# The Anticompetitiveness of a Private Information Exchange of Prices\*

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#### Abstract

Competitors privately sharing *price intentions* is universally prohibited under antitrust/competition law. In contrast, there is no common well-accepted treatment of competitors privately sharing *prices*. This paper is the first to show that a private exchange of prices results in higher prices for consumers.

<sup>\*</sup>I appreciate the comments of David Myatt, Dan Stone, and Juuso Toikka. This paper is a revision of SSRN Working Paper 3621073. A motivating case used in this paper is the EU trucks cartel for which I acted as a consulting expert for plaintiffs in Germany and The Netherlands.

#### 1 Introduction

Firms sharing price intentions towards agreeing to the prices they will charge is universally condemned. In most jurisdictions, such conduct is prohibited because it so fundamentally interferes with the competitive process and lacks any credible procompetitive benefits. Express communications among firms about their price intentions is unlawful by the per se rule in the United States (under Section 1 of the Sherman Act) and by object in the European Union (under Article 101 of the Treaty of the Functioning of the European Union).<sup>1</sup>

While competition law is unequivocal regarding the sharing of *price intentions*, there is not a common well-accepted position with respect to the sharing of *prices*. In the U.S., the exchange of prices - and even an agreement among firms to exchange prices - is not outright prohibited:

The exchange of price data ... among competitors does not invariably have anticompetitive effects; indeed such practices can in certain circumstances increase economic efficiency and render markets more, rather than less, competitive. For this reason, we have held that such exchanges of information do not constitute a per se violation of the Sherman Act.<sup>2</sup>

The sharing of prices is evaluated under the rule of reason. In the market for corrugated containers, firms were found guilty because the sharing of prices was shown to have an effect on prices.<sup>3</sup> In the EU, the exchange of prices comes under Article 101 as a concerted practice. To establish a concerted practice,

it suffices for those concerned to inform each other of the amount of charges actually imposed by them or contemplated for the future; for the object or effects of such contacts is to influence the level of the charges imposed by the competitor or, at least, to eliminate uncertainty on the part of the competitor as to the level of charges imposed by the first party.<sup>4</sup>

The EU is less tolerant than the U.S. when it comes to firms sharing prices. The European Commission has taken the position that "mere attendance at a meeting where an undertaking discloses its confidential pricing plans to its competitors is likely to be caught by Article 101(1)." This view has been put into their guidelines which state: "information exchange can constitute a concerted practice if it reduces strategic uncertainty ... because it reduces the independence of competitors' conduct and diminishes their incentives to compete."

The lack of a well-accepted treatment of the sharing of prices is at least partly due to the absence of a well-established theory of harm. This missing theory of harm is exemplified

<sup>&</sup>lt;sup>1</sup>For background information on the economics and law of price fixing, I refer the reader to Motta (2004) for the EU and Kaplow (2013) for the U.S..

<sup>&</sup>lt;sup>2</sup> United States v. United States Gypsum Co., 438 U.S. 422, 441 n. 16 (1978)

<sup>&</sup>lt;sup>3</sup> United States v. Container Corp. of Am., 393 U.S. 333 (1969).

<sup>&</sup>lt;sup>4</sup>Case 172/80, Züchner v. Bayerische Vereinsank, Judgment, E.C.R., 2027 (14 July 1981)

<sup>&</sup>lt;sup>5</sup>Whish and Bailey (2018), p. 117.

<sup>&</sup>lt;sup>6</sup>European Commission, "Guidelines on the Applicability of Article 101 of the Treaty on the Functioning of the European Union to Horizontal Co-operation Agreements," 2011, para. 61.

by the EU's view that the exchange of prices is potentially harmful because it "reduces strategic uncertainty." However, there is no theoretical or empirical bases upon which to conclude that less "strategic uncertainty" among firms implies higher prices for consumers. In the case of the U.S., an agreement to share prices is not per se illegal because it is claimed there can be procompetitive benefits, though there are no economic studies showing either the procompetitive benefits or the anticompetitive harm from such an information exchange.

The relevance of information exchanges regarding prices is exemplified by the recent trucks cartel case in the EU.<sup>7</sup> It was documented that executives of truck manufacturers regularly met and shared gross list prices. A gross list price is a price used for internal purposes and is part of the pricing process that eventually determines the prices charged to dealers and final purchasers. While the European Commission ruled that the truck manufacturers were in violation of Article 101, private litigation is currently seeking to determine the amount of harm to final purchasers. That amount hinges on the theory of harm which depends on what information was actually conveyed at those meetings. Claims range from executives exchanging gross list prices to discussing gross list prices to agreeing on gross list prices.

