Analysis of Equations of State for Neutron Star Modeling and Simulation

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College of the Holy Cross

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  - Quantum Hadrodynamics and the QHD-I parameter set

Introduction

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- extract information about maximum max and radius allowed by the EoS

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- *Note*: works well with tabulated EoSs; an interpolating function is often used

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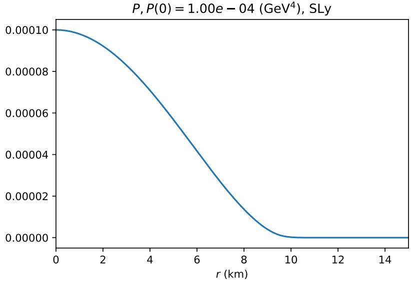

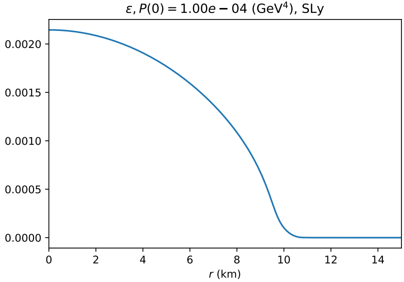

Figure: Example static solution for $P_0 = 10^{-4}\,\text{GeV}^4$ for an EoS called "SLy."

Static Solutions: $M(R)$ and $M(P_0)$ diagrams

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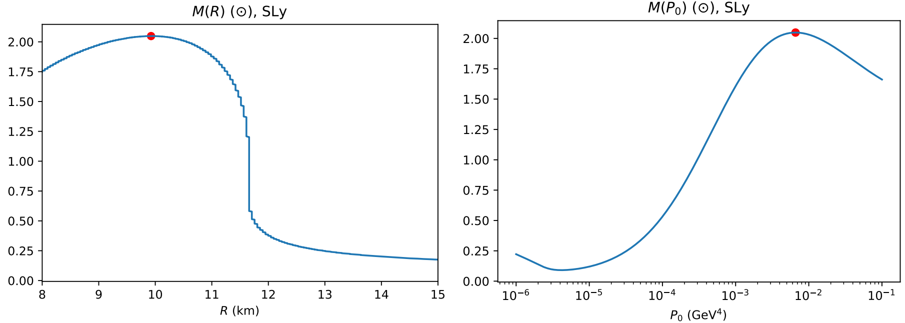
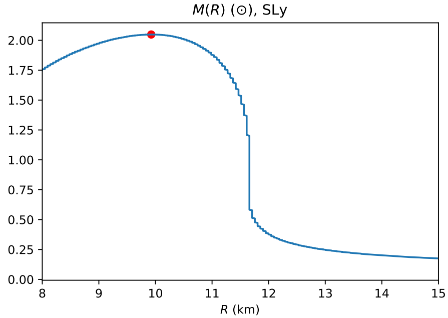
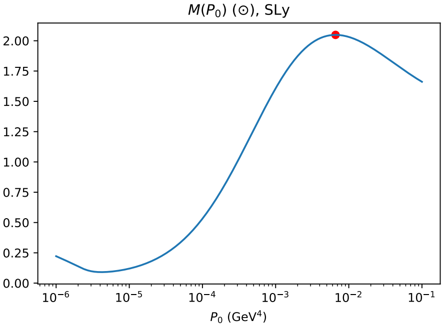


Figure: Example curves for EoS "SLy." $1\odot = 1.989 \times 10^{30}$ kg (solar mass)

Critical Values of P, R, and M





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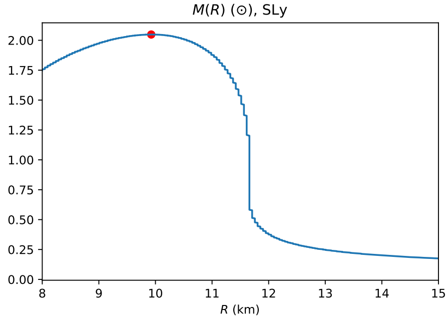

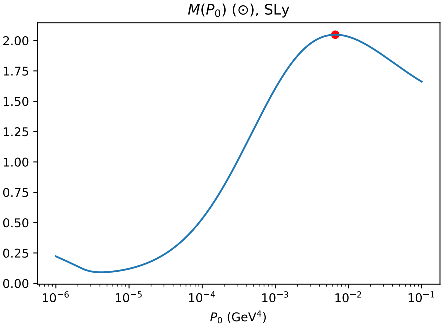

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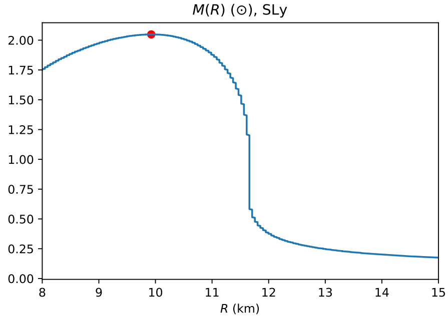

