MaxFlow

Formal Statement of the Max Flow Algorithm

Step 0: (Initialization)

- **Give each edge a feasible flow, ensure that flow is conserved at each node other than the *s* and *t* (This may be done by assigning a zero flow to each edge)**
- **Make a list of edges/edge capacities for scanning**

Step 1: Label node *s* with the label (*, ∞), and ensure that no other node is labeled

Step 2:

- **Scan through the edges sequentially FROM THE TOP OF YOUR SCAN LIST until one edge (*i, j*) is found for which:**
  - **A) node *i* is labeled and node *j* is not labeled and $f(i, j) < u(i, j)$ (forward edge) or**
  - **B) node *j* is labeled and node *i* is not labeled and $f(i, j) > 0$ (reverse edge)**
  - **If no such arc exists in the entire scan list go to Step 5, otherwise go to Step 3**

Step 3:

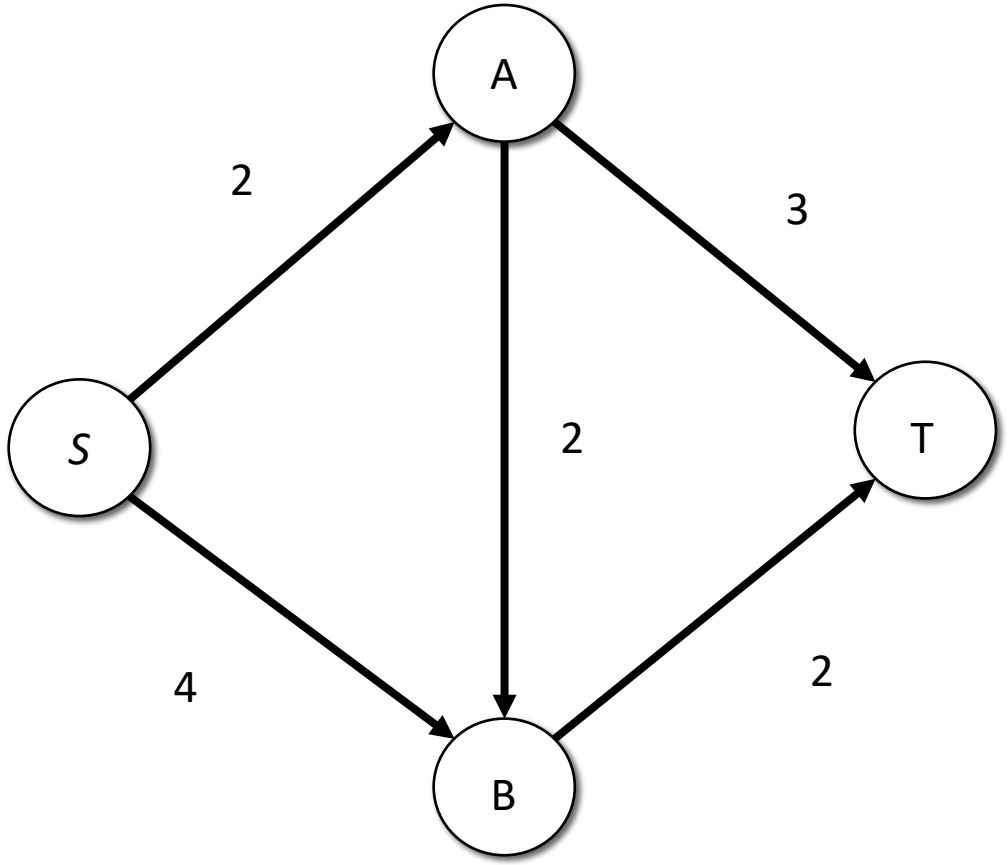
- **if A) in Step 2 was true, label node *j* with the label (a_j, b_j) where $a_j = i$, $b_j = min(b_i, u(i, j) - f(i, j))$**
- **if B) in Step 2 was true, label node *i* with the label (a_i, b_i) where $a_i = -j$, $b_i = min(b_j, f(i, j))$**
- **If node *t* is labeled you found a Flow Augmenting Chain go to step 4, otherwise go to step 2**

Step 4: (A Flow Augmenting Chain has been found)

- **Increase the flow on each of the edges of the flow-augmenting chain by the amount b_t.**
- **If a node is labeled (*a, b*), then increase the flow on the edge going to it by b_t**
- **If a node is labeled (*-a, b*) then decrease the flow on the edge going to it by b_t**
- **Examine all labels in the chain, increasing or decreasing flow, always changing by b_t.**
- **Go to Step 1**

Step 5: The optimal flow has been found. Stop.

