UNIVERSITY OF BIRMINGHAM

Reducing the Shot Noise of Cosmic Explorer using an Unstable Optomechanical Filter

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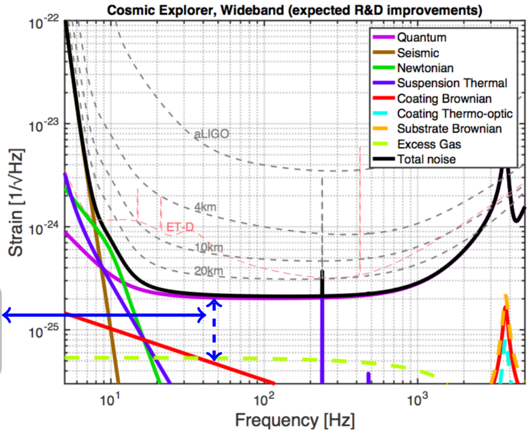
Motivation: Room for Improvement



B. P. Abbott et al. *Classical and Quantum Gravity*, 34(4):044001, 2017

Noise margin

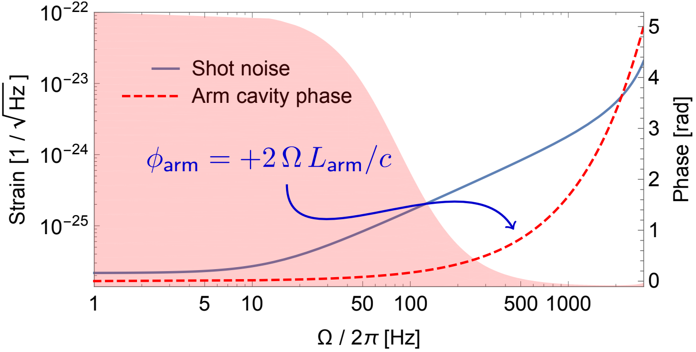
$L = 40\,\mathrm{km}$
10 dB squeezing

Science case:
Increase BBH merger detection rate

At $> \sim$ 20 Hz we are dominated by shot noise

Motivation: What Limits High Frequency Sensitivity?

- Signal sidebands *gain phase* $\propto \Omega$ relative to the carrier due to positive dispersion of arm cavities
- $\therefore$ lower sideband frequencies *constructively* interfere (resonant enhancement within arm cavities)
- higher sideband frequencies *destructively* interfere



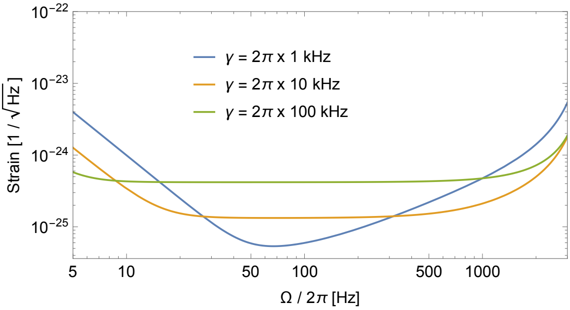
Motivation: Bandwidth-Sensitivity Tradeoff

- An obvious approach: increase bandwidth of arm cavities
- However, we lose peak sensitivity!
- Is there a way to improve bandwidth without sacrificing peak sens.
- . . . or improve peak sens. without sacrificing bandwidth?
- Can we do more than injecting squeezed vacuum?

Diminishing returns!!

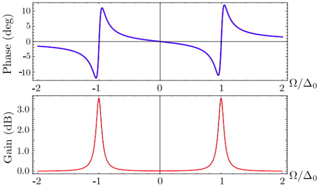
How to compensate positive dispersion of arm cavities?

