

## CSCE-633

Machine Learning Homework #4: CNN

### Question 1: Convolution Operation

Given the following  $7 \times 7$  input matrix and  $3 \times 3$  filter:

**Input Matrix:**

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 5 & 2 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Filter:**

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

#### Part 1: Without max pooling and with a stride of 1

**Calculation approach:**

For convolution, we slide the  $3 \times 3$  filter across the input matrix by the stride value.

The output dimensions are calculated as:

$$\text{Output size} = \frac{\text{Input size} - \text{Filter size}}{\text{Stride}} + 1 = \frac{7 - 3}{1} + 1 = 5$$

So the output will be a  $5 \times 5$  matrix.

**Example calculation for position (0,0):**

The filter overlaps with the top-left  $3 \times 3$  region of the input:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Element-wise multiplication and sum:

$$\begin{aligned} \text{Output}[0, 0] &= (0 \times 1) + (0 \times 0) + (0 \times -1) \\ &\quad + (0 \times 1) + (0 \times 0) + (2 \times -1) \\ &\quad + (0 \times 1) + (3 \times 0) + (1 \times -1) \\ &= 0 + 0 + 0 + 0 + 0 + (-2) + 0 + 0 + (-1) \\ &= -3 \end{aligned}$$

**Example calculation for position (0,1):**

Shifting the filter right by 1, it now overlaps with columns 1-3:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Element-wise multiplication and sum:

$$\begin{aligned}
 \text{Output}[0, 1] &= (0 \times 1) + (0 \times 0) + (0 \times -1) \\
 &\quad + (0 \times 1) + (2 \times 0) + (4 \times -1) \\
 &\quad + (3 \times 1) + (1 \times 0) + (1 \times -1) \\
 &= 0 + 0 + 0 + 0 + 0 + (-4) + 3 + 0 + (-1) \\
 &= -2
 \end{aligned}$$

Following this process for all positions yields the complete output matrix.

**Final Output (5×5):**

$$\begin{bmatrix} -3 & -2 & 2 & 4 & 1 \\ -7 & -1 & 6 & 4 & 1 \\ -5 & 0 & 3 & 3 & 2 \\ -5 & -5 & 1 & 5 & 4 \\ -1 & -6 & -3 & 5 & 4 \end{bmatrix}$$

## Part 2: Without max pooling and with a stride of 2

**Calculation approach:**

With stride 2, the filter moves 2 positions at a time (both horizontally and vertically).

The output dimensions are:

$$\text{Output size} = \frac{7-3}{2} + 1 = 3$$

So the output will be a  $3 \times 3$  matrix.

**Example calculation for position (0,0):**

Same as Part 1, this gives us  $\text{Output}[0,0] = -3$ .

**Example calculation for position (0,1):**

Now with stride 2, the filter shifts to columns 2-4 (skipping column 1):

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Element-wise multiplication and sum:

$$\begin{aligned}
 \text{Output}[0, 1] &= (0 \times 1) + (0 \times 0) + (0 \times -1) \\
 &\quad + (2 \times 1) + (4 \times 0) + (1 \times -1) \\
 &\quad + (1 \times 1) + (1 \times 0) + (0 \times -1) \\
 &= 0 + 0 + 0 + 2 + 0 + (-1) + 1 + 0 + 0 \\
 &= 2
 \end{aligned}$$

**Final Output (3×3):**

$$\begin{bmatrix} -3 & 2 & 1 \\ -5 & 3 & 2 \\ -1 & -3 & 4 \end{bmatrix}$$

## Part 3: With max pooling and with a stride of 1

**Note on ambiguity:** The problem statement says "stride of 1" but the image shows "stride = 2" for max pooling. To be thorough, I provide both interpretations below.

**Step 1: Convolution with stride 1**

First, perform convolution with stride 1 (same as Part 1):

$$\begin{bmatrix} -3 & -2 & 2 & 4 & 1 \\ -7 & -1 & 6 & 4 & 1 \\ -5 & 0 & 3 & 3 & 2 \\ -5 & -5 & 1 & 5 & 4 \\ -1 & -6 & -3 & 5 & 4 \end{bmatrix}$$

### Interpretation A: Max pooling with stride = 1

If we interpret "stride of 1" as applying to the max pooling operation:

Max pooling takes the maximum value from each  $3 \times 3$  region with stride 1.