All of the Addressees exchanged gross price lists and information on gross prices [which] constituted commercially sensitive information. ... [T]he Addressees participated in meetings involving senior managers of all Headquarters [where] ... the participants discussed and in some cases also agreed their respective gross price increases.<sup>8</sup>

While it is commonly recognized that there is harm if it can be shown they *agreed* on prices, that is not the case when all that can be shown is that they *exchanged* prices.

Motivated by these issues, this research project addresses two questions. First, is it harmful to consumers when firms privately share their prices? Second, if it is harmful, should the sharing of prices be subject to the per se rule (by object) or rule of reason (by effect)? This paper addresses the first question by developing a theory of harm associated with the private sharing of prices and delivers conditions when such an information exchange raises prices. Thus, the first question is answered in the affirmative. The second question will be addressed in a companion policy paper where I argue for the following per se prohibition: Competitors are prohibited from privately communicating prices relevant to transactions that have not yet occurred.

In order to be able to draw general policy conclusions, it is important that the theory of harm is robust and relies on some fundamental forces. Towards that end, a parsimonious model is developed with two distinctive features which I believe are ubiquitous and compelling. The first feature is that firms (or, more specifically, their executives) are not sharing transaction prices. They are instead sharing prices that may ultimately affect the prices that consumers pay but are not necessarily the final prices that will be put before consumers. In the trucks case, manufacturers shared list prices, and list prices would be expected to affect

 $<sup>^7\</sup>mathrm{Commission}$  Decision of 19.7.2016 relating to a proceeding under Article 101 of the TFEA and Article 53 of the EEA Agreement, AT.39824 - Trucks

<sup>&</sup>lt;sup>8</sup> *Ibid*, para. 46, 51.

dealer prices which would then affect the prices paid by final purchasers. Or consider executives of retail chains sharing posted prices. Again, the prices faced by consumers could be different from the posted price through the offering of discount coupons or rebates. The critical property is that the prices shared may influence the transaction price but, at the time of the information exchange, those transaction prices are not yet determined. The second feature is that the executives who are sharing prices may be able to influence transaction prices but do not have full control over them. This feature differs from the standard economic model of the firm, which has a single decision-maker choosing prices, but is very much consistent with actual practice. Generally, prices are the result of a process within a firm involving various employees with different responsibilities. As this second feature is crucial to the theory of harm, Section 3 reviews some of the evidence supporting it.

The way those two features are encompassed in the model of this paper is stylized but the insight it yields is intuitive and robust (in the sense that it only relies on some general assumptions). The sharing of prices by firms is captured with a two-stage structure. Firms first choose prices and then, after sharing those prices, can change the price at some cost. To relate this structure to, for example, the trucks case, suppose an executive chooses a list price. In the absence of sharing list prices, a list price would lead to some final price for purchasers according to the firm's internal pricing process. When executives of truck manufacturers share their list prices, knowledge of other manufacturers' list prices may induce an executive to intervene in the internal process by which the list price affects the final purchaser price. However, such intervention is costly to the executive as other employees must be convinced of the price change or be incentivized to make it. That cost to changing price captures the limited ability of the executive to influence the final price. Though the structure is stylized, in that it subsumes the internal pricing process through an adjustment cost for price, it has the appealing feature that it is not dependent on a particular modeling of that internal pricing process.

The main finding is that a private information exchange of prices by competitors is harmful to consumers when the cost of adjusting price is neither too small nor too large which can be interpreted as the colluding executives having some but not full control over the final price. The intuition is as follows. When executives privately share prices, they are given the opportunity to effectively change the prices they will be offering to consumers before consumers have an opportunity to transact. However, this sharing of prices would be of no consequence when it is near-costless to change prices. In that case, the information exchange is approximately cheap talk so each executive would simply set the final price to maximize profits regardless of what was learned about rivals' prices due to sharing. Sharing of prices would also have no effect when it is very costly to change prices. For example, suppose executives share list prices and, after sharing them, each executive is either unable to change its list price or finds it very difficult to intervene in the process determining the discounts provided off of the list price. Again, the information exchange would have no effect. However, when the executive has some limited control - as reflected in a moderately-valued adjustment cost to price - sharing prices is anticompetitive. When they have an agreement

<sup>&</sup>lt;sup>9</sup>Of course, express communication which is cheap talk can allow coordination on prices. That is the usual avenue for harm and is interpreted as firms discussing and agreeing to prices. Our analysis focuses on when firms are only sharing prices.

to share prices, an executive will set a supracompetitive price because the executive knows that, should s/he learn that other firms have set lower prices, they will have the opportunity and means to respond by lowering price. This anticipation that other firms would respond to a low price incentivizes executives to select and share supracompetitive prices.

