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Problem2a:

Proof:

$$R_x = \frac{1}{n}\left[\begin{array}{cccc} \sum_{i=1}^{n} r_{xi} & \sum_{i=1}^{n} r_{xi} & \cdots & \sum_{i=1}^{n} r_{xi} \end{array}\right] = \left[\begin{array}{cccc} \bar{r_x} & \bar{r_x} & \cdots & \bar{r_x} \end{array}\right]$$
$$R_y = \frac{1}{n}\left[\begin{array}{cccc} \sum_{i=1}^{n} r_{yi} & \sum_{i=1}^{n} r_{yi} & \cdots & \sum_{i=1}^{n} r_{yi} \end{array}\right] = \left[\begin{array}{cccc} \bar{r_y} & \bar{r_y} & \cdots & \bar{r_y} \end{array}\right]$$

Normalized Cosine Similarity:

$$C(x,y) = \cos(r_{xs} - R_x, r_{ys} - R_y)$$

$$= \frac{(r_{xs} - R_x)\cdot(r_{ys} - R_y)}{\|r_{xs} - R_x\|\cdot\|r_{ys} - R_y\|}$$

$$= \frac{\left[\begin{array}{cccc} r_{xs1} - \bar{r_x} & r_{xs2} - \bar{r_x} & \cdots & r_{xsn} - \bar{r_x} \end{array}\right]\cdot\left[\begin{array}{cccc} r_{ys1} - \bar{r_y} & r_{ys2} - \bar{r_y} & \cdots & r_{ysn} - \bar{r_y} \end{array}\right]}{\|\left[\begin{array}{cccc} r_{xs1} - \bar{r_x} & r_{xs2} - \bar{r_x} & \cdots & r_{xsn} - \bar{r_x} \end{array}\right]\|\cdot\|\left[\begin{array}{cccc} r_{ys1} - \bar{r_y} & r_{ys2} - \bar{r_y} & \cdots & r_{ysn} - \bar{r_y} \end{array}\right]\|}$$

$$= \frac{(r_{xs1} - \bar{r_x})(r_{ys1} - \bar{r_y}) + (r_{xs2} - \bar{r_x})(r_{ys2} - \bar{r_y}) + \cdots + (r_{xsn} - \bar{r_x})(r_{ysn} - \bar{r_y})}{\sqrt{(r_{xs1} - \bar{r_x})^2 + (r_{xs2} - \bar{r_x})^2 + \cdots + (r_{xsn} - \bar{r_x})^2}\sqrt{(r_{ys1} - \bar{r_y})^2 + (r_{ys2} - \bar{r_y})^2 + \cdots + (r_{ysn} - \bar{r_y})^2}}$$

$$= \frac{\sum_{s\in S_{xy}} (r_{xs} - \bar{r_x})(r_{ys} - \bar{r_y})}{\sqrt{\sum_{s\in S_{xy}} (r_{xs} - \bar{r_x})^2}\sqrt{\sum_{s\in S_{xy}} (r_{ys} - \bar{r_y})^2}}$$

$$= P(x,y)$$