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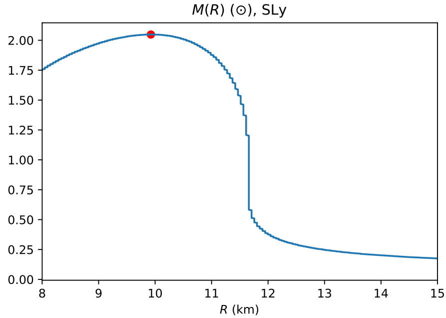

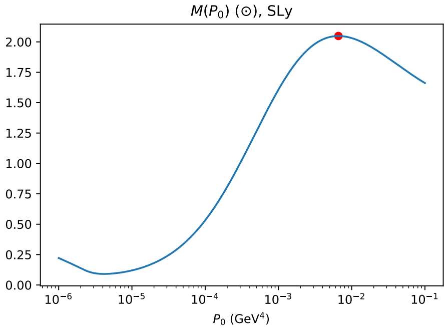

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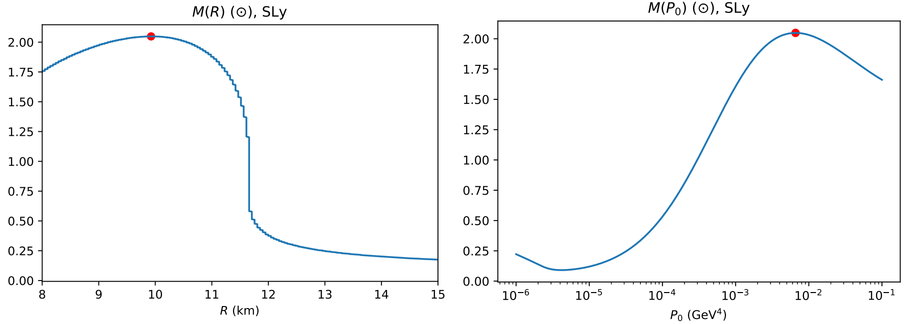

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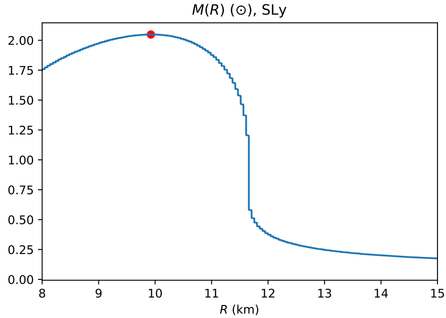

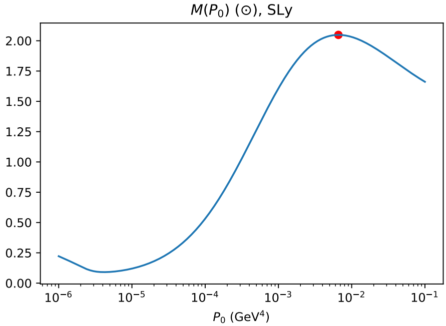

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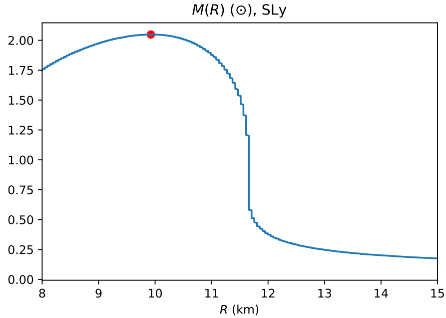

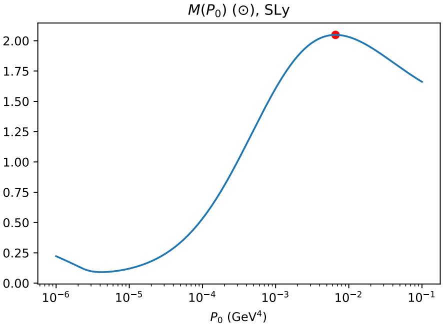

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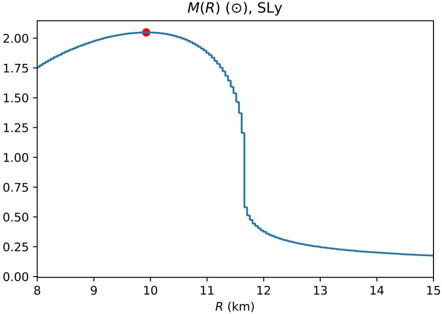

