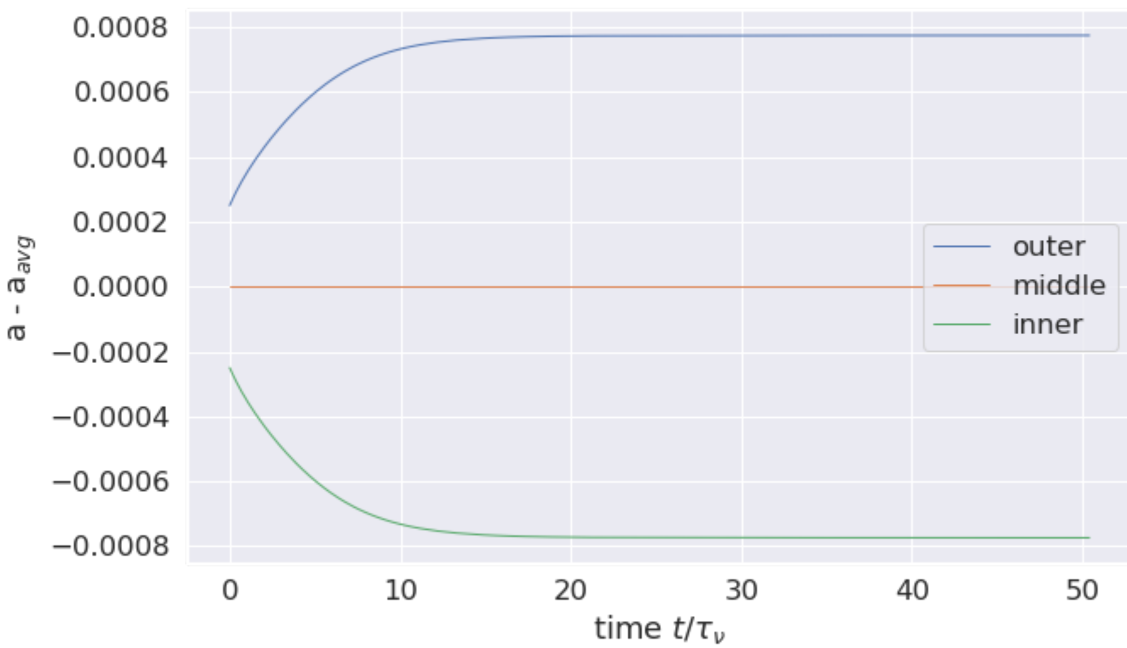
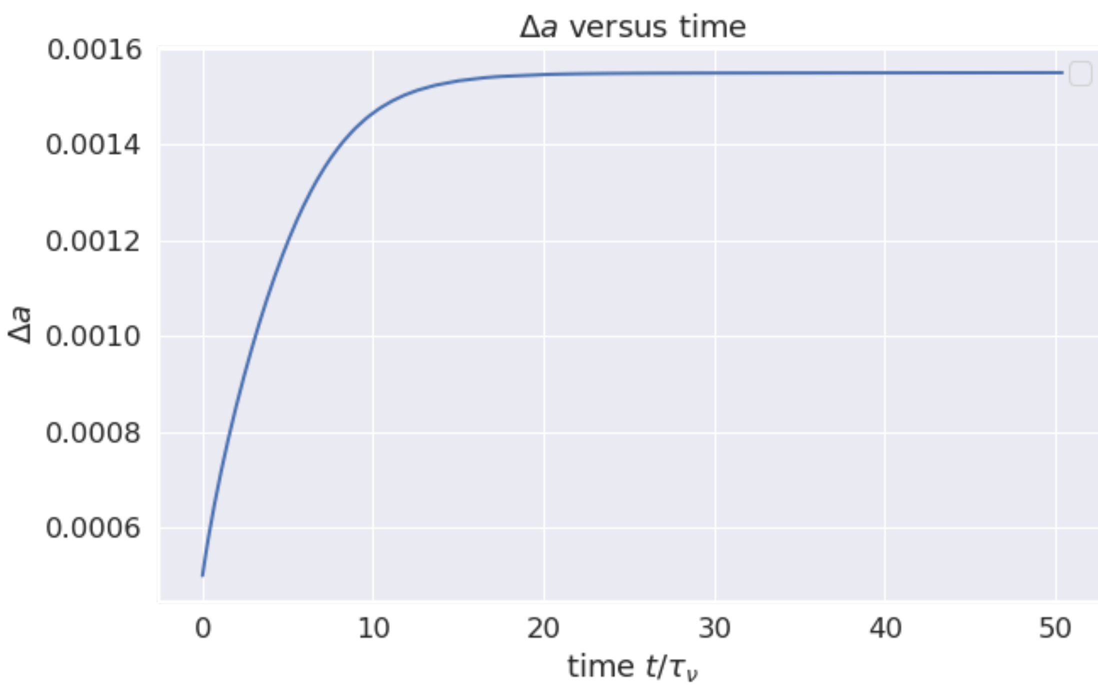
Results from an epi_int_lite Nbody simulation that computes the dynamical evolution of a viscous and self-gravitating planetary ringlet over time. This simulated ringlet is initially narrow with small eccentricity and zero eccentricity gradient.

