Introduction to Set Theory Tutorial

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Welcome to my first tutorial session on Set Theory! My name is Joe. I want this to be an open, judgement-free space to ask questions and learn. I'd really appreciate any questions you have at any stage, so please, please feel free to interrupt me!

I'd like to begin by saying Set Theory as a separate mathematical discipline is a relatively modern branch of mathematics. It is also quite abstract, and it could take some time for it to sink in fully. It took me some time to appreciate why it exists and its inherent beauty, hopefully I can help make it easier for you to understand.

I know that this will be the first time many of you will have been exposed to Set Theory, so this tutorial sheet should be approached in conjunction with a brief slideshow I have prepared going through the very fundamentals of Set Theory. If you haven't yet got a copy of this please reach out!

I'd like to group people together and give you some time to work things out. Just remember that I'm here at any stage if you need a hand at all!

(a) Are the following statements True or False:

    (i) For all sets X, $\emptyset \subseteq X$ (reminder: check for $X = \emptyset$)

    (ii) $|\emptyset| = 0$

    (iii) $|\{\emptyset\}| = 0$

    (iv) If $A \subseteq B$ and $B \subseteq A$ then $A = B$

    (v) There exits a set X such that $|X| < 0$

(b) We define the <u>Cartesian Product</u> between two sets A and B as

$$A \times B = \{(x, y) \text{ where } x \in A \text{ and } y \in B\}$$

    (i) What is $A \times B$ where $A = \{1, 2, 3\}$ and $B = \{10, 11\}$?

    (ii) What is $C \times D$ where $C = \{\{1, 2\}, 3\}$ and $B = \{\emptyset, \{\emptyset\}\}$?

(c) What does $\mathbb{R} \times \mathbb{R}$ represent? Where have you seen this set before? (*note*: $\mathbb{R} \times \mathbb{R}$ is also often called $\mathbb{R}^2$)

(d) Express $|A \times B|$ in terms of $|A|$ and $|B|$

(e) We define the <u>Set Difference</u> between two sets A and B as

$$A - B = \{x \text{ where } x \in A \text{ and } x \notin B\}$$

Calculate the following Set Differences:

    (i) Let $A = \{1, 2, 3\}$ and B be the set of even numbers. What is $A - B$

    (ii) What is $\mathbb{Z} - \mathbb{N}$

(*note*: Set Difference is also commonly denoted by $A \setminus B$)

(f) We define the <u>Complement</u> of a set A with respect to a particular superset B (recall that a superset means that $A \subseteq B$)

$$A^{'} = \{x \text{ where } x \in B \text{ and } x \notin A\}$$

    (i) Let $A = \{1, 2, 3\}$ and $B = 1, 2, 3, 4, 5$, what is the complement of A with respect to B?

    (ii) What is the complement of $\mathbb{Q}$ with respect to the superset $\mathbb{R}$?

(*note*: the Complement of a set is also commonly denoted A^c)

(*question*: are Set Difference and Complement the same? If not, how do they differ?)

(g) **(Bonus)** Research and provide a definition of how "relations" and "functions" are defined in Set Theory. What is the difference between a relation and a function?