Unstable Filter Update: Thermal Noise and Controllability

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November 15, 2018

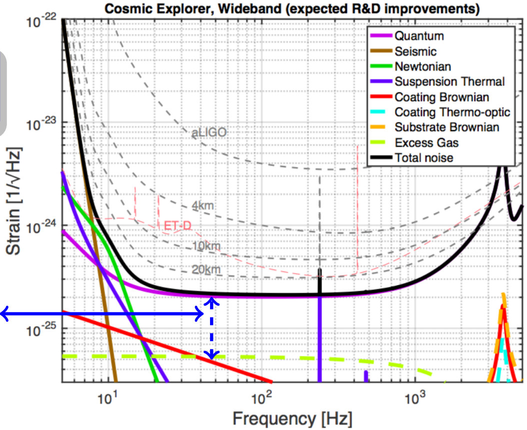
DCC: LIGO-G1801642-v5

Outline

1. Motivation, unstable filter, new transmission readout design
2. Analysis of thermal noise for transmission readout
3. Solution to control of unstable filter

Motivation: Room for Improvement

$L = 40\,\mathrm{km}$
10 dB squeezing



Noise margin

Science case:

→ Increase BBH merger detection rate

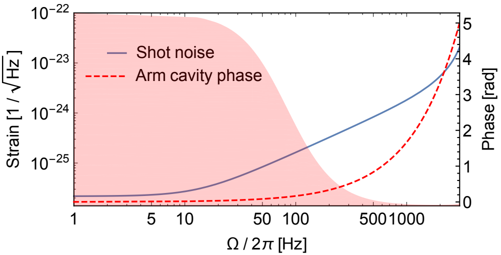
→ Study high freq neutron star physics

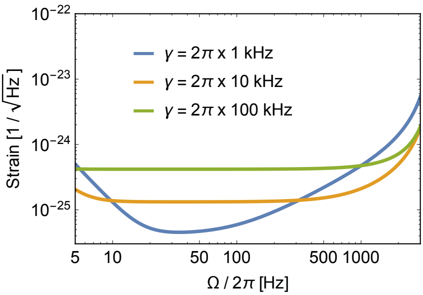
B. P. Abbott et al. Exploring the Sensitivity of Next Generation Gravitational Wave Detectors. 2016

At $> \sim$ 20 Hz we are dominated by shot noise

Motivation: Positive dispersion, Mizuno's theorem

- Sensitivity *decreases with frequency* due to positive dispersion of arm cavities (phase $\propto \Omega$)
- Cavity *enhances* $\Omega \approx 0$ and *suppresses* $\Omega > \gamma$



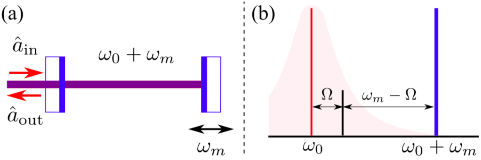


- Clearly "*just increase detector bandwidth* γ!"
- But, Mizuno limit [2] tells us: peak sens. is *inverse* to γ
- Can directly improve peak sens. via squeezing
- Can we *directly improve* γ? ...

Solution: Negative Dispersion via Unstable Filter

- Use a *negative dispersion medium* (phase $\propto -\Omega$) to *cancel* phase gained in arm cavities
- One possible example: the unstable optomechanical filter

Optical cavity resonant at ω_0 w/ mechanically suspended mirror, pumped with an additional laser at $\omega_0 + \omega_m$



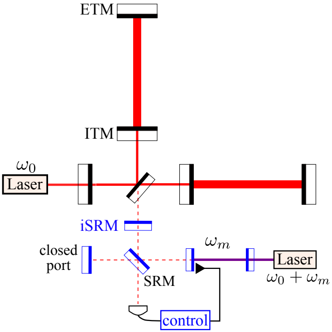
H. Miao, Y. Ma, C. Zhao, and Y. Chen. Enhancing the Bandwidth of Gravitational-Wave Detectors with Unstable Optomechanical Filters. *Physical Review Letters*, 115(21):1–5, 2015

In resolved sideband regime ($\Omega \ll \gamma_f \ll \omega_m$), $a_{\rm out} \approx e^{-2i\Omega\tau} a_{\rm in}$ ✓