As will become clear, the theory is an adaptation of a well-understood strategic pricing mechanism exemplified by most-favored-customer clauses. From a purely theoretical perspective, the paper's contribution is modest. From the practical perspective of understanding firm conduct and providing guidance for courts, the paper's contribution is more significant. The paper's insight is in identifying how this strategic mechanism is relevant to a firm's internal pricing process and how it can be used to evaluate information exchanges.<sup>10</sup>

Section 2 discusses related research. Section 3 offers some evidence regarding company pricing processes which motivates the model. Section 4 provides the simplest structure for showing that a private information exchange of prices can be harmful to consumers. Section 5 shows that the conclusions from that simple model are general as the anticompetitive effect of price sharing holds for the canonical price game with differentiated products. For the case when firms share list prices, Section 6 provides a more explicit model which links list prices to the prices paid by consumers. Section 7 concludes.

### 2 Literature Review

The model of this paper assumes firms make sequential price decisions and, due to a cost to adjusting price, the initial price constrains the final price. This strategic pricing mechanism was originally identified in most-favored-customer (MFC) clauses where the "initial" price is the price charged to customers in one period and the "final" price is the price charged to customers in a later period (Cooper, 1986; Salop, 1986). A MFC clause binds a firm to charging all of its customers the lowest price that any of its customers receive. Thus, if a firm's later price is lower than its earlier price, it must reimburse the difference to its earlier buyers. That is the "cost" to setting a lower "final" price. As described later, the cost of changing price in our model has a different structure from that in the MFC clause literature.

There is some related research assuming a two-stage pricing structure in which list prices are chosen in stage 1 and final prices (or, equivalently, discounts off of list price) in stage 2. In García Díaz, Hernán González, and Kujal (2009), the selection of prices in stage 2 is assumed to be costless, while it is costly in this paper's model and that cost is crucial to the results. There are also some papers, such as Raskovich (2007) and Gill and Thanassoulis (2016), examining how list prices with discounts for some buyers can be used for purposes of price discrimination. Price discrimination is not present in this paper's model. Lubensky (2017) and Harrington and Ye (2019) consider list (or manufacturer recommended) prices as cheap talk signals of cost where transaction prices are retail prices and negotiated prices, respectively. List prices affect search which then impacts transaction prices. In contrast, the model here is one of complete information without search.

Finally, there are two papers that find firms privately sharing prices is procompetitive.

<sup>&</sup>lt;sup>10</sup>As some evidence of its practical relevance, the working paper comprising this theory (Harrington, 2020) was influential and widely quoted in a recent decision by the Amsterdam District Court (12 May 2021) regarding customer damage litigation against members of the trucks cartel.

Myatt and Ronayne (2019) consider a variant of the model in Varian (1980) with list and final prices. It is shown that an information exchange of list prices - so each firm know its rival's list price at the time it chooses its final price (which is its list price less the discount) - can lower average transaction prices. Though sharing list prices does benefit consumers, this theory is *not* an argument against prohibiting the private sharing of list prices. For if there was such a prohibition, the firms could avoid it by instead *publicly* sharing their list prices and the same procompetitive effect arises. Myatt and Ronayne (2019) then provide a rationale for allowing firms to publicly share list prices.

Andreu, Neven, and Piccolo (2020) consider a duopoly setting in which each firm comprises two levels: an upper division which is interested in maximizing the firm's profit, and a lower division which is interested in maximizing a weighted average of profit and output. The list price comes out of the upper division and, after learning the list price, the lower division decides on the discount to offer. Consumers do not observe list prices. When firms share their list prices - so a lower division knows its rival's list price when it chooses the discount - average discounts can be higher compared to the absence of such an information exchange. From this result, the authors conclude: "agreements according to which firms disclose list prices to their competitors should be presumed neither as anti-competitive nor as pro-competitive." However, the theory is not credible due to some unorthodox and unrealistic modelling assumptions. To begin, it is assumed list prices are determined by an arbitrary stochastic process: equalling the consumer's valuation with probability 1/2 and the firm's cost with probability 1/2. It is also assumed that a consumer's purchasing decision depends on a firm's discount and not on the price that a consumer would actually pay. This assumption has the counterfactual property that a firm with a higher net price would have more demand than its rival as long as the firm's discount is higher. Given these assumptions are contrary to actual practice and common sense, the theory is not a sound basis for drawing conclusions about the effect of a private information exchange.

