Negative and other exponent rules

Now we'll look at a few more exponent rules, starting with the negative exponent rule.

Negative exponent rule

The **negative exponent rule** tells us that, if the exponent is negative, it can be change into a positive exponent by taking the reciprocal.

$$a^{-m} = \frac{1}{a^m} \text{ and } a^m = \frac{1}{a^{-m}}$$

For example,

$$2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

And we can still use all the rules we learned earlier for addition and subtraction, and multiplication and division with exponents, even when we have negative. In fact, let's do one more example where we apply the quotient rule, even with a negative exponent.

Example

Simplify the rational expression.

$$\frac{x^3}{x^{-2}}$$

It doesn't matter that the exponent in the denominator is negative. We can still use the quotient rule since the bases are the same, and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{x^3}{x^{-2}} = x^{3-(-2)} = x^{3+2} = x^5$$

We also know that, when we have the quotient of two power terms, and the exponents are equal, that we can rewrite the expression as

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

as long as b is non-zero. Similarly,

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Let's do an example with these rules.

Example

Rewrite the expression.

$$\frac{x^3}{y^3}$$

The base of the expression in the numerator and the base of the expression in the denominator are not the same, so we can't use the quotient rule for exponents. But the exponents are the same, so

$$\frac{x^3}{y^3} = \left(\frac{x}{y}\right)^3$$

Let's do an example with negative exponents.

Example

Rewrite the expression.

$$\frac{x^{-5}}{y^{-5}}$$

The base of the expression in the numerator and the base of the expression in the denominator aren't the same, so we can't use the quotient rule for exponents. But the exponents are the same, so

$$\frac{x^{-5}}{y^{-5}} = \left(\frac{x}{y}\right)^{-5}$$

Then to make the exponent positive, we can take the reciprocal of the base.

$$\left(\frac{x}{y}\right)^{-5} = \left(\frac{y}{x}\right)^5$$

Zero exponent rule

The **zero exponent rule** tells us that, if a real number has an exponent equal to 0, then its value is equal to 1.

$$a^0 = 1$$

For example,

$$2^0 = 1$$