JOVIAL JOE

1ES17CS016

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DAA Revision Test

Q 1.) Proove that clique is np-complete

Q 2.) what is cook's theorem?

A 2.) Cook's theorem

- cook's theorem states that the boolean satisfiability problem is NP-complete.
- In other words any problem in NP can be reduced in polynomial time by deterministic Turing Machine to the problem of determining whether a Bolean formula is satifiable or not. (SAT)

A 2.) Poof that a clique is NP-complete

Steps: 1. show that the clique problem is in NP

2. 3-CNF can be reduced to clique decision problem.

Clique : is a subgraph which is complete in it self.

Problem: where the given sub graph is a cliquee or not?

Part ~~1~~

- This can be easily verified.
- Suppos a graph G is given with V, & E as vertex & Edge set respectively, then another sub graph is given say S with vertices V_s & E_s.
- With and adjecency matrix of the subgraph S, we can easily verify that whether the sub graph S is a clique or not.
- This can be done in polinomal time since to check n vertics it would require $O(n)$ ~~time~~ range.
- Therefor this problem of ~~finding~~ "determining wheter a given subgraph is a clique or not" is in NP.

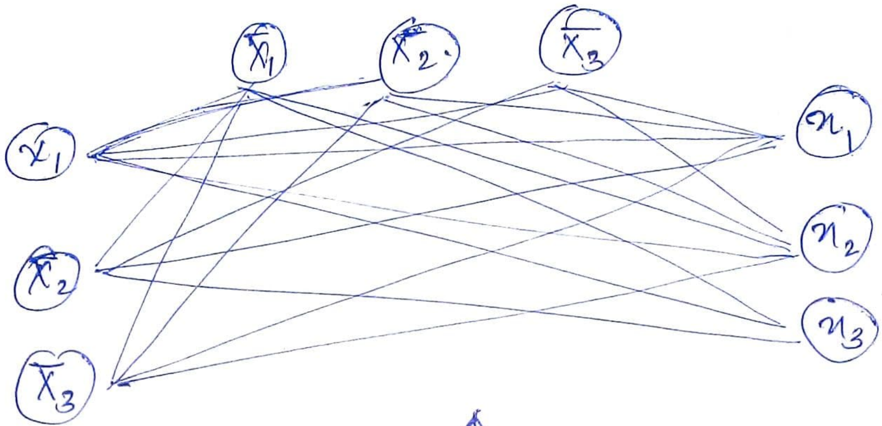
Part 2

- ~~We know~~ The 3-CNF (conjunction Normal Form) is a know circuit satisfiability problem and is NP-Complete.

Now we have ϕ a 3-CNF ~~prob~~

$$\phi = (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_1 \vee \bar{x}_3)$$

$k=3$



convert to a graph problem.

~ ϕ has satisfying argument iff G has a clique of size k

~ $\phi = C_1 \wedge C_2 \wedge C_3$

Proof from LHS

~ x_i & $\bar{x}_i$ are not connect since they are opposite of each other.

~ We see that x_1, x_2, x_3 are mutually connected and therefore the graph ~~are~~ have a clique.

Proof from RHS

~ Given that a clique exists in a graph we just need to check if ϕ is satisfied

$\Rightarrow$ if x_1, x_2 & x_3 are 1 substituting

$\Rightarrow \phi = (1 \vee 0 \vee 0) \wedge (0 \vee 1 \vee 1) \wedge (1 \vee \cancel{0} \vee 1)$

$= 1$ (3-CNF)

~ Hence the function is also satisfied.

~ Using induction this transformation can be extended to
k - size.

~ Hence the ~~B~~-CNF problem can be reduced to
clique decision problem. Hence proved
that clique decision problem is NP complete