### 3 Evidence on the Internal Pricing Process

Our approach is rooted in two implicit assumptions. First, the colluding executives have a large influence on the prices that they will be sharing. In our model, this will be approximated by assuming they have full control over them. Second, after sharing prices, those executives have limited influence when it comes to either changing the prices that were shared or influencing the internal pricing process that translates those prices (such as list prices) into the prices faced by consumers (which encompass any discounts off of the list price). In our model, this second assumption is modelled by assuming an executive can change the subsequent prices but only at a cost. This cost captures having to go outside of standard protocol - such as changing the list price which the firm had already decided upon - or exerting pressure on other employees who have more authority over the subsequent pricing process - such as sales managers who control discounts. Towards substantiating these claims about the internal pricing process, we draw on several case studies. While our theory does not exclusively pertain to when executives share list prices, that is the most likely application of it and industrial markets have been the focus of the studies we have been able to

<sup>&</sup>lt;sup>11</sup>Quotation is from the abstract of Andreu, Neven, and Piccolo (2020).

find. Consequently, our discussion will consider the internal process by which list prices and discounts are determined.

"Research indicates that the pricing of products is a costly and complex activity" <sup>12</sup> because it encompasses many employees from different parts of the organization who bring in different expertise and information.

One approach that many companies find effective is to establish a multidisciplinary pricing council as a venue for offering input, discussing issues, and setting policies. Such a council, typically headed by the executive leading the pricing organization, may include representatives from different functions, geographies, business units, product lines, or any other stakeholder group that plays a significant role in pricing.<sup>13</sup>

Authority over the list price may reside at a high level where the marketing division plays an important role, while discounts are apt to be controlled by the sales division.

The pricing activities were run by a vice president [and] the pricing director and the sales director worked for him. The pricing director managed the pricing manager and several pricing analysts who prepared the price list and reviewed pricing decisions in the field. The sales director managed the sales force . . . As in many industrial settings, the firm used both list and negotiated prices, so the pricing process worked sequentially from marketing to sales. Pricing activities began with a price list, which was set annually . . . The marketing group set list prices, standard discount structures, and procedures for handling exceptions. The sales group then negotiated discounts for individual bids. <sup>14</sup>

That this pricing process is costly in terms of resources is well documented. The following statement refers to the adjustment of price that occurs on a routine (e.g., annual) basis.

The managerial costs of price adjustment increase with the size of the adjustment because the decision and internal communication costs are higher for larger price changes. First, the increased costs occur because more people are involved. . . . Second, the increased costs occur because larger price changes lead to more internal discussions. . . . Third, the increased cost occurs because larger price changes lead to more attention and controversy. <sup>15</sup>

The determination of a list price can take several months which means that an executive could find it difficult, though still possible, to change the list price. The executive may have the authority to do so but find it costly because it means going outside of standard protocol. Again, the reference in the ensuing passage is to the routine change in the list price.

<sup>&</sup>lt;sup>12</sup>Hallberg (2017), p. 179.

<sup>&</sup>lt;sup>13</sup>Simonetto et al (2012), p. 845.

<sup>&</sup>lt;sup>14</sup>Zbaracki and Bergen (2010), p. 958.

<sup>&</sup>lt;sup>15</sup>Zbaracki et al (2004), p. 524.

Changing the list price takes place over a period of several months. ... Once the list-price changes are determined, they must be communicated to the sales force. This requires group meetings with members of the pricing team, senior managers, territory managers, and the field sales force. ... The internal communication costs, therefore, involve the time and the effort for pricing managers need to spend informing the sales force about the motives behind the price change. <sup>16</sup>

Given the length of time to decide on a new list price and then inform and explain it to other company employees, it could be a costly task for an executive to subsequently change it.

If an executive believed that the firm's new list price was not competitive, an alternative to lowering the list price is to have larger discounts off of the list price. However, it could prove costly for the executive to make such a change when the standard operating procedure gives authority over discounts to other members of the organization.

