# Quantum Mechanics 2

## Wave Theory of Matter

### Wave-Particle Duality

Any matter can behave as a particle (e.g. photoelectric effect, showing the particle-nature of light) or as a wave (e.g. diffraction, showing the wave-nature of light). Whether you observe something as a particle or as a wave is dependent on how you *measure it.*

In general,

- Particle-like describes interactions
- Wave-like describes motion

Particle-like and wave-like quantities are connected by the plank constant h,

$$E = h\upsilon$$
$$p = \frac{h}{\lambda}$$

These equations both relate particle and wave quantities. For example, energy (particle-like) is related to frequency (wave-like), and momentum (particle-like) is related to wavelength (wave-like)

These equations were formulated by de Broglie, for which he won a nobel prize in physics.

### Wavefunctions

The wavefunction of a particular state (this means anything, as in, the state of some matter, but could be momentum or energy etc.) contains all the information possible to know about something. We can extract information from the wavefunction to find information about it such as its momentum.

For a free particle, we might expect its wavefunction to look something like

$$\Psi(x,\ t) = A\sin 2\pi\left(\frac{x}{\lambda} - \upsilon t\right)$$

Which represents a sine wave travelling in the positive x-direction.

We know from wave theory that we can write the wavefunction as,

$$\Psi(x,\ t) = A\sin(kx - \omega t)$$

Where,

$$k = \frac{2\pi}{\lambda}$$
$$\omega = 2\pi\upsilon$$

This allows us to rewrite de Broglie's relations in very useful forms,

$$E = h\upsilon = \hbar\omega$$
$$p = \frac{h}{\lambda} = \hbar k$$

In general the wavefunction will be complex, so we cannot directly measure it as it has no physical manifestation, unlike classical waves which are entirely real and measurable.

An example of a complex wavefunction could be,

$$\Psi(x, t) = A\cos(k x - \omega t) + i A \sin(k x - \omega t)$$
$$\Psi(x, t) = A e^{i(k x - \omega t)}$$

The second line is obtained by using Euler's formula.

Since this is complex there is no convenient real way to interpret this, so what do we do?

## Probability Interpretation

This interpretation of wavefunctions was formulated by Max Born, and it often called the Born interpretation.

Classicaly, we describe light as oscillating E and B fields,

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

This is given in one dimension. An example solution to this wave equation is as follows,

$$E(x, t) = E_0 \sin(k x - \omega t)$$

We know from wave theory that we can write the power density (the energy per unit area per unit time) as being proportional to the square of the amplitude, but what about in quantum mechanics.

In QM, light is described as a stream of discrete "packets" of energy called photons. Our energy density can be described as follows,

$$\text{power density} = n \hbar \omega$$

Where n in the average number of photons per unit area per unit time. By observing this power density, and the classical picture, we can see that,

$$n \propto (\text{amplitude})^2$$

Where amplitude is of the wave function.

At very low intensities, where n << 1, photons will arrive intermittently, instead of in a constant stream. Our wave picture with the amplitude and the wave function can describe the *average* power, but cannot predict the arrival of individual photons.