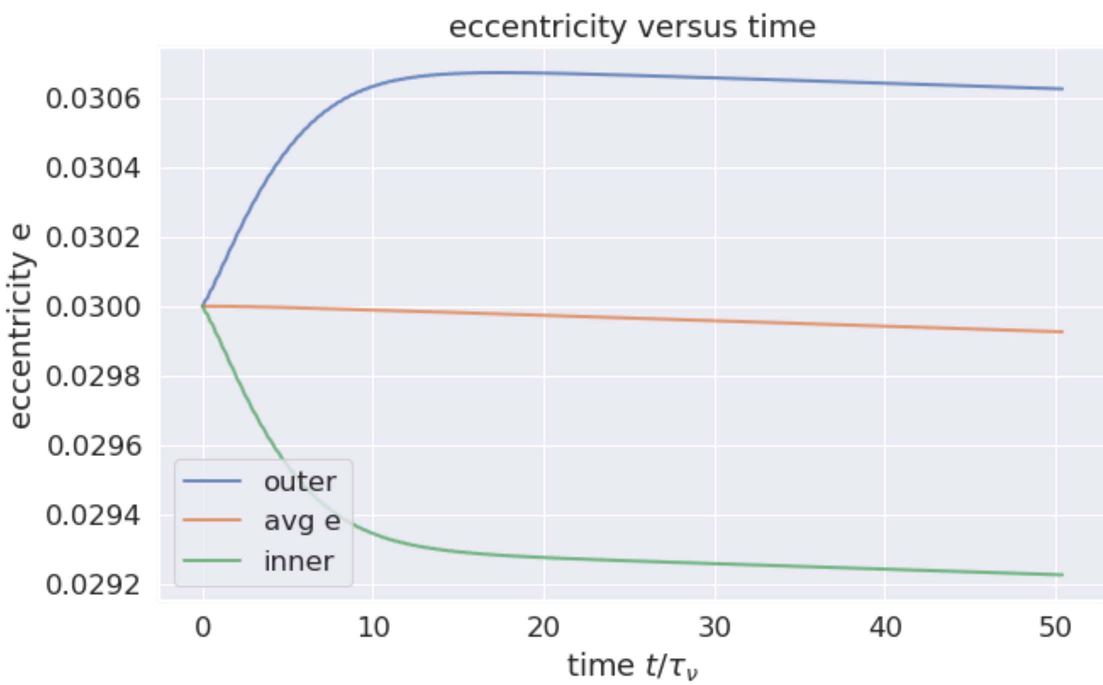
Plot below shows the ringlet's inner, outer, and mean semimajor axis a versus time t. The ringlet's initial radial spreading is driven by viscosity that causes the ringlet to spread over the viscous timescale $\tau_\nu = \Delta a^2/12\nu$.