[T]wo key dimensions of the organizational structure of pricing authority [are] the vertical delegation of authority over tactical pricing decisions within sales and the horizontal dispersion of authority over strategic pricing decisions across sales, marketing, and finance.<sup>17</sup>

Three different set-ups regarding pricing authority were identified: (1) pricing authority held by a sales and marketing manager, (2) pricing authority held by key account managers or internal sale reps, and (3) pricing authority held by external sales reps.<sup>18</sup>

[In our sample], 61% of the firms [gave] limited pricing authority to their salespeople. Here, salespeople are allowed to set prices within a pre-specified range. . . . 11% [gave] their salespeople with full pricing authority. In these cases, salespeople are given the freedom to set any price above marginal cost. 19

There are two takeaways from these case studies. First, list prices are determined by a lengthy process involving various employees. While it is quite plausible that a high-ranking executive would have a large influence on the list price, it would be difficult, though presumably possible, for them to later change the list price which emerged from that process. Second, the authority for setting discounts off of the list price often lies with employees who are distinct from those involved in the setting of list prices. It could then be difficult, though presumably possible, for a high-ranking executive to intervene in the setting of discounts.

## 4 Analysis of a Model with Two Prices

Consider a duopoly in which firms offer differentiated products and choose prices.  $\pi_i(p_1, p_2)$  denotes the profit of firm  $i \in \{1, 2\}$  where prices are  $(p_1, p_2)$ . Departing from the standard formulation,  $p_i$  is to be interpreted as the price that is to be shared. If that price proves to be

<sup>&</sup>lt;sup>16</sup>Zbaracki et al (2004), pp. 517, 519,

<sup>&</sup>lt;sup>17</sup>Homburg, Jensen, and Hahn (2012), p. 49

<sup>&</sup>lt;sup>18</sup>Hallberg (2017), p. 185.

<sup>&</sup>lt;sup>19</sup>Hansen, Joseph, and Krafft (2008), p. 95.

the price that consumers face then it is consistent with the standard formulation. However,  $p_i$  could instead be, say, a list price, and the price at which consumers transact may be that list price or something less due to the offering of discounts. In that case,  $\pi_i(p_1, p_2)$  is to be interpreted as the profit that firm i expects to receive when firms' list prices are  $(p_1, p_2)$ . Hence,  $\pi_i(p_1, p_2)$  implicitly embeds the process by which a list price is translated into the price a consumer faces. A more foundational approach would explicitly model that process but, in doing so, generality would be lost because results would be tied to the particular specification of that process. In order to derive general results appropriate for drawing policy recommendations and judicial guidance, I have sought to derive results based on a minimal set of assumptions.

Suppose each firm has two possible prices,  $\{L, H\}$ , and H > L. The profit function is symmetric across firms:

$$\pi_1(p', p'') = \pi_2(p'', p'), \ \forall (p', p'') \in \{L, H\}^2.$$

The low price strictly dominates the high price:

$$\pi_1(L, p_2) > \pi_1(H, p_2), \ \forall p_2 \in \{L, H\},$$

but each firm earns higher profit when both price high compared to when both price low:  $\pi_1(H, H) > \pi_1(L, L)$ . Finally, the profit function has increasing differences:<sup>20</sup>

$$\pi_1(H, H) - \pi_1(L, H) > \pi_1(H, L) - \pi_1(L, L).$$
 (1)

As each firm has a dominant strategy, the unique Nash equilibrium is for both firms to set a low price. That is the competitive outcome when there is no information exchange.

Let us now modify the game so that firms share prices. In the first stage, firms simultaneously choose initial prices, where the initial price for firm i is denoted  $p_i^I \in \{L, H\}$ . In the second stage, firms share their initial prices and then simultaneously choose final prices, denoted  $p_i^F$  for firm i. Consumers' transactions are based on the final price. If firms are sharing posted prices then the initial price is the posted price and the final price is the posted price less any discount or rebate. The interpretation is more subtle when firms are sharing list prices in a market where prices are negotiated. In that situation, the list price influences transaction prices but is not itself the transaction price. We can think of the initial price as the original list price selected by the firm. After executives share those list prices, an executive can choose to effectively change its list price where the final price selected in stage 2 is the new "effective" list price. This could mean literally changing the list price or, in situations where the executive is incapable of changing the list price, it could mean intervening in the internal pricing process that maps the list price to the price faced by a consumer so it is "as if" the list price equalled the final price. Whether the executive is changing the list price or altering the process mapping the list price into the transaction price, it is assumed this price adjustment is costly to the executive. k > 0 is the cost incurred when the final price differs from the initial price. This cost to changing the price is the critical departure from previous models and is motivated by the evidence reviewed in Section 3.

