Shape Recognition and Classification
Using Fourier Descriptors

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Abstract

This paper shows a method for shape recognition and classification, which is useful for searching a database of shapes. The method explores three suitable properties of shape analysis to develop Shape Signatures and Fourier descriptors. Fourier descriptors of two similar shapes are compared, to give a reasonable measure of the effectiveness of the Shape Signatures.

I. **Introduction**

Shape recognition has been a growing field of study since digital computers brought about its inception in the 20th century. However, the general technique that accomplishes this task was developed many years before such modern machinery became available. In 1822, French mathematician Joseph Fourier developed a method of mapping a function to a set of coefficients of a sinusoidal function. This allowed naturally occurring models to be optimally represented mathematically, while preserving a precise approximation to the original data.

Fourier's transform also preserves an important axiom of inequality: trichotomy. One quantified representation of a shape will either be less than, equal to, or greater than the quantified representation of another shape. This can be extended to imply that given a shape, one can find another shape most like the shape provided. In modern terminology, this means that a database and search engine of shapes can be developed.

In order to construct this application, a reliable method must be found which accurately maps the shape of an object to a quantifiable vector, while minimizing both computation time and the effects of translation, scaling, and rotation. Several techniques have been developed throughout the history of scientific computing. This paper will investigate three elementary methods, and analyze the reliability of the methods.

II. Derivation of Shape Signatures

In the study of linear algebra, a *basis* is a spanning set; every vector of a given vector space can be represented by some linear combination of the elements of the basis. The study of shape classification can be constructed using an analogy to the study of linear algebra.

Let the "vector space" be the universe of two-dimensional objects. The "basis" of this set of objects is referred to as the set of s*hape signatures*. An object is represented by a single shape signature and a combination of translation, rotation, and uniform scaling. Thus, two objects with a different location, rotation, and size may have some linear multiple of the same shape signature, if the objects have the same base shape.

A shape signature is simply a function which maps a finite subset of natural numbers to some corresponding property of the object being described. There are many methods of defining a shape signature for a set of objects. This paper will explore three shape signatures: *complex coordinates*, *centroid distance*, and *curvature*.

1. *Complex Coordinate*

The simplest shape signature, *complex coordinate*, takes the 2-dimensional xy coordinates of the object and maps it to a single complex number:

$$z(n) = x(n) + iy(n)$$

This shape signature does not account for translation, scaling, or rotation, so this signature is limited, but is very effective in describing the minute details of the image. Because it does not account for translation, scaling, and rotation, this shape signature is often used in the field of digital signal processing, which includes audio and video filtering.

2. *Centroid Distance*

The next shape signature, *centroid distance*, is invariant to translation, but still does not account for rotation, and definitely does not account for scaling. It involves a calculation to find the *centroid* of the object, which is the average of the coordinates:

$$x_c = \frac{1}{N}\sum_{n=0}^{N-1} x(n), y_c = \frac{1}{N}\sum_{n=0}^{N-1} y(n)$$

Once the centroid (x_c, y_c) is found, the shape of the object can be represented as the distance from each point to the centroid.

$$r(n) = \sqrt{[x(n) - x_c]^2 + [y(n) - y_c]^2}$$

3. *Curvature*

The shape signatures discussed thus far have not handled rotation gracefully. To resolve the issue, the differentiation of successive boundary angles provides a decent shape signature which accounts for both rotation and translation:

$$K(n) = \theta(n) - \theta(n-1)$$

where

$$\theta(n) = \arctan \frac{y(n) - y(n-\delta)}{x(n) - x(n-\delta)}$$

Small values of δ will generate the most detailed shape signature, but many data points are required. This signature handles translation and rotation, but still does not account for scaling.

III. **Derivation of Fourier Descriptors**

Fourier Descriptors are used in order to derive meaningful, quantifiable information from the shape signature functions. A Fourier descriptor is a set of coefficients which directly describes the shape of the original object. The discrete Fourier transform of shape signature function $s(n)$ is given by:

$$u_n = \frac{1}{N} \sum_{t=0}^{N-1} s(t) e^{\frac{-2i\pi nt}{N}}, n = 0, 1, ..., N-1$$

The meaning of the descriptors depends on the signature used. In the case of the complex coordinates function, the first element in the set is used to show the location of the object, the second element shows the general size of the object, and every other element in the set is required to show the shape of the object. The location of the object, u_0, is not important, so it is removed. The second element is then used to normalize the size of the object, thus making this shape descriptor scale-invariant. The Fourier descriptor now becomes

$$\mathbf{f} = \left[\frac{|u_2|}{|u_1|}, \frac{|u_3|}{|u_1|}, ..., \frac{|u_{N-1}|}{|u_1|}\right]$$

For the centroid distance and curvature functions, the first element is used to normalize the general size of the object, the lower-order elements in the set describe the general shape of the object, and the higher-order elements describe the finer details of the object. The Fourier descriptor becomes

$$\mathbf{f} = \left[\frac{|u_1|}{|u_0|}, \frac{|u_2|}{|u_0|}, ..., \frac{|u_{N/2}|}{|u_0|}\right]$$

IV. **Analysis**

Two images of submarines will be used to compare the effectiveness of the three shape signatures. The first step is to outline the images and plot data points along the perimeter (*Appendix A*). Next, the shape signatures $z(n)$, $r(n)$, and $k(n)$ are computed (*Appendix B*). From here, the Fourier transforms must be computed (*Appendix C*), and finally the Fourier descriptors are derived from the Fourier transforms (*Appendix D*).

Assume a situation where Submarine Two is in a database of images, and Submarine One is given as search criteria. In a real-world application of such a database, the Fourier descriptors would be the only information held in the database about the shape of the object. The purpose of the search is to minimize the distance between Fourier descriptors. The effectiveness of each shape signature's Fourier descriptor can now be calculated. Let Submarine One be the *model shape* f_m, or the model against which the database is searched. Let Submarine Two be the *data shape* f_d, the shape in the database. The distance between the Fourier descriptors of each submarine is calculated as follows:

$$d = \sqrt{\sum_{n=0}^{N_c} |f_m^n - f_d^n|^2}$$

where N_c is the number of Fourier descriptors necessary to complete a thorough search. The numbers retrieved by the searches do not necessarily correlate to any meaningful unit of measure. A successful search will return the lowest differences, even though the quantities may be extremely high or extremely low.

