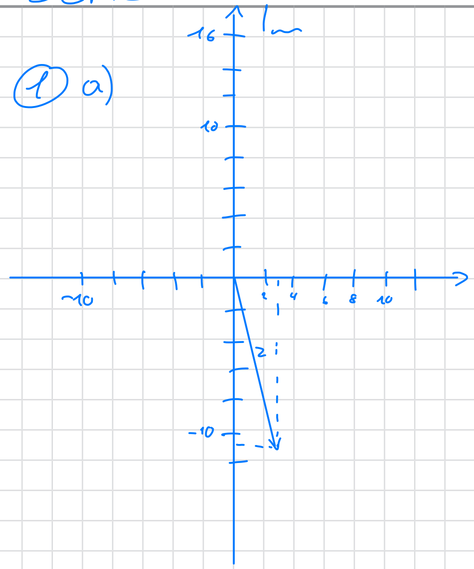
Serie 11

① a)



$z = 3 - 11i$ $\quad 3 = r \cdot \cos\varphi \quad \varphi = 1.31$

$z = \sqrt{130}(\cos(\varphi) + i \cdot \sin(\varphi)) - 11 = \quad r \cdot \sin\varphi$

$\varphi = 74{,}74 \quad r = \sqrt{3^2 + 11^2} = \sqrt{130}$

$z = \sqrt{130}\, e^{-i \cdot 1{,}31}$

$z = \sqrt{130}(0.26 - i \cdot 0.96)$

$z^* = 3 + 11i$

$z^* = \sqrt{130}(0.26 + i \cdot 0.96)$

$z^* = \sqrt{130}\, e^{i \cdot 1{,}31}$

b) $z = 4[\underbrace{\cos(-40°) + i\sin(-40°)}]+ 2e^{i30°} - 3 + 1.5i$

$= 4 \cdot e^{i \cdot 40°} + 2e^{i30°} - 3 + 1.5i$

$= 3.064 - i \cdot 2.57 + \sqrt{3} + i1 - 3 + 1.5i$

$z = 1{,}796 - \frac{7}{100} i$

$z^* = 1{,}79 + \frac{7}{100} i$

c) $z_1 = \frac{2+i}{1-2i} = \frac{2+(-2)}{5} + i\frac{5}{5}$

$= 0 + i1$

$z_1^* = 0 - i1 \qquad z_2 = 2e^{-i\frac{\pi}{3}} = 1 - i\sqrt{3}$

$z_3 = 2\sqrt{3} + i \cdot 2 \qquad z_1^* \cdot z_3 = 2 + i \cdot 2\sqrt{3}$

$$\frac{z_1^* \cdot z_3}{0.5\, z_2} = \frac{2 + i \cdot 2\sqrt{3}}{\frac{1}{2} - i\frac{1}{2}\sqrt{3}} = \left(\frac{1+3}{\frac{1}{4}+\frac{3}{4}}\right) + i\left(\frac{\sqrt{3}-\sqrt{3}}{\frac{1}{4}+\frac{3}{4}}\right) = \frac{4}{1} + i\frac{0}{1} = \underline{\underline{4}}$$

d) $(1 - \sqrt{2}i)^3 = (\sqrt{3}\, e^{-i0.955})^3 = \underline{\underline{\sqrt{3}^3 e^{-i\,2.865}}}$

$r = \sqrt{x^2 + y^2} = \sqrt{3}$

$x = \sqrt{3} \cdot \cos(\varphi)$

$\frac{1}{\sqrt{3}} = \cos(\varphi) \qquad \varphi = -0.955$

$y = \sqrt{3} \cdot \sin(\varphi)$

$\frac{-\sqrt{2}}{\sqrt{3}} = \sin(\varphi)$

② $z^4 + 4z^2 + 16 = 0 \qquad u = z^2$

$u^2 + 4u + 16 = 0$

$\sqrt{-48} = \sqrt{-1 \cdot 48} = \underbrace{\sqrt{-1}}_{i} \cdot \sqrt{48}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{-4 \pm \sqrt{16-64}}{2} = \frac{-4 \pm \sqrt{48}i}{2} = \frac{-4 \pm 4\sqrt{3}i}{2}$

$\rightarrow \frac{-4 + 4\sqrt{3}i}{2} \Rightarrow r = \sqrt{(-4)^2 + (4\sqrt{3})^2} = 8$

$x = r \cdot \cos(\varphi)$

$\cos^{-1}\left(\frac{x}{r}\right) = \varphi = \frac{2}{3}\pi$

$\frac{8e^{i\cdot\frac{2}{3}\pi}}{2} = \underline{\underline{4e^{i\cdot\frac{2}{3}\pi}}} = u \qquad z = \underline{\underline{\sqrt{4e^{i\frac{2}{3}\pi}}}}$

$\underline{\underline{\sqrt{4e^{-i\cdot\frac{2}{3}\pi}}}}$

$\rightarrow \frac{-4 - 4\sqrt{3}i}{2} = r = \sqrt{(-4)^2 + (\sqrt{48})^2} = 8$

$\frac{8e^{i\cdot\frac{2}{3}\pi}}{2} = \underline{\underline{4e^{-i\cdot\frac{2}{3}\pi}}} = u$