency parity content, respectively). They are *not* basis-independent measures of von Neumann entropy or quantum coherence, and should not be interpreted as such.

Interpretation (interface diagnostics). E and Q are functions of the measurement record rather than of the underlying density matrix ρ: they quantify temporal and spectral structure in the classical bitstring sequence produced by repeated quantum measurements. Neither constitutes a POVM element or a basis-independent observable; both operate at the interface where quantum dynamics produce time-ordered classical data. In this sense, E and Q diagnose *measurement stability* and *coherence retention footprints* in the record, not coherence amplitudes within ρ. This is analogous in spirit to record-level indicators used in quantum-trajectory analysis, decoherence diagnostics, and randomized-benchmarking decay fits: empirically useful, but defined in the post-measurement domain.

Terminology safety note. For clarity, the quantities E and Q are *record-space diagnostics*, not observables on the underlying density matrix (ρ). They summarize temporal and spectral structure in the classical measurement transcript produced by repeated quantum measurements, and therefore depend on acquisition order and experimental context. Throughout this paper, references to "coherence," "entropy balance," or "quantum structure" should be understood in this record-space sense: as indicators of how quantum dynamics manifest in, and survive into, classical data streams. No claim is made that E or Q constitute basis-independent measures of coherence, entropy, or other intrinsic properties of (ρ).

Parameter	Symbol	Range	Interpretation
Purity	P	$[2^{-n}, 1]$	Concentration of outcomes
Structure	S	$[0, 1]$	Deviation from maximum entropy
Entropy Balance	E	$[0, 1]$	Shot-to-shot temporal stability
Spectral Parity Index	Q	$[0, 1]$	High-frequency spectral content

Remark 2.1 (UCP interpretation deferred). Optional UCP correspondences (e.g., $P \leftrightarrow \mathsf{L}$) are confined to Section 7. All operational results are UCP-independent.

Remark 2.2 (Normalization). For comparison across system sizes, normalize P as $\tilde{P} = (P - 2^{-n})/(1 - 2^{-n})$ to obtain $\tilde{P} \in [0, 1]$.

2.4 Invariance and Stability

Proposition 2.1 (Bit relabel invariance). *For any permutation π of qubit labels, (P, S, E, Q) computed from the permuted record $\{\pi(\mathbf{b}^{(k)})\}$ equal those from the original record. Thus QSV features depend only on the empirical distribution and parity sequence, not on qubit ordering.*

Proposition 2.2 (Depolarizing stability). *Let $\mathcal{D}_p(\rho) = (1 - p)\rho + p\,\frac{\mathbb{I}}{2^n}$ act before measurement.*

Then

$$P(\mathcal{D}_p(\rho)) \geq (1-p)^2 P(\rho) + \frac{p^2}{2^n}, \tag{6}$$

$$S(\mathcal{D}_p(\rho)) \leq (1-p)S(\rho) + p, \tag{7}$$

$$E(\mathcal{D}_p(\rho)) \geq (1-p)E(\rho), \tag{8}$$

$$Q(\mathcal{D}_p(\rho)) \geq (1-p)^2 Q(\rho). \tag{9}$$

Hence small depolarizing rates contract QSV multiplicatively; the QSV product is $(1-p)^6$-stable up to $O(p/2^n)$ terms.

Proof sketch. Permutation invariance follows because P, S depend only on the histogram $\mathbf{f}$ and entropy thereof, E on single-qubit marginals and their autocorrelations, and Q on the parity spectrum, all symmetric under relabel. Depolarizing mixes $\mathbf{f}$ with uniform noise, yielding the stated bounds by convexity of ℓ_2 norm (for P), concavity/continuity of Shannon entropy (for S), linearity of autocorrelations (for E), and quadratic scaling of high-frequency power (for Q). □

3 Complementarity and No-Go Theorems

This section clarifies what Quantum Eye does and does not claim, with explicit reference to quantum no-go theorems.

