# Analysis of Equations of State for Neutron Star Modeling

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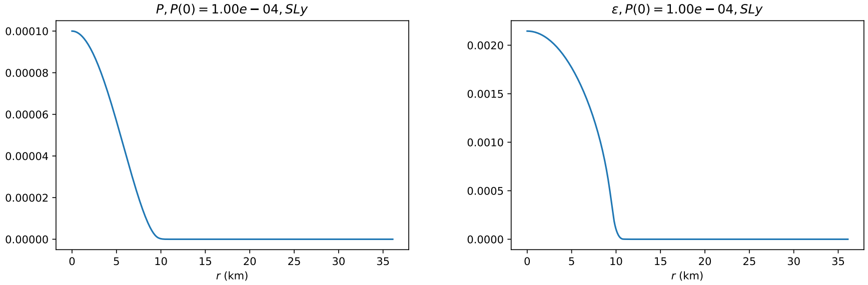


Figure: Example static solution for $P_0 = 10^{-4}$ for an EoS called "SLy."

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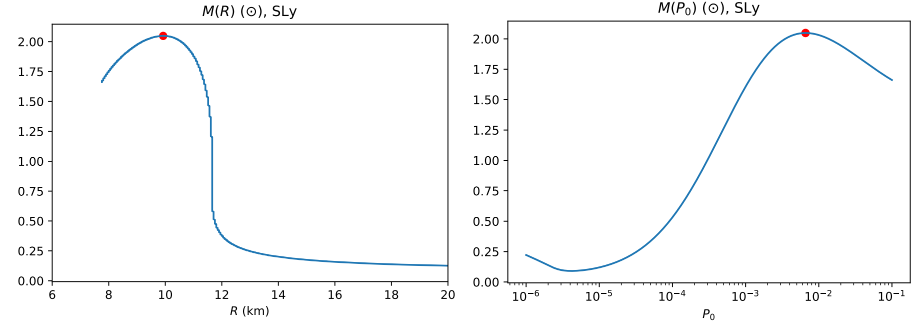
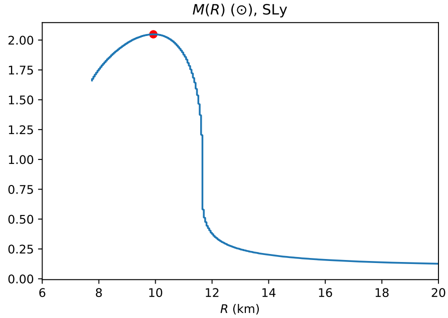
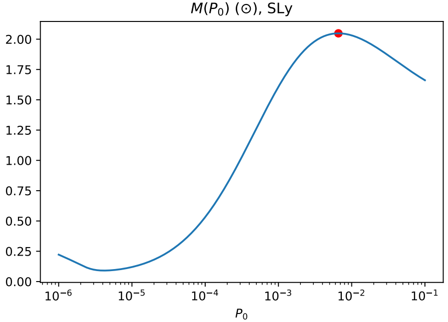



Figure: Example curves for EoS "SLy." $1 \odot = 1.989 \times 10^{30}$ kg (solar mass)

# Critical Values of $P$, $R$, and $M$



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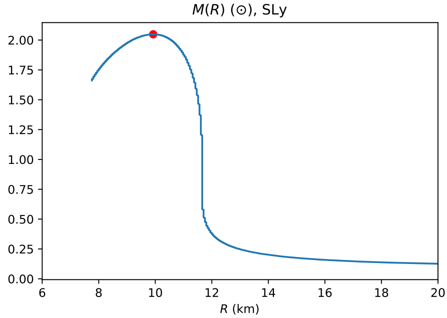


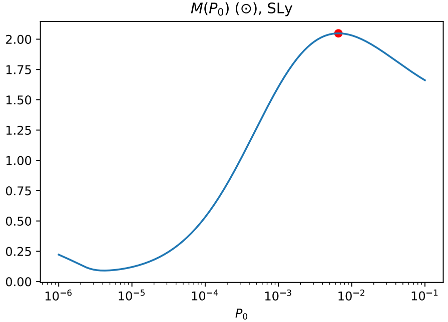


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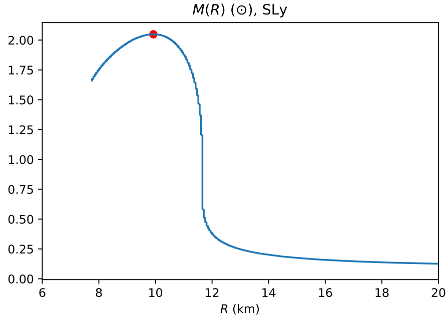