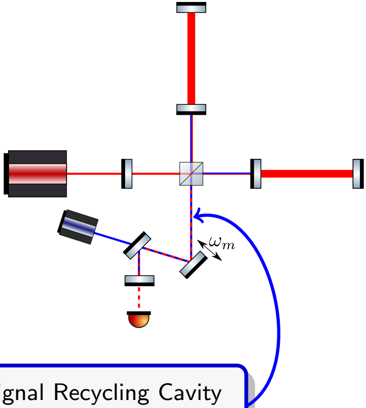
(for more details, see supp. slides or my talk in March: LIGO-G1801642)

Refresher: Transmission-readout design

Previous design



Proposed new design

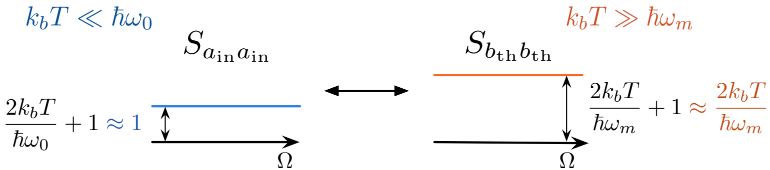


Signal Recycling Cavity
becomes unstable filter

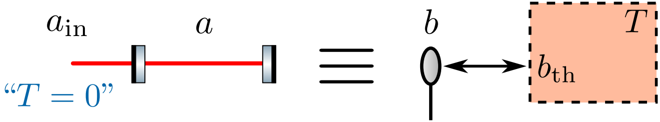
Thermal noise of unstable filter

Mechanical mirror coupled to thermal heat bath at temp T

Spectrum of thermal fluctuations *flat* (just like optical vacuum)



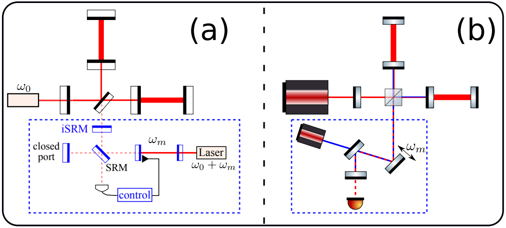
Coupling of vacuum light to *cavity mode* formally equivalent to coupling of thermal noise to *mirror mode*:

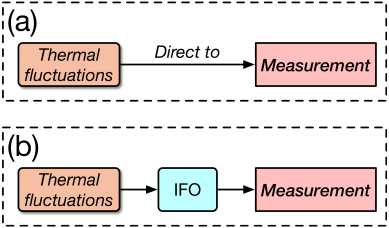


Can circumvent using an *all-optical* realization

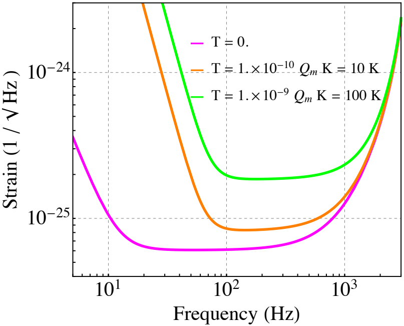
See: [1] Yanbei Chen and Yiqiu Ma, in preparation; [2] Naoki Yamamoto et al. in preparation

Transmission-readout thermal noise





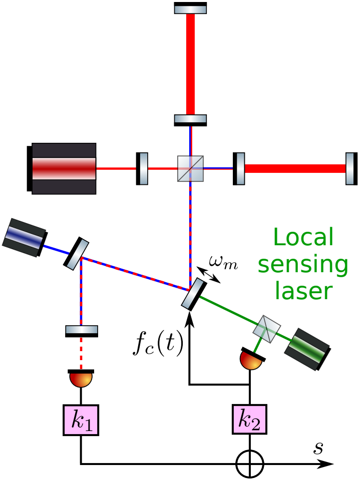
Total quantum noise



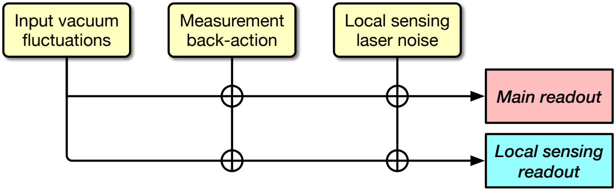
- Thermal noise fluctuations *fully shaped* by interferometer
- Couples *indirectly* through mirror
  ∴ extra filtering via mirror mechanics
- *Amplifies* (thermal) radiation-pressure noise
  Suppresses (thermal) shot noise