Negative dispersion

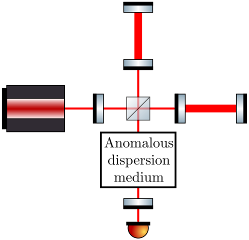


Negative (Anomalous) Dispersion

- Want to compensate for *time delay* of light across arm cavities with equal *time advancement*
- $\phi_{\text{arm}} = 2i\Omega L_{\text{arm}}/c$, want medium with $\phi = -2i\Omega L_{\text{arm}}/c$
- Use white light cavity effect by having anomalous dispersion medium
- One example: double-pumped atomic gain medium
- *but* introduces too much noise due to amplification process
- Unstable optomechanical filter introduces no new fundamentally unsurpassable noise sources
- Why unstable? otherwise bound by Kramers-Kronig relations



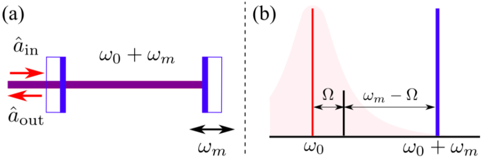
Y. Ma et al. Quantum noise of a white-light cavity using a double-pumped gain medium. *Physical Review A*, 92(2):023807, August 2015



Negative Dispersion using Unstable Optomechanical Filter

Mirror resonance at ω_m
Pump at $\omega_0 + \omega_m$

Cavity resonance at ω_0
Probe at $\omega_0 \pm \Omega$



H. Miao et al. *Physical Review Letters*, 115(21):1–5, 2015

Negative damping rate
$\gamma_{\text{opt}} \propto P_{\text{pump}}$

Single-mode and rotating-wave approx, and $\gamma_{\text{filter}} \gg \Omega$
$\Omega \ll\sim$ FSR $\qquad \Omega \ll\sim \omega_m$

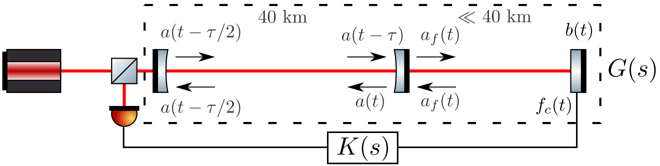
Negative dispersion

Ignoring heat bath coupling

$$\hat{a}_{\text{out}} \approx \frac{\Omega + i\gamma_{\text{opt}}}{\Omega - i\gamma_{\text{opt}}}\hat{a}_{\text{in}} \approx -\exp\left(-\frac{2i\Omega}{\gamma_{\text{opt}}}\right)\hat{a}_{\text{in}} \rightarrow -\exp\left(-\frac{2i\Omega L_{\text{arm}}}{c}\right)\hat{a}_{\text{in}}$$

Set $\gamma_{\text{opt}} = c/L_{\text{arm}}$

Feedback Control

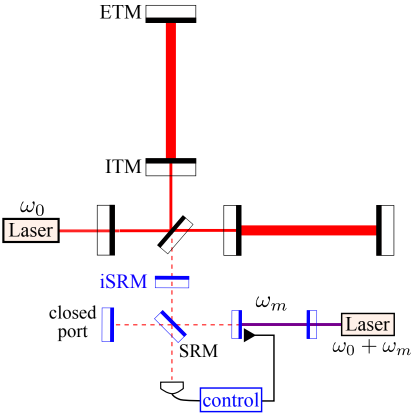


- Want to assess controllability, observability, and stability, of above bandwidth-broadening setup
- Analyse using *state-space approach* of control theory
- In original analysis, Haixing only considered the states a_f, a, b (filter, arm, mirror)
- Neglected time delay between control signal applied to oscillator f_c and detected output field a: $f_c(t) = K * a(t-\tau)$
- Also neglected effect of second-order term $\ddot{b}$ (with corresponding state $\dot{b}$).
- Found that the system is controllable, observable, and a stabilising controller can be made

Feedback Control Pt. 2

- What if we do not neglect time delay and $\ddot{b}$?
- To do this look at transfer function of the open-loop system
- Haixing result: phase margin of 70 degrees at 17 kHz ✔
- If we consider the time delay: phase margin of 7 degrees at 3 kHz, *only marginally stable but still okay*
- Consider $\ddot{b}$ (by including state $\dot{b}$): get an unstable pole at $2\omega_m$! ✗
- Furthermore, system no longer observable if we include $\dot{b}$!
- System unstable to higher frequency perturbations
- Investigation ongoing