Remark 3.1 (Non-identifiability from single-basis counts)*.* Single-basis outcome frequencies are not informationally complete. In general, there exist distinct states $\rho \neq \sigma$ with identical Z-basis histograms ($\mathrm{diag}(\rho) = \mathrm{diag}(\sigma)$), hence identical counts-derived quantities such as P and S. Therefore, Quantum Eye should be read as an inference/prediction method under constraints, not as a general-purpose tomography or "complete state recovery" claim.

3.1 What Quantum Eye Does

1. **Computes signatures from single-basis data:** All four parameters (P, S, E, Q) are computed from a single measurement basis (e.g., Z).

2. **Predicts statistics in other bases:** Given (P, S, E, Q) and Z-basis frequencies, we predict $\langle O \rangle$ for operators O in the X or Y basis.

3. **Provides quality filtering:** The QSV criterion identifies runs where predictions are reliable.

3.2 What Quantum Eye Does NOT Do

1. **No joint incompatible measurements:** We never measure X and Z simultaneously on the same qubit. All data comes from a single basis per run.

2. **No individual outcome prediction:** We predict expectation values $\langle O \rangle$, not the outcome of a specific measurement. This is analogous to predicting a mean, not a single sample.

3. **No violation of no-signaling:** Cross-basis predictions require classical post-processing of local data; no faster-than-light information transfer is implied.

4. **No violation of uncertainty relations:** The uncertainty in our predictions respects Heisenberg bounds. High-precision Z-basis data yields bounded-precision X-basis predictions.

3.3 Analogy: Classical System Identification

Consider a classical random variable X with known mean μ and the constraint that $X \geq 0$. From μ alone, one can bound $\mathrm{Var}(X)$ (Chebyshev-type arguments). This is not "measuring variance directly"—it is inferring statistics from constraints.

Quantum Eye operates similarly: given Z-basis frequencies and the constraint that the state is a valid quantum state (positive semidefinite ρ), the set of compatible density matrices is restricted. The QSV parameters further constrain this set, enabling bounded prediction of X-basis statistics.

3.4 Formal No-Go Compliance

Proposition 3.1 (Complementarity Compliance (modeled))**.** *Let $\hat{Z}$ and $\hat{X}$ be non-commuting observables. Quantum Eye produces an estimator $\langle\hat{X}\rangle$ from Z-basis data with an empirical bootstrap uncertainty $\Delta_{\mathrm{boot}}\langle\hat{X}\rangle$ computed from resampled shot records. Under the standard uncertainty relation for states consistent with the observed Z-histogram and assuming no additional side information, the predicted expectation should satisfy*

$$\Delta_{\mathrm{boot}}\langle\hat{X}\rangle \geq \frac{|\langle[X,Z]\rangle|}{2\Delta Z}$$

where ΔZ is the empirical Z-basis standard deviation. A full derivation for the Quantum Eye estimator is not provided here; this statement is a modeled compliance condition rather than a proven bound.

4 Simulator Validation: Adversarial Protocol

This paper adopts a **simulator-only adversarial validation posture**: we make only claims supported by deterministic artifacts (fixed seed lists, configs, and causal ablations) and we include explicit null-space failures.

4.1 Protocol summary and artifacts

All results in this section come from a counts-only simulator suite run with:

- seed list $\{0, 1, \ldots, 9\}$ (no best-of selection),
- $n = 2$ qubits, $N = 2048$ shots, and a representative noise level (when applicable),
- logged version context (Python/Qiskit/etc.) and git SHA.

The protocol document and artifacts are stored in the Quantum Eye experiment repo under: `ADVERSARIAL_VALIDATION_PROTOCOL.md` and `artifacts/`.

4.2 Causal validation via frequency ablations

We test whether *frequency structure* is causally responsible for discrimination by applying ablations that preserve norms/energy while destroying structure: `bin_shuffle`, `phase_scramble`, `band_drop_low`, `band_drop_high`.

The task is a mechanism probe: match a counts-derived frequency signature to the correct element in a small reference library using overlap. Let the metric be overlap(signature, correct reference).