Controlling the setup via Local Sensing

- Recap: Haixing attempted to construct stabilizing controller for unstable filter [3] but neglected time delay of control signal which significantly reduces phase margin
- New idea (by Denis Martynov): use local sensing laser to stabilize unstable oscillator
- However, local sensing laser imparts additional noise
- But, *can cancel this noise* by combining readouts optimally

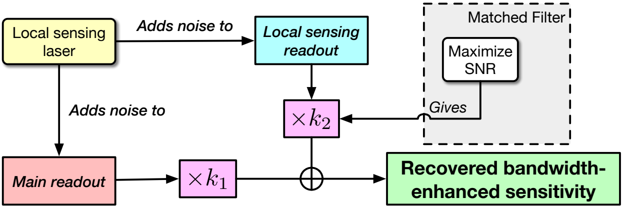


Recovering Bandwidth-Enhanced Sensitivity using Matched Filter



Take linear combination s of main readout signal s_1 and local sensor readout signal s_2

By combining w/ optimal k_1, k_2, can recover enhanced sensitivity



Normalization specifies k_1 given k_2

Recovering Original Sensitivity: Results

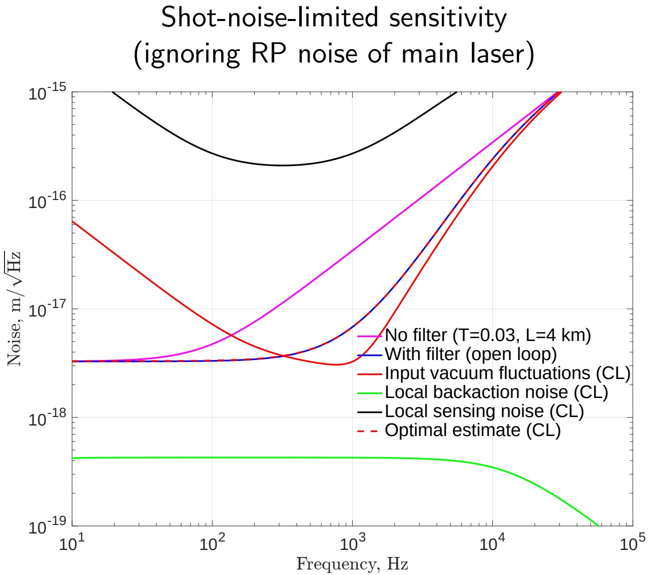
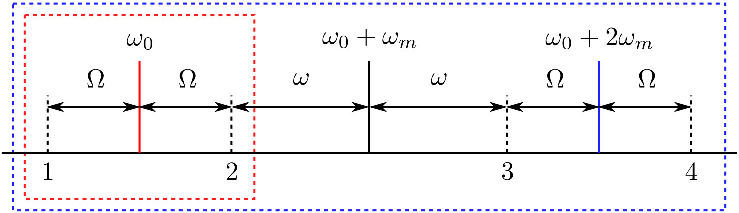


Figure: Courtesy of Denis Martynov

Next Steps

Some more stuff to be done:

- Optimization of parameters in detuned operation of SRC
- Design local sensing control scheme for transmission-readout setup
- Precise formulation involving *all sidebands* ("four-photon formalism")



Supplementary Slides. . .

Controlling the Unstable Filter using Matched Filter

- Write main readout signal (s_1) and local sensor readout signal (s_2) as:

$$s_1 = \xi_0 x_0 + \Sigma_i\, \xi_i n_i, \quad s_2 = \eta_0 x_0 + \Sigma_i\, \eta_i n_i, \quad i = 1 \ldots 3,$$

- x_0 is GW signal, n_1, n_2, n_3 are the noises due to vacuum, back-action, local sensing laser
- Let us combine our readouts via some linear combination $s = k_1 s_1 + k_2 s_2$
- First, assume k_i normalized w.r.t GW signal coefficients: $k_1 \xi_0 + k_2 \eta_0 = 1$
- Then, minimize inverse SNR w.r.t k_2, giving

$$k_2^* = -\frac{\Sigma_i (\xi_i^*/\xi_0^*)(\eta_i - \eta_0 \xi_i/\xi_0) S_{ii}}{\Sigma_i (\eta_i - \eta_0 \xi_i/\xi_0)(\eta_i^* - \eta_0^* \xi_i^*/\xi_0^*) S_{ii}}, \quad i = 1 \ldots 3,$$