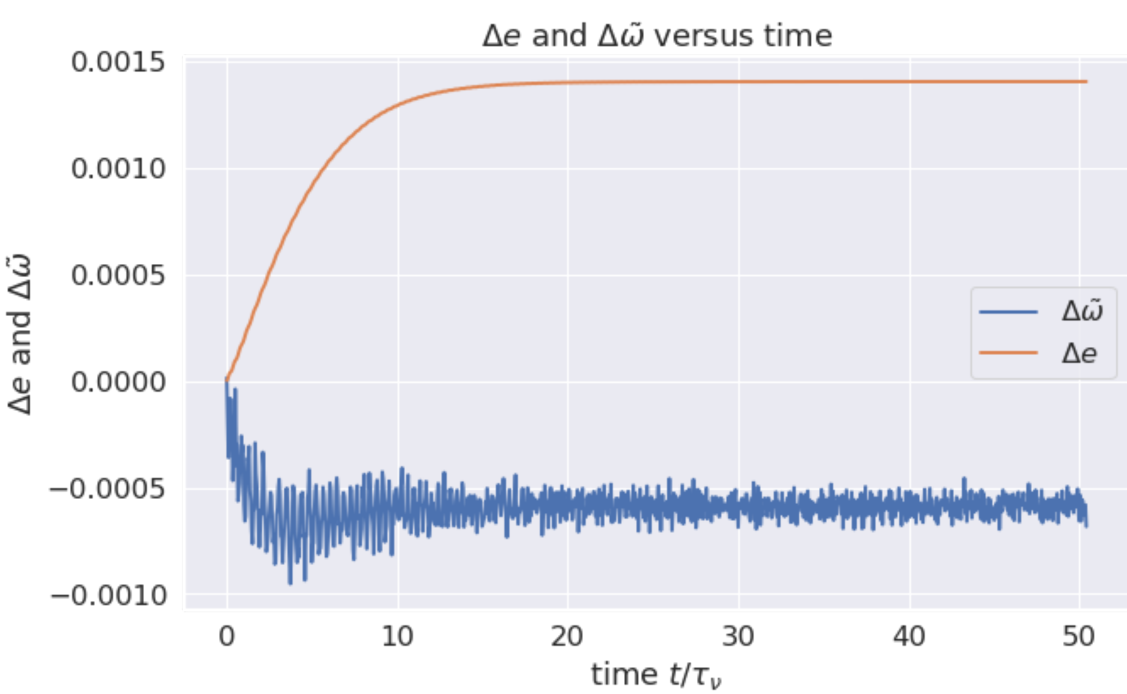
The same ringlet's semimajor axis width Δa versus time, noting that this ringlet has ceased its radial spreading (for reasons to be detailed below) after time $t \sim 20\tau_\nu$



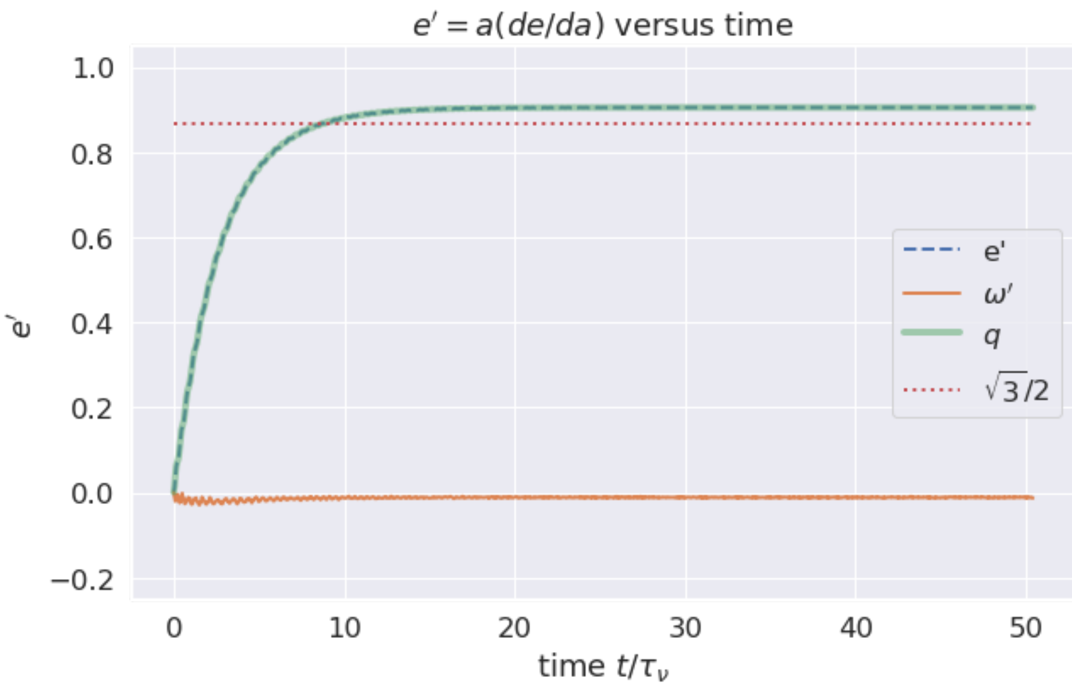
The ringlet's self gravity also causes the ringlet' eccentricity e to evolve over time such that its outer edge gets more eccentric at the expense of the inner edge, ie ringlet self gravity causes its eccentricity gradient $e' = a(de/da)$ to grow over time.