Condition	Mean true overlap	Mean gap (full−ablated)
Full signature	0.7961	—
Bin shuffle	0.0108	0.7853
Phase scramble	0.3607	0.4354
Band drop (low-pass)	0.4781	0.3180
Band drop (high-pass)	0.3258	0.4703

These values are aggregated over 10 seeds and taken directly from the `frequency_ablation_causality` artifact summary (`run_id=fa5ea4fa9fc7`).

4.3 Bounded identifiability: same Z marginals, different recoverable invariant

To separate "decorative" transforms from real mechanism, we construct a pair with identical single-qubit Z marginals but differing in a counts-derivable correlation pattern. A baseline using only single-qubit marginals is blind (chance), while the full counts-only signature discriminates:

Method	Accuracy (10 seeds)
Baseline (single-qubit Z marginals)	0.5
Bin shuffle ablation	0.5
Phase scramble ablation	0.6
Full counts-only signature	1.0

These values are taken from the `identifiability_sameZ_different_invariant` artifact summary (`run_id=10e8b8535ea2`).

4.4 Explicit null-space failures (honesty cases)

Single-basis counts are not informationally complete, so null spaces are unavoidable. We include explicit failures where distinct states have identical Z counts, hence identical counts-only signatures, but different X/Y structure:

- $|{+}{+}\rangle$ vs $|{+}{-}\rangle$: signature overlap ≈ 1.0, forced-choice accuracy $= 0.5$ (chance), while the X/Y expectation L1 difference is 2.0.
- $|{+}{+}\rangle$ vs $|{+}i, {+}\rangle$: signature overlap ≈ 1.0, forced-choice accuracy $= 0.5$ (chance), while the X/Y expectation L1 difference is 2.0.

These results are taken from the `nullspace_honesty_cases` artifact summaries (`run_id=1320cf4a92df` and `run_id=1e2fe5e5b3b4`).

5 The QSV Criterion: Empirical Validation

This section presents the QSV criterion as an **empirical filter**, not a theorem of quantum mechanics.

Criterion 5.1 (Quantum State Validity (QSV) — Empirical). A measurement record passes the QSV filter if:

$$\mathrm{QSV} = P \cdot S \cdot E \cdot Q > \tau \tag{10}$$

where $\tau > 0$ is a threshold. In this paper we do not claim a universal choice of τ; if used as a gate, τ should be calibrated per (noise model, qubit-count, shot regime) and reported as part of the experimental configuration.

Calibration plan. If τ is used operationally, re-fit it per simulator regime and report the calibration split. In the adversarial suite reported here, we avoid best-of selection and instead score mechanisms directly via overlap and explicit falsifiers (Section 4).

Remark 5.1 (Epistemic Status: Not a Theorem). The QSV criterion is an **empirically validated heuristic**, not a mathematical theorem. It has been tested on:

- simulated noisy circuits (adversarial suite artifacts, Section 4)

The criterion may fail for:

- Exotic states not in the training distribution
- Highly non-Markovian noise
- Systems with strong temporal correlations ($E \approx 0$)

5.1 Performance Metrics (simulator posture)

This paper's primary empirical validation is simulator-only and mechanism-focused: causal ablations (frequency structure destroyed while preserving norms/energy), bounded identifiability tests, and explicit null-space failures (Section 4). We do not report hardware AUC/accuracy tables here.

5.2 Failure Modes

1. **Borderline cases:** States with QSV $\in [0.05, 0.15]$ have unreliable classification. We recommend abstaining in this range.
2. **False positives:** Structured classical noise (e.g., periodic bit-flip) can produce high QSV despite not being a valid quantum state.
3. **False negatives:** Highly mixed entangled states may have low Q despite genuine quantum correlations.

6 Theoretical Framework

This section provides the mathematical foundation for cross-basis prediction.

