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Q:) Proof that vertex cover is NP complete.

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Ans:

Given: (problem) Let G be a graph with V & E vertex & edge set the problem is to find a positive integer k such that there is a ~~sub~~ vertex subset $V_1$. The question is to find whether this subset $V_1$ exists such that every edge in the graph G is connect to some vertex in $V'$. (This is a decision - problem)

Part 1: proof that vertex cover is in NP.

- Here the certificate is a subset $V_1$.
- we have to check if this is a vertex cover of the given graph.
- For that we will loop through all the vertice $v \in V_1$
- As we visit each vertex we remove adjacent edges from it and each time we check that if the count is k and the edge set E is empty or not.

~ Now since the subset $V'$ may contain $n$ vertices and we traverse over it 1 time each $\Rightarrow$ $n$ time at the maximum then the complexity of this verification is linear

$\Rightarrow$ This certificate can be verified in polynomial time.

$\therefore$ The vertex cover problem is in the class NP.

Part 2: Proof that vertex cover is NP Hard.

~ To prove vertex cover is NP hard we just need to reduced any known NP-Hard problem to vertex cover.

~ One of the known NP-Hard problem is Clique problem.

Proof from LHS

~ Assume that there is a clique of size $k$ in $G$.

~ Let the set of vertices in the clique be $V' \Rightarrow |V'| = k$.

~ In the complent graph $G'$, let an edge be $(u, v)$ then at least on of $u$ or $v$ must be in the $G'$ set $V - V'$.

~ If both u & v were in the set V', then the edge (u, v) would belong to V',

~ That would mean that the edge (u, v) is in G.

~ This is not possible sin (u, v) is in G'.

∴ All the edges in G' are covers by the vertic in the set V - V'.

Proof from RHS

~ Now assum athat a vertex cover V'' of size |V|-k is in G.

~ This mens that all the edges in G' are conneld to some vertex in V''.

~ Thus if we pick (u, v) from G', both of them cannot be outside the set V''.

~ That implies that for all the edges (u, v), bothe u & v are outside the set V'' are in G.

~ This concluded that these edges constitute a clique of size k.

~ Therefore we can say that there is a clique of size k in graph G if and only if there is a vertex cover of size |V| - k in G'.

~ Thus the vertexcover is NP-Hard and therefore vertex cover is NP complete.