Analysis of Equations of State for Neutron Star Modeling

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Solve an *initial value problem*. A solution to the above system is called a *static solution*.

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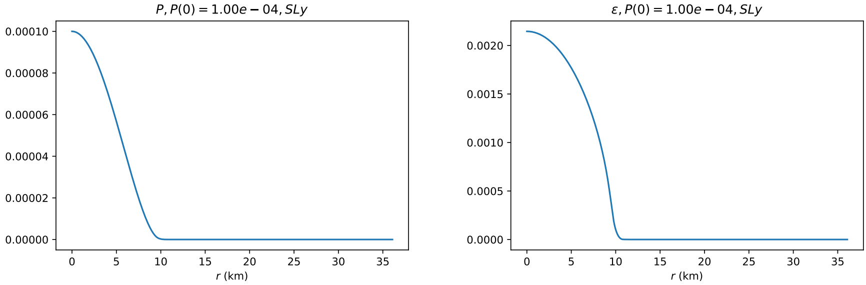

Figure: Example static solution for $P_0 = 10^{-4}$ for an EoS called "SLy."

Static Solutions: $M(R)$ and $M(P_0)$ diagrams

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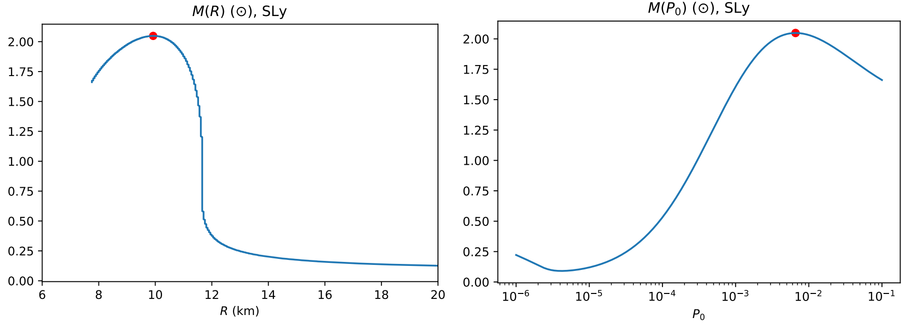
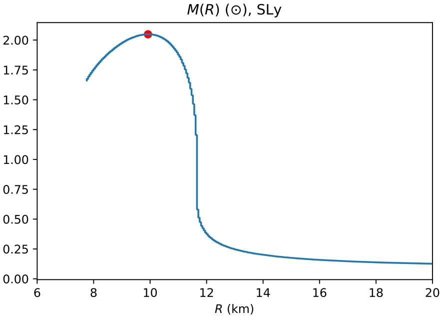
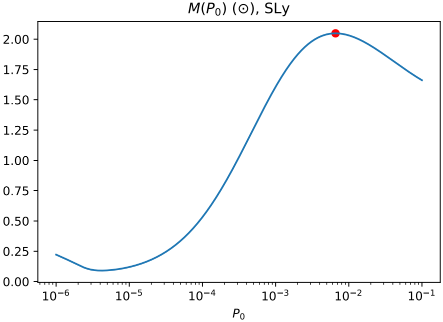


Figure: Example curves for EoS “SLy.” $1 \odot = 1.989 \times 10^{30}$ kg (solar mass)

Critical Values of P, R, and M





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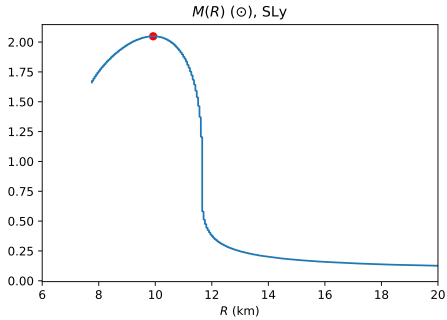