6.1 Frequency Signatures

Definition 6.1 (Frequency Spectrum)**.** The frequency spectrum of the parity sequence $m^{(k)} = (-1)^{\sum_j b_j^{(k)}}$ is:

$$F_\omega = \frac{1}{N} \sum_{k=1}^{N} m^{(k)} e^{-2\pi i \omega k/N} \tag{11}$$

Proposition 6.1 (Heuristic spectral–QSV relationships)**.** *The QSV parameters relate to spectral properties:*

$$P \propto |F_0|^2 + \textit{(higher harmonics)} \tag{12}$$

$$S = 1 - H_{spectral}/H_{\max} \tag{13}$$

$$E \approx 1 - |\textit{spectral drift}| \tag{14}$$

$$Q = \textit{(high-frequency power)/(total power)} \tag{15}$$

where $H_{spectral}$ is the entropy of $|F_\omega|^2$ normalized.

Status. *These relationships are interpretive and are not used as proofs; causal validation is provided empirically via frequency ablations (Section 4).*

6.2 Cross-Basis Prediction (Conjectural Roadmap)

Heuristic 6.1 (Cross-Basis Statistics (not proved)). For a state ρ with QSV parameters (P, S, E, Q) and Z-basis frequencies $\mathbf{f}$, the expectation value of observable O in basis B' can be predicted (heuristically):

$$\langle \hat{O} \rangle_{B'} = \mathcal{T}(\mathbf{f}; P, S, E, Q; B \to B') \tag{16}$$

where $\mathcal{T}$ is a transformation with prediction error:

$$|\langle \hat{O} \rangle - \langle O \rangle| \leq C \cdot \frac{\|O\|}{\sqrt{N}} \cdot g(P, S, E, Q) \tag{17}$$

for constant C and function g decreasing in each argument.

Proof sketch. The QSV parameters constrain the density matrix ρ:

1. P bounds $\mathrm{Tr}(\rho^2)$, constraining eigenvalue spread.
2. S bounds the Shannon entropy of the measured single-basis distribution, constraining concentration/spread of the diagonal in that basis (not the von Neumann entropy of ρ).
3. E ensures temporal consistency (no drift artifacts).
4. Q quantifies coherence, distinguishing quantum from classical.

Given $\mathbf{f}$ (diagonal of ρ in Z-basis) and these constraints, the set of compatible ρ is bounded. Standard semidefinite programming arguments yield the error bound. □

Remark 6.1 (Prediction Algorithm). In practice, $\mathcal{T}$ is computed via constrained optimization:

$$\langle \hat{O} \rangle = \max_{\rho \in \mathcal{C}(\mathbf{f}, P, S, E, Q)} \mathrm{Tr}(\rho\, O) \tag{18}$$

where $\mathcal{C}$ is the constraint set. For Bell states, closed-form approximations suffice.

Remark 6.2 (Status and evidence). The cross-basis mapping is heuristic: no complete proof is provided that single-basis Q determines cross-basis behavior. Evidence is empirical and limited to simulator-only adversarial tests in small qubit/shots/noise regimes (Section 4); higher-qubit validation remains open.

Wall-off statement: No claims in Sections 4–5 rely on the conjectural mapping described in this subsection.

7 UCP Interpretation

Note: This section is optional. All preceding results are UCP-independent and can be understood as device-agnostic data analysis.

7.1 Mapping to Constraint Modes

The QSV parameters map to UCP constraint modes:

QSV	UCP	Correspondence
P (Purity)	L	Logical consistency: the state is well-defined, not maximally mixed
S (Structure)	C	Computational closure: the state has computable, non-trivial structure
E (Entropy)	I	Information conservation: temporal stability, no information loss to drift
Q (Coherence)	U	Unity: the state exhibits genuine quantum correlations

7.2 Simplex Embedding

Definition 7.1 (Normalized QSV Vector)**.** Define the normalized QSV vector:

$$\mathbf{X}_{\text{QSV}} = \frac{(P, S, E, Q)}{P + S + E + Q} \in \Delta^3 \tag{19}$$

Proposition 7.1 (REAL Manifold)**.** *QSV-valid states lie on the REAL manifold:*

$$\kappa_{QSV} = \frac{P \cdot S \cdot E \cdot Q}{(P + S + E + Q)^4} \tag{20}$$

States with higher QSV product have larger κ, indicating greater "constraint integrity."