<sup>&</sup>lt;sup>20</sup>This property commonly holds for price games with differentiated products. See Chapter 6 in Vives (1999).

Firm 1's payoff function is

$$\begin{cases} \pi_1 \left( p_1^F, p_2^F \right) & \text{if } p_1^F = p_1^I \\ \\ \pi_1 \left( p_1^F, p_2^F \right) - k & \text{if } p_1^F \neq p_1^I \end{cases}$$

with firm 2's payoff function analogously defined. Gross profits are based on final prices. This structure gives substance to the idea that, at the time that executives share prices, there is some level of commitment to those prices - as captured by the cost k to changing them - but the prices faced by consumers are not yet determined - in that the price which affects a firm's demand and profits can be changed.<sup>21</sup>

For the two-stage game, first note that it is a subgame perfect equilibrium (SPE) outcome for both firms to set low and final initial prices. Given low initial prices, it is strictly dominant for a firm to set a low final price. Turning to the initial stage and given firm 1 expects  $p_2^I = L$ , its payoff is  $\pi_1(L, L)$  from  $p_1^I = L$  and is max  $\{\pi_1(L, L) - k, \pi_1(H, L)\}$  from  $p_1^I = H$ , which is strictly lower. Thus, as in the case when initial prices are not shared, it is an equilibrium outcome for both firms to set low final prices.

Under certain conditions, it is also a SPE outcome for both firms to choose high initial prices. Consider the following symmetric strategy profile: i)  $p_1^I = H$ ; ii)  $p_1^F = H$  if  $(p_1^I, p_2^I) = (H, H)$ , and  $p_1^F = L$  if  $(p_1^I, p_2^I) \neq (H, H)$ . Thus, a firm sets a high initial price and does not change it when both firms set high initial prices, but chooses a low final price otherwise (and, in particular, when the other firm chose a low initial price).

In establishing conditions whereby this strategy pair is a SPE, consider the four possible stage 2 subgames.

- 1.  $(p_1^I, p_2^I) = (H, H)$ .  $(p_1^F, p_2^F) = (H, H)$  is a stage 2 Nash equilibrium (NE) if and only if (iff)  $\pi_1(H, H) \ge \pi_1(L, H) k$  which, after rearranging, is  $k \ge \pi_1(L, H) \pi_1(H, H)$ .
- 2.  $(p_1^I, p_2^I) = (L, H)$ . Given  $(p_1^F, p_2^F) = (L, L)$ , firm 1's price is trivially optimal and firm 2's price is optimal iff  $\pi_2(L, L) k \ge \pi_2(L, H)$  or  $\pi_2(L, L) \pi_2(L, H) \ge k$ . By symmetry, this condition is equivalent to  $\pi_1(L, L) \pi_1(H, L) \ge k$ . The analysis is the same when  $(p_1^I, p_2^I) = (H, L)$ .
- 3.  $(p_1^I, p_2^I) = (L, L)$ .  $(p_1^F, p_2^F) = (L, L)$  is trivially a NE.

Combining the conditions in cases #1 and #2, the strategy projection is a NE for all subgames iff:

$$\pi_1(L, H) - \pi_1(H, H) \le k \le \pi_1(L, L) - \pi_1(H, L).$$
 (2)

Re-arranging (1), one can see that the right-hand side expression of (2) strictly exceeds the left-hand side expression and, furthermore, the left-hand side expression is strictly positive. Hence, there is a set of values for k such that (2) is satisfied.

Turning to stage 1, given  $p_2^I = H$ ,  $p_1^I = H$  yields firm 1 a payoff of  $\pi_1(H, H)$  and  $p_1^I = L$  yields a payoff of  $\pi_1(L, L)$ . Thus,  $p_1^I = H$  is optimal because  $\pi_1(H, H) > \pi_1(L, L)$ . By

<sup>&</sup>lt;sup>21</sup>As we are modelling the incentives of the executive, this payoff specification is appropriate if the executive's compensation is proportional to profits.

symmetry, it applies as well to firm 2. We conclude that if k satisfies (2) then the strategy pair is a SPE.