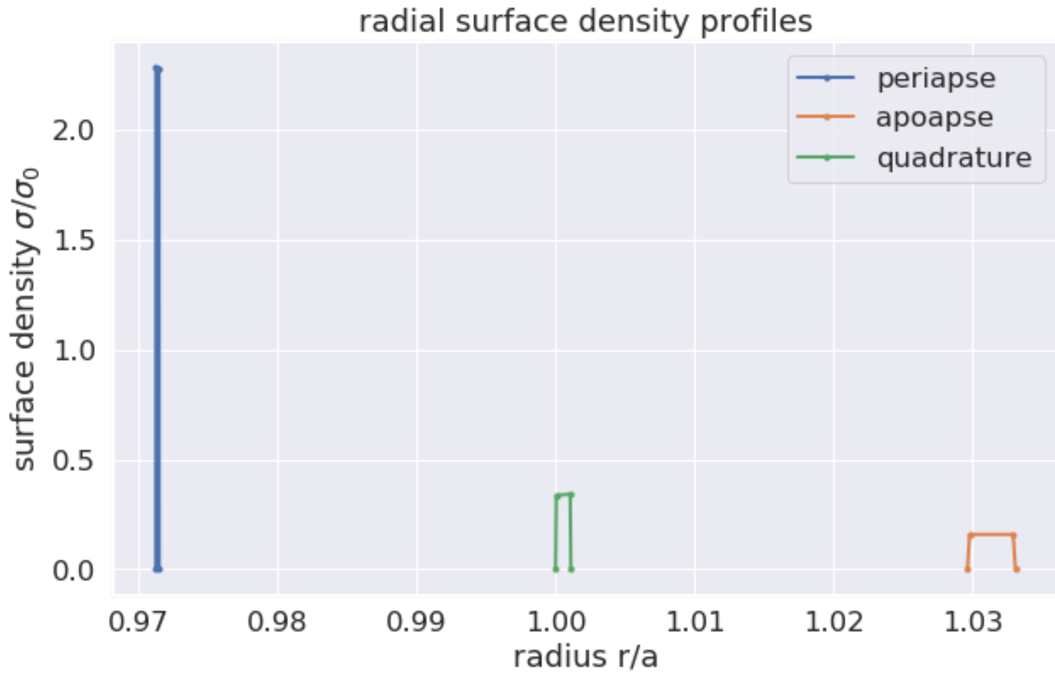
Plot below again shows that ringlet self gravity causes the eccentricity difference $\Delta e = e_{outer} - e_{inner}$ to initially grow over time. Ringlet viscosity (which is due to collisions among individual ring particles) also causes the ringlet's longitudes of periapse to develop a small trailing twist such that $\tilde{\omega}$ at ringlet's outer edge trails behind the inner edge. This viscous twist of the ringlet's longitudes of periapse $\tilde{\omega}$ also allows ringlet self gravity to exert a torque on itself, which is also assessed below.