The output dimensions after max pooling are:

$$\text{Output size} = \frac{5 - 3}{1} + 1 = 3$$

So the final output will be a  $3 \times 3$  matrix.

#### Example: Position (0,0)

The top-left  $3 \times 3$  region is:

$$\begin{bmatrix} -3 & -2 & 2 \\ -7 & -1 & 6 \\ -5 & 0 & 3 \end{bmatrix}$$

The maximum value in this region is  $\max(-3, -2, 2, -7, -1, 6, -5, 0, 3) = 6$ .

#### Example: Position (0,1) - Moved right by 1

Shifting right by stride 1, we get columns 1-3:

$$\begin{bmatrix} -2 & 2 & 4 \\ -1 & 6 & 4 \\ 0 & 3 & 3 \end{bmatrix}$$

The maximum value is  $\max(-2, 2, 4, -1, 6, 4, 0, 3, 3) = 6$ .

Note: This pool overlaps 2 columns with the previous pool.

#### Final Output (Interpretation A) - $3 \times 3$ :

$$\begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \\ 3 & 5 & 5 \end{bmatrix}$$

### Interpretation B: Max pooling with stride = 2 (as shown in image)

If we follow the image which shows "stride = 2" for max pooling:

Max pooling takes the maximum value from each  $3 \times 3$  region with stride 2.

The output dimensions after max pooling are:

$$\text{Output size} = \frac{5 - 3}{2} + 1 = 2$$

So the final output will be a  $2 \times 2$  matrix.

#### Example: Position (0,0)

Same top-left  $3 \times 3$  region as Interpretation A, maximum value = 6.

#### Example: Position (0,1) - Moved right by 2

Shifting right by stride 2, we get columns 2-4:

$$\begin{bmatrix} 2 & 4 & 1 \\ 6 & 4 & 1 \\ 3 & 3 & 2 \end{bmatrix}$$

The maximum value is  $\max(2, 4, 1, 6, 4, 1, 3, 3, 2) = 6$ .

Note: This pool does NOT overlap with the previous pool.

**Final Output (Interpretation B) -  $2 \times 2$ :**

$$\begin{bmatrix} 6 & 6 \\ 3 & 5 \end{bmatrix}$$

Note: Python code was written to compute and verify these results, as manual computation is prone to errors. The examples shown demonstrate understanding of the hand calculation process.

Key concepts demonstrated: - Output size determines the dimensions of our output matrix - Stride determines how much we move the filter across the input matrix - Max pooling takes the maximum value from each pooling window - Pools can overlap depending on the stride value (stride=1 overlaps, stride=2 does not)

Due to ambiguity in the problem statement (text says "stride of 1" while the image shows "stride = 2" for max pooling), both interpretations were computed to ensure completeness.

## Question 2: Image Classification using CNN

The required functions for the SUN397Dataset and CNN model were implemented in the submitted Python code.

### Strategy and Implementation

The final model employed a \*\*5-layer CNN architecture\*\* chosen after iterative testing found 3-layer designs to underfit the data. The model was trained using the **Adam** optimizer and a **ReduceLROnPlateau** learning rate scheduler. The entire training process required approximately 8 hours to complete 30 epochs on a CPU.

### Dataset Analysis

The SUN397 dataset subset consisted of **6,042 images** distributed across **4 classes**:

- airport-terminal (label: 0)
- bedroom (label: 1)
- dining-room (label: 2)
- living-room (label: 3)

Labels were assigned alphabetically as specified in the assignment requirements.

The dataset normalization statistics were computed across all training images:

- **Mean (R, G, B):** 0.5286, 0.4668, 0.4109
- **Std (R, G, B):** 0.2157, 0.2236, 0.2264

However, the normalization check failed on Gradescope, and it was difficult to determine why the computed values did not match the expected results.

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### Results

- **Final Accuracy:** After 30 epochs, the training set achieved an accuracy of approximately **78.00%**, successfully surpassing the **75%** target.
- **Submission:** The final model weights were saved to the required **model.pt** file.

### Use of AI and External Resources

Claude.ai was used to assist with the following aspects of this assignment:

The following prompts were used to get assistance with the assignment:

- "Explain a standard starting point for CNN values? "
- "Why is my 3 layer network under performing?"
- "What is an Adam Optimizer?"
- "Provide a structure for print statements to aid in locally testing code."