The *complex coordinate* Shape Signature will be the first signature to compare. The Fourier descriptor of this signature is scale-invariant, and it is very effective with describing details of images, so it should fare well. The Euclidean distance for the *complex coordinate* Shape Signature is calculated here:

$$d_z = \sqrt{\sum_{n=0}^{57} |f_m^n - f_d^n|^2} = 2.8122$$

The *centroid distance* Shape Signature is the next signature to compare. The Fourier descriptor of this signature is both translation- and scale-invariant, so it should do just as well as the *complex coordinate* signature. The Euclidean distance for the *centroid distance* Shape Signature is calculated here:

$$d_r = \sqrt{\sum_{n=0}^{29} |f_m^n - f_d^n|^2} = 4.0570$$

The *curvature* Shape Signature is the last signature to compare. Although the Fourier descriptor of this signature is both translation- and scale-invariant, the signature is only useful in recognizing general shapes, because a relatively small amount of data points was taken. The Euclidean distance for the *curvature* Shape Signature is calculated here:

$$d_k = \sqrt{\sum_{n=0}^{29} |f_m^n - f_d^n|^2} = 29.2078$$

V. **Results**

Given 60 data points on two images of similar submarines (*Appendix A*), the Euclidean distance between the *complex coordinate* Fourier descriptors was 2.8122, the Euclidean distance between the *centroid distance* Fourier descriptors was 4.0570, and the Euclidean distance between the *curvature* Fourier descriptors was 29.2078.

Further detail on the data points can be found in *Appendix A*. Information regarding Shape Signatures can be found in *Appendix B*, Fourier transforms are in *Appendix C*, and Fourier descriptors are in *Appendix D*.

I.Conclusion

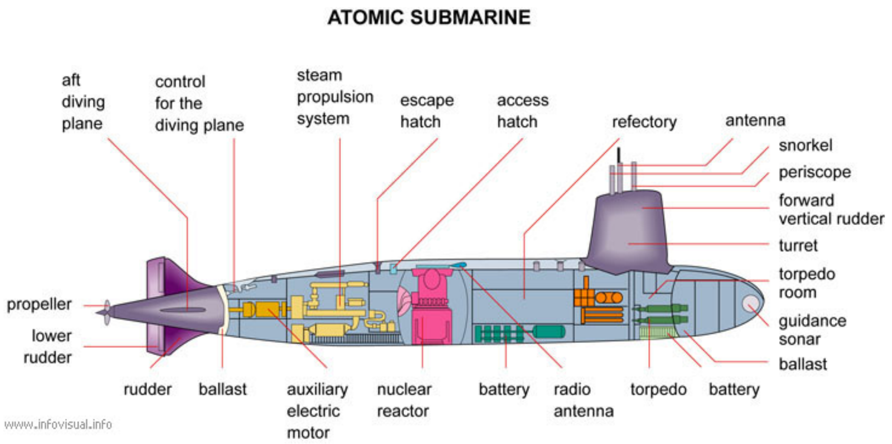
Based on the results, it is likely that the *complex coordinate* Shape Signature will provide the best search results in a naïve search application. This assumption agrees with research conducted by D. S. Zhang and G. Lu, which showed that the *complex coordinate* Shape Signature provided drastically better search results, the *centroid distance* Shape Signature provided mediocre results, and the *curvature* Shape Signature provided inadequate search results.

Appendix A – Image Outline and Data Points

Submarine One

y

x



553	56
549	41
529	31
507	24
486	21
464	19
441	18

418 16

396 16

372 16

348 16

326 16

303 17

279 15

256 14

233 14

210 14

186 15

163 17

138 20

115 23

92 24

1

70

5

48 10

42 34

19 38

13 52

31 53

44 72

57 88

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256 86

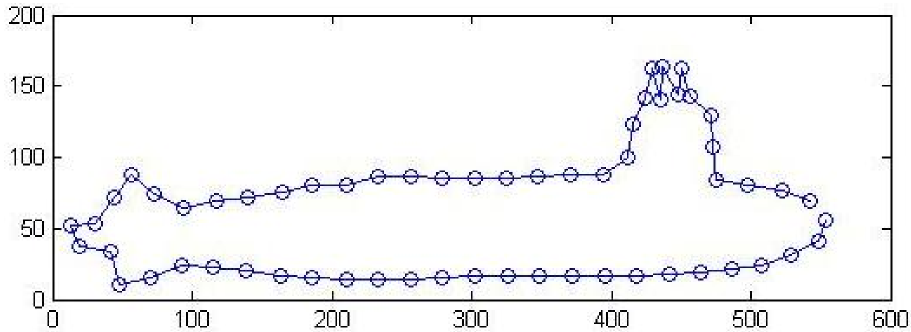
279 85

302 85

325 85

347 87

8



371

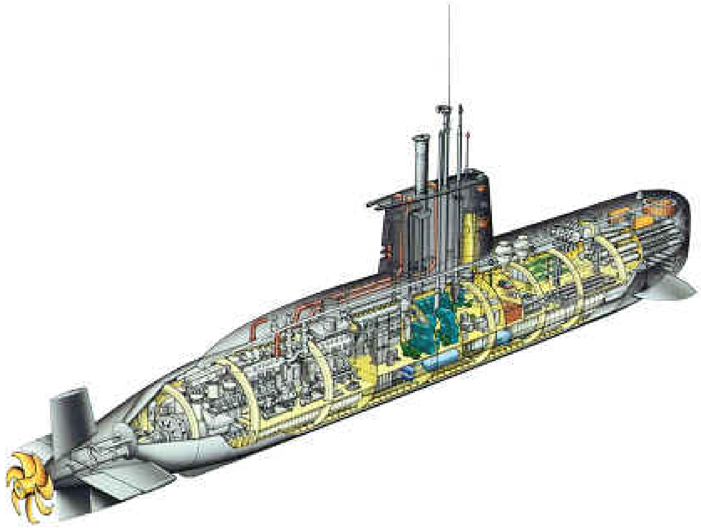
8

394 88

412 100

416 123

424	142
429	163
436	141
437	164
448	144
450	163
456	143
471	130
473	107
475	84
498	80
522	77
542	69