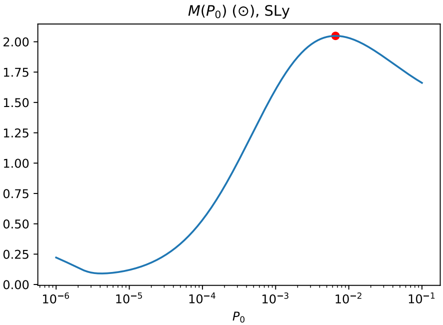

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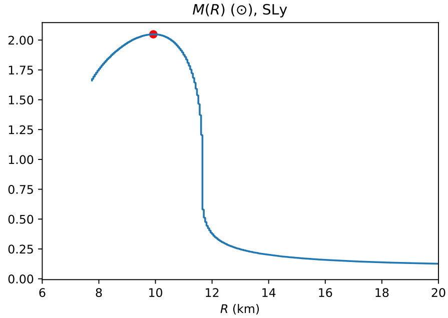

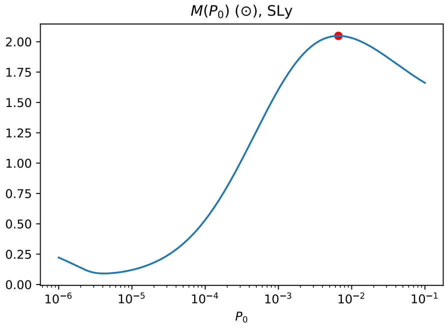

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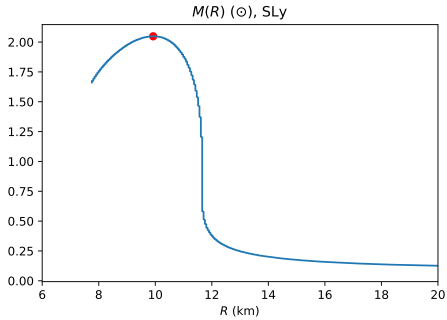

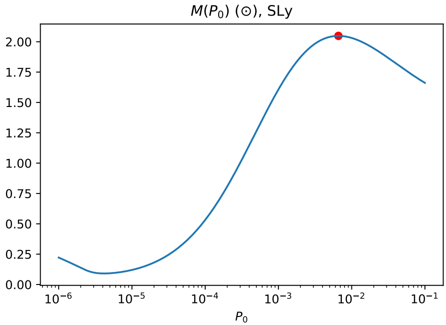

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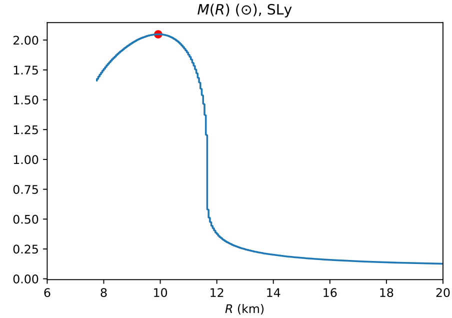

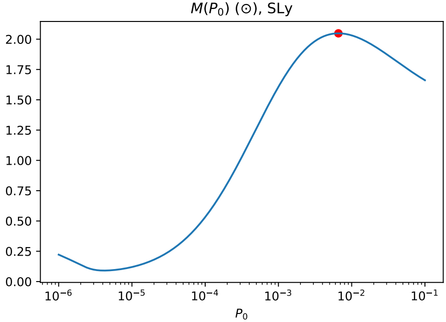

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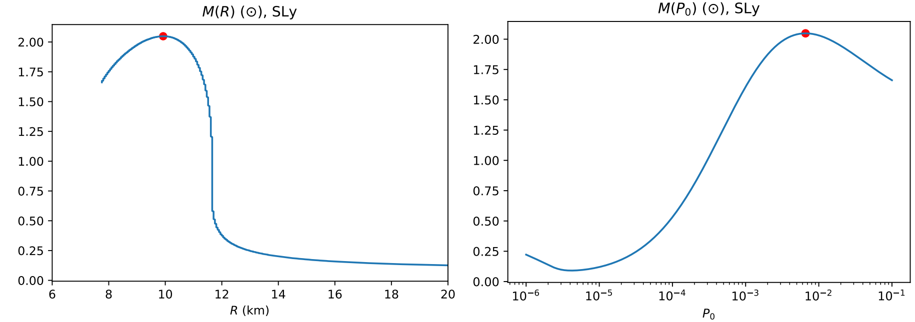