Example



Scan order

C(S,A) = 2

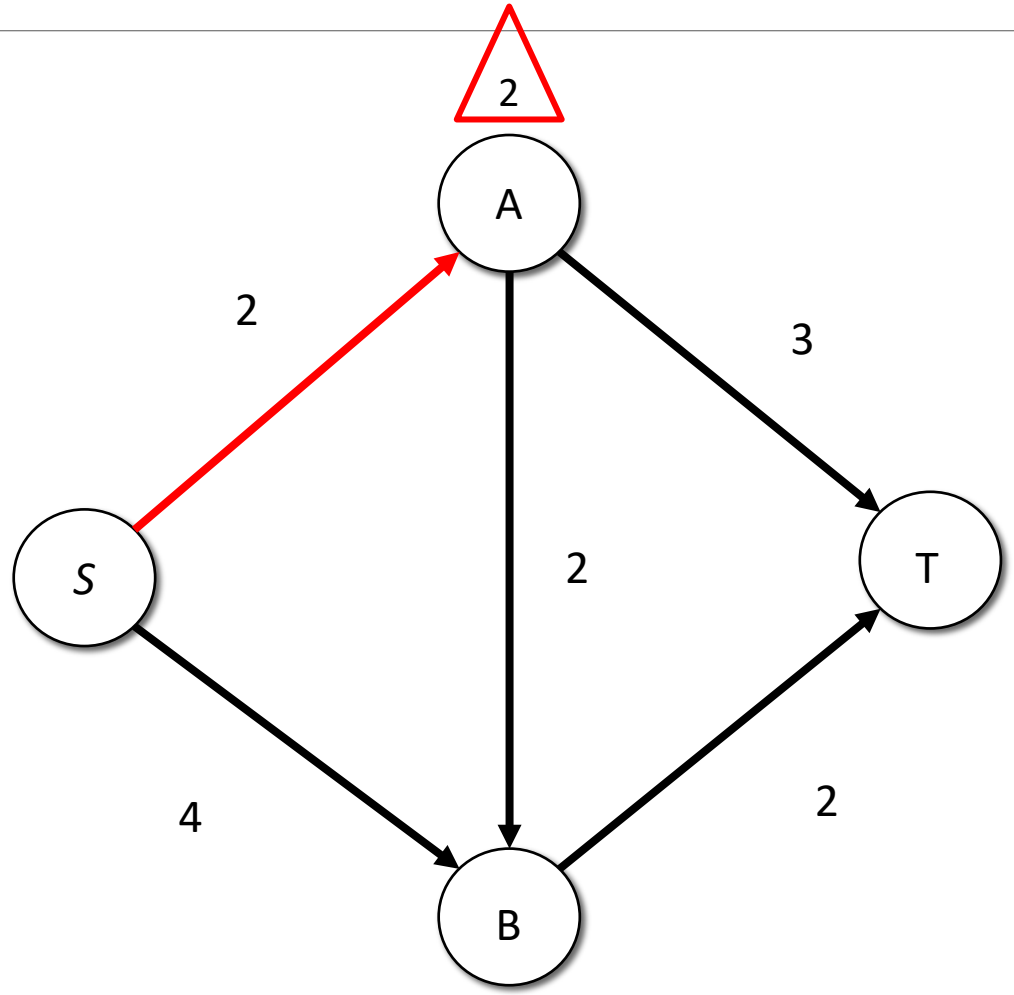
C(A,T) = 3

C(S,B) = 4

C(B,T) = 2

C(A,B) = 2

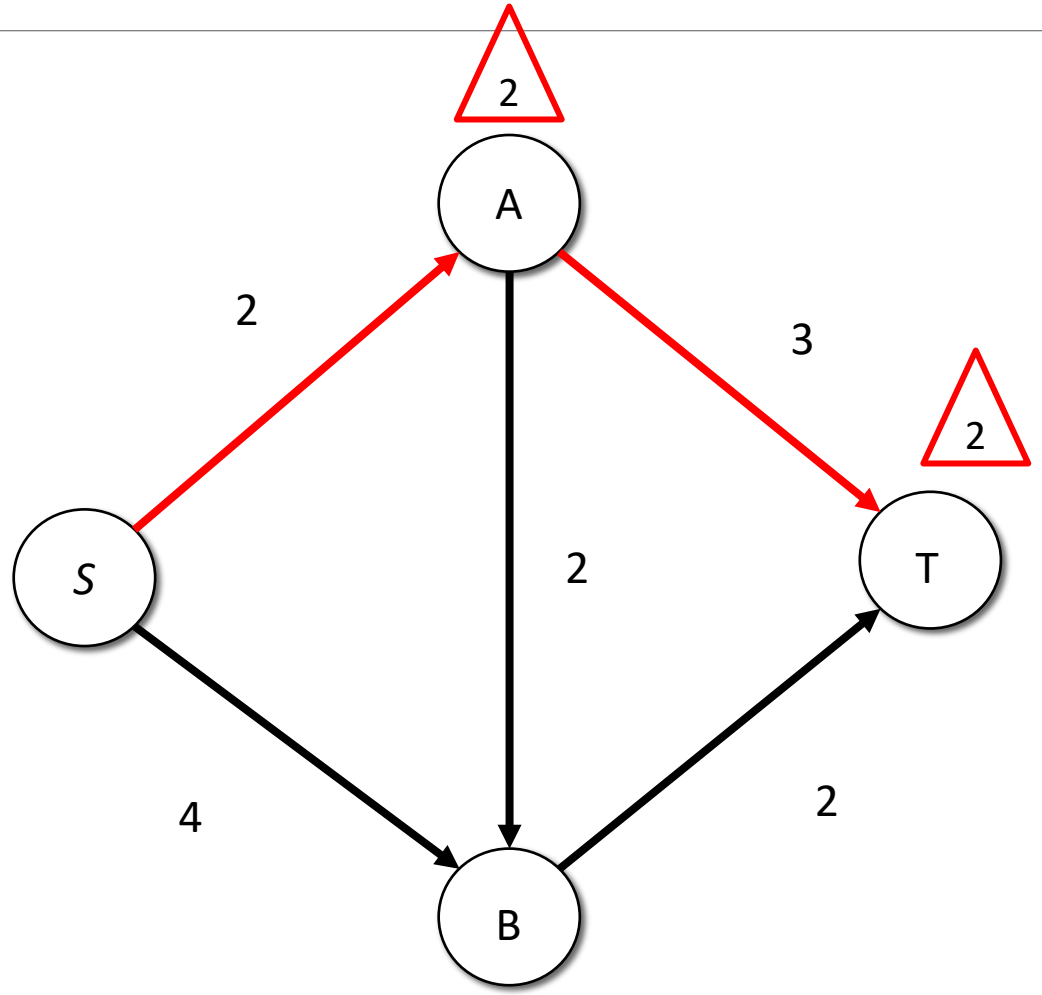
Example



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

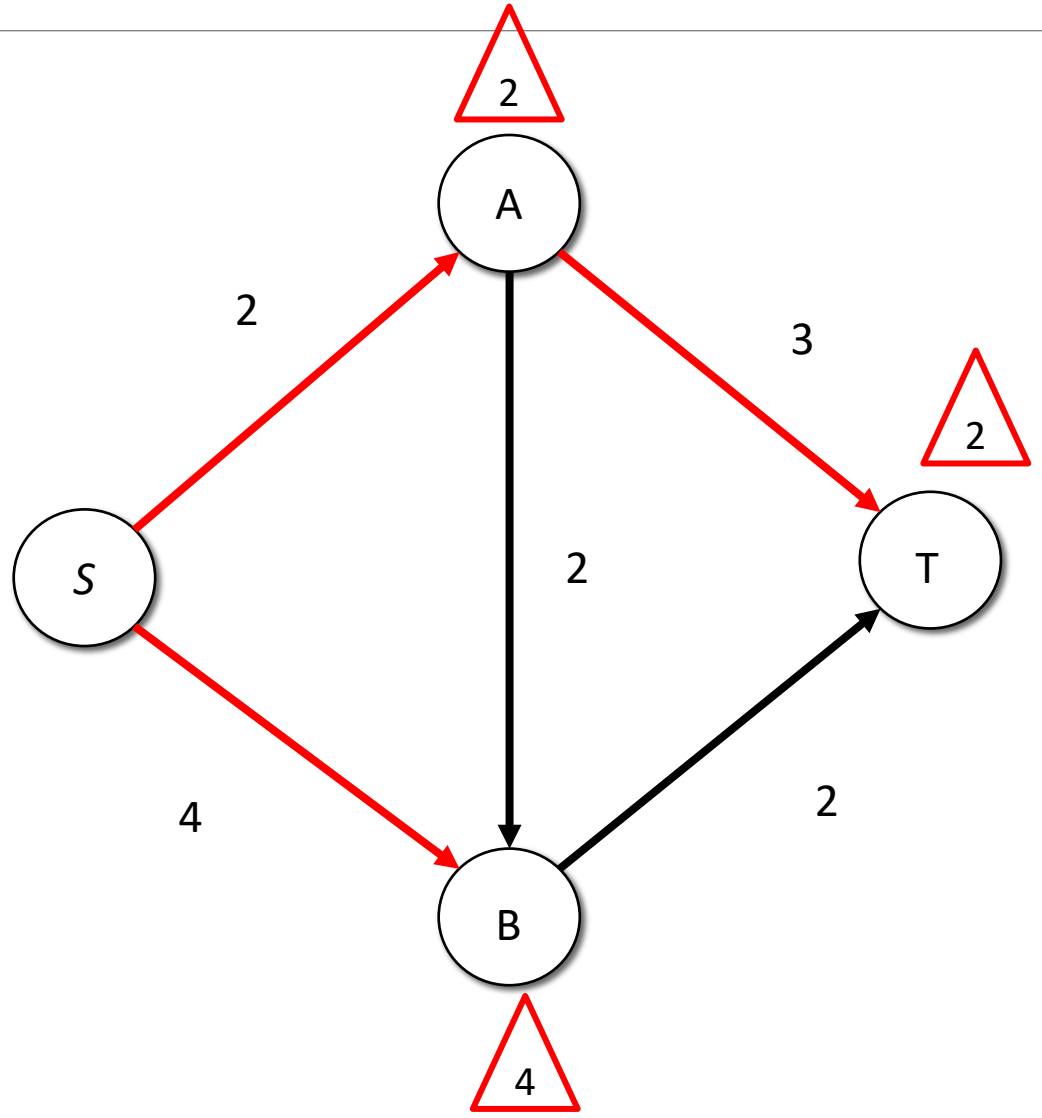
Example