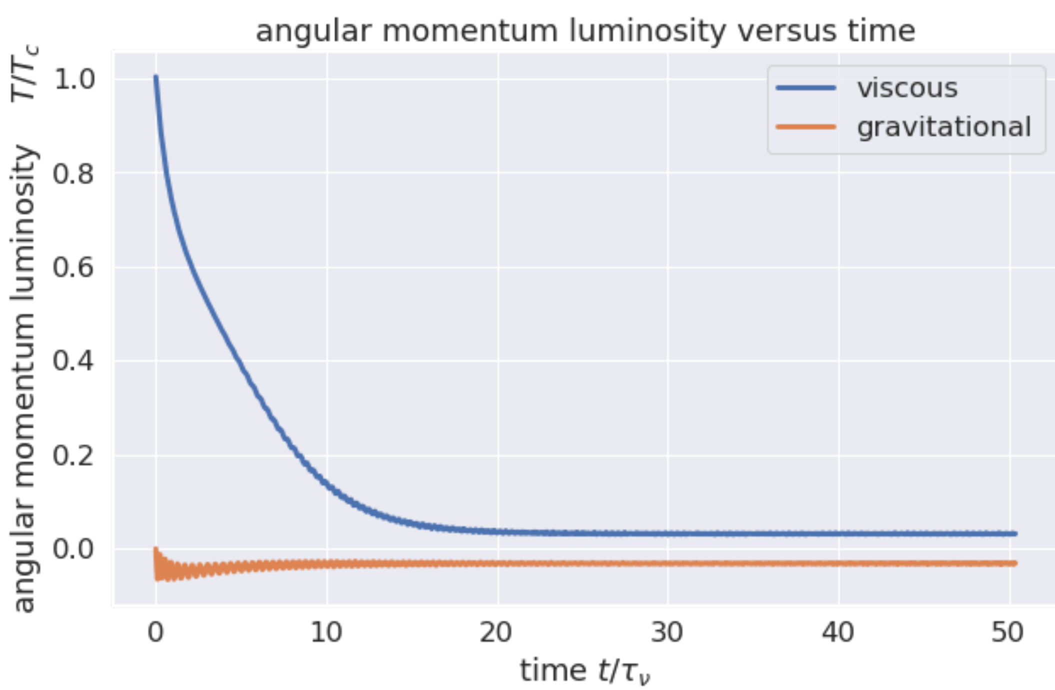
Self gravity causes ringlet's eccentricity gradient $e' = a(de/da)$ to increase to just above red line at $e' = \sqrt{3}/2 = 0.866$, at which point angular momentum flux reversal (which is derived in Borderies et al 1982 for non-gravitating ringlet) inhibits any further radial spreading. This simulation shows that ringlet self-gravity alters the angular momentum flux reversal threshold just above that derived in BGT1982.



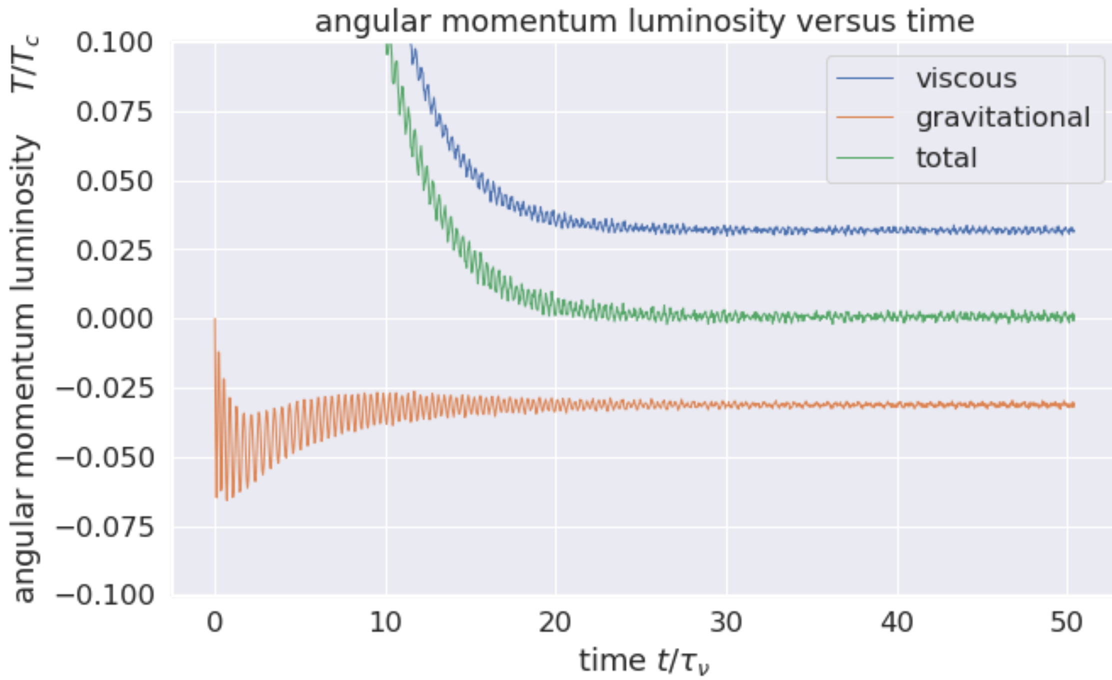
Radial cuts of ringlet surface density at the end of the simulation, when the viscous ringlet's eccentricity gradient e' is large such that angular momentum flux reversal has halted the ringlet's radial spreading. That large eccentricity gradient also results in a high contrast in the ringlet surface density, between periapse (where the ringlet is radially pinched) and apopase (where the ringlet radially broadened). Higher resolution simulations (such as that analyzed in this Jupyter notebook at https://github.com/joehahn/epi_int_lite/blob/master/runs/self_confining/number_of_streamlines/9/number_of_streamlines.ipynb (https://github.com/joehahn/epi_int_lite/blob/master/runs/self_confining/number_of_streamlines/9/number_of_streamlines.ipynb)) also show that these self-confining ringlets are sharp-edged, which is another property of all well-studied narrow eccentric planetary ringlets.