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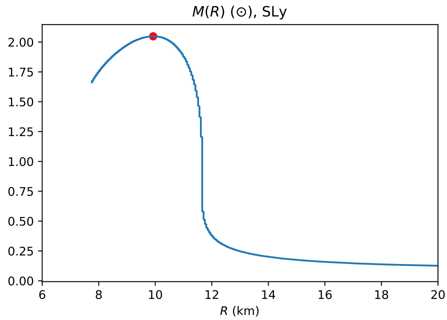

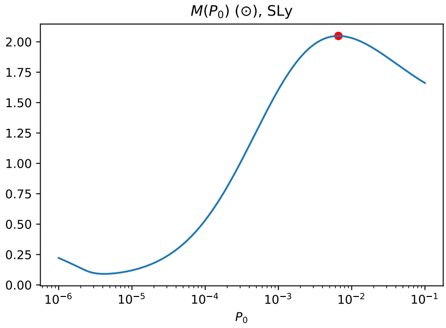

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From $\mathscr{L}$, we can determine ϵ and P, the EoS we desire.

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$$\phi_0 = \frac{g_\phi}{m_\phi^2}\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{k^2 m^*}{\sqrt{k^2+m^{*2}}},$$

$$V_0 = \frac{g_v}{m_v^2}\frac{k_f^3}{3\pi^2},$$

$$\epsilon = \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{\pi^2}\int_0^{k_f} dk\,k^2\sqrt{k^2+m^{*2}},$$

$$P = -\frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{3}\left(\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{k^4}{\sqrt{k^2+m^{*2}}}\right).$$

where $m^* = (M - g_\phi\phi)$, the *reduced mass*.

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- Using those values, calculate P and ϵ and store in a table

$M(R)$ and $M(P_0)$ Curves for QHD-I

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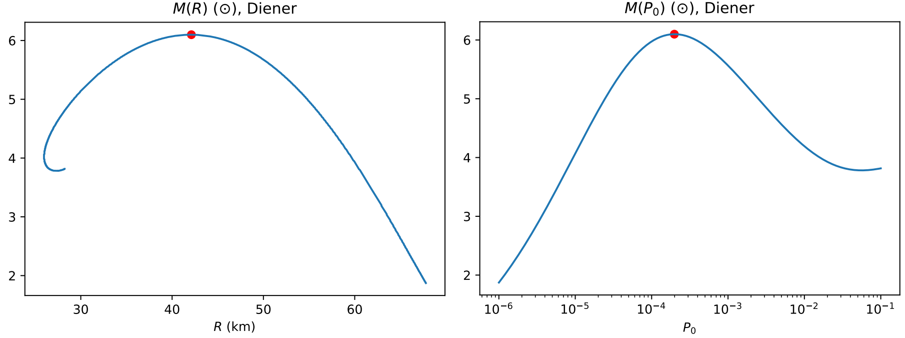

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

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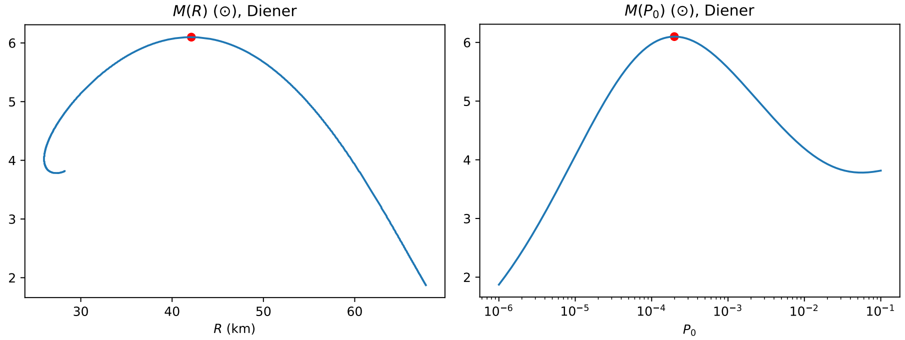

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

These curves give

$$M_{\max} = 6.1 \odot, \quad R_{\max} = 42.1\,\text{km}, \quad P_{\text{crit}} = 1.98 \times 10^{-4}.$$

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- We use the QHD-I parameter set and RMF simplifications to solve a system of equations and generate an equation of state

Thanks

- Prof. Ben Kain

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- My family