Investment Management Course Notes

Joe Hollander

Summer 2024

1 Fundamentals of risk and returns

Compounding returns with different return rates:

$$(1 + r_1)(1 + r_2) - 1.$$

Compounding returns with the same return rate:

$$((1 + r)^t - 1).$$

To compare different time period standard deviations multiply (or divide) by the square root of the number of time periods. The sharpe ratio:

$$\frac{R_p - R_f}{\sigma_p}.$$

Pandas standard deviation method uses the sample standard deviation and not the population standard deviation. Similar to the sharpe ratio, the calmar ratior is a risk adjusted return where risk is measured by drawdown. Use index method to_period to convert from datetime to period. Use series method cummax to find the highest value for each timestep. Drawdown is then:

$$\frac{Value - PreviousPeak}{PreviousPeak}.$$

2 Beyond The Gaussian Case

Skew and kurtosis are the third and fourth moments of a distribution respectively. A distribution with a greater than three kurtosis is considered fat tailed.

The Jarque Bera is a test that determines whether or not a sample distribution fits a normal distribution based on its skew and kurtosis. If the test is close to zero it signals that the distribution is close to normal. The test:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K - 3)^2\right).$$

Semi-deviation is the volatility of below-average or below-zero returns:

$$\sigma_{semi} = \sqrt{\frac{1}{N} \sum_{R_t \leq \bar{R}} (R_t - \bar{R})^2}.$$

Value at risk (VaR) represents the maximum "expected" loss over a time period. A 99% one month VaR gives the maximum loss excluding the 1% of worst cases. The VaR is also typically expressed as a positive number. The conditional value at risk (CVaR) is the expected loss beyond VaR:

$$CVar = -E(R \,|\, R \leq -VaR).$$

Setting ddof as zero for pandas standard deviation calculates the population standard deviation.

There are four methods to calculate VaR. The historical methodology calculates the VaR from the historical outcomes. The parametric gaussian methodology assumes a gaussian distribution. In this methodology the VaR is simple to calculate:

$$VaR_\alpha = -(\mu + z_\alpha \sigma),$$

where z_α is the α-quantile of the normal distribution with mean zero and standard deviation one. The gaussian is almost inaccurate. A parametric non-gaussian distribution doesn't assume gaussian. The Cornish-Fisher VaR is a semi-parametric approach. The expansion states:

$$\tilde{z_\alpha} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(K - 3) - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2,$$

where $\tilde{z_\alpha}$ is the updated quantile.

Use numpy's percentile for historical VaR and scipy.stats' ppf function for gaussian, parametric, and Cornish-Fisher VaR.

3 Optimization and the Efficient Frontier

The return of a portfolio is equal to the weighted average of the return of the components. The volatility of a portfolio, however, depends on the correlation:

$$\sigma^2(w_a, w_b) = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{A,B}.$$

The efficient frontier is the boundary of regions created from the assets with a given correlation. Each point on this line represents the best return for each volatility.

4 Implementing Markowitz

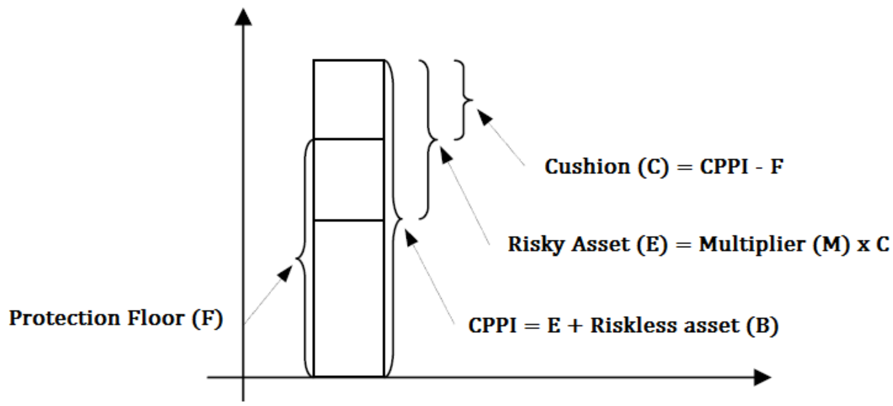
The capital market line is the tangent line from the risk free rate to the efficient frontier. The portfolios from this line have the highest sharpe ratio. This maximum sharpe ratio portfolio (MSR) also has no exposure to unrewarded risks.

As a result of estimation errors and misleading expected returns, some use the global minimum variance (GMV), which is the nose of the efficient frontier. Even small estimation errors result in large portfolio changes.

5 CPPI and Drawdown constraints

You cannot diversify out all risk. Dynamic hedging allows for upside exposure with less downside exposure. Correlation often rises when returns fall.

The CPPI procedure allows for the construction of convex payoffs. The risky asset is a multiplier multiplied by the cushion. The CPPI is the riskless asset plus the risky asset. The cushion is then the CPPI minus the protection floor set in place. Therefore, as the cushion decreases, more of the portfolio is riskless assets, and as the cushion increases, more of the portfolio is risky assets.



Gap risk occurs when trading discretely.

Given a max drawdown contraint

$$V_t > \alpha M_t,$$

where: V_t is the value of the portfolio, M_t is the peak of the portfolio between time 0 and time t, and $1 - \alpha$ is the maximum acceptable drawdown. Then choosing a multiplier multiplied by $M_t\alpha$ will also provide a convex payoff.

A cap can also be used to reduce risk taking passed a value. In this system, if the floor is closer, the distance to the floor is used while if the ceiling cap is closer, the distance to the ceiling is used.

Instead of constructing a new DataFrame using an array, use the DataFrame method reindex_like.

We can model the return process of a stock S_t with risk-free rate r, sharpe ratio λ, and volatility σ:

$$\frac{dS_t}{S_t} = (r + \sigma\lambda)dt + \sigma dW_t.$$

For discrete time:

$$\frac{S_{t+dt} - S_t}{S_t} = (r + \sigma\lambda)dt + \sigma\sqrt{dt}\,\xi_t.$$

6 Monte Carlo

A more general model of a return process with the same variables:

$$\frac{dS_t}{S_t} = \left(r_t + \sqrt{V_t}\,\lambda_t^S\right)dt + \sqrt{V_t}\,dW_t^S.$$

We can also define the risk-free rate and variance in terms of brownian motion:

$$dr_t = a(b - r_t)dt + \sigma_r dW_t^r$$
$$dV_t = \alpha(\bar{V} - V_t)dt + \sigma_V\sqrt{V_t}\,dW_t^V$$

Here, b is the long term mean of the risk-free rate. Both of these processes are mean reverting.

In a CPPI system, raising the risky-asset multiplier when the market is less volatile and vice-versa results in less breaches of the floor and less need for rebalancing more frequently.

7 Asset-Liability Management

The funding ratio $F_t = A_t/L_t$ and surplus $S_t = A_t - L_t$ is what really matters for asset-liability management.

The present value of a set of liabilities is

$$PV(L) = \sum_{i=1}^{k} B(t_i)L_i,$$

and if the yield curve is flat, the price of a pure discount bond is

$$B(t) = \frac{1}{(1+r)^t},$$

where r is the annual rate of interest.

Liability-hedging portfolios attempt to match the cashflows of the liability side in order to pay liabilities in the future. Often, cash-flow matching is not feasible or practicle, so one may use factor exposure matching. These factor exposure matching portfolios often use bonds to gain similar exposure to interest rates that affect liabilities.

The Cox Ingersoll Ross (CIR) model that is used to model interest rates:

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}\,dW_t,$$

where b is the long term mean of the interest rate. T