Investment Management Course Notes

Joe Hollander

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1 Fundamentals of risk and returns

Compounding returns with different return rates:

$$(1 + r_1)(1 + r_2) - 1.$$

Compounding returns with the same return rate:

$$((1 + r)^t - 1).$$

To compare different time period standard deviations multiply (or divide) by the square root of the number of time periods. The sharpe ratio:

$$\frac{R_p - R_f}{\sigma_p}.$$

Pandas standard deviation method uses the sample standard deviation and not the population standard deviation. Similar to the sharpe ratio, the calmar ratior is a risk adjusted return where risk is measured by drawdown. Use index method to_period to convert from datetime to period. Use series method cummax to find the highest value for each timestep. Drawdown is then:

$$\frac{Value - PreviousPeak}{PreviousPeak}.$$

2 Beyond The Gaussian Case

Skew and kurtosis are the third and fourth moments of a distribution respectively. A distribution with a greater than three kurtosis is considered fat tailed.

The Jarque Bera is a test that determines whether or not a sample distribution fits a normal distribution based on its skew and kurtosis. If the test is close to zero it signals that the distribution is close to normal. The test:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K - 3)^2\right).$$

Semi-deviation is the volatility of below-average or below-zero returns:

$$\sigma_{semi} = \sqrt{\frac{1}{N} \sum_{R_t \leq \bar{R}} (R_t - \bar{R})^2}.$$

Value at risk (VaR) represents the maximum "expected" loss over a time period. A 99% one month VaR gives the maximum loss excluding the 1% of worst cases. The VaR is also typically expressed as a positive number. The conditional value at risk (CVaR) is the expected loss beyond VaR:

$$CVar = -E(R \,|\, R \leq -VaR).$$

Setting ddof as zero for pandas standard deviation calculates the population standard deviation.

There are four methods to calculate VaR. The historical methodology calculates the VaR from the historical outcomes. The parametric gaussian methodology assumes a gaussian distribution. In this methodology the VaR is simple to calculate:

$$VaR_\alpha = -(\mu + z_\alpha \sigma),$$

where z_α is the α-quantile of the normal distribution with mean zero and standard deviation one. The gaussian is almost inaccurate. A parametric non-gaussian distribution doesn't assume gaussian. The Cornish-Fisher VaR is a semi-parametric approach. The expansion states:

$$\tilde{z_\alpha} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(K-3) - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2,$$

where $\tilde{z_\alpha}$ is the updated quantile.

Use numpy's percentile for historical VaR and scipy.stats' ppf function for gaussian, parametric, and Cornish-Fisher VaR.

3 Optimization and the Efficient Frontier

The return of a portfolio is equal to the weighted average of the return of the components. The volatility of a portfolio, however, depends on the correlation:

$$\sigma^2(w_a, w_b) = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{A,B}.$$

The efficient frontier is the boundary of regions created from the assets with a given correlation. Each point on this line represents the best return for each volatility.