07.04.20

Tutorial-

Q.) Consider 5 items along with their respective weights and values.

$I = \langle I_1, I_2, I_3, I_4, I_5 \rangle$

$w = \langle 5, 10, 20, 30, 40 \rangle$

$v = \langle 30, 20, 100, 90, 160 \rangle$

The capacity of knapsack is $W = 60$. Find the solution to the ~~Factorial~~ Fractional Knapsack Problem.

Sol

Given.

Items	I_1	I_2	I_3	I_4	I_5
Weights	5	10	20	30	40
Values	30	20	100	90	160

This is a fractional knapsack problem, hence, fraction or part of I_i can be picked.

Following are the three ways to approach it:

(i) Selection of items with largest Profit.

∴ The items will be arranged as

~~$I_5(160), I_4(30), I_3(20), I_2(10), I_1(5)$~~

$I_5(160), I_3(100), I_4(90), I_1(30), I_2(20)$

But the maximum ~~cost~~ weight is 60 that can be carried by the knapsack

$$\therefore \quad \frac{I_5(160)}{40} + \frac{I_3(100)}{20} \Rightarrow \begin{array}{l}\text{Value/Profit} = 260 \\ \text{Weight} = 60\end{array}$$

weight → 40

(ii) Selecting in the increasing order of weight

Item's can be arranged in the increasing order of weight as:

$I_1(5)$, $I_2(10)$, $I_3(20)$, $I_4(30)$, $I_5(40)$

∴ Since, the limit of knapsack wight is 60, the Items taken are as.

$$\frac{I_1(5)}{30},\ \frac{I_2(10)}{20},\ \frac{I_3(20)}{100},\ \frac{I_4(25)}{\left(\frac{25}{30}\times \cancel{90}^{3}\right)} \Rightarrow \begin{array}{l}\text{Profit} = 225\\ \text{Weight} = 60\end{array}$$

Profit →

(iii) Selecting items with ~~highest~~ least profit isto weight ratio.

$I_1 \Rightarrow \frac{5}{30} = 0.1667$ $\quad I_2 = \frac{10}{20} = 0.5$ $\quad I_3 = \frac{20}{100} = 0.2$

$I_4 = \frac{30}{90} = 0.3334$ $\quad I_5 = \frac{40}{160} = 0.25$

$$\cancel{\frac{I_2(10)}{20}},\ \frac{I_1(5)}{30},\ \frac{I_3(20)}{100},\ \frac{I_5(35)}{\left(\frac{35}{\cancel{40}}\times \cancel{160}^{4}\right)} \Rightarrow \begin{array}{l}\text{Profit} = 270\\ \text{Weight} = 60\end{array}$$

Profit →

∴ The maximum profit obtained is when we take I_1, I_3 and $\frac{7}{8}$th of I_5 which equals to 270.