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

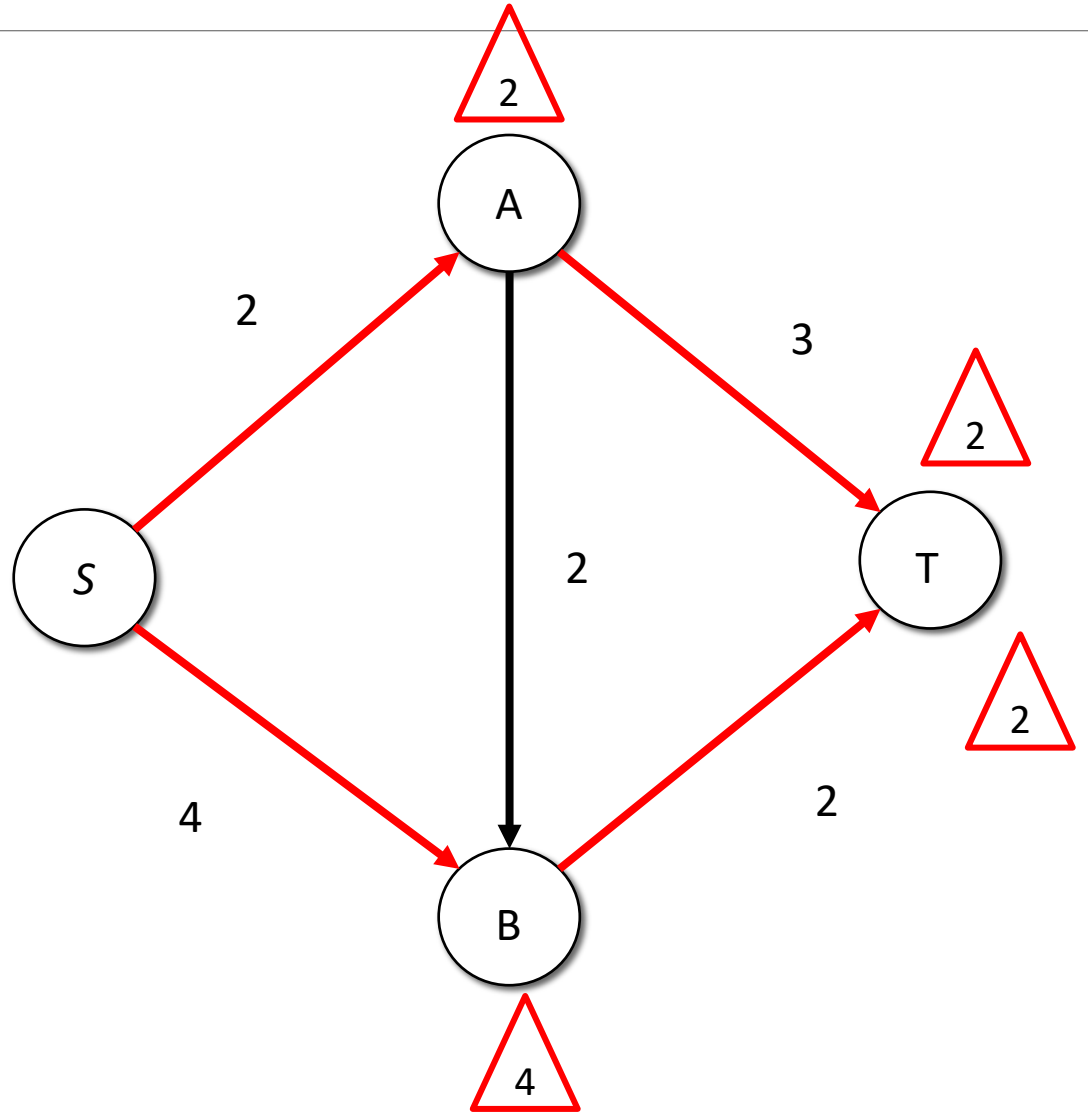
Example



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

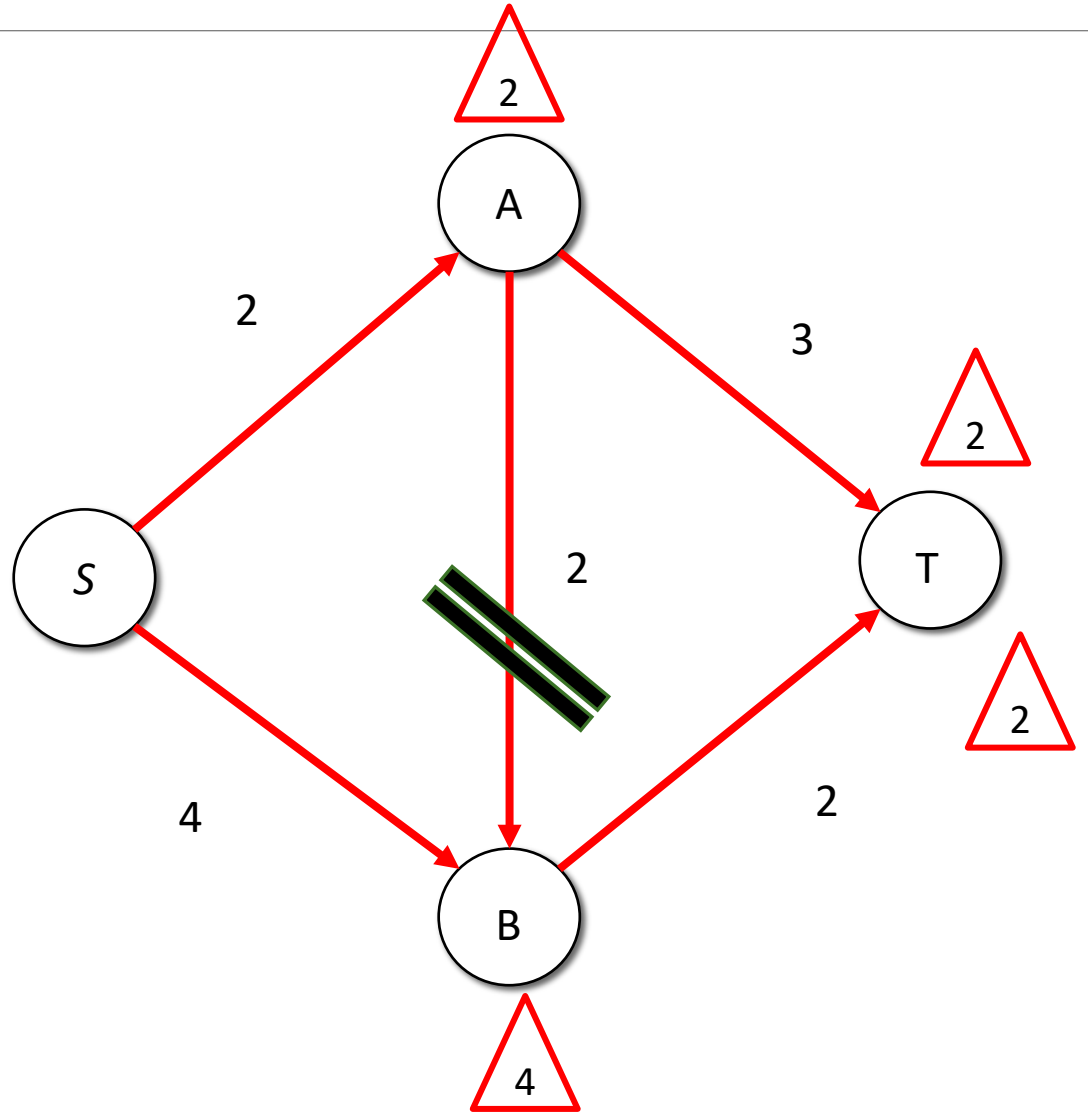
Example



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

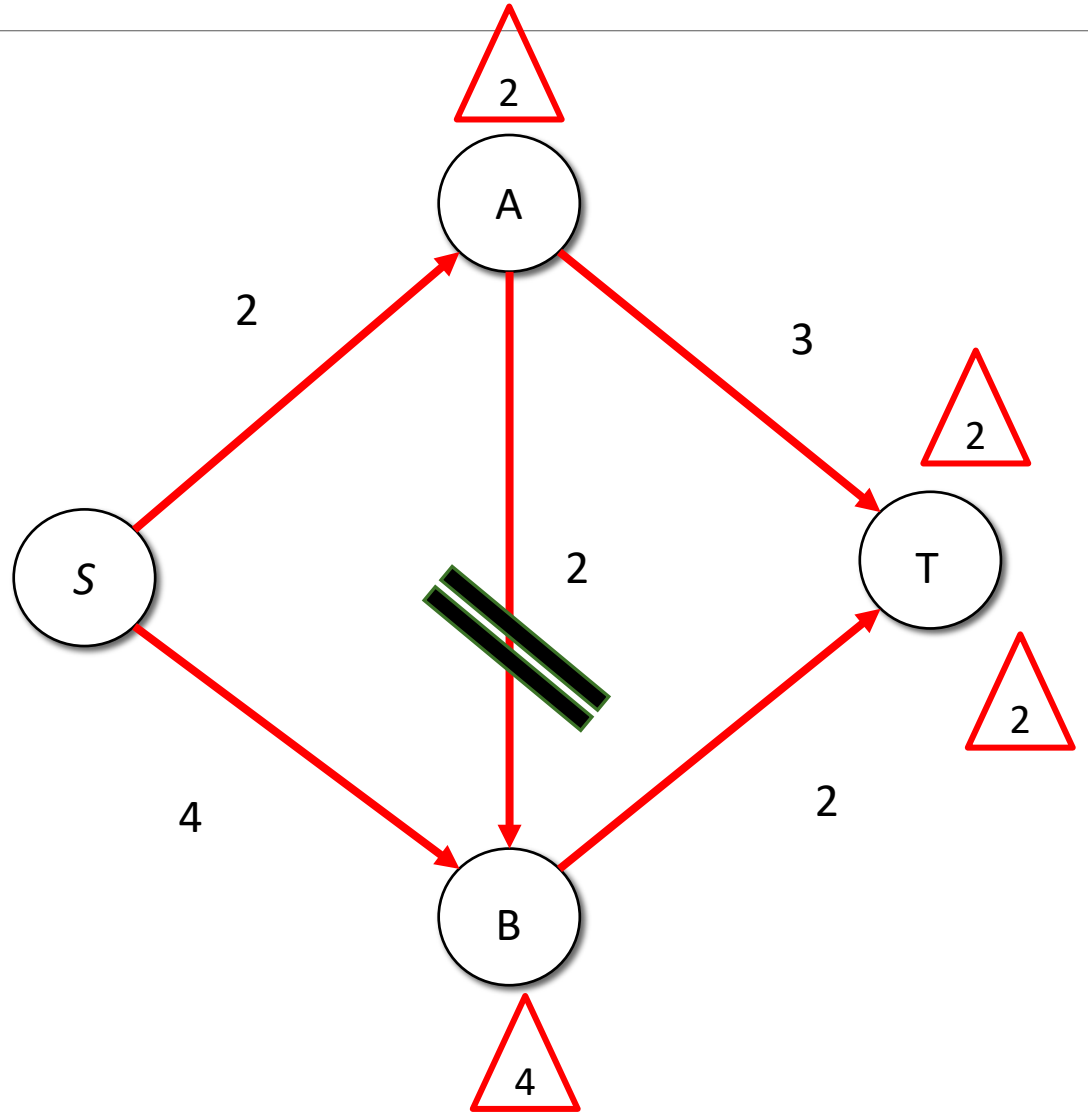
Example