- where $S_{ii}, i = 1 \ldots 3$ is the PSD of each noise term

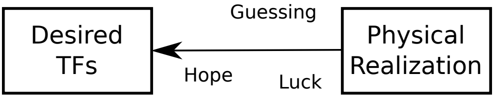
Network Synthesis of Arbitrary Quantum Systems

- New theoretical synthesis framework by Hendra Nurdin (Uni. of New South Wales) et. al. from Quantum control community [4]
- For classical n-dof systems can easily synthesize equivalent circuit if described as n coupled ODEs, using integrators along with feedback
- Can we do same thing for n-dof dynamical *quantum* systems?
- Many reasons why this is much harder:
  - Have to preserve $[q(t), p(t)] = i\hbar$, $\forall t$ for each dof
  - Variables are quantum & stochastic rather than classical & deterministic
  - Finally, the resulting realizations are complicated to implement
- Main theorem: can split n-dof system into 1-dof "open oscillators" and a direct interaction Hamiltonian (supp. slides)

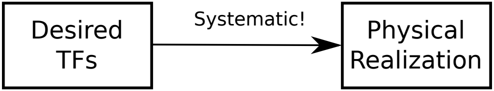
Network Synthesis: Motivation

Say we want system with desired TFs...

Old way: guess various possibilities



New way: systematic synthesis approach



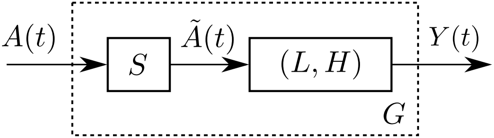
$\therefore$ could build a general interferometer toolkit:
Describe the behaviour you want, and can systematically construct interferometer with that behaviour

Network Synthesis: Brief summary

- Consider desired n degree of freedom system with, state $x = [q_1, p_1; \ldots; q_n, p_n]^T$, m input noise fields $A = [A_1, \ldots, A_m]^T$, m output fields $Y = [Y_1, \ldots, Y_m]^T$,
- Write out system in state-space formalism,

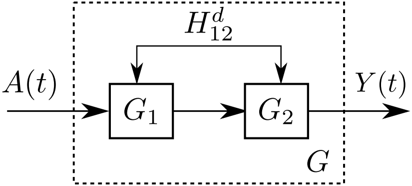
$$dx(t) = Ax(t)dt + B\left[dA(t),\, dA(t)^*\right]^T$$
$$dY(t) = Cx(t)dt + DdA(t)$$

- Describe system as a "n degree of freedom generalized open oscillator" $G = (S, L, H)$, S scattering matrix, $L = Kx$ coupling of system to the noise fields $A(t)$, $H = \frac{1}{2}x^T Rx$ is system Hamiltonian



Network Synthesis: Brief summary part 2

- Can infer (S, L, H) from (A, B, C, D)
- **Main theorem:** can split n-dof gen. open oscillator G into n 1-dof open oscillators G_j and a direct interaction Hamiltonian H^d
- Problem becomes one of
  - Implementing each 1-dof generalized open oscillator G_j
  - Implementing the direct interaction Hamiltonian H^d
- (More) trivial after we've decomposed the system in this way

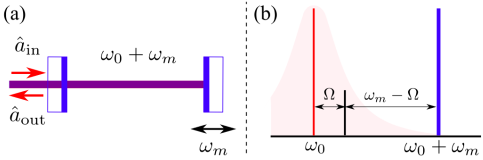


Supp. slides from Sonoma...

Negative Dispersion using Unstable Optomechanical Filter

Mirror resonance at ω_m
Pump at $\omega_0 + \omega_m$

Cavity resonance at ω_0
Probe at $\omega_0 \pm \Omega$



H. Miao, Y. Ma, C. Zhao, and Y. Chen. Enhancing the Bandwidth of Gravitational-Wave Detectors with Unstable Optomechanical Filters. *Physical Review Letters*, 115(21):1–5, 2015

Negative damping rate
$\gamma_{\rm opt} \propto P_{\rm pump}$

Single-mode and rotating-wave approx, and $\gamma_{\rm filter} \gg \Omega$
$\Omega \ll\sim$ FSR $\quad\quad \Omega \ll\sim \omega_m$