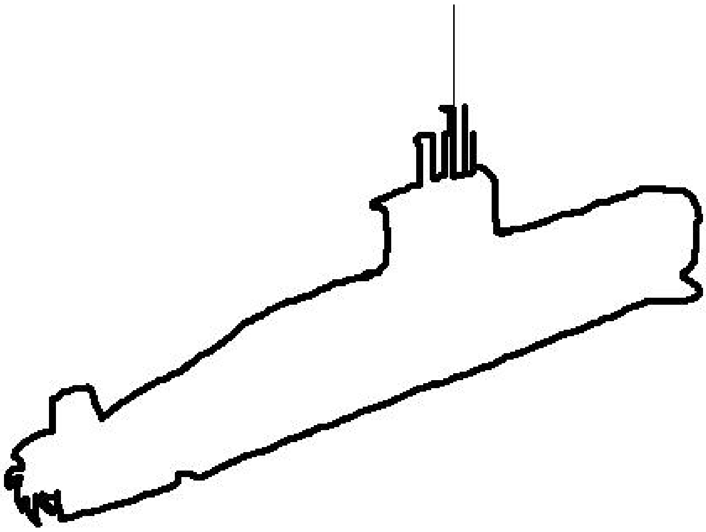
Submarine Two

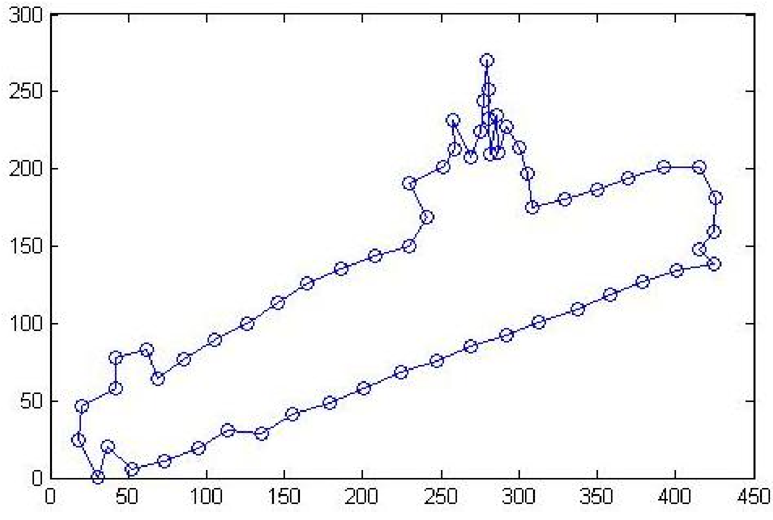
x	y
426	181
425	159

415	148
425	138
401	134
379	127
358	119
337	109
313	101
292	92
269	85
247	76
224	68
201	58
179	49
155	41
135	29
114	31
95	19
73	11
52	6
37	20
31	1
18	25
20	47
42	58
42	78

62	83
69	64
86	77
105	89
126	100
146	113
164	126
186	135
208	144
230	150
241	169
230	191
251	201
259	213
258	232
269	208
275	224
277	244
279	270
280	251
280	233
281	210
286	235
287	211
292	227

300	214
305	197
308	175
329	180
350	187
370	194
392	201
415	201



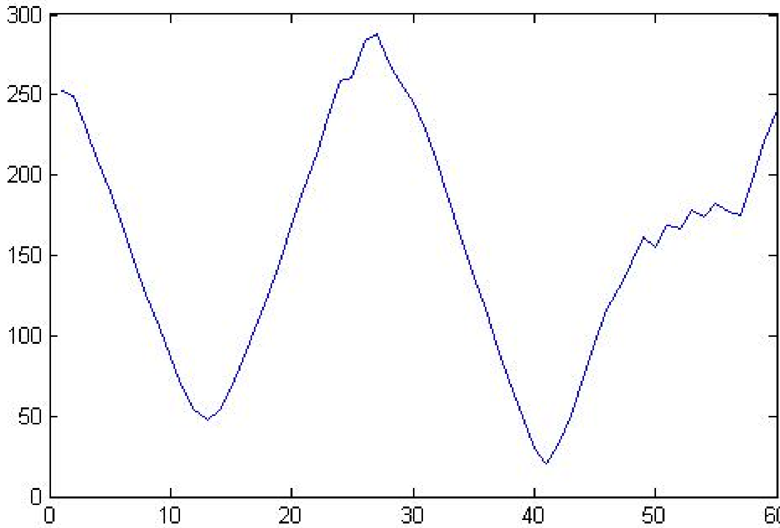


Appendix B – Shape Signatures

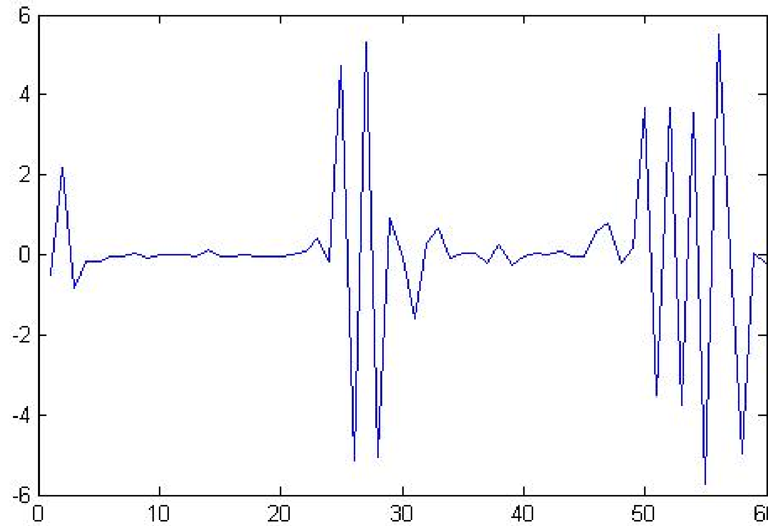
Submarine One - Shape Signatures

z	r	k
553 + 56i	251.9878	-0.488
549 + 41i	248.9459	2.1787
529 + 31i	230.2753	-0.8465
507 + 24i	209.7642	-0.1556
486 + 21i	189.8613	-0.1662
464 + 19i	169.0421	-0.0512
441 + 18i	147.3255	-0.0472
418 + 16i	126.4517	0.0433
396 + 16i	106.4548	-0.0867
372 + 16i	85.7662	0
348 + 16i	67.313	0
326 + 16i	54.3473	0
303 + 17i	47.3695	-0.0435
279 + 15i	54.0777	0.1266
256 + 14i	67.6163	-0.0397
233 + 14i	84.7223	-0.0435
210 + 14i	104.1238	0
186 + 15i	125.2729	-0.0416
163 + 17i	146.0338	-0.0451
138 + 20i	169.0662	-0.0327
115 + 23i	190.6837	-0.0103
92 + 24i	213.0035	0.0863
70 + 15i	236.3559	0.4318
48 + 10i	258.9151	-0.1648
42 + 34i	260.9192	4.7339
19 + 38i	283.3762	-5.1296
13 + 52i	288.4138	5.2895
31 + 53i	270.3876	-5.0618
44 + 72i	257.2643	0.9152
57 + 88i	245.2944	-0.0823
73 + 74i	228.3547	-1.6073
94 + 64i	207.1503	0.2744
117 + 69i	184.2091	0.6585
140 + 72i	161.3323	-0.0844
165 + 76i	136.6489	0.029
186 + 80i	116.2109	0.0296
210 + 80i	92.4866	-0.1882
233 + 86i	71.5113	0.2552
256 + 86i	50.0796	-0.2552
279 + 85i	30.2941	-0.0435
302 + 85i	20.6841	0.0435
325 + 85i	31.5584	0
347 + 87i	51.1469	0.0907
371 + 88i	73.7505	-0.049
394 + 88i	95.8188	-0.0416
412 + 100i	116.4467	0.588
416 + 123i	128.9663	0.8106
424 + 142i	145.3418	-0.2263
429 + 163i	161.4953	0.1648
436 + 141i	155.1203	3.6834
437 + 164i	168.4894	-3.4931
448 + 144i	167.068	3.6879
450 + 163i	178.5817	-3.7493
456 + 143i	173.6864	3.5379
471 + 130i	182.102	-5.7179
473 + 107i	177.0674	5.5132
475 + 84i	174.9589	0
498 + 80i	197.4724	-4.9713
522 + 77i	221.2129	0.0478
542 + 69i	240.8952	-0.2562