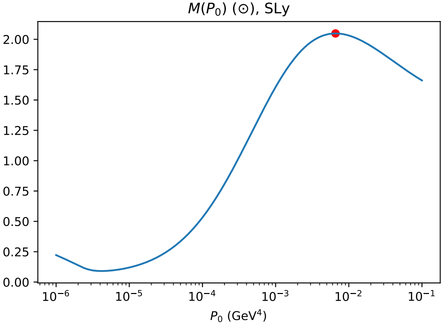

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- SLy: $M_{\max} = 2.05\,\odot$, $R_{\max} = 9.93\,\mathrm{km}$, and $P_{\mathrm{crit}} = 6.59 \times 10^{-3}\,\mathrm{GeV}^4$.

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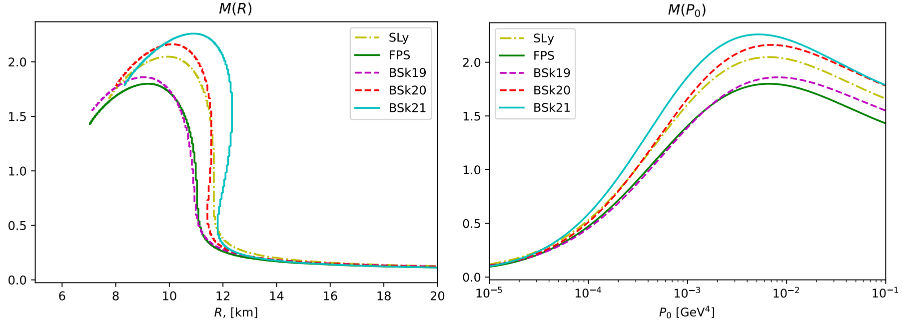

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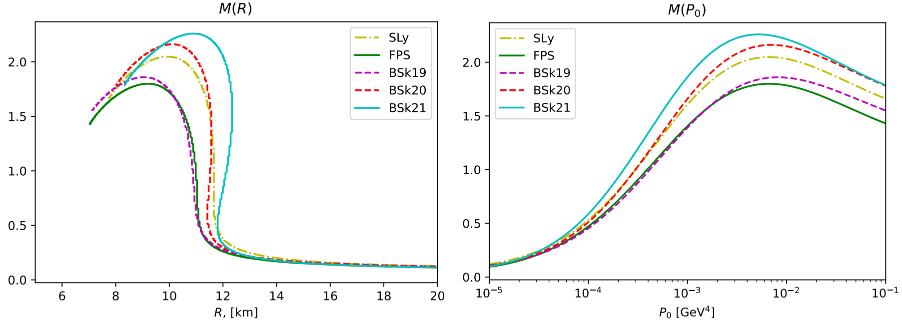

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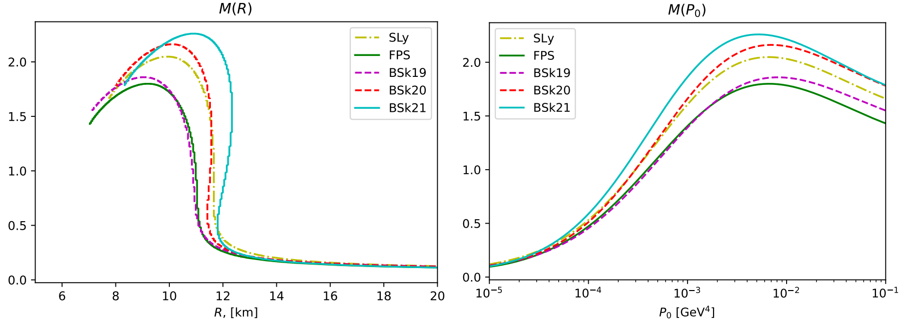

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Critical values of the extreme EoSs:

- FPS: $R_{\text{max}} = 9.20\,\text{km}$, $M_{\text{max}} = 1.80\,\odot$, $P_{\text{crit}} = 6.67 \times 10^{-3}\,\text{GeV}^4$.
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  - Extensive studies of realistic, analytical EoSs from [3, 5]; SLy family