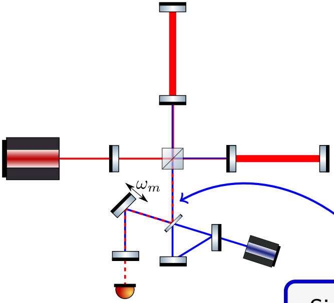
Original Reflection-Readout design



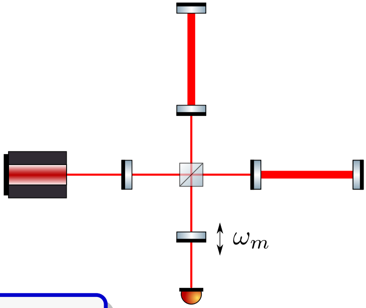
H. Miao et al. Towards the design of gravitational-wave detectors for probing neutron-star physics, 2017

Proposed New Transmission-Readout Design

Proposed new "theorist's" design



Simplified scheme

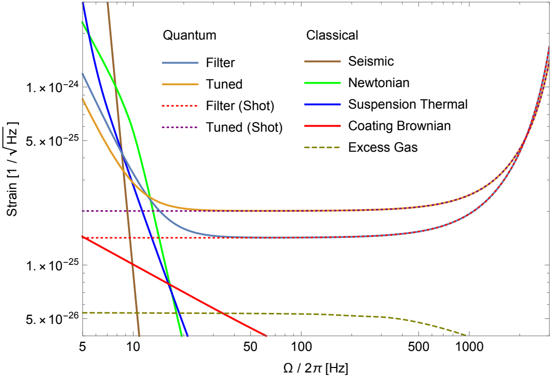


Signal Recycling Cavity becomes unstable filter

Find that effective bandwidth of setup $\sim \frac{c}{2\sqrt{2}}\left(\frac{T_{\mathsf{ITM}}T^2_{\mathsf{SRM}}}{L_{\mathsf{arm}}L^3_{\mathsf{SRC}}}\right)^{\frac{1}{4}}$.

Results: Peak Sensitivity Improvement, match CE peak

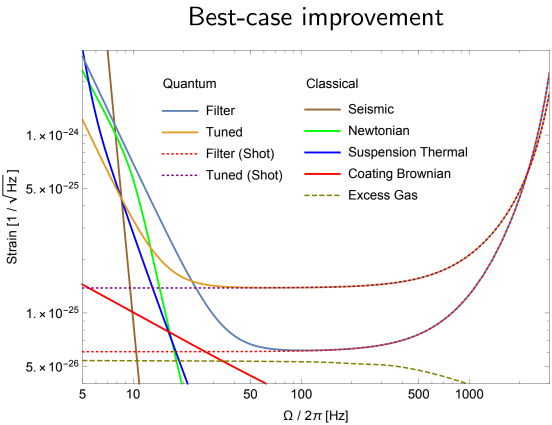
Approximately match bandwidth by setting effective bandwidth to arm cavity bandwidth



CE params
10 dB squeezing
$L_{\text{arm}} = 40\,\text{km}$
$L_{\text{SRC}} = 20\,\text{m}$
$T_{\text{ITM}} = 0.045$
$T_{\text{SRM}} = 0.0015$
$m_{\text{oscill.}} = 10\,\text{mg}$
$\omega_m = 2\pi \times 1\,\text{kHz}$
$P_{\text{SRC}} = 237.9\,\text{W}$

In the ideal case get $\sim 4\,\text{dB}$ improvement from 15 to 1000 Hz

Results: Peak Sensitivity Best-Case Improvement



CE params
10 dB squeezing
$L_{\text{arm}} = 40\,\text{km}$
$L_{\text{SRC}} = 20\,\text{m}$
$T_{\text{ITM}} = 0.045$
$T_{\text{SRM}} = 0.00035$
$m_{\text{oscill.}} = 10\,\text{mg}$
$\omega_m = 2\pi \times 1\,\text{kHz}$
$P_{\text{SRC}} = 237.9\,\text{W}$