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

Example

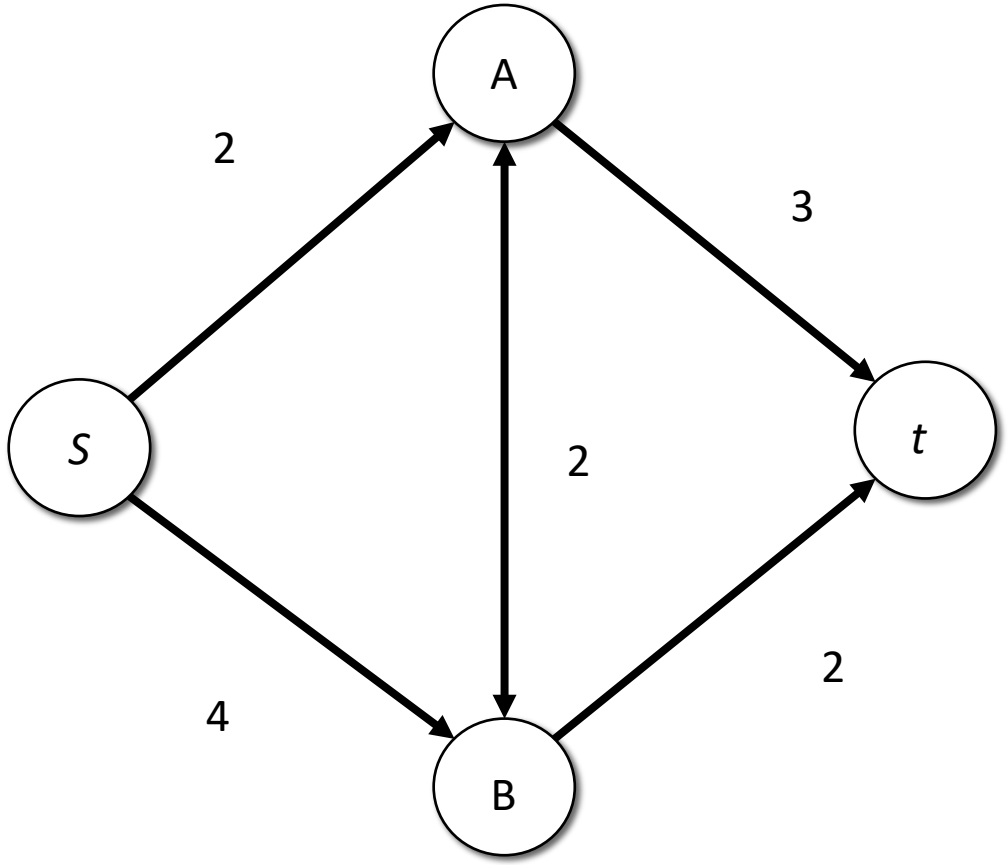


Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

Maxflow = 4

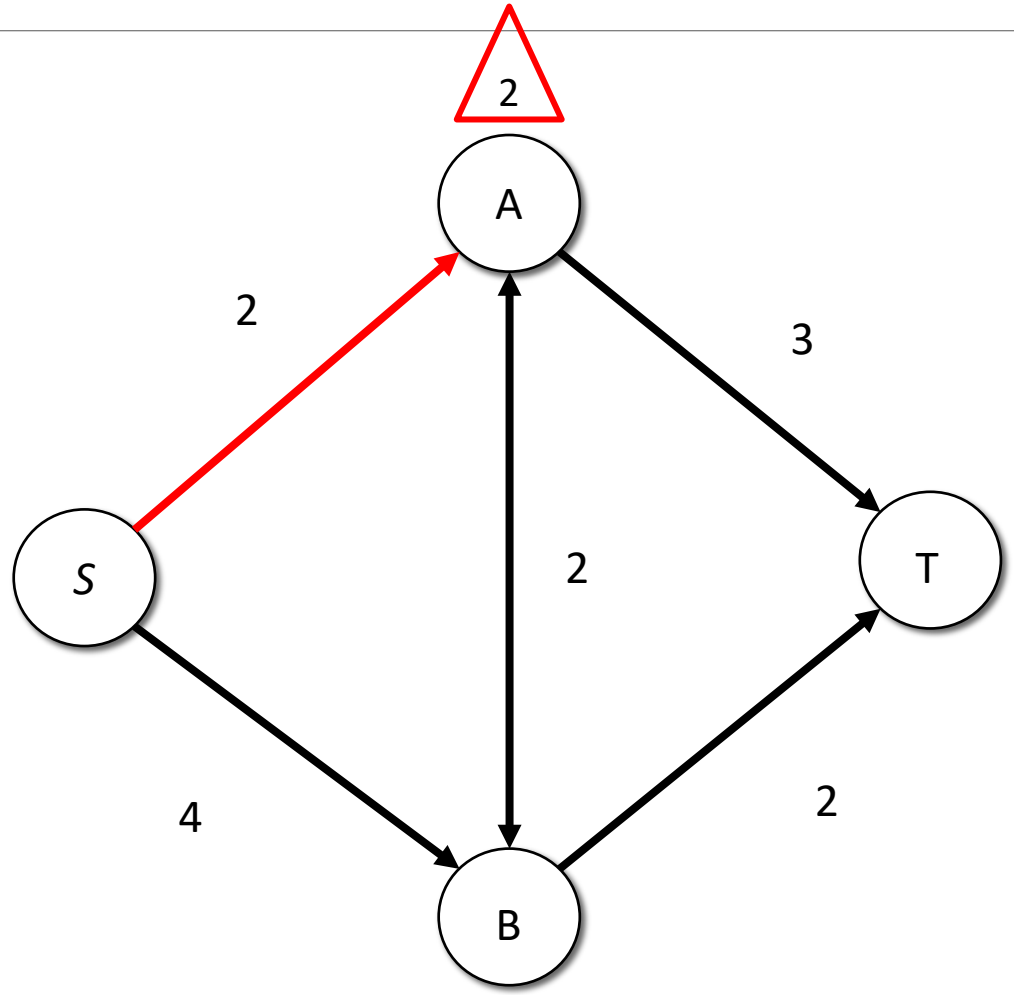
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,B) = 2
C(B,T) = 2
C(A,T) = 3
C(S,B) = 4

