PRINCIPLES OF QUANTUM MECHANICS

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1. Mixed States

If $\psi_n(x)$ is a solution of the time independent Schrödinger equation, then $\psi_n(x)$ is an eigenfunction of the Hamiltonian operator with an energy eigenvalue. The full time-dependant form of the wavefunction is given by,

$$\Psi_n(x,t) = \psi_n(x)e^{-\frac{iE_n t}{\hbar}}$$

When we find the probability density, multiply the wavefunction by its complex conjugate. Since the exponential is negative when we take the complex conjugate, they cancel to give,

$$|\Psi_n|^2 = \psi_n^*\psi_n$$

Therefore since ψ_n has no time dependence, neither does the probability density $|\Psi_n|^2$.

Now we consider a mixed state of wavefunctions,

$$\Phi(x,t) = \Psi_m(x,t) + \Psi_n(x,t)$$

By taking the probability density of this,

$$\begin{aligned} \|\Phi\|^2 &= (\Psi_m + \Psi_n)^*(\Psi_m + \Psi_n) \\ &= \Psi_m^*\Psi_m + \Psi_n^*\Psi_n + \Psi_m^*\Psi_n + \Psi_n^*\Psi_m \end{aligned}$$

The two cross terms at the end of the second line we call the interference terms, which we define as z and z^* such that,

$$\begin{aligned} z &= \Psi_m^*\Psi_n = \psi_m^*\psi_n e^{-i(E_n-E_m)t/\hbar} \\ z^* &= \Psi_m^*\Psi_n = \psi_m^*\psi_n e^{i(E_n-E_m)t/\hbar} \end{aligned}$$

We see that this satisfies $z + z^* = 2Re\{z\}$.