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N-body simulations of the Self–Confinement of Viscous Self–Gravitating Narrow Eccentric Planetary Ringlets

Joseph M. Hahn,[1] Douglas P. Hamilton,[2] Thomas Rimlinger,[2] and Lucy Luu[2]

[1]*Space Science Institute*
[2]*University of Maryland*

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ABSTRACT

The following shows how narrow eccentric planetary ringlets can evolve into a self–confining state.

Keywords: editorials, notices — miscellaneous — catalogs — surveys — update, me

1. INTRODUCTION

Narrow eccentric planetary ringlets have properties both interesting and not well understood: sharp edges, sizable eccentricity gradients, and a confinement mechanism that inhibits radial spreading due to ring viscosity. Prevailing ringlet confinement mechanisms include: unseen shepherd satellites (reference), periapse pinch (ref), self gravity (ref), and self-confinement (ref). This study uses N-body simulations to show how narow self–gravitating ringlets can evolve into a self-confining state.

2. RINGLET CONFINEMENT MECHANISMS

This section explains the pros and cons of the various ringlet confinement mechanisms, and then motivates the possibility that ringlets are self confining. That possibility is explored further via numerical simulations using the epi_int_lite N-body integrator.

3. EPI_INT_LITE

Epi_int_lite is a child of the epi_int N-body integrator that was used to simulate the outer edge of Saturn's B ring that is sculpted by satellite perturbations (Hahn & Spitale 2013). The new code is very similar to its parent but differs in three significant ways: (*i.*) epi_int_lite is written in python and recoded for more efficient execution, (*ii.*) epi_int_lite uses a more accurate drift step for unperturbed motion around an oblate planet (detailed in Appendix A), and (*iii.*) epi_int_lite uses the $C = 1$ approximation that is justified below (Appendix B).

Otherwise epi_int_lite's treatment of ring self–gravity and viscosity are identical to that used by the parent code; see Hahn & Spitale (2013) for additional details. The epi_int_lite source code is available

Corresponding author: Joseph M. Hahn
jhahn@spacescience.org

at https://github.com/joehahn/epi_int_lite, and the code's numerical quality is assessed in Appendix C where the output of several numerical experiments are compared against theoretical predictions.

Calculations performed by epi_int_lite use natural units with gravitation constant $G = 1$, central primary mass $M = 1$, and the ringlet's inner edge has initial radius $r_0 = 1$, and so the ringlet masses m_r and radii r quoted below are in units of M and r_0. Converting code output from natural units to physical units requires choosing physical values for M and r_0 and multiplying accordingly, and when this text does so it assumes the primary's mass is Saturn's, $M = 5.68 \times 10^{29}$ gm, and a typical ring radius of $r_0 = 1.0 \times 10^{10}$ cm.

Describe streamlines and forces are computed from such. Many of the runs described below employ only $N_s = 2$ streamlines, so that the model output can be benchmarked against theoretical treatments that also treated the planetary ringlet as two gravitating rings (e.g. BGTXX).

4. N-BODY SIMULATIONS OF VISCOUS GRAVITATING RINGLETS

This Section describes a suite of N-body simulations of narrow viscous gravitating planetary ringlets, to highlight the range of initial ringlet conditions the do evolve into a self-confining state, and those that do not.