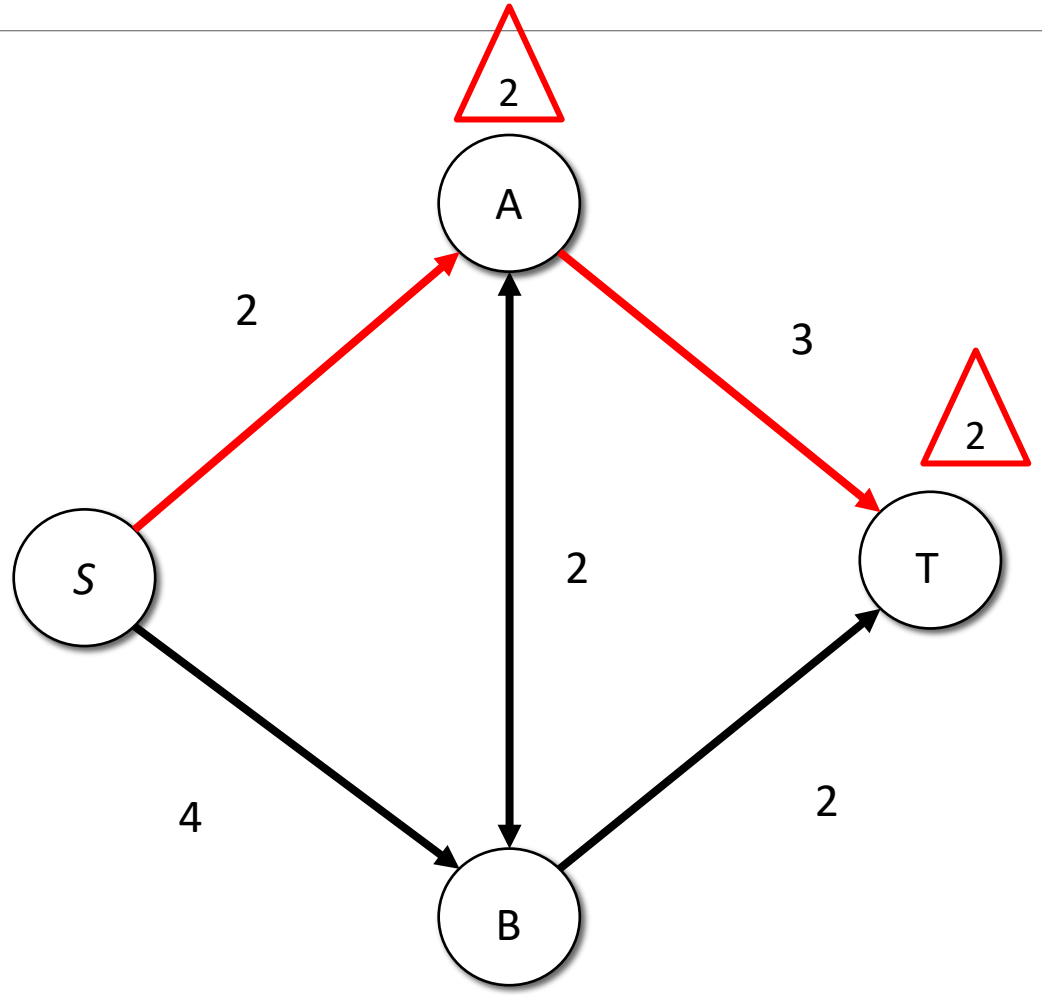
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

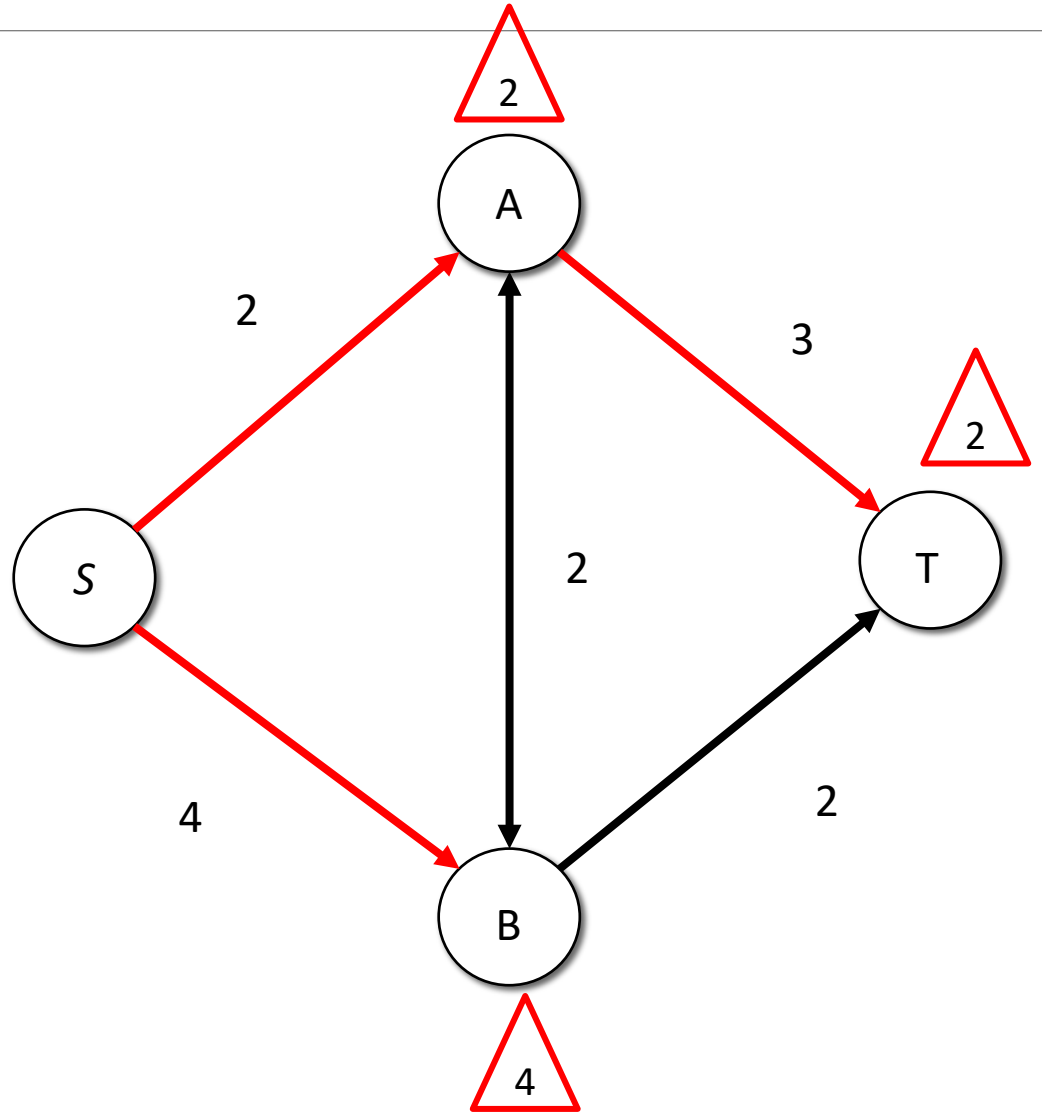
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

