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An Integrated Six Degree-of-Freedom Trajectory Simulator for Hybrid Sounding Rockets

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In this paper a set of mathematical models of a generic hybrid rocket mission is developed for flight performance prediction purposes. This includes a hybrid rocket physical model, recovery system model, accurate atmospheric and gravitational models, a CFD based aerodynamics database and a hybrid motor combustion model. The integration of these mathematical models within a 6-DOF numerical trajectory simulation code based on Newtonian rigid-body dynamics results in a useful hybrid rocket design and evaluation tool, the Six Degree-of-freedom Performance Simulator (SDPS). This software tool is used to swiftly predict relevant flight performance parameters and perform Monte Carlo dispersion analyses on key design parameters. Variations to rocket geometric and physical parameters are then performed to improve performance. As a test case, flight performance uncertainties due to aerodynamics and wind variations on a small hybrid rocket were predicted and analysed. Results suggest that it is possible to measure and quantify uncertainties in general flight performance.

Nomenclature

A	=	Reference Area, m^2
A,B,C	=	Vehicle Moments of Inertia, $kg.m^2$
C_D	=	Drag Coefficient
C_L	=	Lift Coefficient
C_l	=	Rolling Moment Coefficient
C_m	=	Pitching Moment Coefficient
C_n	=	Yawing Moment Coefficient
C_Y	=	Side-Force Coefficient
D,E,F	=	Vehicle Products of Inertia, $kg.m^2$
$F_x^b,F_y^b,F_z^b,\boldsymbol{F}_T^b$	=	Propulsion Forces, N
g, $\boldsymbol{g}^b$	=	Gravitational Acceleration, m/s^2
L	=	Reference Length, m
M_x^b,M_y^b,M_z^b	=	Propulsion Moments, N.m
p,q,r	=	Angular Velocities in Body Frame, rad/s
u, v, w	=	Translational Velocities in Body Frame, m/s
V_a	=	Aerodynamic Velocity, m/s
$\boldsymbol{V}_k$	=	Kinematic Velocity, m/s
$\boldsymbol{X}$	=	State Vector
α_a	=	Angle of Attack, rad
β_a	=	Sideslip Angle, rad
θ	=	Elevation Angle, rad
φ	=	Azimuth Angle, rad
ρ	=	Air Density, kg/m^3
Ω	=	Body-Frame Angular Velocity, rad/s

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I. Introduction

RECENT advances in hybrid propulsion technologies have allowed for smaller, cheaper and more flexible sounding rockets. These rockets are typically single stage vehicles designed for re-usability and low-cost along with reliability and acceptable flight performance. Hybrid propulsion systems offer greater reliability than sophisticated liquid rocket motors whilst ensuring greater levels of control and safety over the relatively simpler solid rocket motors. However, in the absence of expensive onboard control systems and control actuation surfaces, small hybrid sounding rockets must be designed with maximum knowledge of unsupervised flight performance. Limiting costs also reduces the scope for testing at the design stage, when the effects of mission uncertainties such as wind on the flight performance are largely unknown. In order to address these factors, it is necessary to develop an integrated modelling and simulation tool which may be used during the design cycle to swiftly predict flight performance of hybrid rockets given relevant geometric and physical parameters. This tool should have the following characteristics:

a) A single computational framework. The flight vehicle's physical model, the hybrid propulsion system model and mathematical models for the aerodynamics of the vehicle must be combined with the models describing the Earth and the atmosphere during flight to predict flight performance.
b) Scalability and flexibility to cover a range of physical vehicle configurations and sizes that the designer may wish to design. The tool should be specific to hybrid propulsion technology for maximum relevance and robustness.
c) Ability to model, quantify and predict uncertainties arising from environmental conditions and technical faults.
d) Graphical and textual outputs of relevant results to facilitate ease of use during the design process.
e) Minimal inputs of geometric and physical data from the user.

This paper describes the development of the Six Degree-of-freedom Performance Simulator (SPDS) software tool and its application to a small hybrid sounding rocket. The tool's development forms part of an ongoing hybrid sounding rocket research program at the University of KwaZulu-Natal (UKZN)[1] with the initial goal of developing the Phoenix-1A sounding rocket. The Phoenix-1A is specified to carry a 1 kg payload to an apogee altitude of 10 km. Several researchers have also proposed small hybrid sounding rockets for sub-orbital or low-earth orbit missions, such as Doran *et al*[2]. Others have demonstrated that multidisciplinary optimization during the rocket design process enables the designer to simultaneously optimize several key design parameters[3]. It has also been shown that a sensitivity analysis based on trajectory simulation leads to a better balance between performance and cost in the final design of multi-mission missiles[4]. There have been several investigations into the means of optimizing rocket trajectories for specific flight performance goals, such as energy minimization[5], altitude and range performance[6], minimizing propellant costs[7] and general flight economy[8]. The Monte Carlo statistical method has been shown to be useful for aerodynamic optimization of rocket airframes[9] and the minimization of impact point error[10]. The majority of these works rely on idealizations of some environmental and technical factors to simplify the analysis and optimize the target variable, such as cost, altitude and so on. However, an accurate multidisciplinary design tool that allows useful design of all the key design variables in a rocket must take into account all relevant factors affecting flight rocket performance. It is for this reason that an integrated simulation tool is developed. Often, it will also be necessary to make small changes to vehicle design to finely vary flight performance in an uncontrolled rocket and such a tool allows this to be done with relative ease.

II. Mathematical Models

Sounding rockets experience propulsion, aerodynamic and gravitational forces. Mathematical models are needed to describe the earth, the atmosphere and the rocket vehicle itself, with special emphasis on the hybrid rocket motor and the recovery mechanism. For simulation purposes, these models need to be computationally efficient. The airframe geometry determines the interaction between the atmosphere and the vehicle and hence an aerodynamics model must be integrated into the simulation environment. Figure 1 displays the relationships between the models and the SDPS simulation tool. The standard ellipsoidal Earth model (World Geodetic System 1984 – WGS 84) suffices as a physical model of the Earth whilst the Standard International Atmosphere model is used in the required altitude range. Due to the ellipsoidal Earth approximation, gravitational acceleration is dependent only on altitude and latitude. The International Standard Atmosphere provides values for atmospheric density, temperature and pressure and is empirical in nature. Both models are easily integrated and computationally inexpensive.