With these parameters we get $\sim 7\,\text{dB}$ improvement from 50 to 500 Hz

Issues & Further Steps

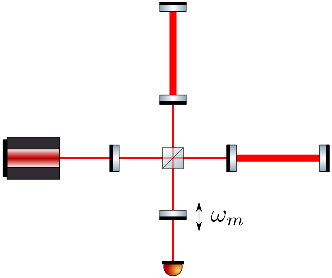
- Preliminary results, still working on full simulation and noise budgeting (thermal noise)
- Analytics past $\sim 3\,\mathrm{kHz}$ (no single-mode approx) use-case for FINESSE (see Phil Jones poster)
- Increasing γ_f further will decrease peak sensitivity, can mitigate this with reflection readout setup
- How do we control this?
- Denis Martynov trying to realise this experimentally and considering control

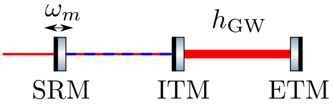
Thanks for listening!

Questions?

Transmission readout setup

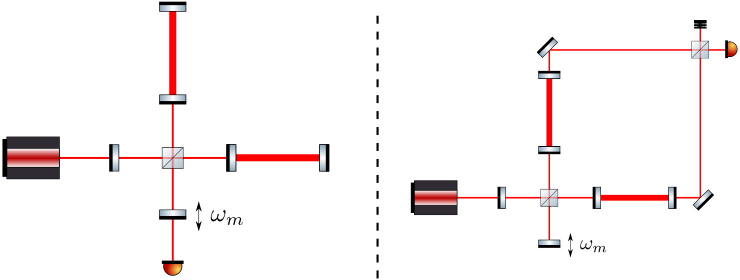
- Pros: More realistic
- Cons: New effective bandwidth only $\gamma_{\text{eff}} \sim \sqrt{\gamma_f \omega_s}$





Alternative setup: Reflection Readout

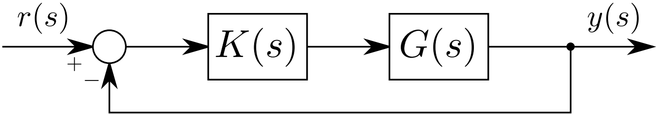
- Pro: Effective bandwidth $\gamma_{\text{eff}} \sim \omega_s$ ✔
- Cons: Another noise injection port (?) ✗
- Cons: Unclear how to inject squeezing ✗



Control Theory Primer

- Fact: can write any set of ODEs as set of first-order differential equations
- Dynamics: $\dot{\vec{x}} = A\vec{x} + B\vec{u}$
- $\vec{x}$ describes set of n system states, $\vec{u}$ describes system inputs, A describes internal system dynamics, B describes input coupling to internal dynamics
- Output coupling: $\dot{\vec{y}} = C\vec{x} + D\vec{u}$
- $\vec{y}$ describes set of outputs, C describes coupling of internal states to outputs, D describes direct feed of inputs into the outputs (often zero)
- System observable if all states are in some way connected to an output, so somehow you can infer the internal state of the system. True if $\text{rank}([B, AB, AB^2, \dots]) = n$.
- System controllable if any set of internal states can be achieved by giving the correct input for a finite amount of time

Feedback Control and Stability

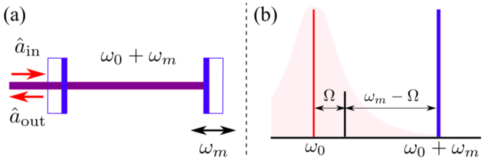


- $r(s)$ is control signal, $y(s)$ is output signal
- Open-loop transfer: $K(s)G(s)$
- Closed-loop transfer: $K(s)G(s)/(1 + K(s)G(s))$
- Closed-loop transfer instability if $K(s)G(s) = -1$, so $|K(s)G(s)| = 1$ (gain of 0 dB), and $K(s)G(s)$ has phase lag of -180°
- Phase margin: difference between closed-loop transfer phase lag and -180° at unity gain frequency
- Gain margin: difference between closed-loop gain and 0 dB at frequency where phase lag is -180°

