

DISTRIBUTIONS

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1. DIRAC DELTA FUNCTION

The Dirac delta function is a distribution that is defined to be zero everywhere except at $x = 0$ where it is infinite. That is,

$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

It is also constrained to the identity,

$$\int_a^b \delta(x) dx = 1 \quad a < 0 < b$$

The Dirac delta function is thus an infinitely tall, infinitely thin spike. We can see that any function integrated over it between a and b is,

$$\int_a^b f(x) \delta(x) dx = f(0)$$

and therefore,

$$f(x) \delta(x) = f(0) \delta(x)$$

2. HEAVISIDE STEP FUNCTION

The Heaviside step function is defined such that,

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

The gradient at the step is therefore infinite, but everywhere else it is zero. The gradient is therefore given by the Dirac delta function,

$$\frac{d\Theta}{dx} = \delta(x)$$