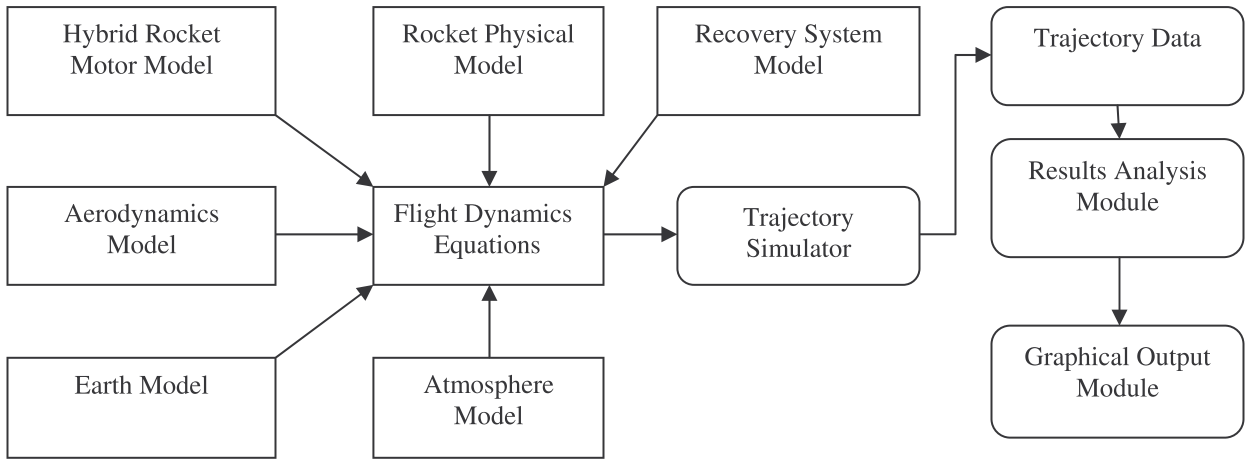


Fig. 1 Information flow in the SDPS from mathematical models to flight performance predictions

The physical structure of the sounding rocket is modelled using a rigid arrangement of constant-density geometric constructs. These may include flat plates, annuli, cylinders, cones, tangent ogives, quadrilateral prisms and point masses. This approach allows simple but accurate analytical evaluations of relevant vehicle physical quantities such as mass, centre-of-mass, moments of inertia and products of inertia. The vehicle geometry is described by a minimal number of geometric parameters such as lengths, diameters and fin sweep angles. Time variation of the motor fuel grain radii and oxidizer fluid length is derived from the hybrid motor performance model to capture the dynamic shifts in center-of-mass and moments-of-inertia during the motor burn. The SPDS software's idealized model of a small hybrid rocket is shown in Figure 2, and can be compared with the CAD rendering of the same in Figure 3. The user is allowed to define any number of components, allowing scalable complexity during design.

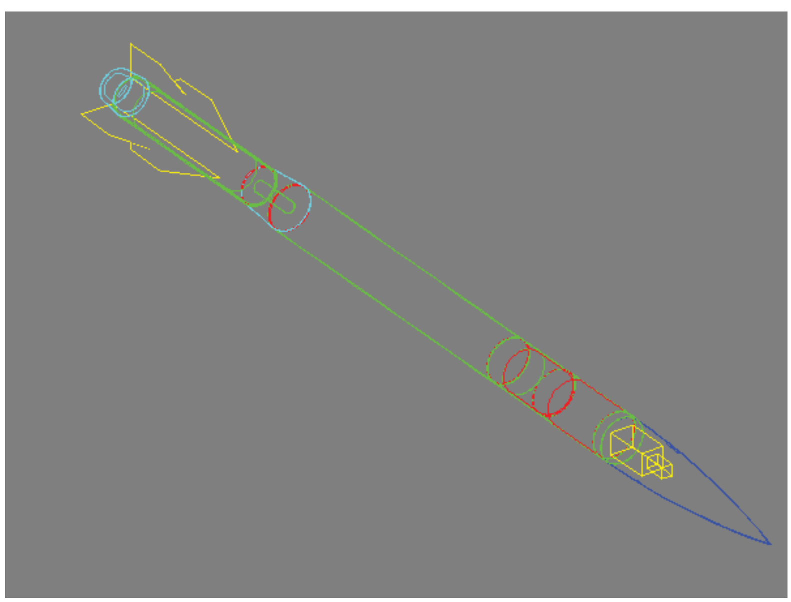


Fig. 2 Rigid bodies such as cylinders, ogives and annuli are used to model the simulated vehicle

The aerodynamic forces and moments experienced by rockets during atmospheric flight are functions of airframe geometry, free stream Mach number, atmospheric density and angle-of-attack. Aerodynamic coefficients are evaluated using experimental tests in wind tunnels, computational fluid dynamics (CFD) codes or empirical correlations derived from CFD and/or experimentation. Small hybrid sounding rockets experience significant periods of transonic and supersonic flight for which sufficient experimental data is not available in the literature, especially for aerodynamic stability derivatives. In this study, the empirical model rocket aerodynamics software RasAero[11] is used to calculate aerodynamic coefficients for angles-of-attack smaller than 15°. Aerodynamic analysis of the UKZN Phoenix-1A hybrid rocket is performed using the StarCCM+ CFD code. The analysis provides a more accurate aerodynamics database for large Mach numbers and angles-of-attack. The database is interpolated and used in the trajectory simulator as an aerodynamic coefficient lookup-table. The table is stored in a file and updated to match changes in airframe geometry during the vehicle design process. The StarCCM+ CFD model employs a coupled, viscous, compressible, turbulent (K-Epsilon) fluid solver for the 3D Navier Stokes equations that govern the flow of air. The external atmospheric density is a function of altitude and is specified from the atmospheric model. Figure 4 shows a typical result.

Fig. 3 Exploded view of an idealized 200 mm calibre 15 km apogee sounding rocket. This model was used as the basis for trajectory simulation tests in the SDPS software

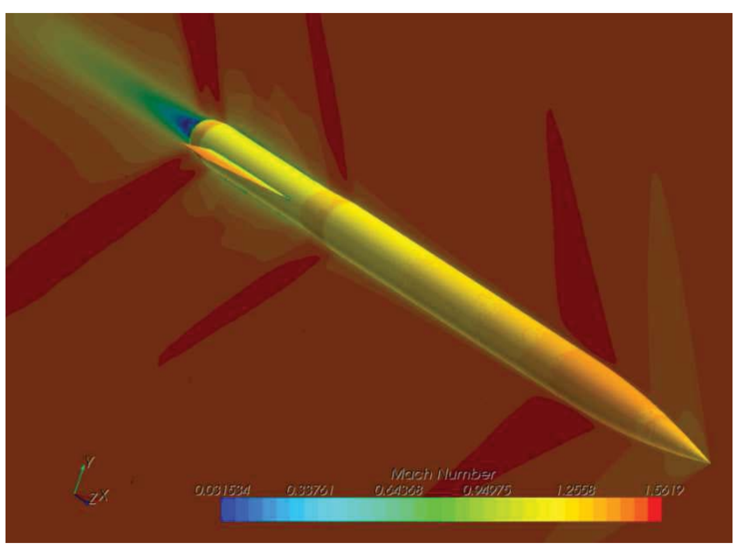


Fig. 4 Mach number distribution surrounding a hybrid rocket airframe at Mach 1.5 and an angle of attack of 0°

The parachute recovery system is modeled in the same computational framework as the rocket vehicle itself, with certain simplifications. The recovery parachutes are described using rigid hemispherical shells for the canopy (after deployment). The rocket is treated as a point mass being supported by the parachute canopy during the recovery phase. Only the drag and side-force coefficients of the canopy are needed to complete the model. The rocket's hybrid propulsion system must be accurately modeled to provide thrust characteristics.