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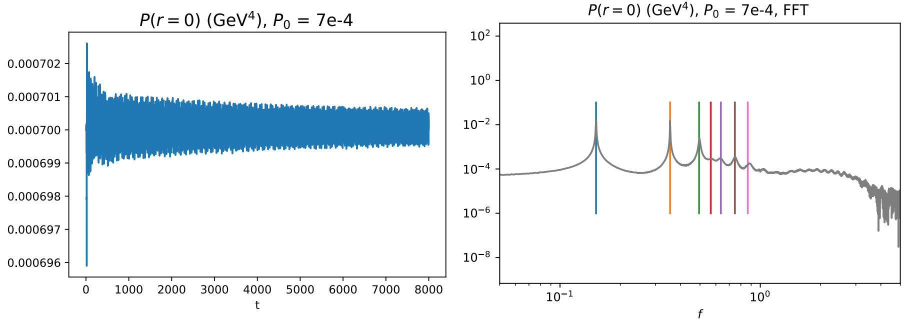

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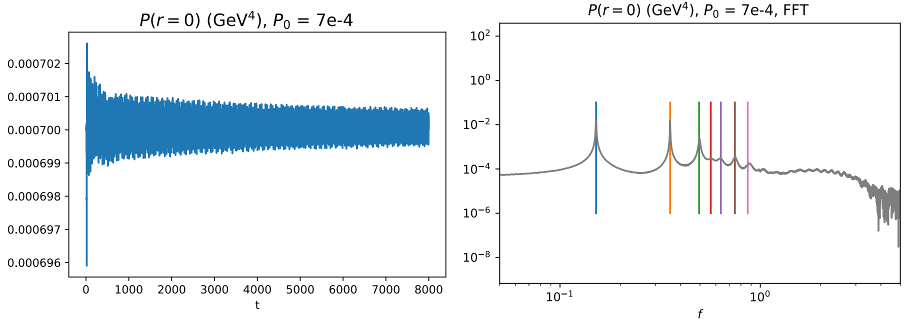

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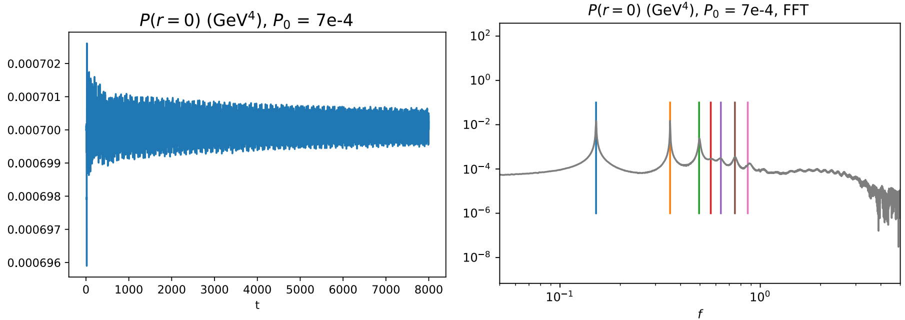

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- Interesting finding: there is a range of unstable static solutions that *migrate* to stable solutions

Derivation and Computation of an Equation of State

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- Considered quite complicated to solve; we introduce some simplifications in the QHD-I model

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where $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$, $\partial_\mu \equiv \partial/\partial x^\mu$. The fields:

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From $\mathcal{L}$, we can determine ϵ and P, the EoS we desire.

QHD-I: Derivation of Equations of Motion

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for $\varphi_\alpha \in \{\phi, V^\mu, \psi\}$, we obtain the equations of motion:

$$
\begin{aligned}
&\partial_\nu \partial^\nu \phi + m_s^2 \phi = g_s \bar{\psi}\psi, \\
&\partial_\mu V^{\mu\nu} + m_\omega^2 V^\nu = g_v \bar{\psi}\gamma^\nu \psi, \\
&[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (M - g_s \phi)]\psi = 0,
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$$\phi \to \langle \phi \rangle = \phi_0, \quad V_\mu \to \langle V_\mu \rangle = V_0, \quad \bar{\psi}\psi \to \left\langle \bar{\psi}\psi \right\rangle, \quad \bar{\psi}\gamma^\mu\psi \to \left\langle \bar{\psi}\gamma^0\psi \right\rangle,$$

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$$\mathcal{L}_{\text{RMF}} = \bar{\psi}[i\gamma_\mu\partial^\mu - g_v\gamma_0 V_0 - (M - g_s\phi_0)]\psi - \frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}m_\omega^2 V_0^2,$$