4.1. *nominal model*

Figure 1 shows the semimajor axis evolution of what is referred to as the nominal model since this ringlet readily evolves into a self-confining state. The simulated ringlet is composed of $N_s = 2$ streamlines having $N_p = 241$ particles per streamline, and the integrator timestep is $\Delta t = 0.5$ in natural units, so the integrator samples the particle orbits $2\pi/\Delta t \simeq 13$ times per orbit, and this ringlet is evolved for 6.7×10^5 orbits, which requires 40 minutes execution time on a 5 year old laptop. The ringlet's mass is $m_r = 2 \times 10^{-9}$, its shear viscosity is $\nu_s = 1 \times 10^{-12}$, and its bulk viscosity is $\nu_b = 1.5\nu_s$. The ringet's initial radial width is $\Delta a = 5 \times 10^{-4}$, its initial eccentricity is $e = 0.03$, and its eccentricity gradient is initially zero. A convenient measure of time is the ringlet's viscous radial spreading timescale, $\tau_\nu = \Delta a^2/12\nu_s$, which is the time for viscosity to double the radial width of an initially narrow circular ringlet (Pringle 1981?). This simulation's viscous timescale is $\tau_\nu = 2.1 \times 10^4$ in natural units so $\tau_\nu/2\pi = 3.3 \times 10^3$ orbit periods. If this ringlet were orbiting Saturn at $r_0 = 1.0 \times 10^{10}$ cm then the simulated ringlet's physical mass would be $m_r = 1.1 \times 10^{21}$ gm which is equivalent to a $R = 64$ km iceball assuming a volume density $\rho = 1$ gm/cm^3, and the ringlet's initial radial width would be $\Delta a = 5 \times 10^{-4} r_0 = 50$ km. This ringlet's orbit period would be $T_{orb} = 2\pi/\sqrt{r_0^3/GM} = 9.0$ hours in physical units, so the ringlet's viscous timescale is $\tau_\nu = 3.4$ years and implies a ring viscosity $\nu_s = \Delta a^2/12\tau_\nu = 1.9 \times 10^4$ cm^2/sec when evaluated in physical units. This ringlet's initial surface density would be $\sigma = m_r/2\pi r_0 \Delta a = 3500$ gm/cm^2, but that shrinks by a factor of 3 as the ringlet's radial width grows (due to viscous spreading) until it settles into the self confining state, see Fig. 2. So our so-called nominal model is probably overdense and overly viscous compared to known planetary rings, but the suite of simulation described in Sections XX show how this outcome scale when other initial masses and viscosities are considered.