Outputs of this model are the time variation of the thrust provided by the hybrid motor and the consumption of the fuel grain and oxidizer fluid which affect overall vehicle mass, moment of inertia and stability. Aerodynamic effects due to the motor jet plume are modeled by coupling motor thrust to the aerodynamic coefficient lookup-table. Genevieve[12] has developed a hybrid rocket motor performance model at UKZN. This model has been integrated into the simulation loop to provide coupling between motor simulation and flight performance evaluation. The motor model thus accounts for the effects of varying external atmospheric pressure on the nozzle exit pressure and motor thrust.

III. Six Degree-of-Freedom Trajectory Simulation

The problem of free rocket trajectory simulation is closed, so that given a set of initial conditions a unique solution is known to exist[13, 14]. The most common approach involves the explicit solution of Newtonian rigid-body equations-of-motion using numerical methods. Cartesian co-ordinates are used to describe the kinematics of rigid bodies and Newton's laws of motion describe the dynamics of the rocket vehicle. Equating the sum of external forces on the vehicle to the product of its mass and its acceleration gives Eq. (1), as per Newton's second law. In angular co-ordinates, the sum of external torques is equated to the product of the inertia tensor and the angular acceleration vector (Eq. (3)). The effects of the curvature of the Earth and its daily rotation are included for generality, according to Zipfel[15]. The inertial (kinematic) velocity of the vehicle is then the sum of the wind velocity and the vehicle aerodynamic velocity. Winds are modeled by supplying the local wind velocity vector as an input and subtracting it from kinematic velocities to calculate aerodynamic velocities.

Aerodynamic and propulsion forces align themselves well to the local frame of the vehicle and hence it is advantageous to formulate the solution in a frame originating at the rocket center-of-mass or body frame. The velocities obtained from integrating the body-frame accelerations must be projected, using transformation matrices, to the inertial frame before the second integration to calculate positions. Quaternion mathematics is used to represent rotational co-ordinates due to its lack of trigonometric singularities. The first term on the R. H. S. of Eq. (1) is the gravitational force; the second is the aerodynamic force and the last term accounts for any other external forces, primarily the motor thrust.

$$m\left(\frac{d\boldsymbol{V}_k}{dt} + \boldsymbol{V}_k \times \boldsymbol{\Omega}\right) = m\boldsymbol{g}^b + \frac{1}{2}\rho \mathrm{A} V_a^2 \boldsymbol{C}_F + \boldsymbol{F}_T^b \qquad \text{Eq. (1)}$$

$$m\begin{pmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{pmatrix} = mg\begin{pmatrix} -sin\theta \\ cos\theta sin\phi \\ cos\theta cos\phi \end{pmatrix} + \frac{1}{2}\rho \mathrm{A} V_a^2 \begin{pmatrix} -C_D cos\alpha_a cos\beta_a - C_y sin\beta_a cos\alpha_a + C_L sin\alpha_a \\ -C_D sin\beta_a + C_y cos\beta_a \\ -C_D cos\beta_a sin\alpha_a - C_y sin\alpha_a sin\beta_a - C_L cos\alpha_a \end{pmatrix} + \begin{pmatrix} F_x^b \\ F_y^b \\ F_z^b \end{pmatrix} \qquad \text{Eq. (2)}$$

$$\begin{pmatrix} +A\dot{p} - F\dot{q} - E\dot{r} + rq(C-B) - Epq + Frp + D(r^2 - q^2) \\ -A\dot{p} + B\dot{q} - D\dot{r} + rp(A-C) - Frq + Dpq + E(p^2 - r^2) \\ -E\dot{p} - D\dot{q} - C\dot{r} + pq(B-A) - Dpr + Erq + F(q^2 - p^2) \end{pmatrix} = \frac{1}{2}\rho \mathrm{AL} V_a^2 \begin{pmatrix} Cl \\ Cm \\ Cm \end{pmatrix} + \begin{pmatrix} M_x^b \\ M_y^b \\ M_z^b \end{pmatrix} \qquad \text{Eq. (3)}$$

The right hand sides of Eq. (1) and Eq. (2) may be evaluated when the rocket's mass, moments of inertia and external forces are known. To calculate these at a given instant, the state of the vehicle must be provided. The complete state consists of the six degrees of freedom and their derivatives, aerodynamic angles and the vehicle fuel geometry. Starting from a given initial state, the next set of acceleration vectors are evaluated at each time step using the 6-DOF equations of motion and the state vectors. The equations of motion are first order nonlinear differential equations in time, hence readily solved using an explicit numerical time integration algorithm. The current implementation uses the Runge-Kutta 4 (RK4) integration algorithm which offers a compromise between accuracy and speed. The termination criterion for the simulation is the return of the vehicle to the surface of the earth or the execution of a maximum number of time steps. The resulting vehicle state vectors are written to file for later analysis. Figure 5 displays the structure of the core simulation loop.