Applying the same simplifications to the equations of motions gives:

$$\begin{aligned}
&m_s^2\phi_0^2 = g_s \left\langle \bar{\psi}\psi \right\rangle \\
&m_\omega^2 V_0 = g_v \left\langle \bar{\psi}\gamma^0\psi \right\rangle \\
&[i\gamma_\mu\partial^\mu - g_v\gamma_0 V_0 - (M - g_s\phi_0)]\psi = 0
\end{aligned}$$

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where $T^{\mu\nu}$ is the energy momentum tensor, given by

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Using $\mathcal{L}_{\text{RMF}}$, we obtain

$$T^{\mu\nu}_{\text{RMF}} = i\bar{\psi}\gamma^\mu \partial^\nu \psi - \eta^{\mu\nu}\left(-\frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_\omega^2 V_0^2\right).$$

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The above expectation values are non-trivial and are derived in [1].

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$$\phi_0 = \frac{g_\phi}{m_\phi^2}\frac{1}{\pi^2}\int_0^{k_f} dk\, \frac{(M - g_\phi\phi_0)k^2}{\sqrt{k^2 + (M - g_\phi\phi_0)}},$$

$$V_0 = \frac{g_v}{m_v^2}\frac{k_f^3}{3\pi^2},$$

$$\epsilon = \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_v^2 V_0^2 + \frac{1}{\pi^2}\int_0^{k_f} dk\, k^2\sqrt{k^2 + m^{*2}},$$

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where $m^* = (M - g_\phi\phi)$, the *reduced mass*. k_f, the Fermi wavenumber, is a free parameter.

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We loop through k_f values until we have a large range of P values

$$P \in [10^{-20}, 10^{-1}](\text{GeV}^4).$$

$M(R)$ and $M(P_0)$ Curves for QHD-I

We use the tabulated values of P and ϵ to solve the TOV equations:

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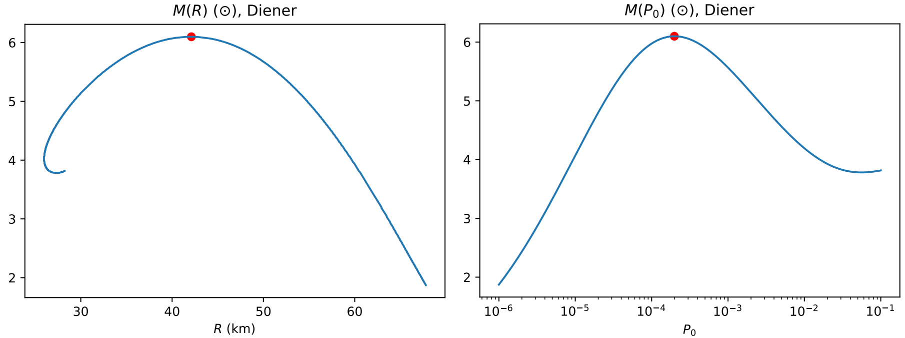

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

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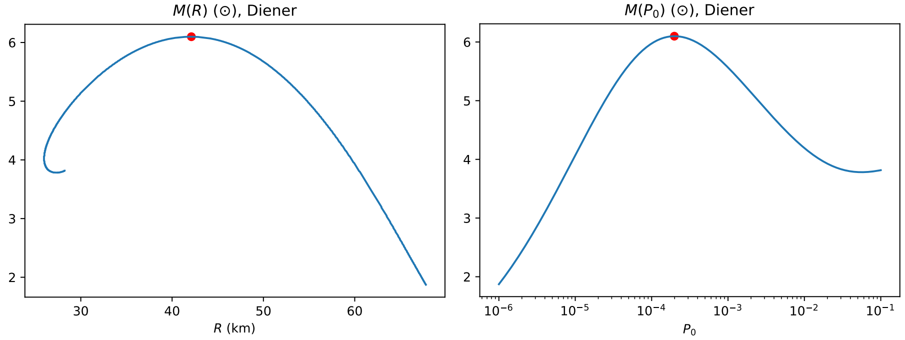

Figure: $M(R)$ and $M(P_0)$ curves for QHD-I EoS.

These curves give

$$M_{\max} = 6.1\,\odot, \quad R_{\max} = 42.1\,\text{km}, \quad P_{\text{crit}} = 1.98 \times 10^{-4}\,\text{GeV}^4.$$

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  - Static solutions are used as initial data
  - We can predict radial oscillation frequencies of neutron stars, which could soon be measurable
- We use the QHD-I parameter set and RMF simplifications to solve a system of equations and generate an equation of state

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