Example - with reverse flow



Scan order

C(S,A) = 2

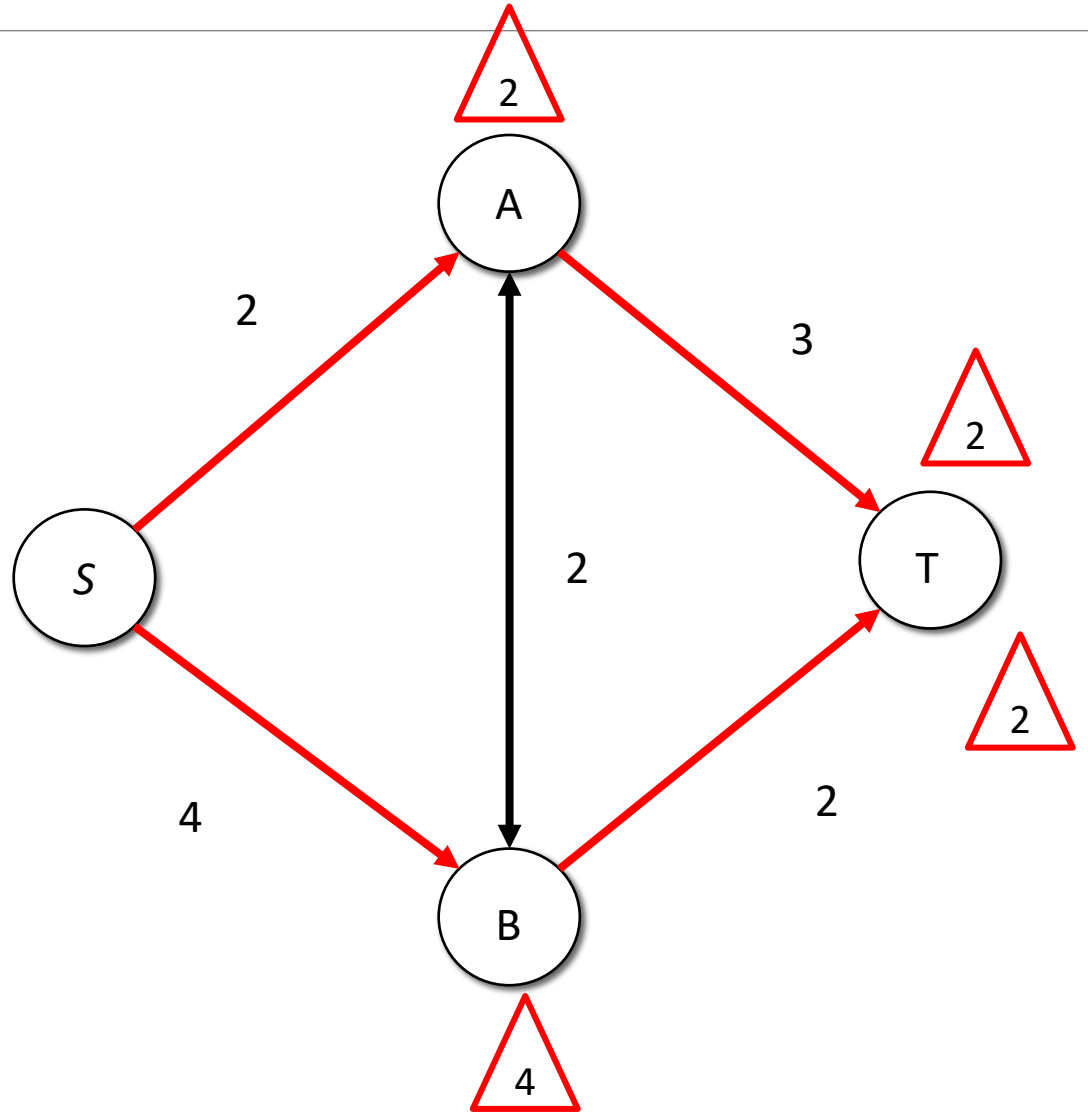
C(A,T) = 3

C(S,B) = 4

C(B,T) = 2

C(A,B) = 2

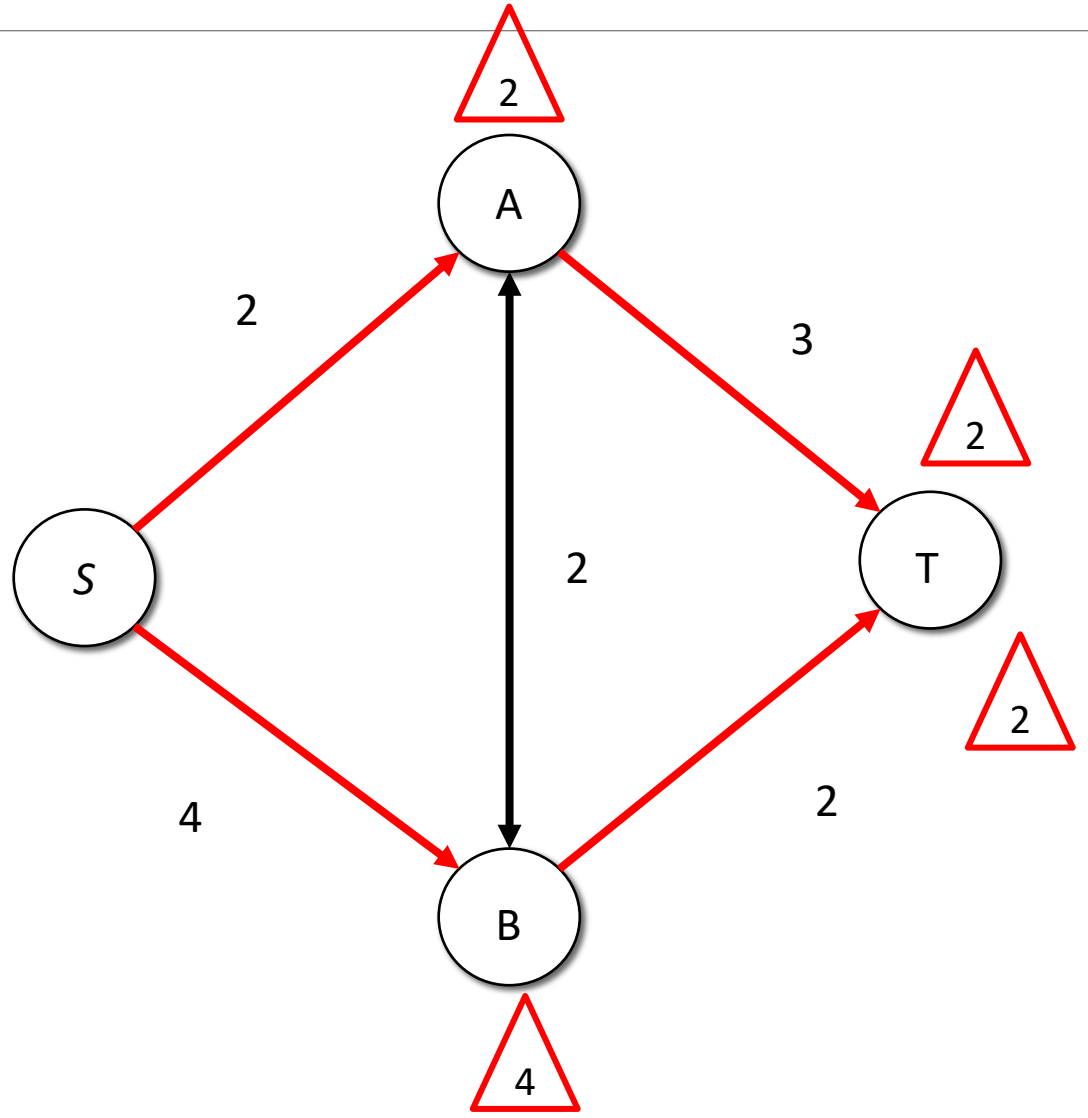
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

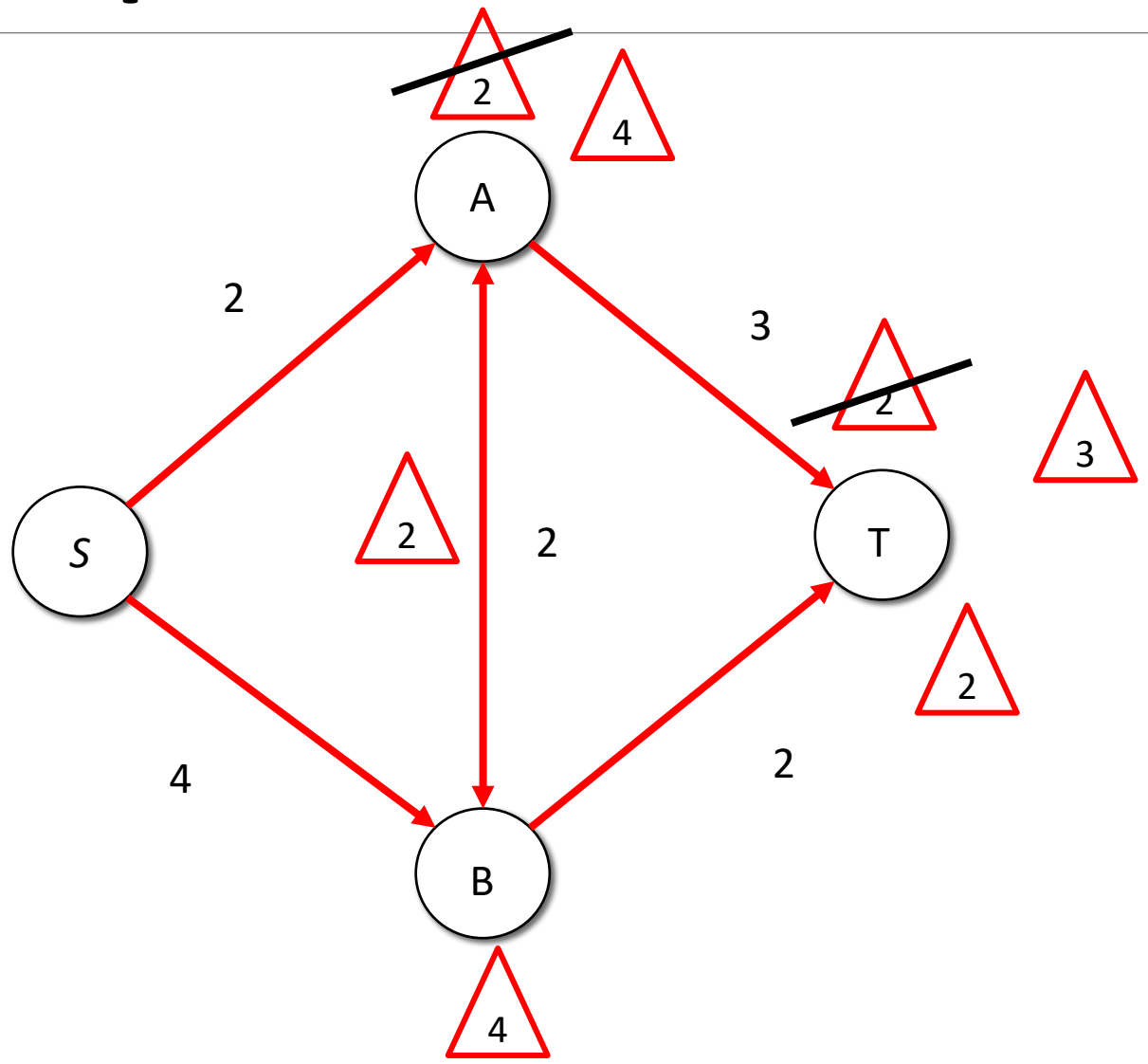
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