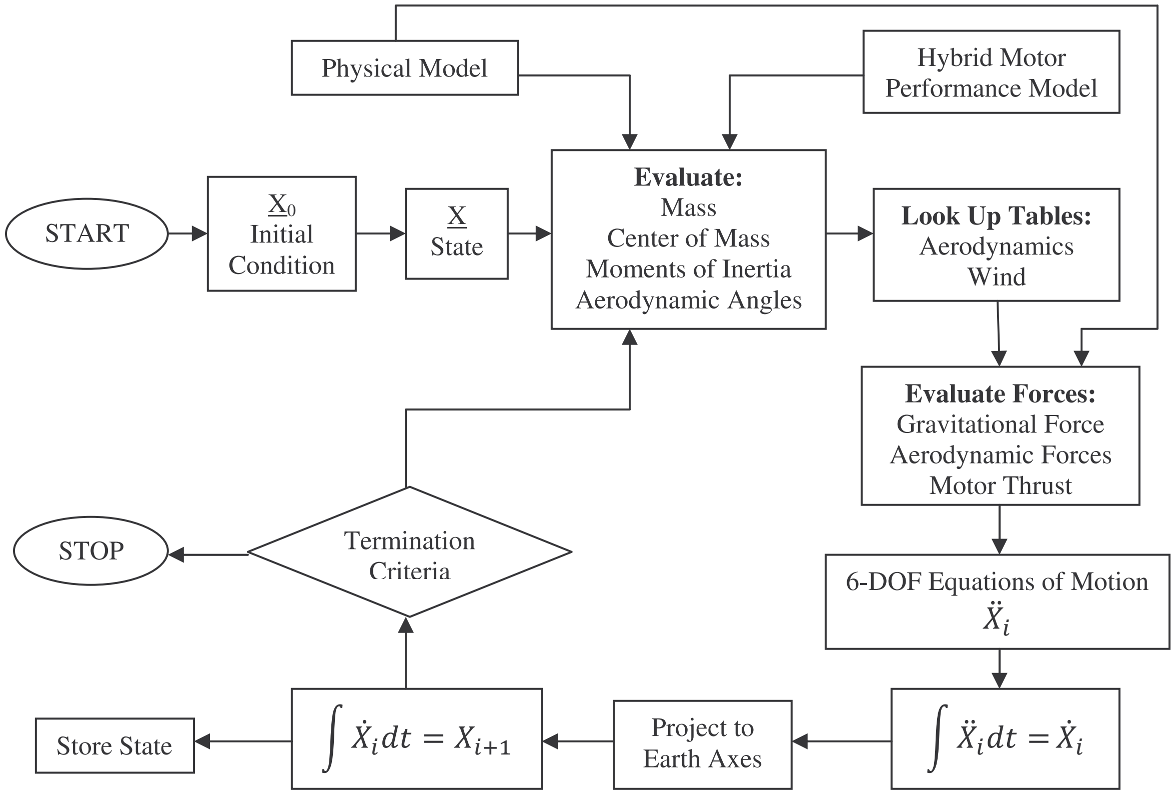
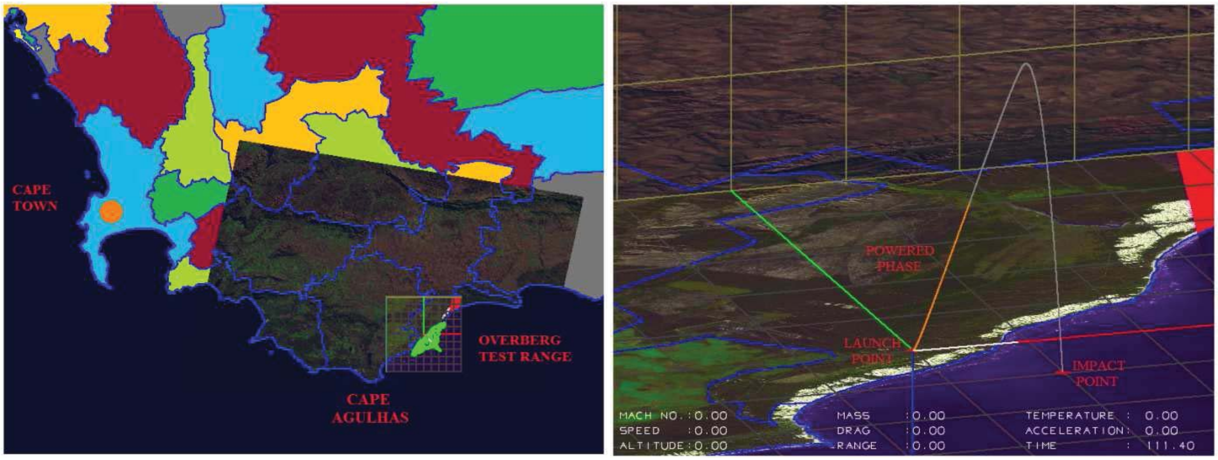


Fig. 5 Core simulation loop of the SDPS tool. Note the explicit nature of the simulation

The treatment of the recovery phase requires modification to the above methods. When enabled, the recovery phase is activated using a specified time-delay (typically 1 s) after the trajectory apogee is reached. The aerodynamic angles are assumed to be zero during the recovery phase, reducing the complexity of the simulation to include only the three translational degrees of freedom. The simulator enforces a step change in the aerodynamic and physical properties of the rocket when the recovery phase is initiated, accounting for the deployment of the parachute and the resulting changes in geometry. Thereafter, only the 3-DOF translational equations of motion are solved and the linear acceleration vectors are integrated to obtain velocities and positions as before.

The SDPS 6-DOF trajectory simulator is implemented in a standalone C++ Windows application with a graphical user interface. The geometric and physical parameters of the vehicle and the nature of its propulsion system are provided by the user. In this framework, a vehicle design consists of a set of internal and external geometric parameters along with the corresponding aerodynamic force and moment coefficient database files. The application generates time plots of the state vectors as well as other pertinent data such as the Mach no., aerodynamic forces and aerodynamic moments.

Geographic Information System (GIS) data such as satellite imagery, district, provincial and national border data, roads and populated places are integrated into the graphical user interface to allow better planning of sounding rocket launches. Visualization of flight trajectories, landing/ impact point with associated uncertainty ranges and wind vectors in this virtual environment make it easier for the designer to evaluate flight risks and adhere to civil / military aviation regulations. The user can navigate the simulation domain in 3D, using the keyboard and/or mouse. Figure 6 shows screenshots from the Windows application.



a) b)

Fig. 6 a) Location of Overberg Test Range in South Africa b) Perspective view of a typical 10 km apogee trajectory generated using the simulation tool. Background GIS data courtesy of NASA and GADM

IV. Monte Carlo Parameter Variation

To predict flight performance with varying environmental conditions and vehicle designs, the simulation tool includes a Monte Carlo probability estimation feature. This feature allows statistically defined random variation of all input parameters to the trajectory simulation, including vehicle internal and external geometry, aerodynamic coefficients, launch conditions, environmental effects and the technical functioning of hardware such as the propulsion system and the recovery system. The Monte Carlo scheme distorts input variables with normally distributed noise of defined standard deviation of amplitude. For the functional state of critical systems, the input is Boolean, with a predefined probability of failure. A large number of flight instances are then tested with the randomly distorted simulation inputs, allowing for statistical evaluation of flight performance uncertainties. Relevant flight characteristics such as extreme state vector values, apogee, ground impact point and burnout velocity from each test instance are stored for later analysis. The Monte Carlo feature can be used repeatedly to optimize a vehicle's flight performance in the presence of uncertainties. Figure 7 shows an example application.

