Analysis of Equations of State for Neutron Star Modeling

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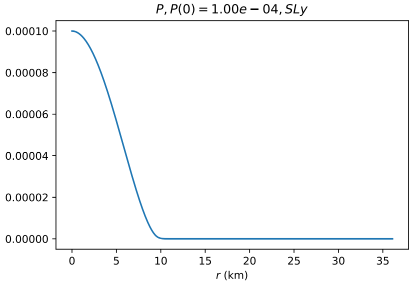

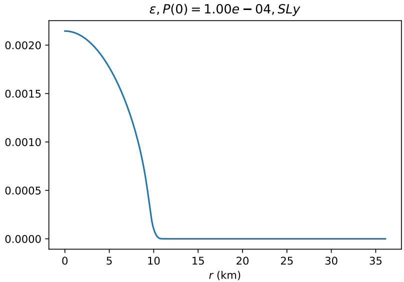

Figure: Example static solution for $P_0 = 10^{-4}$ for an EoS called "SLy."

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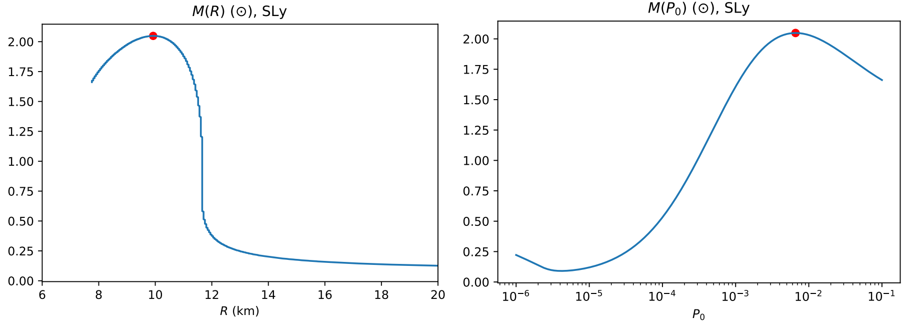
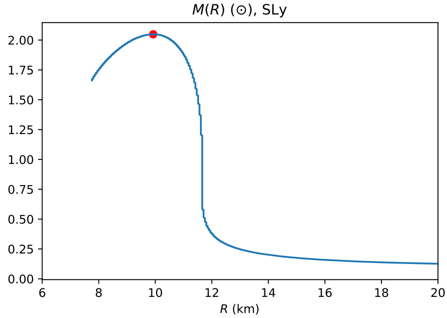
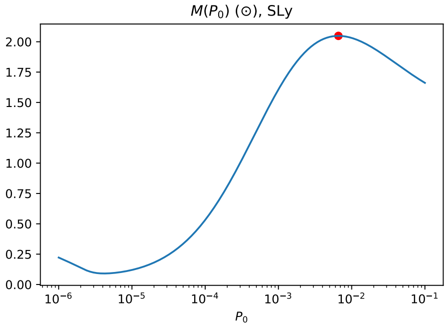


Figure: Example curves for EoS "SLy." $1 \odot = 1.989 \times 10^{30}$ kg (solar mass)

Critical Values of P, R, and M





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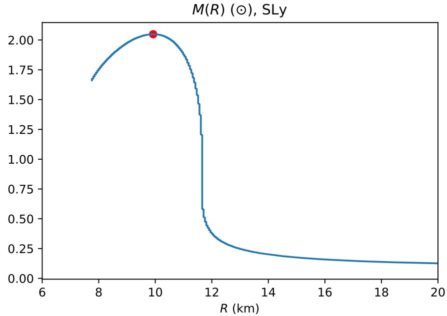

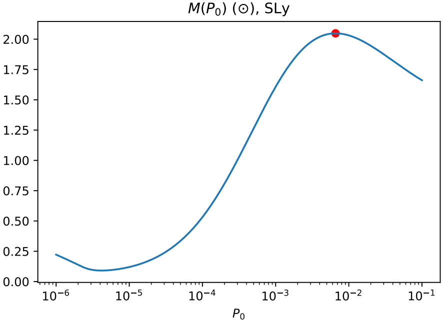

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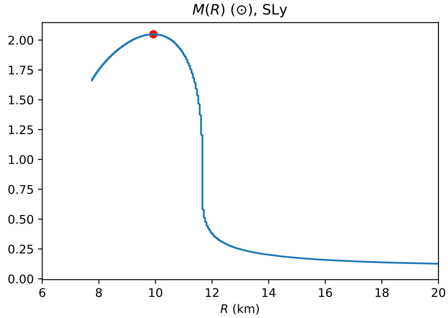

