

Table 1: Increasing Order Of Growth

Type	Big-Oh	Asymptotic Order
Logarithmic:	$O(\log n)$	$\log n$
Poly Logarithmic:	$O((\log n)^2), O((\log n)^3), \dots$	$(\log n)^2 \leq (\log n)^3 \leq (\log n)^4 \leq \dots$
Fractional Power:	$O(n^c) \text{ for } 0 < c < 1$	$n^{0.1} \leq n^{0.2} \leq n^{0.3} \leq \dots$
Linear:	$O(n)$	n
$n \log n$ time:	$O(n \log n)$	$n \log n \leq n(\log n)^2 \leq n(\log n)^3 \leq \dots$
Polynomial Time:	$O(n^a) \text{ for } a > 1$	$n^2 \leq n^3 \leq \dots$
Exponential Time:	$O(2^n)$	$1.5^n \leq 2^n \leq 3^n \dots$

Definitions

Big O Notation: Will give us an upper bound for function $f(n)$ when n is very large/asymptotically. This is used extensively to describe the worst case scenario for the number of operations used by an algorithm. The notation used in the definition is: $O(g(n))$ which is read “big-oh of g of n ”

Definition:

Let $f(n) : N \rightarrow R+$. For a given function $g(n)$, we say that $f(n)$ is $O(g(n))$ if there are constants C and k such that:
 $f(n) \leq Cg(n)$ for all $n > k$

$$\log_2 n \leq n \text{ for all } n \geq 1$$

This inequality is true for all logarithms for any base $b > 0$

For a base $b > 1$ and any exponent $a > 0$ we have that $\log_b n \leq n^a$ for large enough n (for $n \geq k$ for some constant k)

Big Omega Notation Ω : The notation Big- ω is defined similar to that of Big-O but for a lower bound.

Definition:

Let $f(n)$ and $g(n)$ be functions on the natural numbers to the positive real numbers. We say that $f(n)$ is $\Omega(g(n))$ if there is a constant C such that $f(n) \geq Cg(n)$ whenever $n > k$

Big Theta Notation θ : Big- θ Notation is used to specify the function $f(n)$ that is sandwiched between multiples of $g(n)$. If being asked for Theta one must prove for Big O AND Big Omega **Definition:**

Let $f(n)$, and $g(n)$ be functions from the natural numbers to the positive reals. If $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$ then we say that $f(n)$ is of the order of $g(n)$ and use the notation $\theta(g(n))$

Formulas

Summation Formula:

$$\sum_{k=1}^n k = 1/2n(n+1) = \theta(n^2)$$

$$\sum_{k=1}^n 10k = 10n(n+1)/2 = \theta(n^2)$$

LOG N Formula: $\log N : 50 * \log n^3 \Leftrightarrow 150 * \log n$