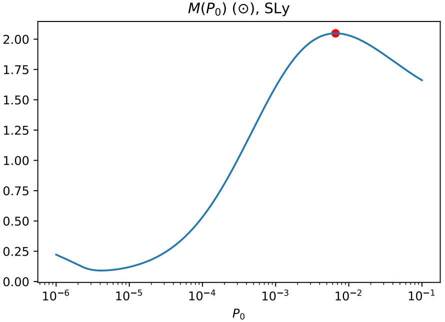


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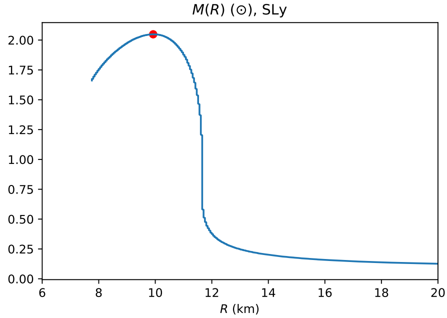


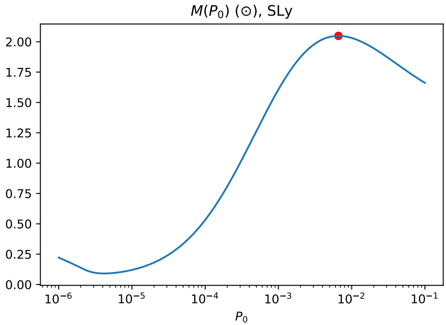


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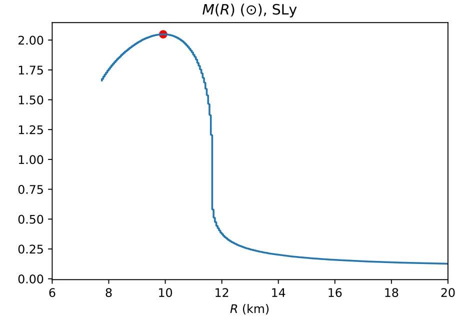


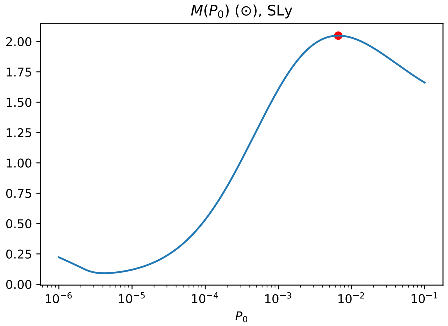


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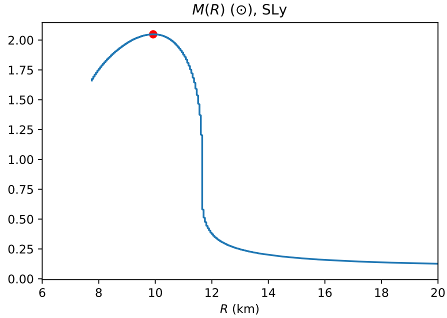


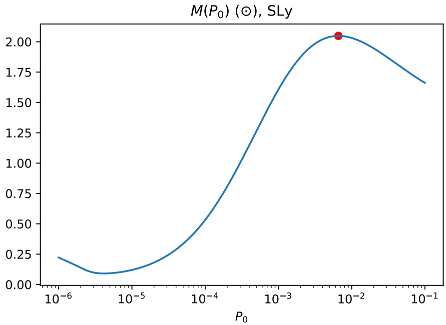


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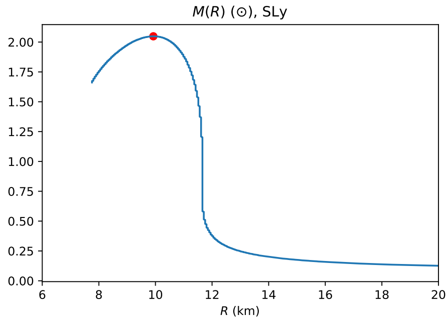


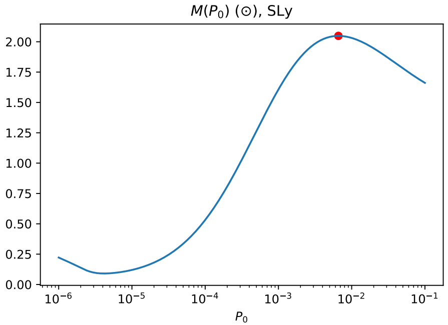


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- For SLy, $M_{\max} = 2.05\,\odot$, $R_{\max} = 9.93\,\text{km}$, and $P_{\text{crit}} = 6.59 \times 10^{-3}$.

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From $\mathcal{L}$, we can determine $\epsilon$ and $P$, the EoS we desire.