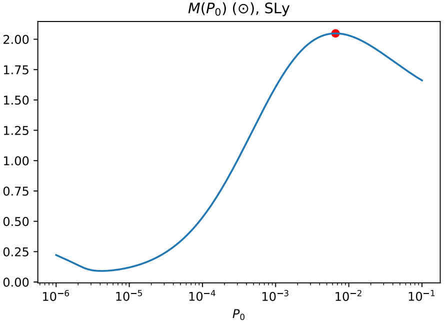

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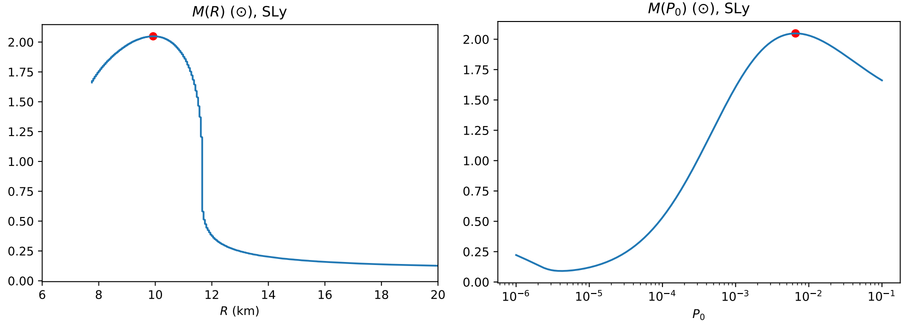

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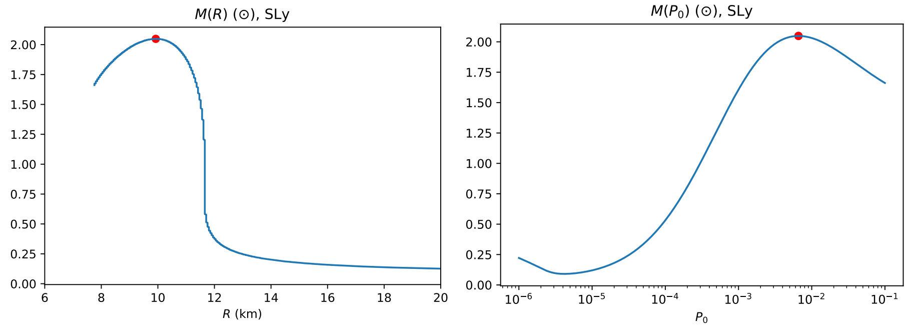

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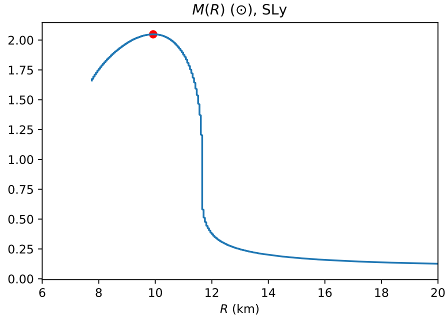

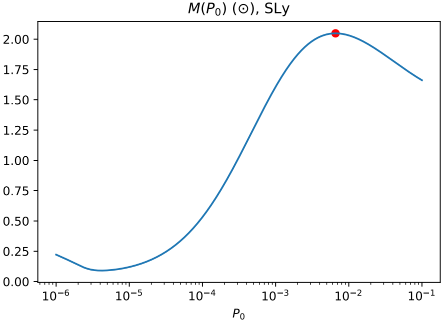

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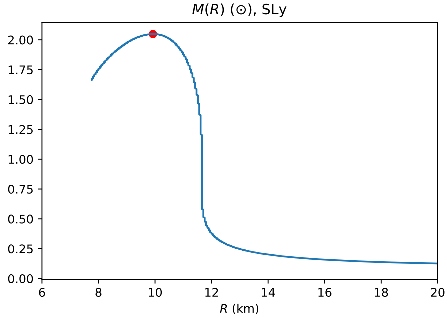

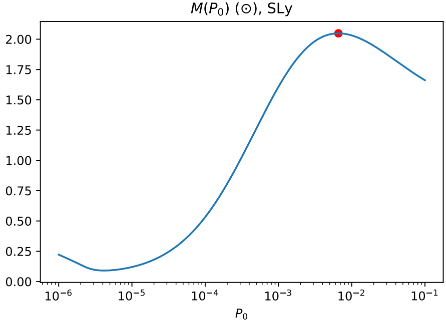

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- For SLy, $M_{\max} = 2.05\,\odot$, $R_{\max} = 9.93\,\text{km}$, and $P_{\text{crit}} = 6.59 \times 10^{-3}$.