Summarizing, both firms choosing a low initial and final price is always a SPE outcome. If k is moderate in value, so that (2) is satisfied, then it is also a SPE outcome for both firms to choose a high initial and final price.<sup>22</sup> What sustains an outcome of high prices is that a firm which instead charges a low initial price expects the other firm to reduce its price in stage 2 to match it. Matching the lower price is optimal as long as the cost of adjusting price is sufficiently low:  $k < \pi_1(L,L) - \pi_1(H,L)$ . However, this raises the possibility that, should both firms set high initial prices, a firm may find it optimal to undercut its rival with a low final price. That will not be profitable as long as the cost of adjusting price is sufficiently high:  $k > \pi_1(L, H) - \pi_1(H, H)$ . As, by increasing differences,  $\pi_1(L,L) - \pi_1(H,L) > \pi_1(L,H) - \pi_1(H,H)$ , it is then more profitable to match a rival's low price than to undercut a rival's high price. If the cost of adjusting price is neither too high nor too long then high prices are consistent with equilibrium. If k is too high then a firm with a high initial price would not lower its final price in response to its rival having a low initial price, in which case firms would initially set a low initial price. If k is too low then, when both firms have set high initial prices, a firm would find it optimal to set a low final price and undercut its rival's price. In anticipation, firms would set low initial prices.

When there are two equilibria, a slight modification of the game allows us to use forward induction and weak dominance to select the equilibrium with high prices. Append the two-stage game with a stage 0 in which firms simultaneously decide whether or not to propose sharing prices. If one or both firms choose do not share prices then the ensuing game is the one-stage game in which they choose prices and realize profits. If both choose share prices then the ensuing game is the two-stage game in which they choose initial prices and then, after having shared those prices, decide on their final prices. Assume each firm incurs a small cost z > 0 when both choose share prices, perhaps due to possible litigation.<sup>23</sup>

By choosing do not share prices, firm i expects to earn  $\pi_i(L,L)$  because firms will be engaging in the one-stage game. If a firm chose share prices and anticipates that (L, L)would ensue - either because it expects the other firm to choose do not share prices or to choose share prices and then price at L - then its payoff is  $\pi_i(L,L)-z$  which is less than when it chooses do not share prices. If it instead anticipates the other firm choosing share prices and then pricing at H, it expects to earn  $\pi_i(H,H)-z$  which exceeds  $\pi_i(L,L)$  (due to z being small). Hence, by forward induction, firm j can infer from firm i having chosen share prices that firm i will price at H in the event that firm j also chose share prices. Of course, do not share prices is (weakly) optimal if a firm expects the other firm to choose do not share prices. More specifically, a SPE outcome for this three-stage game is do not share prices and price at L. However, action share prices weakly dominates action do not share prices. By choosing share prices, a firm earns  $\pi_i(H,H)-z$  when the other firm chooses share prices and earns  $\pi_i(L,L)$  when the other firm chooses do not share prices, while by choosing do not share prices, it earns  $\pi_i(L,L)$  whether the other firm chooses share prices or do not share prices. We conclude that, when they have the opportunity to share prices, firms will do so and subsequently set high prices.

<sup>&</sup>lt;sup>22</sup>It can also be shown that there are no other SPE outcomes.

<sup>&</sup>lt;sup>23</sup> All that is required for the forward induction argument is  $0 < z < \pi_i(H, H) - \pi_i(L, L)$ .

### 5 General Analysis

The theory of harm derived in the previous section tells the following compelling story. Executives of competing companies are sharing prices such as list or posted prices. Such prices influence the prices that consumers pay. After conducting the information exchange, it is costly but feasible for an executive to either change the price that was shared or to change how that price determines the prices faced by consumers. Due to the anticipation of sharing prices and that rival firms are able, to some limited extent, to respond to the prices that are shared, firms are incentivized to choose supracompetitive prices and, consequently, consumer pay supracompetitive prices. Having shown such an information exchange is anticompetitive for the simple case when there are just two prices, I now extend it to the canonical price-setting duopoly game with differentiated products. Proofs are in the Appendix.