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$$\mathcal{L}_{\mathrm{RMF}} = \bar{\psi}[i\gamma_\mu \partial^\mu - g_v \gamma_0 V_0 - (M - g_s \phi_0)]\psi - \frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_\omega^2 V_0^2,$$

Determining $\phi_0$, $V_0$, $\epsilon$, and $P$: $\quad \varphi_\alpha \in \{\phi_0, V_0, \psi\}$

$$\partial_\nu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\nu \varphi_\alpha)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi_\alpha} = 0, \quad T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_\alpha)} \partial^\nu \varphi_\alpha - \mathcal{L} \eta^{\mu\nu}.$$

# RMF Simplifications

We introduce the *Relativistic Mean Field* (RMF) simplifications. We treat the meson fields as their average value:

$$\phi \to \langle \phi \rangle = \phi_0, \quad V_\mu \to \langle V_\mu \rangle = V_0,$$

where $\phi_0$ and $V_0$ are constants. This allows us to simplify $\mathcal{L}$ considerably:

$$\mathcal{L}_{\text{RMF}} = \bar{\psi}[i\gamma_\mu \partial^\mu - g_v \gamma_0 V_0 - (M - g_s \phi_0)]\psi - \frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_\omega^2 V_0^2,$$

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$$\epsilon = \left\langle T^{00} \right\rangle, \quad P = \left\langle T^{ii} \right\rangle$$

# Resulting Equations

From above, we obtain the following equations:

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$$\phi_0 = \frac{g_\phi}{m_\phi^2}\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{k^2 m^*}{\sqrt{k^2+m^{*2}}},$$

$$V_0 = \frac{g_v}{m_v^2}\frac{k_f^3}{3\pi^2},$$

$$\epsilon = \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{\pi^2}\int_0^{k_f} dk\,k^2\sqrt{k^2+m^{*2}},$$

$$P = -\frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{3}\left(\frac{1}{\pi^2}\int_0^{k_f} dk\,\frac{k^4}{\sqrt{k^2+m^{*2}}}\right).$$

where $m^* = (M - g_\phi \phi)$, the *reduced mass*.

# Producing the Equation of State

- Equations have one *free parameter*, $k_f$

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- Equations have one *free parameter*, $k_f$; loop through values
- During each iteration, calculate $\phi_0$ and $V_0$; use *rootfinding* for $\phi_0$
- Using those values, calculate $P$ and $\epsilon$ and store in a table

# $M(R)$ and $M(P_0)$ Curves for QHD-I

We use the tabulated values of $P$ and $\epsilon$ to create an *interpolated function*

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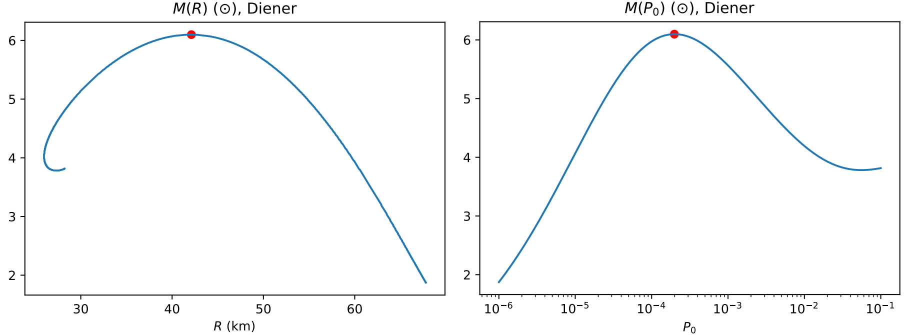


Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

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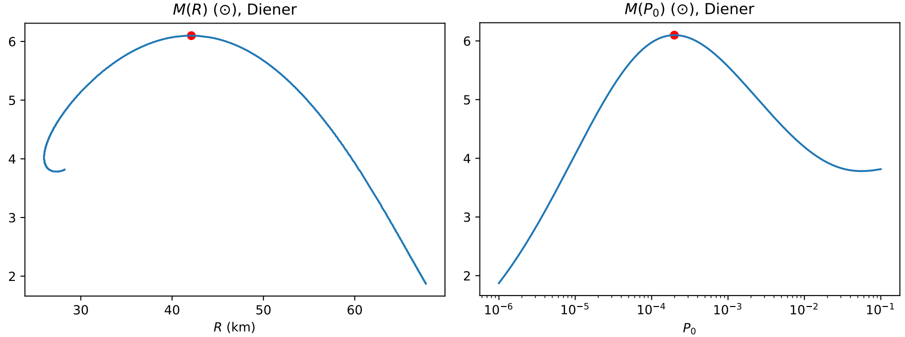


Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

These curves give

$$M_{\max} = 6.1\,\odot, \quad R_{\max} = 42.1\,\text{km}, \quad P_{\text{crit}} = 1.98 \times 10^{-4}.$$

# Conclusion

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- An equation of state is a relationship between energy density and pressure within a neutron star
- We use the TOV equations to predict the maximum mass and radius that a given EoS will produce
- We use the QHD-I parameter set and RMF simplifications to solve a system of equations and generate an equation of state

# Thanks

- Prof. Ben Kain

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- Holy Cross Physics Department

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- Prof. Ben Kain
- Holy Cross Physics Department
- My family