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From $\mathcal{L}$, we can determine ϵ and P, the EoS we desire.

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RMF Simplifications

We introduce the *Relativistic Mean Field* (RMF) simplifications. We treat the interactions (exchange of mesons) as their average values:

$$\phi \to \langle \phi \rangle = \phi_0, \quad V_\mu \to \langle V_\mu \rangle = V_0,$$

where ϕ_0 and V_0 are constants. This allows us to simplify $\mathcal{L}$ considerably:

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$$\phi_0 = \frac{g_\phi}{m_\phi^2}\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{(M - g_\phi\phi_0)k^2}{\sqrt{k^2 + (M - g_\phi\phi_0)}},$$

$$V_0 = \frac{g_v}{m_v^2}\frac{k_f^3}{3\pi^2},$$

$$\epsilon = \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{\pi^2}\int_0^{k_f} dk\,k^2\sqrt{k^2 + m^{*2}},$$

$$P = -\frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{3}\left(\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{k^4}{\sqrt{k^2 + m^{*2}}}\right).$$

where $m^* = (M - g_\phi\phi)$, the *reduced mass*.

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From above, we obtain the following equations:

$$\phi_0 = \phi_0(k_f, \phi_0), \quad V_0 = V_0(k_f),$$
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To produce the EoS: (repeat the following)

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- calculate ϕ_0 and V_0; use *rootfinding* for ϕ_0
- Using those values, calculate P and ϵ and store in a table

$M(R)$ and $M(P_0)$ Curves for QHD-I

We use the tabulated values of P and ϵ to solve the TOV equations:

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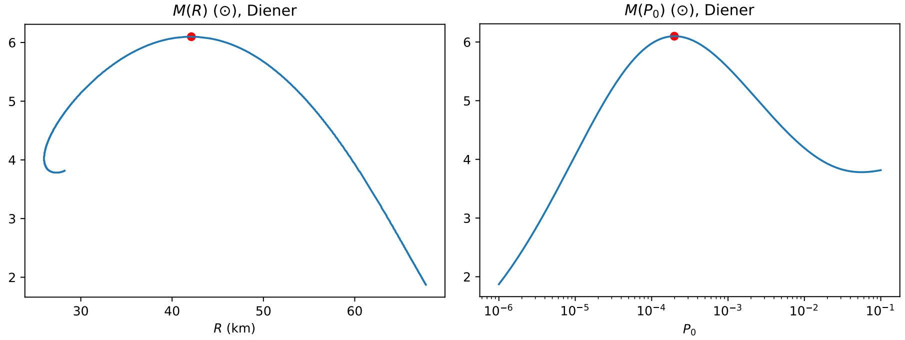

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

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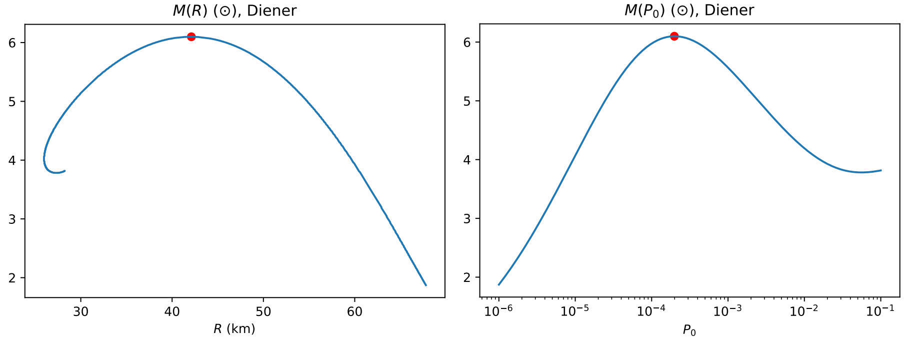

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

These curves give

$$M_{\text{max}} = 6.1\,\odot, \quad R_{\text{max}} = 42.1\,\text{km}, \quad P_{\text{crit}} = 1.98 \times 10^{-4}.$$

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- We use the TOV equations to predict the maximum mass and radius that a given EoS will produce
- We use the QHD-I parameter set and RMF simplifications to solve a system of equations and generate an equation of state

Thanks

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- My family

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