7.3 UCP Design Motivation

The Quantum Eye method was originally designed using UCP principles:

- P corresponds to the L mode: is the state logically coherent?
- S corresponds to C: does the state have computational structure?
- E corresponds to I: is information preserved over time?
- Q corresponds to U: does the state exhibit quantum unity?

However, the resulting method and its validation are entirely empirical and do not require accepting UCP as a physical principle.

8 Conclusion

We have established Quantum Eye I:

1. **Raw-Data Definitions:** (P, S, E, Q) computed directly from bitstrings (Section 2).
2. **Empirical QSV:** $P \cdot S \cdot E \cdot Q > \tau$ as an empirical heuristic filter (Section 5).

3. **Adversarial Simulator Validation:** causal frequency ablations, bounded identifiability, and explicit null-space failures (Section 4).

4. **Complementarity Safe:** No forbidden measurements; predictions are statistical (Section 3).

5. **Theoretical Foundation:** Frequency signatures and constrained optimization (Section 6).

Paper 8 (Quantum Eye II) extends these methods to 100-qubit systems via golden-ratio sampling.

Falsifiers and Limitations

This paper would be **invalidated** if:

1. **QSV ceases to track prediction quality out of regime:** If, under comparable simulator-validated regimes and tasks (same inference target and scoring), QSV fails to correlate with prediction quality on new platforms, the method's generality is questionable.

2. **Cross-basis predictions fail:** If predicted $\langle X \rangle$ values differ from measured values by $> 3\sigma$ on $> 20\%$ of runs, the prediction framework is flawed.

3. **Complementarity violation found:** If any aspect of Quantum Eye is shown to violate a quantum no-go theorem, the method is fundamentally incorrect.

4. **Frequency signatures are non-reproducible:** If (P, S, E, Q) values vary by $> 20\%$ across identical state preparations, the definitions are not robust.

5. **QSV product is uncorrelated with state quality:** If states with high QSV fail cross-basis predictions at the same rate as low-QSV states, the criterion has no predictive value.

Known Limitations:

- Single-basis measurement statistics are not informationally complete; therefore, any method operating on them necessarily admits nontrivial null spaces. Our approach explicitly demonstrates and respects these limits.

- Simulator-only in this manuscript; extending conclusions to hardware requires a separate, explicitly documented hardware layer with raw distributions and uncertainty.

- Bell and GHZ states are highly symmetric; performance on generic entangled states is less characterized.

- The empirical QSV threshold $\tau = 0.1$ may need recalibration for different noise profiles.

- No formal guarantee that single-basis Q determines cross-basis behavior; the cross-basis mapping is heuristic.

- Sensitivity to ordering and calibration: reordering shot transcripts or miscalibrated τ can invalidate Q and QSV comparisons.

A Local Usage Notes

This paper uses the standard UCP notation established in Papers 1–2:

- $\mathbf{X} = (\mathsf{L}, \mathsf{C}, \mathsf{I}, \mathsf{U}) \in \Delta^3$ — constraint vector on the 3-simplex
- $\mathsf{L} + \mathsf{C} + \mathsf{I} + \mathsf{U} = 1$ — UCP additive invariant
- $\mathsf{L} \cdot \mathsf{C} \cdot \mathsf{I} \cdot \mathsf{U} = \kappa$ — REAL multiplicative invariant

Paper-specific notation:

- P, S, E, Q — counts-derived Quantum Eye signatures (purity, structure, temporal stability, parity-spectrum proxy)
- QSV $= P \times S \times E \times Q$ — quantum state validity criterion
- ϕ — golden ratio sampling parameter

For complete UCP/REAL definitions, see Papers 1–2 and the canonical glossary in UCT Final.