Centroid Distance



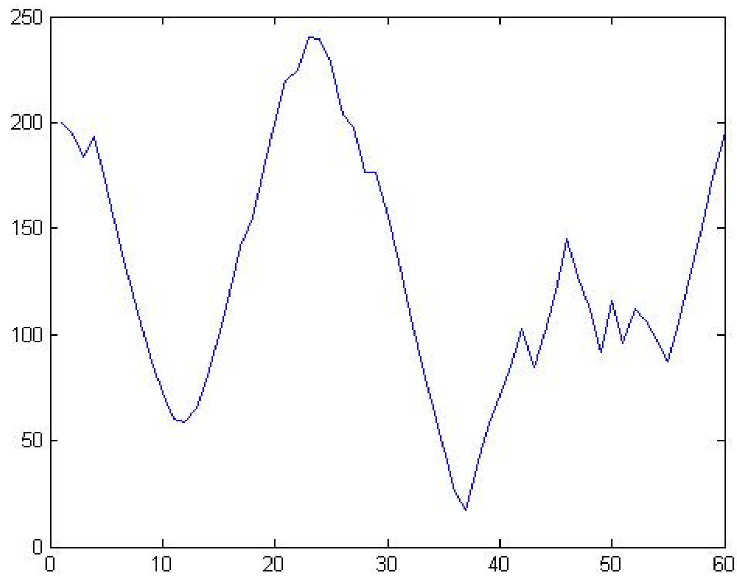
Curvature



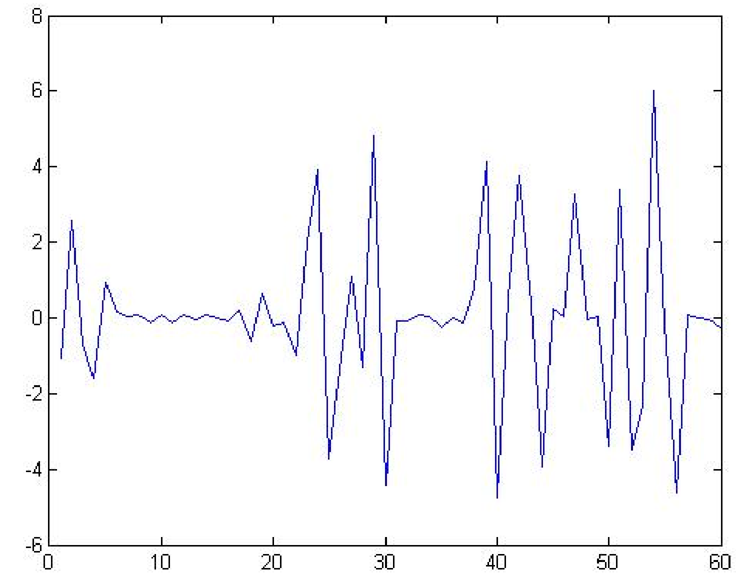
Submarine Two - Shape Signatures

z	r	k
426 + 181i	200.2416	-1.068
425 + 159i	195.1042	2.5933
415 + 148i	183.9582	-0.6924
425 + 138i	193.3907	-1.6184
401 + 134i	169.322	0.9505
379 + 127i	147.425	0.1429
358 + 119i	127.052	0.0559
337 + 109i	107.9394	0.0804
313 + 101i	87.2589	-0.1227
292 + 92i	72.736	0.0831
269 + 85i	60.5232	-0.1095
247 + 76i	58.6841	0.0929
224 + 68i	65.105	-0.0536
201 + 58i	80.7099	0.0754
179 + 49i	98.8578	-0.0218
155 + 41i	119.4992	-0.0666
135 + 29i	141.7427	0.2187
114 + 31i	155.5059	-0.6354
95 + 19i	177.7601	0.6583
73 + 11i	199.9478	-0.2145
52 + 6i	219.8325	-0.115
37 + 20i	224.9258	-0.9847
31 + 1i	240.0115	2.0158
18 + 25i	239.2678	3.9439
20 + 47i	228.3545	-3.7287
42 + 58i	203.844	-1.0165
42 + 78i	197.3991	1.1071
62 + 83i	176.7981	-1.3258
69 + 64i	176.5749	4.8204
86 + 77i	155.9505	-4.4125
105 + 89i	133.9925	-0.0895
126 + 100i	110.6119	-0.0808
146 + 113i	87.9077	0.0939
164 + 126i	68.0092	0.0491
186 + 135i	45.7437	-0.2372
208 + 144i	26.2626	0
230 + 150i	17.4315	-0.1221
241 + 169i	37.525	0.7797
230 + 191i	58.3743	4.13
251 + 201i	71.0272	-4.7316
259 + 213i	84.8665	0.5384
258 + 232i	102.7764	3.7822
269 + 208i	84.0842	0.3772
275 + 224i	101.0997	-3.9301
277 + 244i	120.2182	0.2591
279 + 270i	145.2718	0.0229
280 + 251i	127.8328	3.2709
280 + 233i	111.376	-0.0526
281 + 210i	91.7342	0.0435
286 + 235i	115.8699	-3.3824
287 + 211i	95.9096	3.3806
292 + 227i	111.9822	-3.4861
300 + 214i	106.2308	-2.2871
305 + 197i	97.5513	6.0176
308 + 175i	87.2798	-0.1505
329 + 180i	108.2246	-4.6142
350 + 187i	130.2028	0.088
370 + 194i	151.312	0.0149
392 + 201i	174.279	-0.0286
415 + 201i	195.6444	-0.3081

