

Dot product:

$$a \cdot b = |a||b|\cos(\theta)$$

Cross product: To find the cross product of a and b find the determinant of 3x3 matrix with unit vectors:  $\mathbf{i}, \mathbf{j}, \mathbf{k}$  for the first row.

$$|a \times b| = |a||b|\sin(\theta)$$

Vector equation of a line starting at point  $P$ ,  $(a, b, c)$ , with parallel vector  $\vec{v}$ ,  $\langle d, e, f \rangle$ , and parameter  $t$ :

$$r(t) = P + t\vec{v} = \langle a + dt, b + et, c + ft \rangle$$

Given a constant point  $P_0$ ,  $(x_0, y_0, z_0)$  and a point on the plane  $P$ ,  $(x, y, z)$ , the vector  $\vec{P_0P}$  must be orthogonal to the normal vector of the plane  $N$ ,  $\langle a, b, c \rangle$ . Therefore,

$$N \cdot \vec{P_0P} = 0$$

and

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

.

Given vector function  $r(t) = \langle f(t), g(t), h(t) \rangle$ ,

$$r'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

and

$$R(t) = \langle F(t), G(t), H(t) \rangle$$

Length of curve from a to b is:

$$\int_a^b |r'(t)| dt$$

,

$$s(t) = \int_0^t |r'(x)| dx$$

. An equation  $y = f(x)$  can be parameterized as  $r(t) = \langle t, f(t) \rangle$ . The unit tangent vector:  $T(t) = r'(t)/|r'(t)|$ . The principle unit normal vector:  $N(t) = T'(t)/|T'(t)|$ . The binomial vector:  $B = T \times N$ . The osculating plane is the one formed by  $T$  and  $N$ . Curvature:

$$\kappa(x) = \left| \frac{dT}{ds} \right| = \frac{|T'(t)|}{|r'(t)|} = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

. Torsion:

$$\gamma(x) = -N \cdot \frac{dB}{ds} = \frac{(r'(t) \times r''(t)) \cdot r'''(t)}{|r'(t) \cdot r''(t)|}$$