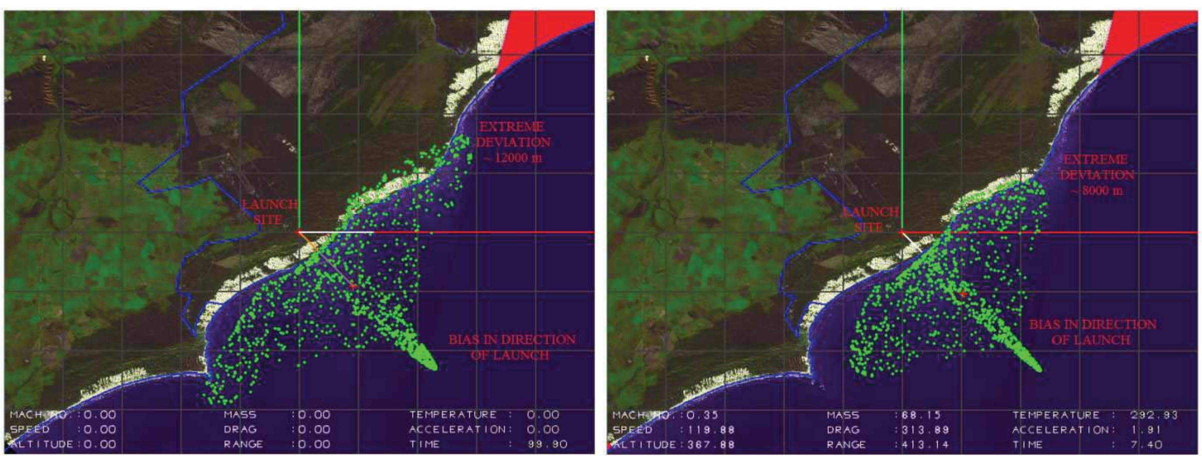


Fig. 7 Simulated Monte Carlo dispersions for two rocket fin configurations, with varying winds

V. Results

For software testing purposes, a small hybrid sounding rocket with a 15 km nominal altitude range was chosen for study. The launch is assumed to occur form the Overberg Test Range near Cape Town, South Africa, at coordinates 34°35' S 20°19' E. The aims of the tests were to validate the SDPS code and characterize the effects of strong winds on the splashdown/impact point of the vehicle. In the event of failure of the parachute recovery systems, it is desirable that the rocket fall into the waters of the Indian Ocean to the south of the testing range. The Overberg area experiences strong winds during certain seasons, leading to uncertainties in the probable landing footprint. The rocket motor is designed to provide a constant thrust of 2500 N for 25 s. Some characteristic properties of this test vehicle and its nominal flight performance are presented in Table 1.

Table 1. Nominal trajectory characteristics of the validation vehicle, a 14.8 km apogee hybrid rocket

Property	Nominal Value
Total Length	4.0 m
Maximum Diameter	200 mm
Maximum Mach No.	1.75
Apogee Altitude	14800 m
Average Thrust	2500 N
Propellant Burnout	25 s
Flight Time (without recovery)	99 s
Empty Mass	36 kg
Oxidizer Mass	30 kg
Fuel Mass	6 kg

Monte Carlo analyses were run on the test vehicle with various input parameters to quantify landing/impact location uncertainties. The primary sources of trajectory variation in unguided sounding rockets are wind and thrust misalignment. In practice, uncertainties also arise from empirically derived aerodynamic coefficients. A Monte Carlo analysis using 500 flight instances with a standard deviation of 10% on all force and moment coefficients in the presence of a time-constant, altitude-invariant unidirectional wind of 28.3 m/s revealed that aerodynamic coefficient inaccuracies do significantly affect flight performance in the presence of strong winds. The standard deviation in apogee altitude was found to be 1151 m, which is 7.8 % of the nominal apogee altitude (see Figure 8).

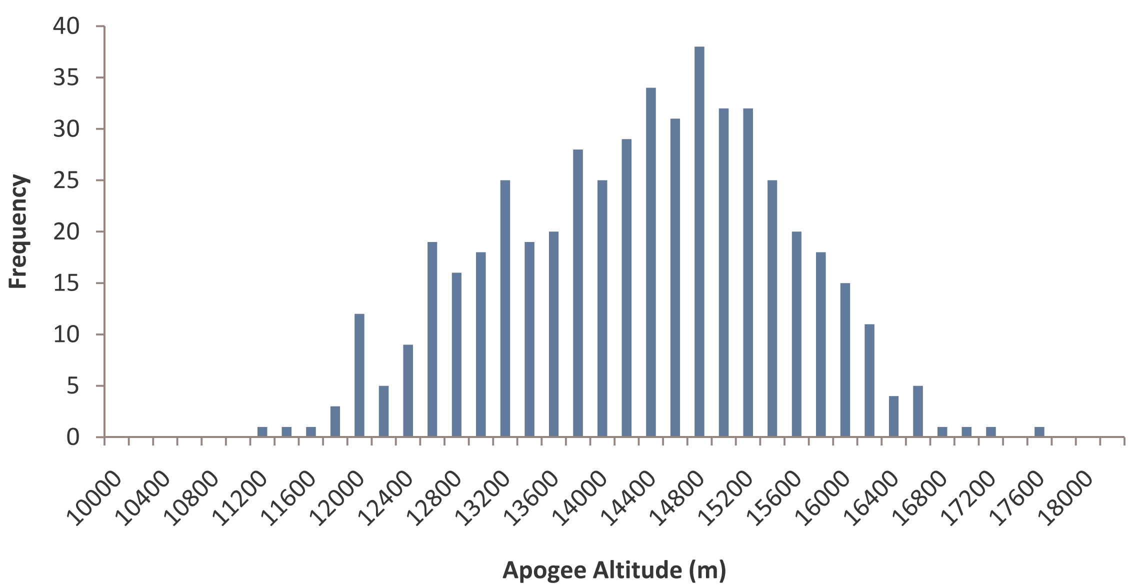


Fig. 8 Monte Carlo dispersion analysis of apogee altitude with a 10% standard deviation on aerodynamic coefficients (500 samples)

A random (i.e. 100% standard deviation) variation on wind direction with fixed aerodynamic coefficients was also run using 500 flight instances, with all other parameters being constant. Results showed a large but fairly even distribution around the nominal landing/impact point. The nominal splashdown point was 13350 m from the launch pad. The mean splashdown deviation distance of 7475 m varied with a standard deviation of 5520 m, suggesting that wind directional variation causes large deviations in splashdown distance and direction (see Figure 9). Consequently, launch azimuth compensation would be necessary if swift location and recovery of the rocket is desired.

