COMP0246 Modelling and Motopn Planning
Lab 1(a) questions: Linear Algebra

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Linear Algebra

1. a. Given an arbitrary 3D rotation matrix,

$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}$$

Prove that $||r_i|| \leq 1$ where $i = 1, 2, ..., 9$.

b. For any rotation matrix $\mathbf{R}$, prove that $\mathbf{R}_{k,\theta} = \mathbf{R}_{-k,-\theta}$, where k is the unit vector defined axis of rotation and θ is the angle of rotation.

c. Given two arbitrary Cartesian coordinate frames a and b, what does each row in a rotation matrix ${}^{a}\mathbf{R}_{b}$ represent?

2. a. Provide a matricial example, i.e. a succession of 3 matrices along the 3 different axes, of gimbal lock for the Y-Z-Y (proper Euler, extrinsic) and x-y-z (Tait-Bryan, intrinsic) rotations. Why do we need to avoid gimbal lock when controlling robotic arms? How is this achieved?

b. Show how to pass from Quaternion representation to rotation matrix representation. (You will need to provide all steps, not just the formula).

c. What rotation representation would you suggest to use in the following cases:

- Nano-robot with very limited memory storage
- Nano-robot with very limited computational power
- Iphone navigation system
- Robotic arm wth 6 DOF

3. a. Prove that a rotation quaternion $\mathbf{q}$ and $-\mathbf{q}$ are equivalent.

b. When do two arbitrary rotation matrices $\mathbf{R}_a$ and $\mathbf{R}_b$ become commutative?