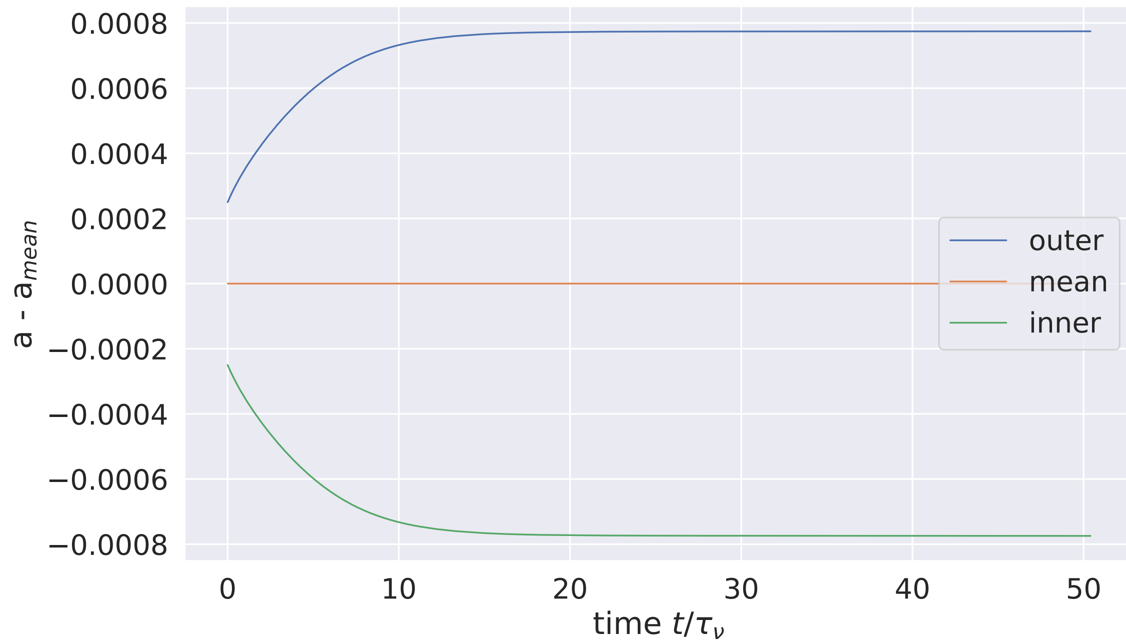

Figure 1. Semimajor axis evolution of the nominal ringlet model having mass $m_r = 2 \times 10^{-9}$, shear viscosity $\nu_s = 1 \times 10^{-12}$, initial width $\Delta a = 5 \times 10^{-4}$ and initial eccentricity $e = 0.03$. Blue and green curves show the outer and inner streamline's semimajor axis relative to their mean a.

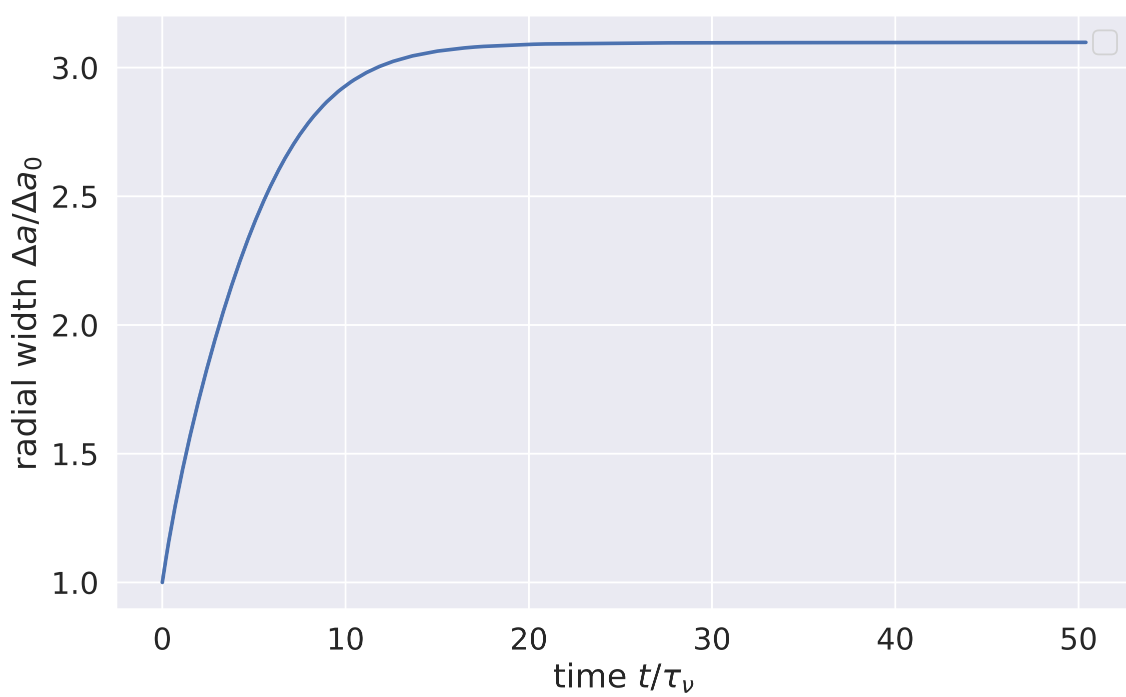

Figure 2. The nominal ringlet's radial width over time in units of its initial radial width Δa_0.

acknowledgments...

APPENDIX

A. APPENDIX A

Derive the more accurate drift step used by epi_int_lite...

B. APPENDIX B

Detail the $C = 1$ approximation used by epi_int_lite, and show that the errors associated with this approximation are negligable...

C. APPENDIX C

Compare epi_int_lite to theoretical predictions

C.1. *radial spreading of viscous viscous*

Show that ringlet viscosity causes circular non-gravitating ringlet to spread at the expected rate...

REFERENCES

Hahn, J. M., & Spitale, J. N. 2013, ApJ, 772, 122