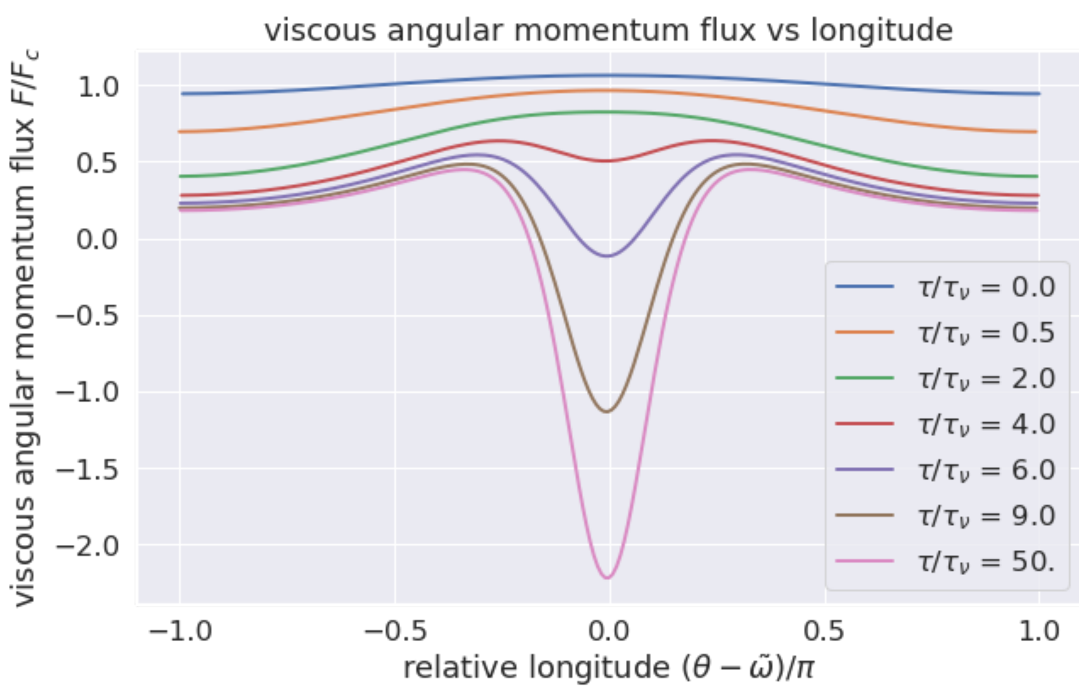
The ringlet's angular momentum luminosity versus time, which is the torque that a ringlet streamline exerts on it exterior neighbor. Note that the viscous angular momentum luminosity is positive (which indicates that viscous effects transmit angular momentum radially outwards through the ring) yet diminishes over time as the ringlet evolves into the self-confining state having large e'. Note also that the ringlet's gravitational angular momentum luminosity is negative (ie ringlet self gravity transmits angular momentum inwards).



The above plot is redrawn below to show that the ringlet's radial spreading has ceased when the ringlet's viscous and gravitational torques are balanced.



The following plot shows the ringlet's viscous angular momentum flux $F(\theta)$ versus longitude θ, at early time (blue curve when e' is small and $F(\theta)$ positive everwhere) and at late times (pink curve when e' is large and radial spreading has halted). Angular momentum flux reversal occurs where $F(\theta) < 0$ near ringlet's periapse. Also note that the viscous torque seen in the above plot is the integral $T = \oint F(\theta)rd\theta$, so that ringlet spreading ceases when e' is large enough and angular momentum flux reversal is strong enough such that that the integrated flux $T = \oint Frd\theta = 0$.



The preceeding plots show that a viscous ringlet's self-gravity can excite the ringlet eccentricity gradient e' sufficiently high such that angular momentum flux reversal is strong enough so that $T = \oint F(\theta)rd\theta = 0$. At this moment there is no net radial transport of angular momentum through the ring, so the ringlet is now self-confining and its radial spreading is halted.