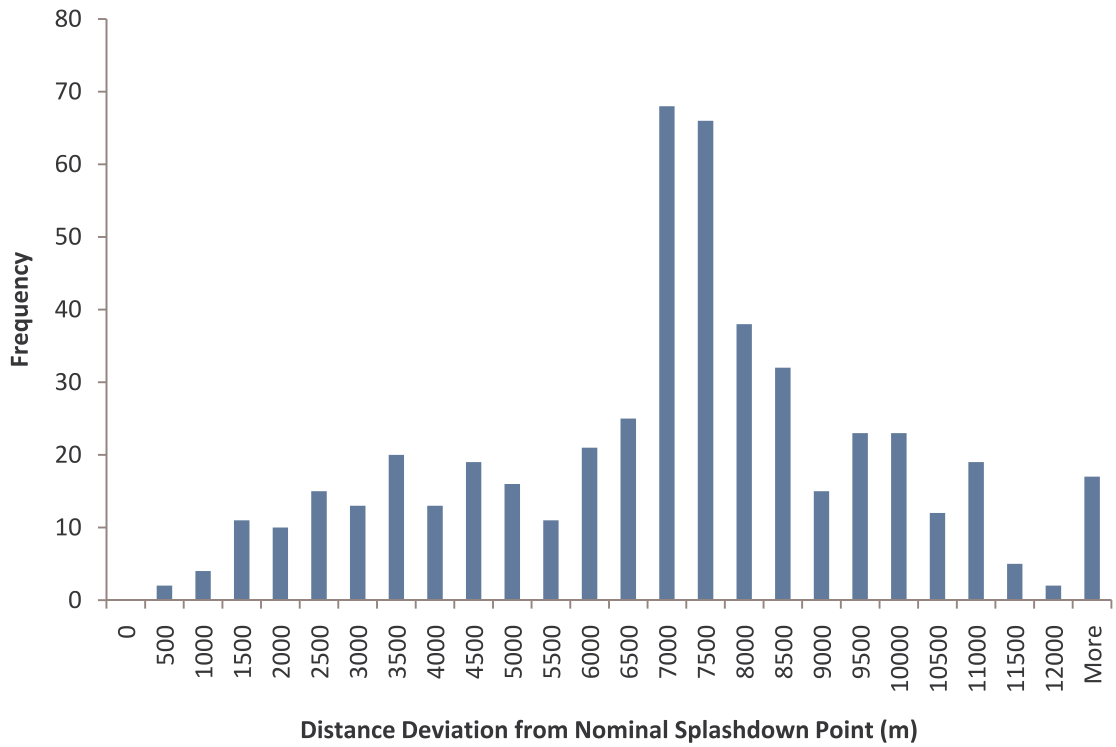


Fig. 9 Monte Carlo dispersion analysis of splashdown distance due to random wind directions (500 samples)

In contrast to the Monte Carlo dispersion analyses, results using monthly mean wind data from the test site at Overberg showed very little deviation from month to month. This implies that the seasonal wind variation is not a significant factor in flight performance at the site when compared to extreme winds that may be encountered on shorter time scales. The ability of the simulator to finely capture wind variation effects was apparent.

Trajectories simulated with the SPDS code were verified against the existing trajectory simulation codes OpenRocket and RasAero. OpenRocket is an open-source 6-DOF software targeted at high power amateur rocketeers. RasAero is a combinational model rocket aerodynamics and flight simulation code with a 3-DOF trajectory solver. For comparison purposes a large model rocket of 2.7 m length featuring a Contrail O600C motor was chosen. To eliminate differences in program functionality, winds and recovery systems were suppressed. The launch angle was set to 15 below the vertical and a time-step size of 0.01 s was used in all three simulations to generate comparable numerical errors. Figure 10 compares the predicted altitude and Figure 11 and Figure 12 compare the predicted velocity and acceleration magnitudes respectively. It was observed that the two 6-DOF codes, SDPS and OpenRocket, agreed better on altitude and range predictions than the 3-DOF code RasAero. As could be expected, aerodynamic coefficients predicted by the RasAero and SDPS codes were found to be in better agreement as these used the very same aerodynamic coefficients database. In all three cases predicted velocities, positions and accelerations were found to agree within an error margin of 10%.

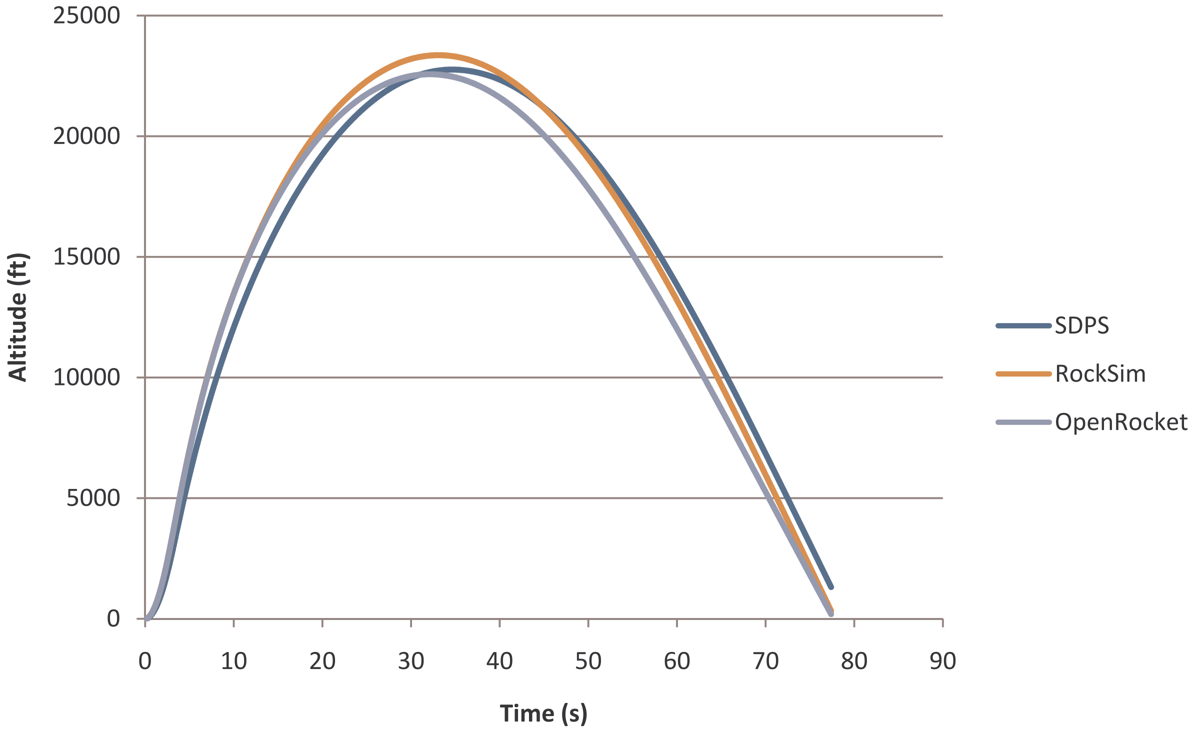


Fig. 10 Altitude comparisons between the OpenRocket, RasAero and the SDPS codes for a large model rocket

