Convolution

Overview

Class Meeting : 20 Jan 2026

- **convolution**
  - **analytical**
  - **discrete**
  - **numerical**
- **formulate in terms of convolution kernels**
- **compute convolutions in one and higher dimension numerically**
- **Gaussian convolution**
- **box convolution**
- **efficiency and benefit of separable kernels**

Convolution

[optional]

- **convolution** is a mathematical **technique used** on two **functions** f **and** g that **produces** a third **function** $f * g$
- $f * g$ is the **integral** of the **product of** the two **functions** after **one** is **reflected** about the **y-axis** and **shifted**
- uses an **average** of local-**adjacent values** based on kernel size

https://en.wikipedia.org/wiki/Convolution

Convolution in 1D

Applications of Convolution	• **signal processing**
	  ○ **filtering** (noise reduction
	  ○ system **modeling**
	  ○ **reverberation** (simulating sound reflections
	• **image processing**
	  ○ **sharpening**
	  ○ edge **detection**
	  ○ feature **extraction**
	• **artificial intelligence**

	○ convolutional **neural networks** • **physics** and **engineering** https://www.fieldbox.ai/seeing-through-computer-vision-convolution-101/
General Mathematical form	$(f * g)(t) = \int_{\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{\infty}^{\infty} f(t-\tau)g(\tau)d\tau$
Properties	• ***f * g = g * f*** • ***f * (g * f) = (f*g) * h*** • ***f * (g + h) = (f * g) + (f * h)*** https://mathworld.wolfram.com/Convolution.html
Discrete Convolution	$(f * g)(i) = \sum_k f(k)g(i-k) = \sum_k f(i-k)g(k)$
Layering	• a **weighted sum** of the convolution **kernel** • can **extract** local **features**
Convolution Kernel	• the **filter** of a matrix **for feature extraction** • **each** convolution **process** has its own **kernel** • **image blurring** example: for **each pixel** in an **image**, the inner **product** of the **pixel** within the local **window** centered on that pixel and the **kernel** is **calculated** • some **kernel types** ○ **asymmetric** ○ **hat** ○ **box** ○ **exponential decay**
Padding	• for when the convolution **range** is **not defined** at edges

	• ***f*** is **padded** with reasonable **data** there are **multiple tactics** available
	    ○ **zeros**
	    ○ **mirror-copy**
	    ○ **repeat** closest **value**
	    ○ **trim** the **range** (usually only with **very large sets**)

Convolution in 2D	
Overview	• two-dimensional convolution is mathematically the same as in 1D • produced 2D integral • often kernels are separable
Kernel Separability	• a function g(x, y), that can be rewritten as a product such that: $g(x, y) = g^x(x)*g^y(y)$ • the result is two 1D kernels • by separating the kernel, each dimension can be convoluted individually • continuous separable $(f*g)(x,y) = \int_{-R}^{R}\int_{-R}^{R} f(x-u, y-v)g(u,v)dudv$ • discrete separable $(f*g)(i,j) = \sum_{I=-k}^{k}\sum_{J=-k}^{k} f(i-I, j-J)g(I,J)$
Sources	• https://mathworld.wolfram.com/Convolution.html • https://evidentscientific.com/en/microscope-resource/tutorials/digital-imaging/processing/convolutionkernels

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