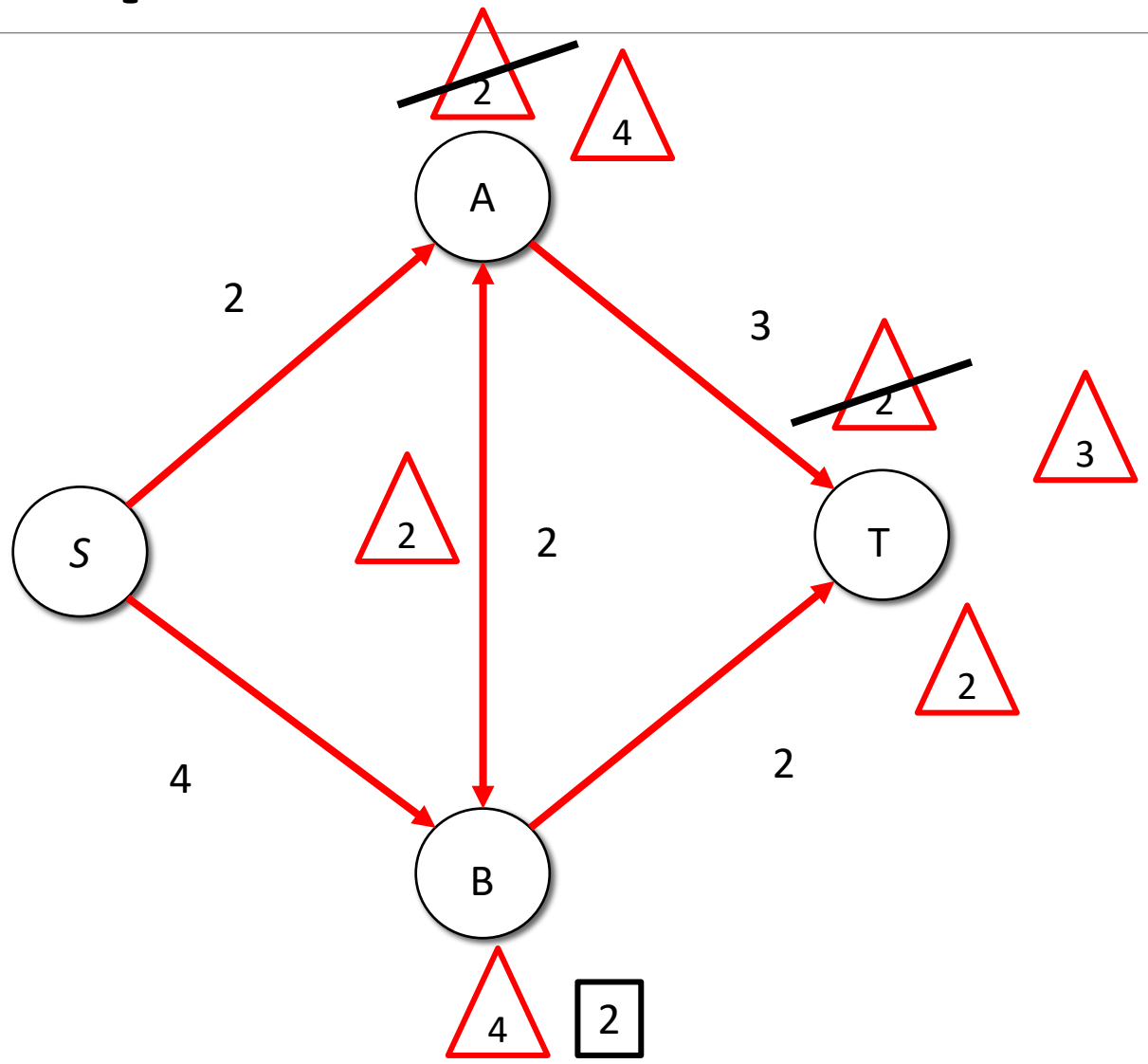
Example - with reverse flow



Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

Example - with reverse flow

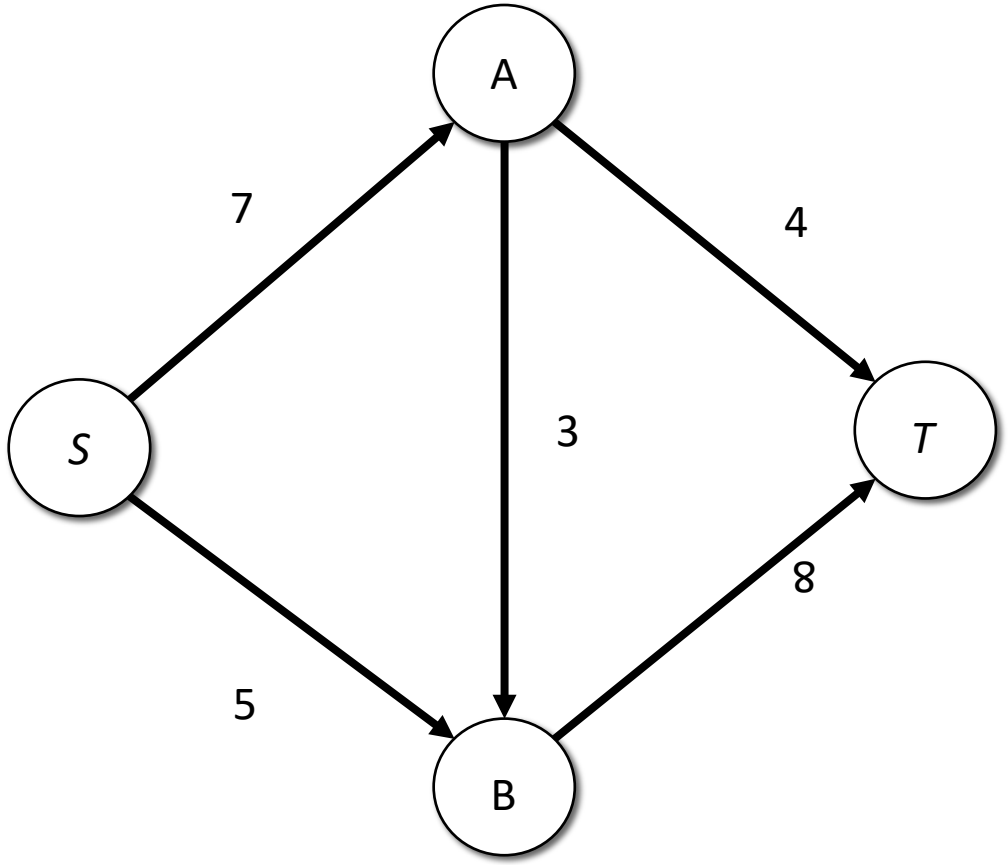


Scan order

C(S,A) = 2
C(A,T) = 3
C(S,B) = 4
C(B,T) = 2
C(A,B) = 2

Maxflow = 5

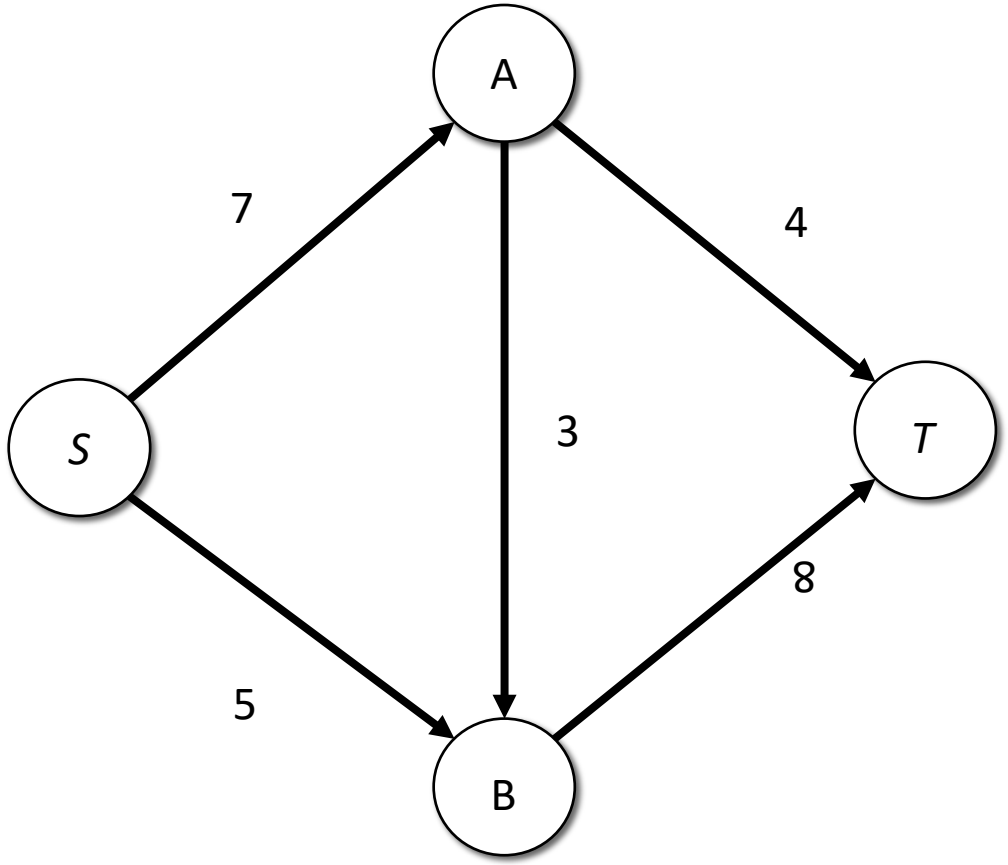
Example



Scan order

C(S,A) = 7
C(A,T) = 4
C(S,B) = 5
C(B,T) = 8