Solution: Unstable Optomechanical Filter

Mirror resonance at ω_m
Pump at $\omega_0 + \omega_m$

Cavity resonance at ω_0
Probe at $\omega_0 \pm \Omega$



H. Miao et al. *Physical Review Letters*, 115(21):1–5, 2015

$$\gamma_{\text{opt}} = \frac{g^2}{\gamma_f}$$

$$\phi_{\text{arm}} = 2i\Omega\, L_{\text{arm}}/c$$

$$\phi_f = -2i\Omega\, L_{\text{arm}}/c$$

Single-mode and rotating-wave approx, and $\gamma_{\text{filter}} \gg \Omega$

$\Omega \ll\sim$ FSR

$\Omega \ll\sim \omega_m$

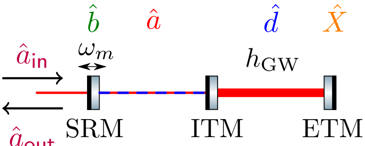
Negative dispersion

Ignoring heat bath coupling

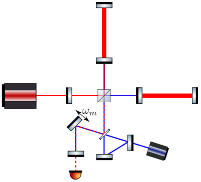
$$\hat{a}_{\text{out}} \approx \frac{\Omega + i\gamma_{\text{opt}}}{\Omega - i\gamma_{\text{opt}}}\hat{a}_{\text{in}} \approx -\exp\left(-\frac{2i\Omega}{\gamma_{\text{opt}}}\right)\hat{a}_{\text{in}}$$

Set to c/L_{arm}

Transmission Readout Hamiltonian Analysis



Simplified setup



Full setup

Interaction Hamiltonian has form of squeezing process

$$H_{\text{int}}^{\text{RWA}} \approx -\hbar g(\hat{a}\hat{b} + \hat{a}^\dagger \hat{b}^\dagger)$$

Sloshing between SRC and arms

$$-i\hbar\omega_s(\hat{d}\hat{a}^\dagger - \hat{d}^\dagger \hat{a})$$

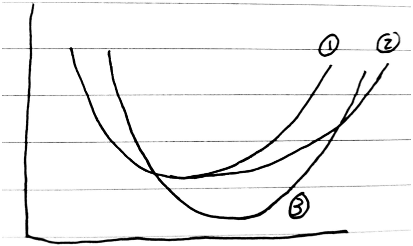
Solve RP for ETM

$$\frac{\hat{P}^2}{2M} + ML_{\text{arm}}\ddot{h}_{\text{GW}}\hat{X} - \hbar G_0(\hat{d} + \hat{d}^\dagger)\hat{X}$$

Write input-output relation relating $\hat{a}_{\text{out}}$, $\hat{a}_{\text{in}}$, and h. *(Single mode approx. . .)*

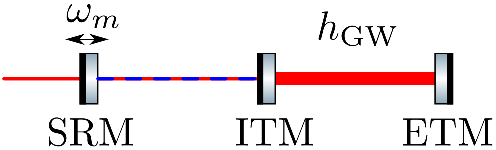
Find that effective bandwidth of setup $\sim \frac{c}{2\sqrt{2}}\left(\frac{T_{\text{ITM}}T_{\text{SRM}}^2}{L_{\text{arm}}L_{\text{SRC}}^3}\right)^{\frac{1}{4}}$.

Improving Peak Sensitivity Process

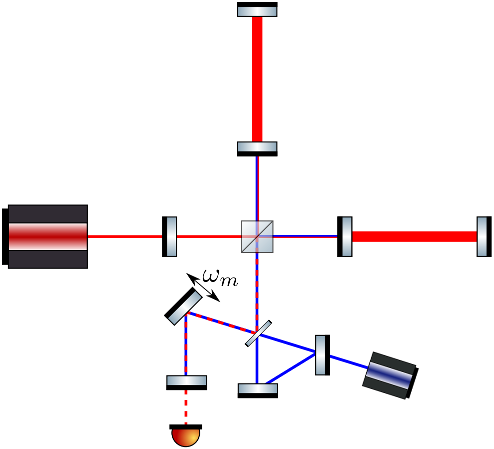


- 1: tuned Michelson sensitivity
- $1 \rightarrow 2$: include unstable filter: improve high frequency sensitivity
- $2 \rightarrow 3$: harness bandwidth-sensitivity tradeoff: by reducing overall bandwidth we improve peak sensitivity
- Tradeoff: low frequency sensitivity gets worse
- Pro: We have not lost much high-frequency sensitivity because of our high frequency bandwidth improvement

System analysed



Full Transmission readout setup



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