Centroid Distance



Curvature



Appendix C – Fourier Transforms

Submarine One - Fourier Transforms

z	r	k
82.8909e+000 + 35.2758e+000i	-2.0617e+000 - 8.1206e+000i	-27.2301e-003 + 21.6522e-003i
-17.2349e+000 - 15.8116e+000i	40.4767e+000 + 23.5085e+000i	-104.8019e-003 + 43.7695e-003i
2.0511e+000 - 9.5292e+000i	-4.6382e+000 - 14.8185e+000i	-29.9596e-003 - 79.3278e-003i
4.9376e+000 - 8.0591e+000i	1.8452e+000 - 8.2299e+000i	-87.9046e-003 - 70.4363e-003i
4.4709e+000 - 1.0080e+000i	3.2083e+000 - 1.7295e+000i	99.7840e-003 - 35.5089e-003i
765.2211e-003 + 1.0552e+000i	-250.6347e-003 +657.3209e-003i	-77.7473e-003 - 70.3739e-003i
896.2290e-003 - 2.1779e+000i	1.5873e+000 - 2.6649e+000i	17.6576e-003 + 28.5972e-003i
1.1575e+000 -653.7713e-003i	1.3954e+000 - 1.6005e+000i	-86.4970e-003 -127.0563e-003i
1.6883e+000 + 85.5440e-003i	458.7923e-003 -324.5796e-003i	-54.4626e-003 - 58.0756e-003i
884.8076e-003 + 1.3380e+000i	121.5692e-003 -843.9317e-003i	-20.9217e-003 -163.8405e-003i
-1.0195e+000 -117.8659e-003i	1.2322e+000 + 83.9350e-003i	-35.3170e-003 -144.3066e-003i
304.7698e-003 +586.1825e-003i	99.8144e-003 + 54.1149e-003i	49.3023e-003 -114.0047e-003i
-139.4598e-003 - 1.3724e+000i	82.6379e-003 +190.6328e-003i	37.7588e-003 -143.9231e-003i
371.6224e-003 + 1.5283e+000i	-63.8415e-003 -415.0446e-003i	50.8613e-003 - 31.0031e-003i
83.3333e-003 -833.3333e-003i	-7.9017e-003 +557.2567e-003i	56.9883e-003 -118.2450e-003i
-693.1669e-003 +987.0066e-003i	189.0321e-003 -244.8542e-003i	124.7847e-003 - 47.0411e-003i
213.6083e-003 -217.1481e-003i	-340.9202e-003 +539.7897e-003i	176.3306e-003 + 39.7490e-003i
-903.2360e-003 +465.5868e-003i	10.9964e-003 +258.9653e-003i	203.8706e-003 +130.9357e-003i
55.9054e-003 + 18.5656e-003i	-883.7926e-003 - 21.0376e-003i	-28.4751e-003 +273.2975e-003i
-380.6624e-003 -119.7756e-003i	184.3342e-003 + 29.6051e-003i	97.9983e-003 +207.4362e-003i
-298.1320e-003 -259.0670e-003i	-122.7671e-003 -320.1173e-003i	-229.8372e-003 +147.7896e-003i
-100.5002e-003 - 65.1484e-003i	-138.0963e-003 +258.6468e-003i	216.4648e-003 +251.3619e-003i

-328.9390e-003 -789.4964e-003i	-79.7800e-003 -203.7590e-003i	-245.6371e-003 + 95.0141e-003i
699.6131e-003 -314.4869e-003i	-379.4644e-003 - 98.5969e-003i	36.7877e-003 +489.2871e-003i
862.4471e-003 -141.9873e-003i	342.4089e-003 -350.7372e-003i	-355.7973e-003 - 89.4911e-003i
469.9707e-003 +823.4820e-003i	220.0883e-003 -250.0195e-003i	-360.0525e-003 +280.2485e-003i
-80.1063e-003 +911.3670e-003i	482.9876e-003 +263.8768e-003i	-70.5205e-003 -567.2715e-003i
-1.1355e+000 +153.0073e-003i	114.0003e-003 +292.1026e-003i	-186.9481e-003 -129.8612e-003i
-355.8750e-003 -582.9902e-003i	-359.3108e-003 +362.5225e-003i	645.2423e-003 -394.9092e-003i
50.0000e-003 - 1.1667e+000i	-64.8983e-003 +199.0613e-015i	120.3533e-003 +4.4235e-015i
776.2429e-003 -112.2681e-003i	-359.3108e-003 -362.5225e-003i	645.2423e-003 +394.9092e-003i
988.0027e-003 +169.1426e-003i	114.0003e-003 -292.1026e-003i	-186.9481e-003 +129.8612e-003i
-274.8097e-003 +928.8034e-003i	482.9876e-003 -263.8768e-003i	-70.5205e-003 +567.2715e-003i
-6.4678e-003 +597.6786e-003i	220.0883e-003 +250.0195e-003i	-360.0525e-003 -280.2485e-003i
-1.0120e+000 -298.5257e-003i	342.4089e-003 +350.7372e-003i	-355.7973e-003 + 89.4911e-003i
-82.5917e-003 - 14.2024e-003i	-379.4644e-003 + 98.5969e-003i	36.7877e-003 -489.2871e-003i
124.8219e-003 -923.3108e-003i	-79.7800e-003 +203.7590e-003i	-245.6371e-003 - 95.0141e-003i
71.0174e-003 +656.4856e-003i	-138.0963e-003 -258.6468e-003i	216.4648e-003 -251.3619e-003i
271.1651e-003 -308.9371e-003i	-122.7671e-003 +320.1173e-003i	-229.8372e-003 -147.7896e-003i
-669.3376e-003 +486.4422e-003i	184.3342e-003 - 29.6051e-003i	97.9983e-003 -207.4362e-003i
-100.2979e-003 -231.6011e-003i	-883.7926e-003 + 21.0376e-003i	-28.4751e-003 -273.2975e-003i
-438.0501e-003 -381.2729e-003i	10.9964e-003 -258.9653e-003i	203.8706e-003 -130.9357e-003i
120.9255e-003 -491.2773e-003i	-340.9202e-003 -539.7897e-003i	176.3306e-003 - 39.7490e-003i
296.6416e-003 +156.7741e-003i	189.0321e-003 +244.8542e-003i	124.7847e-003 + 47.0411e-003i
-83.3333e-003 -200.0000e-003i	-7.9017e-003 -557.2567e-003i	56.9883e-003 +118.2450e-003i
-90.1439e-003 +474.6797e-003i	-63.8415e-003 +415.0446e-003i	50.8613e-003 + 31.0031e-003i
523.2953e-003 -871.5408e-003i	82.6379e-003 -190.6328e-003i	37.7588e-003 +143.9231e-003i