Negative dispersion

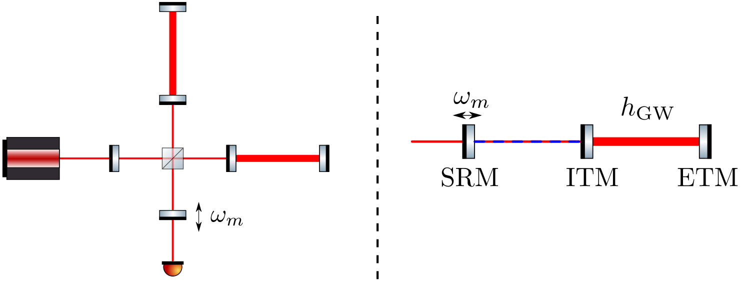
Ignoring heat bath coupling

$$\hat{a}_{\rm out} \approx \frac{\Omega + i\gamma_{\rm opt}}{\Omega - i\gamma_{\rm opt}}\hat{a}_{\rm in} \approx -\exp\left(-\frac{2i\Omega}{\gamma_{\rm opt}}\right)\hat{a}_{\rm in} \rightarrow -\exp\left(-\frac{2i\Omega L_{\rm arm}}{c}\right)\hat{a}_{\rm in}$$

Set $\gamma_{\rm opt} = c/L_{\rm arm}$

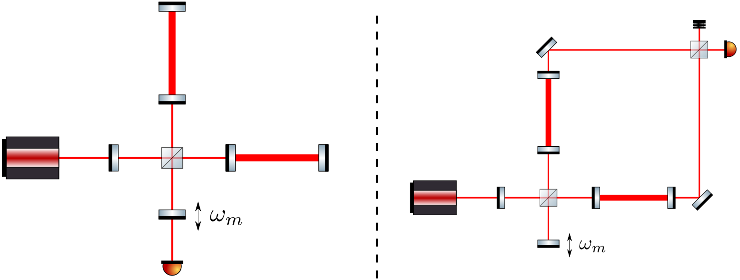
Transmission readout setup

- Pros: More realistic
- Cons: New effective bandwidth only $\gamma_{\text{eff}} \sim \sqrt{\gamma_f \omega_s}$



Alternative setup: Reflection Readout

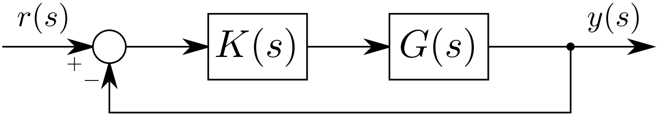
- Pro: Effective bandwidth $\gamma_{\text{eff}} \sim \omega_s$ ✔
- Cons: Another noise injection port (?) ✗
- Cons: Unclear how to inject squeezing ✗



Control Theory Primer

- Fact: can write any set of ODEs as set of first-order differential equations
- Dynamics: $\dot{\vec{x}} = A\vec{x} + B\vec{u}$
- $\vec{x}$ describes set of n system states, $\vec{u}$ describes system inputs, A describes internal system dynamics, B describes input coupling to internal dynamics
- Output coupling: $\dot{\vec{y}} = C\vec{x} + D\vec{u}$
- $\vec{y}$ describes set of outputs, C describes coupling of internal states to outputs, D describes direct feed of inputs into the outputs (often zero)
- System observable if all states are in some way connected to an output, so somehow you can infer the internal state of the system. True if $\text{rank}([B, AB, AB^2, \ldots]) = n$.
- System controllable if any set of internal states can be achieved by giving the correct input for a finite amount of time

Feedback Control and Stability

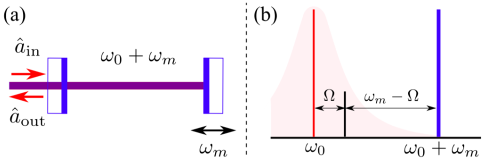


- $r(s)$ is control signal, $y(s)$ is output signal
- Open-loop transfer: $K(s)G(s)$
- Closed-loop transfer: $K(s)G(s)/(1+K(s)G(s))$
- Closed-loop transfer instability if $K(s)G(s) = -1$, so $|K(s)G(s)| = 1$ (gain of 0 dB), and $K(s)G(s)$ has phase lag of -180°
- Phase margin: difference between closed-loop transfer phase lag and -180° at unity gain frequency
- Gain margin: difference between closed-loop gain and 0 dB at frequency where phase lag is -180°