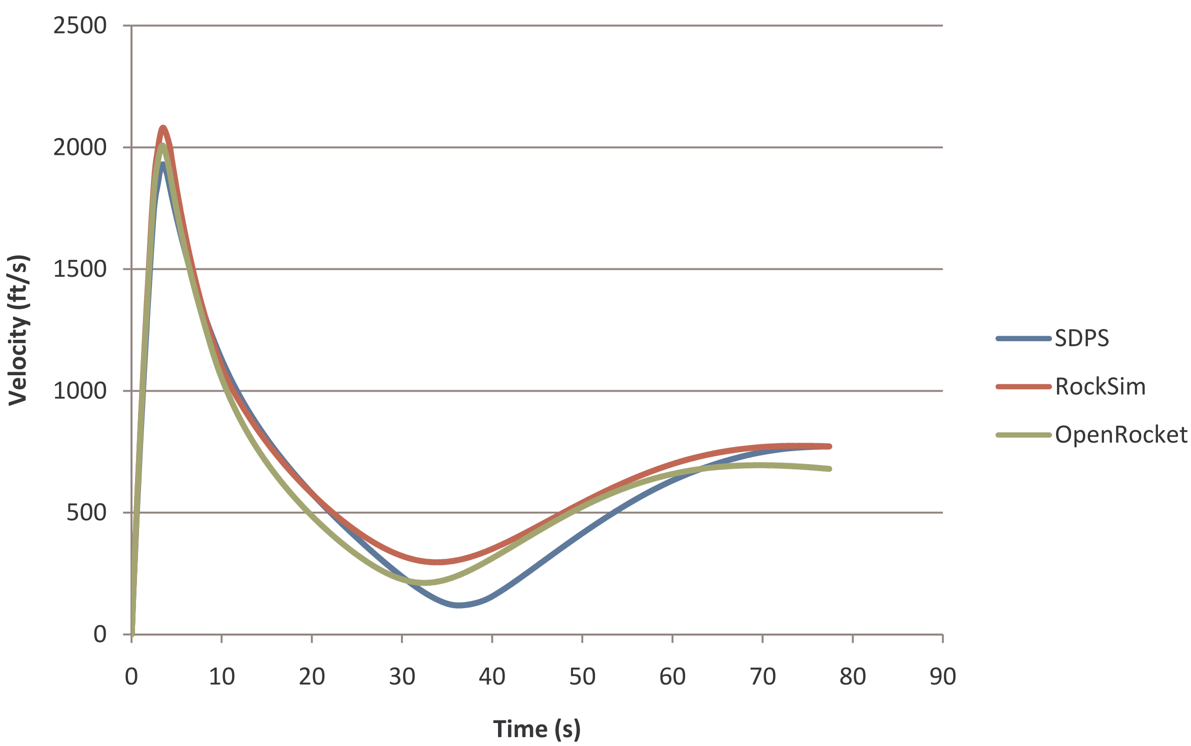


Fig. 11 Velocity magnitude comparisons between the OpenRocket, RasAero and the SDPS code for a large model rocket

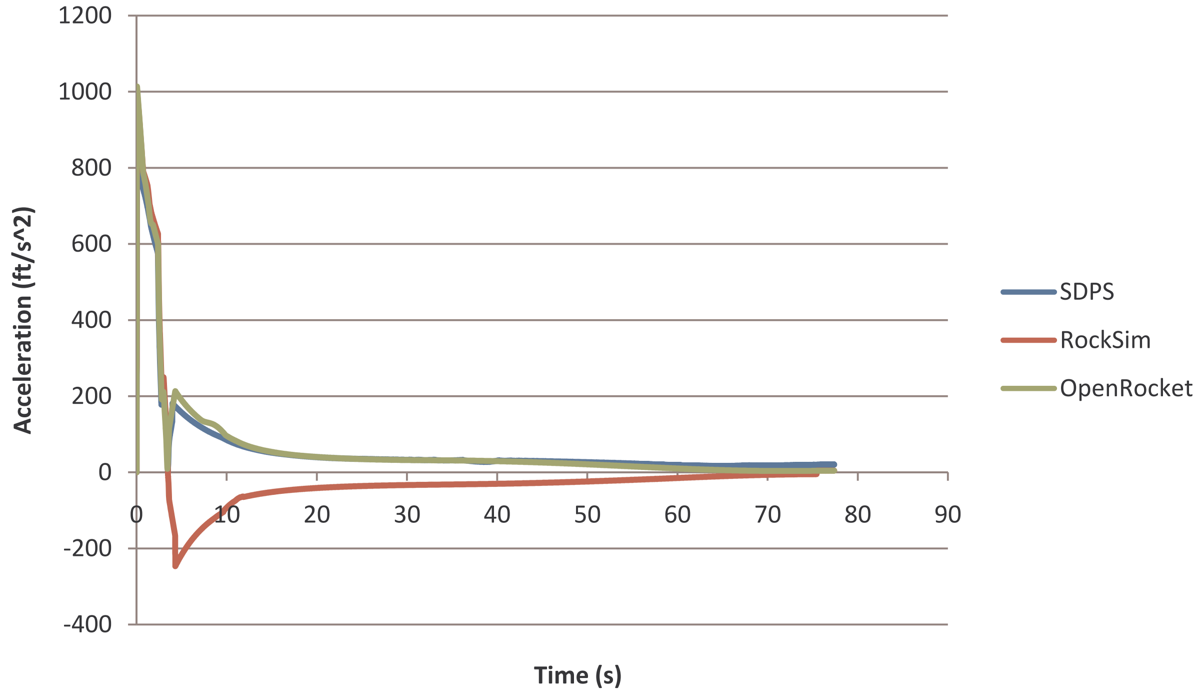


Fig. 12 Acceleration magnitude comparisons between the OpenRocket, RasAero and the SDPS code for a large model rocket

The UKZN Phoenix sounding rocket program aims to develop reusable sounding rockets. Wind drift during the recovery phase is important in the prediction of landing footprints. Several parachute drag coefficients were tested to determine the degree of delay that may be expected during parachute assisted descent. Figure 13 shows altitude histories for several parachute drag coefficients for the test case hybrid rocket (nominal apogee of 14800 m).