1.0949e+000 +575.8402e-003i	99.8144e-003 - 54.1149e-003i	49.3023e-003 +114.0047e-003i
925.7943e-003 + 1.4987e+000i	1.2322e+000 - 83.9350e-003i	-35.3170e-003 +144.3066e-003i
365.1924e-003 + 1.8287e+000i	121.5692e-003 +843.9317e-003i	-20.9217e-003 +163.8405e-003i
-908.8714e-003 + 1.3435e+000i	458.7923e-003 +324.5796e-003i	-54.4626e-003 + 58.0756e-003i
1.5969e+000 + 1.8220e+000i	1.3954e+000 + 1.6005e+000i	-86.4970e-003 +127.0563e-003i
-392.8404e-003 +869.5647e-003i	1.5873e+000 + 2.6649e+000i	17.6576e-003 - 28.5972e-003i
1.3594e+000 + 4.6938e+000i	-250.6347e-003 -657.3209e-003i	-77.7473e-003 + 70.3739e-003i
-521.2928e-003 -818.1410e-003i	3.2083e+000 + 1.7295e+000i	99.7840e-003 + 35.5089e-003i
2.9853e+000 + 4.6416e+000i	1.8452e+000 + 8.2299e+000i	-87.9046e-003 + 70.4363e-003i
13.2180e+000 - 3.9719e+000i	-4.6382e+000 + 14.8185e+000i	-29.9596e-003 + 79.3278e-003i
1.6861e+000 + 17.7128e+000i	40.4767e+000 - 23.5085e+000i	-104.8019e-003 - 43.7695e-003i
138.8425e+000 - 25.1749e+000i	-2.0617e+000 + 8.1206e+000i	-27.2301e-003 - 21.6522e-003i
301.1500e+000 + 64.3333e+000i	155.5095e+000 + 3.5995e-012i	3.3333e-006 -6.7186e-015i

Submarine Two - Fourier Transforms

z	r	k
28.2484e+000 + 47.6445e+000i	-633.7319e-003 - 15.6402e+000i	-61.6612e-003 -5.0725e-003i
10.9222e+000 - 34.2017e+000i	14.3076e+000 + 20.7911e+000i	-64.8139e-003 - 43.6820e-003i
9.1511e+000 -2.6005e+000i	9.9356e+000 - 22.4156e+000i	57.2009e-003 - 66.0460e-003i
1.6250e+000 -220.2465e-003i	4.8313e+000 -4.7326e+000i	-67.6055e-003 -3.4070e-003i
2.3897e+000 -351.3140e-003i	-1.0531e+000 -236.2312e-003i	57.7889e-003 - 36.9786e-003i
-1.0285e+000 +988.3749e-003i	1.6218e+000 -1.9918e+000i	-8.5394e-003 +3.0124e-003i
1.1132e+000 +893.0395e-003i	199.9535e-003 +1.7992e+000i	-91.2901e-003 - 40.3470e-003i
521.5267e-003 +1.2756e+000i	1.8961e+000 -1.6057e+000i	86.0010e-003 -132.7480e-003i
-1.2829e+000 +4.2773e+000i	1.7283e+000 +1.0370e+000i	111.3795e-003 -181.3210e-003i

-2.0484e+000 -628.8675e-003i	-288.4692e-003 +2.7160e+000i	231.2992e-003 +157.2745e-003i
5.3380e-003 -147.6011e-003i	-1.5385e+000 -296.9555e-003i	76.4628e-003 + 16.6721e-003i
1.1351e+000 -441.5643e-003i	767.8263e-003 -370.3297e-003i	-273.4304e-003 +189.4580e-003i
383.3171e-003 +1.2478e+000i	590.3853e-003 -533.5908e-003i	221.1522e-003 -215.4155e-003i
-391.6754e-003 -200.0512e-003i	488.3869e-003 +735.5145e-003i	-172.7855e-003 +335.4639e-003i
250.0000e-003 -1.4833e+000i	-379.3533e-003 -566.1000e-003i	-324.0200e-003 +235.8500e-003i
1.8385e+000 - 15.5634e-003i	217.1016e-003 -1.5615e+000i	-412.1834e-003 -318.8845e-003i
-259.3732e-003 +898.6467e-003i	1.1414e+000 -105.9450e-003i	-90.9751e-003 - 24.8224e-003i
238.5416e-003 -771.8569e-003i	384.9616e-003 - 58.9224e-003i	-189.9889e-003 - 69.8804e-003i
-416.7943e-003 +406.5284e-003i	-171.2125e-003 -327.0724e-003i	24.0316e-003 -458.5899e-003i
-412.5215e-003 +652.0726e-003i	39.4592e-003 +577.5798e-003i	416.9458e-003 +224.6369e-003i
-649.2564e-003 -1.3381e+000i	-825.9062e-003 - 49.9526e-003i	176.4123e-003 +130.2512e-003i
410.8545e-003 +619.9181e-003i	-385.3111e-003 -267.1571e-003i	-54.1311e-003 +324.0403e-003i
-87.4990e-003 +575.8194e-003i	161.2506e-003 -222.3520e-003i	313.3492e-003 +103.7406e-003i
-430.5664e-003 +206.1649e-003i	52.2137e-003 +603.3494e-003i	-388.8413e-003 +297.1610e-003i
-6.3351e-003 +601.3140e-003i	33.7310e-003 +216.1862e-003i	205.2961e-003 -112.1814e-003i
-505.6242e-003 -349.5922e-003i	-115.6533e-003 +597.1153e-003i	-141.0812e-003 - 66.0094e-003i
-345.1072e-003 + 85.4370e-003i	-286.7459e-003 +134.2732e-003i	183.8273e-003 - 45.0737e-003i
1.5256e+000 -589.8719e-003i	-689.7709e-003 +106.9836e-003i	78.7046e-003 +6.8526e-003i
897.2325e-003 -128.8506e-003i	1.0214e+000 -1.2418e+000i	167.1455e-003 +261.9712e-003i
716.6667e-003 +1.9500e+000i	1.1032e+000 +176.5886e-015i	-439.3933e-003 -6.8461e-015i
-844.6011e-003 - 52.6033e-003i	1.0214e+000 +1.2418e+000i	167.1455e-003 -261.9712e-003i
-611.3190e-003 -922.0718e-003i	-689.7709e-003 -106.9836e-003i	78.7046e-003 -6.8526e-003i
292.1481e-003 -536.5860e-003i	-286.7459e-003 -134.2732e-003i	183.8273e-003 + 45.0737e-003i
282.9192e-003 + 25.3001e-003i	-115.6533e-003 -597.1153e-003i	-141.0812e-003 + 66.0094e-003i