B Technical Note: On the Non-Injectivity of Single-Basis Inference Maps

B.1 Setup: the Z-compatible state set

Fix an n-qubit computational basis and an observed Z-basis outcome histogram $\mathbf{f} = (f_{\mathbf{s}})_{\mathbf{s}\in\{0,1\}^n}$ as in Definition 2.2. Define the convex feasibility set

$$\mathcal{C}_Z(\mathbf{f}) \;=\; \{\rho \succeq 0 \;:\; \mathrm{Tr}(\rho) = 1,\ \mathrm{diag}_Z(\rho) = \mathbf{f}\}\,.$$

This set contains all density matrices consistent with the observed single-basis statistics; it is generally high-dimensional for $n \geq 1$.

B.2 Counts-only feature maps

In this paper, the operational features are computed from a single-basis measurement record (Definitions 2.1–2.6):

$$F(\text{record}) = (P, S, E, Q).$$

Critically, F is **counts-only**: it depends only on the observed Z-basis transcript (histogram plus shot-order structure used by E and Q). Therefore, for any two underlying states $\rho_1, \rho_2 \in \mathcal{C}_Z(\mathbf{f})$ that generate identical Z-basis transcripts (in distribution, or deterministically under a construction), we will have the same computed feature vector and signature.

B.3 The implied inference map and its limitation

Many claims in the literature can be abstracted as an inference map of the form

$$T:\ (\mathbf{f}, P, S, E, Q) \;\mapsto\; \widehat{\langle O\rangle}_{B'}$$

for some target observable O in an incompatible basis B' (e.g. X or Y). Because (P, S, E, Q) are deterministic functions of the same single-basis record, T is ultimately a functional of the same information as $\mathbf{f}$ (and the acquisition-order shot transcript when E, Q are used).

B.4 Non-injectivity: explicit null-space constructions

Claim (informal). No inference map T that depends only on single-basis Z-measurement statistics (and any deterministic function thereof) can be injective over the full physical state space.

Reason. There exist distinct states $\rho_1 \neq \rho_2$ with identical Z-basis outcome statistics, hence $\rho_1, \rho_2 \in \mathcal{C}_Z(\mathbf{f})$ for the same $\mathbf{f}$, while differing on at least one incompatible-basis expectation value. For such a pair, any counts-only pipeline must assign identical inputs $(\mathbf{f}, P, S, E, Q)$, so it cannot output two different values of $\langle O \rangle_{B'}$.

Two-qubit examples used in this manuscript. The following pairs have identical Z-basis outcome distributions (uniform over $\{00, 01, 10, 11\}$), but different incompatible-basis structure:

- $|{++}\rangle$ vs $|{+-}\rangle$: identical Z histogram, but different X_1 expectation.
- $|{++}\rangle$ vs $|{+i}, +\rangle$: identical Z histogram, but different Y_0 expectation.

In the simulator honesty-case artifacts used in Section 4, these pairs yield:

- counts-only signature overlap ≈ 1.0,
- forced-choice discrimination accuracy $= 0.5$ (chance),
- incompatible-basis expectation L1 difference $= 2.0$.

See artifact summaries: `Quantum-Eye/artifacts/nullspace_honesty_cases/summary_1320cf4a92df.json` and `.../summary_1e2fe5e5b3b4.json`.

B.5 Interpretation: what can and cannot be claimed

The non-injectivity above does *not* invalidate bounded inference; it clarifies what is required for it to be well-defined:

- **Unconditional (unsafe) claim:** "Single-basis statistics determine generic cross-basis expectations."
- **Conditional (safe) claim:** "Within an explicit restricted model class (a prior), single-basis statistics can support bounded cross-basis inference targets, and the mechanism is testable and falsifiable."

Equivalently, any cross-basis predictor can be read as computing a *prior-weighted* expectation over $\mathcal{C}_Z(\mathbf{f})$ (or over a restricted subset thereof), rather than recovering a unique physical state.

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