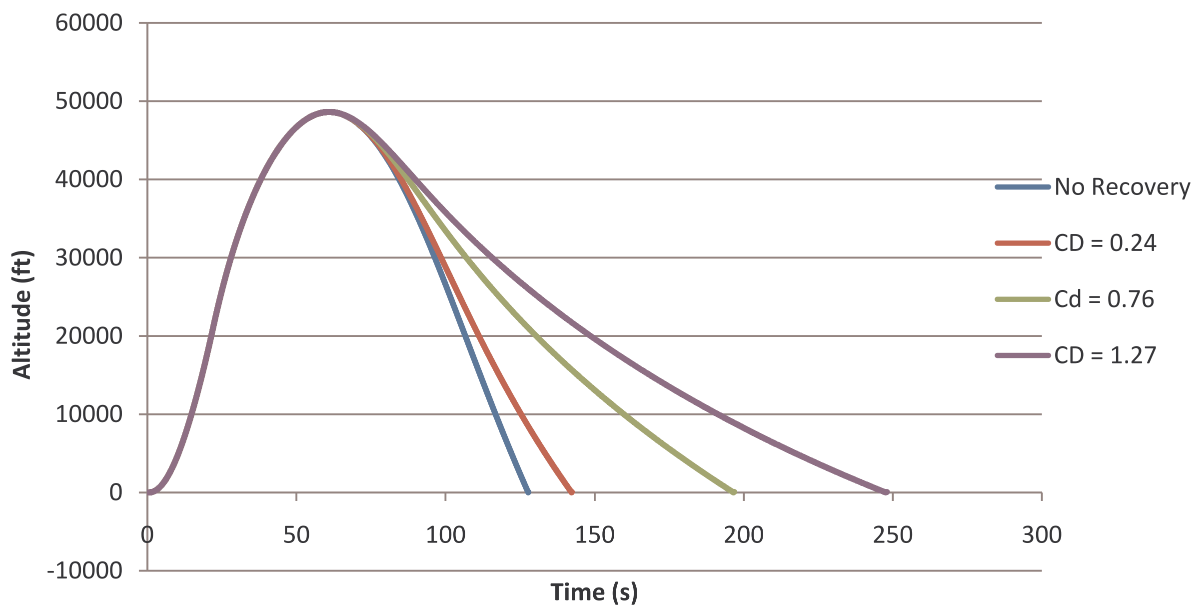


Fig. 13 Effect of parachute drag coefficient on recovery descent time for a small hybrid sounding rocket

Numerical stability issues were encountered during the first few seconds of simulations featuring high velocity winds. Due to the initial vehicle velocity being low, swift winds induced large aerodynamic angles and could result in inaccurate trajectory behavior. The use of an explicit forward marching time integration algorithm also implied that these instabilities could adversely affect trajectory accuracies in the latter stages of the simulations. The instabilities were artificially suppressed by setting aerodynamic angles to zero for the first second of flight. Alternatively, winds could be suppressed in the first second of flight to achieve the same effect.

Numerical accuracy was investigated by varying the simulation time-step size. The solutions were observed to become virtually time-step independent for time-steps shorter than 0.002 s using the explicit Euler integrator. The same was observed for time-steps shorter than 0.01s using the RK4 integration method. Figure 14 displays the deviation in calculated apogee altitude resulting from a decrease in the time-step size, using Euler integration. The result from using a time-step of 0.001 s is considered the reference value. Note the sharp drop in error at a time-step size of approximately 0.002s.

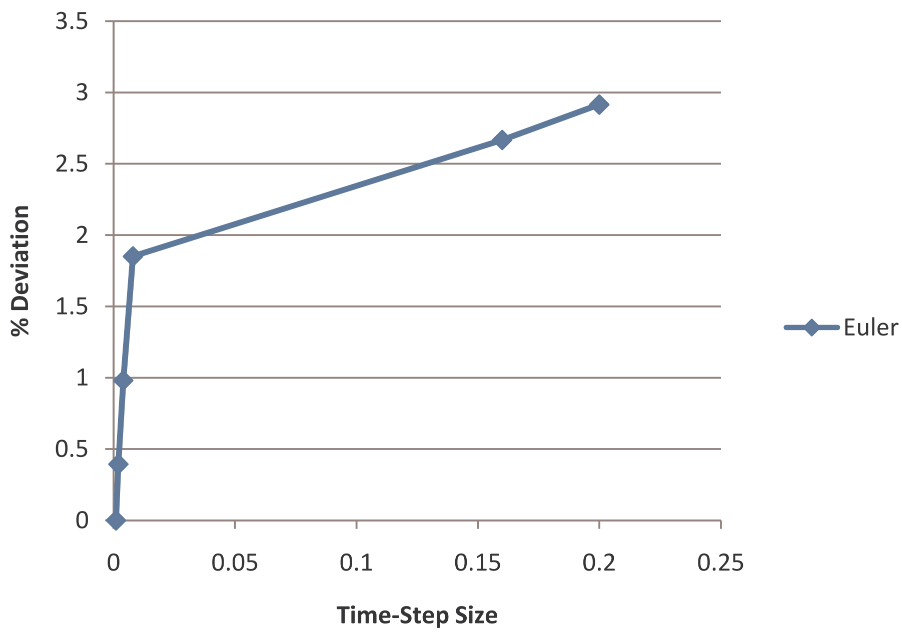


Fig. 14 Effects of varying time-step size on apogee altitude error.

VI. Conclusion

A comprehensive hybrid rocket flight performance prediction software has been implemented and tested. The SDPS code has been verified against existing 3-DOF and 6-DOF rocket simulators and trajectory predictions compare favorably. Numerical discretization errors in the results have also been quantified, placing known bounds on accuracy. Monte Carlo dispersion simulations allow for meaningful sounding rocket design despite inherent system uncertainties. The results from individual trajectory predictions and the statistical dispersion tests both give insight into the relationships between flight performance and vehicle design. The use of integrated GIS information in the software enables visualization of results within a global framework. It is apparent that the integration and coupling of relevant mathematical models into a single simulation leads to greater reliability and accuracy in flight performance prediction. Future research may extend the SDPS tool to allow direct optimization of all design variables using pre-defined constraints on flight performance. Verification of the code against actual flight data is planned upon the launch of the UKZN Phoenix program's Phoenix-1A sounding rocket in late 2011.

Acknowledgments

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