Serie 9

① a) $\|A\| = 3$

$\|A^{-1}\| = 20001$

$\text{cond}(A) = \|A\| \cdot \|A^{-1}\| = \underline{\underline{60'003}}$

b) $\tilde{x} = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 10^{-4} & 0 & 10^{-4} & \varepsilon \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 10^{4}\cdot\varepsilon \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1-10^{4}\cdot\varepsilon \end{array}\right) = \begin{pmatrix} -1+(2\cdot 10^{4}\cdot\varepsilon) \\ 1 \\ 1-10^{4}\cdot\varepsilon \end{pmatrix}$

$\tilde{x} - x = \begin{pmatrix} 2\cdot 10^{4}\cdot\varepsilon \\ 0 \\ -10^{4}\cdot\varepsilon \end{pmatrix}$

$\frac{\|\tilde{x} - x\|}{\|x\|} \leq 0.01$ ($2\cdot 10^{4}\cdot\varepsilon$ über dem Zähler, 1 unter dem Nenner)

$2\cdot 10^{4}\cdot\varepsilon \leq 0.01$

$\varepsilon \leq 5\cdot 10^{-7}$

$x_1 + 2\cdot(1-10^{4}\cdot\varepsilon) = 1$

$x_2 = 1$

$x_3 = 1-10^{4}\cdot\varepsilon$

$x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

c) $\tilde{x} = \begin{pmatrix} -0.99 \\ 1 \\ 0.995 \end{pmatrix}$

d) $\tilde{x} = (A = \tilde{b}) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1e^{7} \\ 0 & 1 & 0 & 1e^{7} \\ 10^{-4} & 0 & 10^{-4} & \varepsilon e^{7} \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 10^{6} \\ 0 & 1 & 0 & 10^{6} \\ 1 & 0 & 1 & \varepsilon\cdot 10^{6} \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 10^{6} \\ 0 & 1 & 0 & 10^{6} \\ 0 & 0 & 1 & -(\varepsilon\cdot 10^{6} - 10^{6}) \end{array}\right)$

$\tilde{x} = \begin{matrix} 10^{6} \\ 10^{6} \\ ? \end{matrix}$