-81.6987e-003 -861.0357e-003i	33.7310e-003 -216.1862e-003i	205.2961e-003 +112.1814e-003i
377.2060e-003 +922.5244e-003i	52.2137e-003 -603.3494e-003i	-388.8413e-003 -297.1610e-003i
-707.0189e-003 -211.8394e-003i	161.2506e-003 +222.3520e-003i	313.3492e-003 -103.7406e-003i
-1.2311e+000 +101.7534e-003i	-385.3111e-003 +267.1571e-003i	-54.1311e-003 -324.0403e-003i
1.0193e+000 -663.2334e-003i	-825.9062e-003 + 49.9526e-003i	176.4123e-003 -130.2512e-003i
-1.2208e+000 +247.9274e-003i	39.4592e-003 -577.5798e-003i	416.9458e-003 -224.6369e-003i
1.1512e+000 +578.6311e-003i	-171.2125e-003 +327.0724e-003i	24.0316e-003 +458.5899e-003i
606.0211e-003 +240.1619e-003i	384.9616e-003 + 58.9224e-003i	-189.9889e-003 + 69.8804e-003i
-391.6243e-003 +2.0729e+000i	1.1414e+000 +105.9450e-003i	-90.9751e-003 + 24.8224e-003i
-264.0607e-003 +475.9435e-003i	217.1016e-003 +1.5615e+000i	-412.1834e-003 +318.8845e-003i
216.6667e-003 -250.0000e-003i	-379.3533e-003 +566.1000e-003i	-324.0200e-003 -235.8500e-003i
75.0037e-003 +408.5536e-003i	488.3869e-003 -735.5145e-003i	-172.7855e-003 -335.4639e-003i
1.1645e+000 +914.3079e-003i	590.3853e-003 +533.5908e-003i	221.1522e-003 +215.4155e-003i
-1.5984e+000 + 79.5416e-003i	767.8263e-003 +370.3297e-003i	-273.4304e-003 -189.4580e-003i
725.3682e-003 -1.3701e+000i	-1.5385e+000 +296.9555e-003i	76.4628e-003 - 16.6721e-003i
2.2817e+000 -571.1325e-003i	-288.4692e-003 -2.7160e+000i	231.2992e-003 -157.2745e-003i
1.6747e+000 +3.3283e+000i	1.7283e+000 -1.0370e+000i	111.3795e-003 +181.3210e-003i
273.4382e-003 -168.2378e-003i	1.8961e+000 +1.6057e+000i	86.0010e-003 +132.7480e-003i
-72.4065e-003 +3.7578e+000i	199.9535e-003 -1.7992e+000i	-91.2901e-003 + 40.3470e-003i
2.1339e+000 +343.3201e-003i	1.6218e+000 +1.9918e+000i	-8.5394e-003 -3.0124e-003i
-168.3013e-003 +3.8444e+000i	-1.0531e+000 +236.2312e-003i	57.7889e-003 + 36.9786e-003i
4.8080e+000 +8.5977e+000i	4.8313e+000 +4.7326e+000i	-67.6055e-003 +3.4070e-003i
2.1733e+000 +680.7102e-003i	9.9356e+000 + 22.4156e+000i	57.2009e-003 + 66.0460e-003i
-1.2126e+000 + 28.2959e+000i	14.3076e+000 - 20.7911e+000i	-64.8139e-003 + 43.6820e-003i
118.9575e+000 +187.5809e-003i	-633.7319e-003 + 15.6402e+000i	-61.6612e-003 +5.0725e-003i

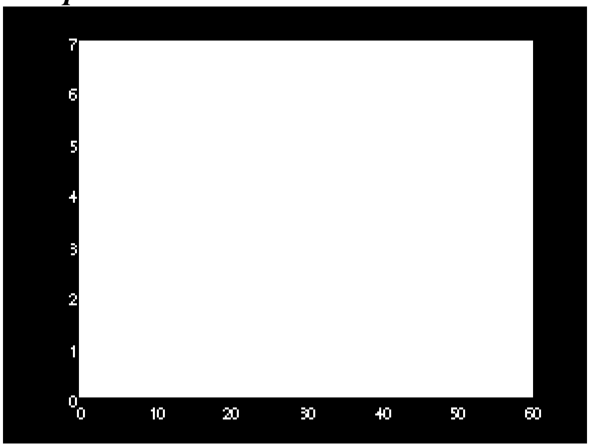
231.6833e+000 +132.6500e+000i 128.4392e+000 +2.8548e-012i -6.6667e-006 +2.1990e-015i

Appendix D – Fourier Descriptors

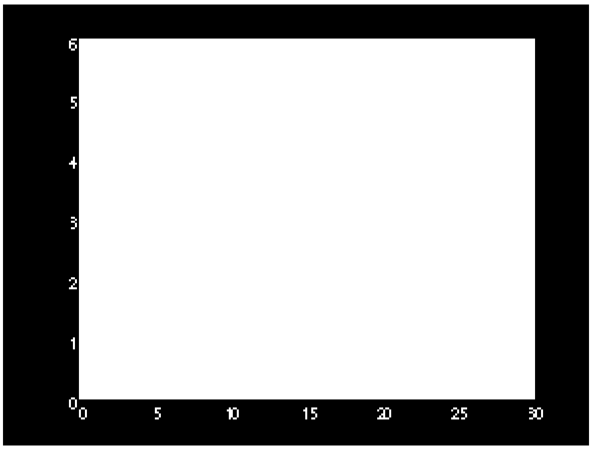
Submarine One - Fourier Descriptors

z	r	k
0.4167526	5.5869000	3.2646000
0.4040919	1.8533000	2.4374000
0.1959508	1.0067000	3.2379000
0.0557289	0.4350286	3.0444000
0.1006907	0.0839660	3.0144000
0.0568390	0.3702212	0.9660836
0.0722754	0.2534358	4.4182000
0.0685817	0.0670788	2.2886000
0.0438802	0.1017694	4.7478000
0.0282472	0.1474148	4.2704000
0.0589781	0.0135519	3.5703000
0.0672481	0.0247994	4.2770000
0.0358068	0.0501213	1.7122000
0.0515665	0.0665195	3.7730000
0.0130232	0.0369212	3.8333000
0.0434464	0.0762021	5.1957000
0.0025186	0.0309373	6.9647000
0.0170619	0.1055172	7.8983000
0.0168868	0.0222836	6.5946000
0.0051207	0.0409219	7.8545000
0.0365675	0.0349962	9.5352000
0.0327951	0.0261179	7.5705000
0.0373703	0.0467959	14.1040000
0.0405383	0.0585047	10.5458000
0.0391157	0.0397568	13.1151000
0.0489853	0.0656909	16.4314000
0.0292028	0.0374258	6.5430000
0.0499266	0.0609223	21.7452000
0.0335336	0.0077461	3.4595000
0.0428566		
0.0414127		
0.0255552		
0.0451129		
0.0035830		
0.0398352		
0.0282318		
0.0175750		
0.0353767		
0.0107908		
0.0248294		
0.0216315		
0.0143452		
0.0092636		
0.0206576		
0.0434636		
0.0528909		
0.0753170		
0.0797301		
0.0693488		
0.1035843		
0.0407961		
0.2089316		
0.0414767		
0.2359536		
0.5900983		
0.7607328		
6.0330000		