C(A,B) = 4

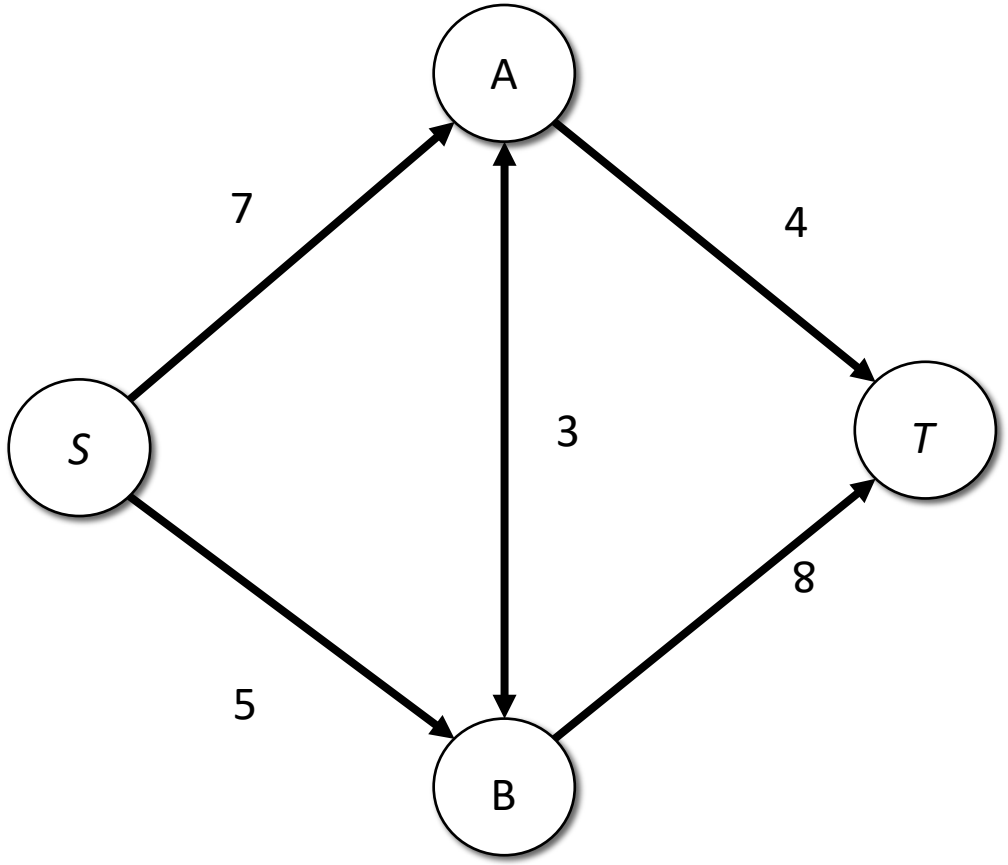
Example



Scan order

C(S,A) = 7
C(A,T) = 4
C(S,B) = 5
C(B,T) = 8
C(A,B) = 4

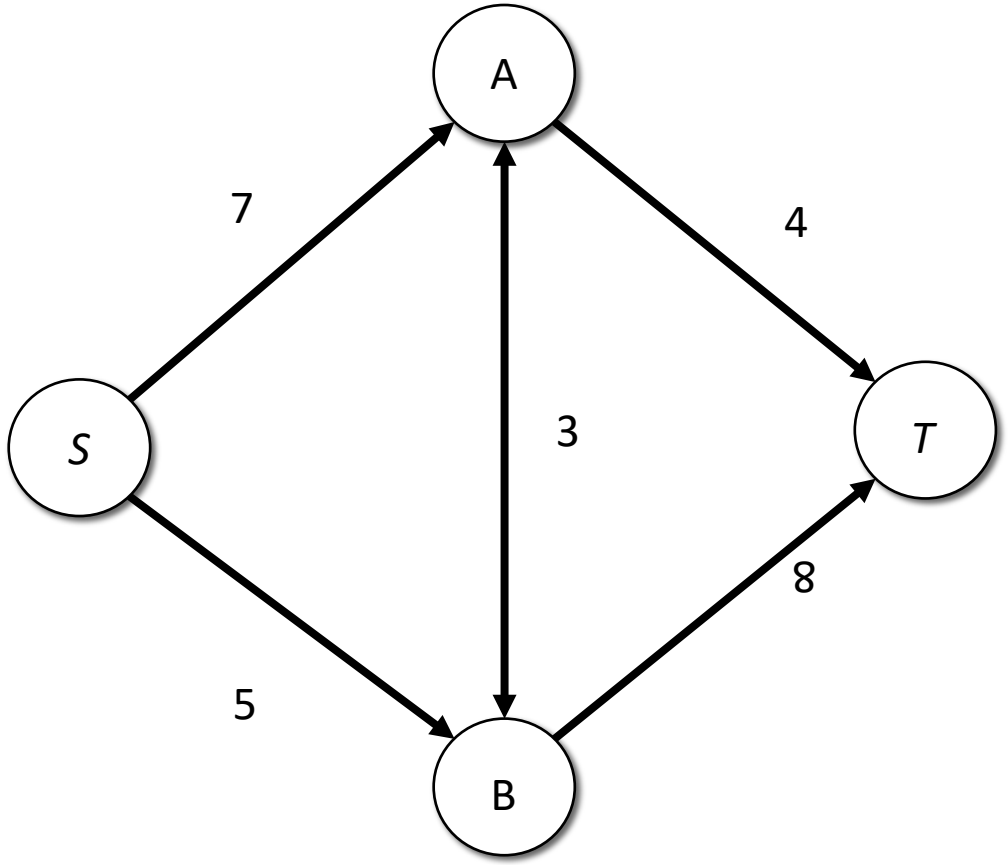
Example - with reverse flow



Scan order

C(S,A) = 7
C(A,T) = 4
C(S,B) = 5
C(B,T) = 8
C(A,B) = 4

Example - with reverse flow



Scan order

C(S,A) = 7
C(A,T) = 4
C(S,B) = 5
C(B,T) = 8
C(A,B) = 4

Maxflow = 8