Solution: Unstable Optomechanical Filter

Mirror resonance at ω_m
Pump at $\omega_0 + \omega_m$

Cavity resonance at ω_0
Probe at $\omega_0 \pm \Omega$



$$\gamma_{\text{opt}} = \frac{g^2}{\gamma_f}$$

$$\phi_{\text{arm}} = 2i\Omega\, L_{\text{arm}}/c$$

$$\phi_f = -2i\Omega\, L_{\text{arm}}/c$$

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Single-mode and rotating-wave approx, and $\gamma_{\text{filter}} \gg \Omega$

$\Omega \ll\sim$ FSR

$\Omega \ll\sim \omega_m$

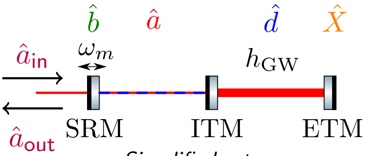
Negative dispersion

Ignoring heat bath coupling

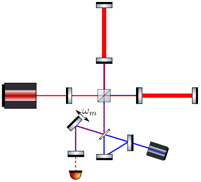
$$\hat{a}_{\text{out}} \approx \frac{\Omega + i\gamma_{\text{opt}}}{\Omega - i\gamma_{\text{opt}}}\hat{a}_{\text{in}} \approx -\exp\left(-\frac{2i\Omega}{\gamma_{\text{opt}}}\right)\hat{a}_{\text{in}}$$

Set to c/L_{arm}

Transmission Readout Hamiltonian Analysis



Simplified setup



Full setup

Interaction Hamiltonian has form of squeezing process

$$H_{\text{int}}^{\text{RWA}} \approx -\hbar g(\hat{a}\hat{b} + \hat{a}^\dagger \hat{b}^\dagger)$$

Sloshing between SRC and arms

$$-i\hbar\omega_s(\hat{d}\hat{a}^\dagger - \hat{d}^\dagger \hat{a})$$

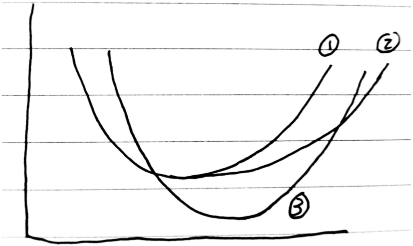
Solve RP for ETM

$$\frac{\hat{P}^2}{2M} + ML_{\text{arm}}\ddot{h}_{\text{GW}}\hat{X} - \hbar G_0(\hat{d} + \hat{d}^\dagger)\hat{X}$$

Write input-output relation relating $\hat{a}_{\text{out}}$, $\hat{a}_{\text{in}}$, and h. *(Single mode approx. . .)*

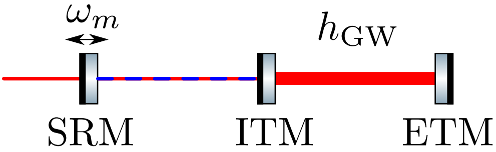
Find that effective bandwidth of setup $\sim \frac{c}{2\sqrt{2}}\left(\frac{T_{\text{ITM}}T_{\text{SRM}}^2}{L_{\text{arm}}L_{\text{SRC}}^3}\right)^{\frac{1}{4}}$.

Improving Peak Sensitivity Process

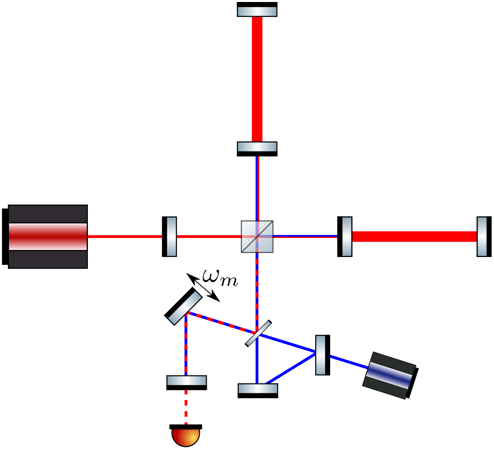


- 1: tuned Michelson sensitivity
- $1 \rightarrow 2$: include unstable filter: improve high frequency sensitivity
- $2 \rightarrow 3$: harness bandwidth-sensitivity tradeoff: by reducing overall bandwidth we improve peak sensitivity
- Tradeoff: low frequency sensitivity gets worse
- Pro: We have not lost much high-frequency sensitivity because of our high frequency bandwidth improvement

System analysed



Full Transmission readout setup



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