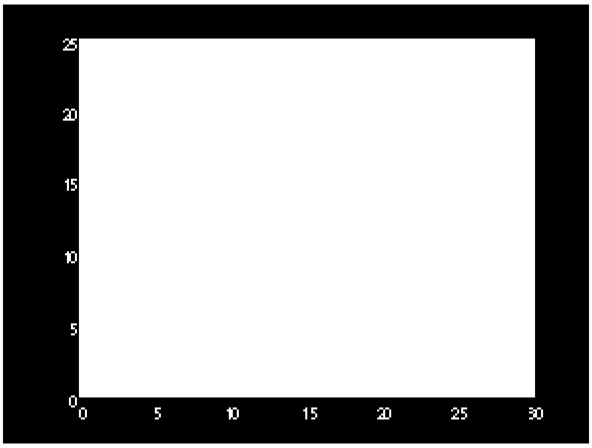
Complex Coordinate



Centroid Distance



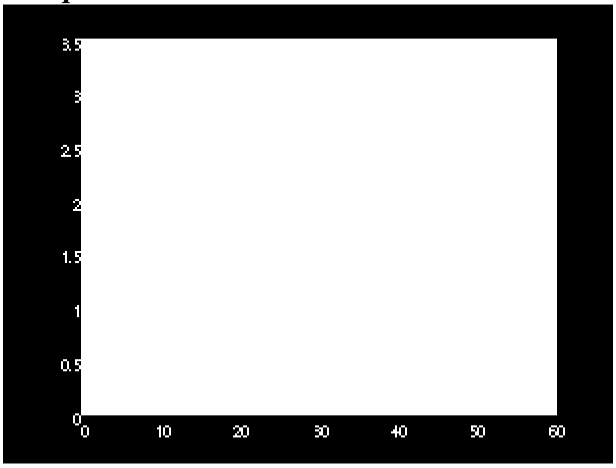
Curvature



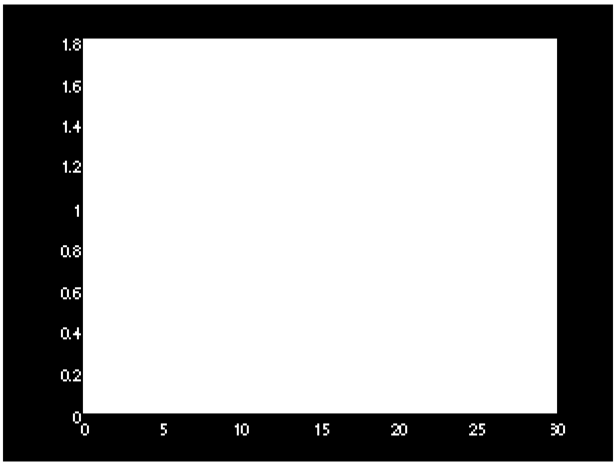
Submarine Two - Fourier Descriptors

z	r	k
0.2649722	1.6124000	1.2633000
0.0456748	1.5664000	1.4122000
0.0672737	0.4320632	1.0941000
0.0397297	0.0689488	1.1089000
0.0397499	0.1640923	0.1463594
0.0383845	0.1156482	1.6132000
0.1243759	0.1587339	2.5565000
0.0596811	0.1287637	3.4395000
0.0041138	0.1744863	4.5209000
0.0339244	0.1001049	1.2649000
0.0363586	0.0544602	5.3767000
0.0122497	0.0508390	4.9900000
0.0418972	0.0564041	6.0991000
0.0512076	0.0435349	6.4776000
0.0260513	0.1007197	8.4232000
0.0225014	0.0732347	1.5242000
0.0162164	0.0248798	3.2719000
0.0214911	0.0235849	7.4224000
0.0414241	0.0369849	7.6550000
0.0207141	0.0528597	3.5443000
0.0162221	0.0299538	5.3101000
0.0132962	0.0175472	5.3350000
0.0167490	0.0386893	7.9100000
0.0171213	0.0139782	3.7813000
0.0099023	0.0388559	2.5176000
0.0455563	0.0202278	3.0592000
0.0252466	0.0445932	1.2769000
0.0578643	0.1027192	5.0227000
0.0235698	0.0704773	7.1019000
0.0308136		
0.0170168		
0.0079115		
0.0240897		
0.0277595		
0.0205572		
0.0344065		
0.0338707		
0.0346968		
0.0358859		
0.0181563		
0.0587576		
0.0151598		
0.0092143		
0.0115694		
0.0412365		
0.0445759		
0.0431781		
0.0655125		
0.1037769		
0.0089420		
0.1046823		
0.0602000		
0.1071779		
0.2743697		
0.0634321		
0.7888343		
3.3133000		

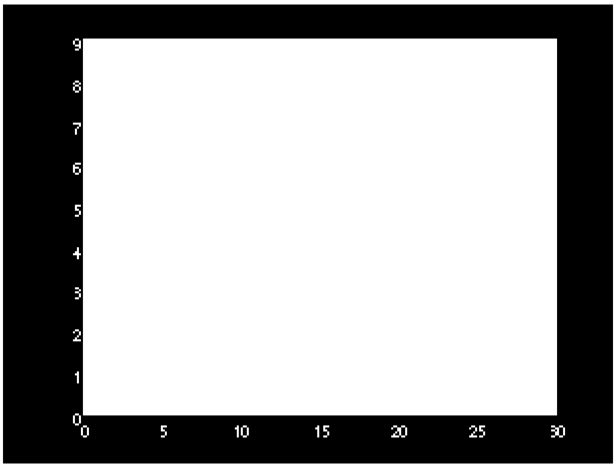
Complex Coordinates



Centroid Distance



Curvature



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