#### 5.1 Firms Do Not Share Prices

Consider a standard symmetric duopoly setting in which firms choose prices and have differentiated products. Firm i's profit function is  $\pi_i(p_1, p_2) : [0, \overline{p}]^2 \to \Re_+$  where  $\overline{p}$  is sufficiently great so as not to constrain equilibrium prices.  $\pi_i(p_1, p_2)$  is assumed to be twice continuously differentiable, strictly concave in  $p_i$ , increasing in  $p_j$ , and has increasing differences in  $(p_1, p_2)$ :

$$\frac{\partial^{2} \pi_{i}\left(p_{1}, p_{2}\right)}{\partial p_{i}^{2}} < 0 < \frac{\partial^{2} \pi_{i}\left(p_{1}, p_{2}\right)}{\partial p_{i} \partial p_{j}} < \left|\frac{\partial^{2} \pi_{i}\left(p_{1}, p_{2}\right)}{\partial p_{i}^{2}}\right|.$$

Note that it is also assumed the marginal effect of own price is more sensitive to own price than to the rival's price. A firm's best response function is represented by  $\psi_i(p_j)$  and is increasing when it delivers an interior optimum:

$$\psi_i'(p_j) = -\frac{\partial^2 \pi_i / \partial p_i \partial p_j}{\partial^2 \pi_i / \partial p_i^2} \in (0, 1).$$

Since  $(\psi_1(p_2), \psi_2(p_1)) : [0, \overline{p}]^2 \to [0, \overline{p}]^2$  is a contraction mapping, there is a unique (symmetric) fixed point:  $p^N = \psi_i(p^N)$ .

The standard model represents the case when firms do not share prices. Thus,  $\pi_i(p_1, p_2)$  is the profit that firm i can expect to earn when prices are  $(p_1, p_2)$ . For example, suppose these are list prices. Then, in the absence of sharing prices, each firm will choose a list price of  $p^N$ , and consumers will pay the transaction prices that result from a list price of  $p^N$ . While the properties assumed on  $\pi_i(p_1, p_2)$  are standard ones, those assumptions are typically made presuming prices are transaction prices. It is not immediate that they should hold when, say, prices are list prices for then  $\pi_i$  embeds an unspecified relationship between list and transaction prices. In order to address this issue, I could have developed a specific structure for how list and transaction prices are connected. However, rather than be tied to a particular specification, I've chosen this more general but reduced form approach. In Section 6, a model is provided which explicitly makes the connection between list and transaction prices, and I show that the assumptions made on  $\pi_i(p_1, p_2)$  are satisfied.

#### 5.2 Firms Share Prices

As in Section 4, the extensive form has two stages. In stage 1, firms simultaneously choose initial prices, where recall  $p_i^I$  is firm i's initial price. In stage 2, initial prices are shared, and firms can, at some cost, have its final price,  $p_i^F$ , be below its initial price,  $p_i^F \leq p_i^I$ . It is assumed that the cost of lowering price is a linear function of the extent of the price reduction, and is represented by  $g\left(p_i^I-p_i^F\right)$  where g>0. Thus, the firm's payoff is  $\pi_i\left(p_1^F,p_2^F\right)-g\left(p_i^I-p_i^F\right)$ .

In motivating this structure, suppose firms are retailers and executives select and share posted prices. After doing so, an executive can exert effort within the firm to persuade, say, the sales manager to offer a discount or rebate so that the final price is less than the initial price. If instead firms are in an industrial market, let us suppose executives are selecting and sharing list prices. Upon learning other firms' list prices, an executive can reconvene the committee which sets the list price and lobby to lower it, or s/he can intervene in the process determining negotiated discounts so it is "as if" the list price is lower. This may require pressuring the sale manager to adopt larger discounts or providing additional financial incentives to sales agents in order to get them to lower the prices that they are offering. The implicit assumption is that the executive who largely determined the choice of the initial price does not have as much control over the subsequent pricing process but can influence it at a cost. Furthermore, the larger is the change the executive wants to make between the initial and final prices, the more difficult - and therefore costly - it will be.

To solve for the subgame perfect equilibria of this two-stage game, we begin by analyzing the stage 2 game. Given initial prices  $(p_1^I, p_2^I)$  from stage 1, firm 1's stage 2 problem is:

$$\max_{p_1^F} \pi_1\left(p_1^F, p_2^F\right) - g\left(p_1^I - p_1^F\right) \text{ subject to } p_1^F \leq p_1^I.$$

Consider firm 1's optimal stage 2 price when the constraint  $p_1^F \leq p_1^I$  is not binding. The associated best response function,  $\phi_1(\cdot)$ , is defined by the first-order condition:

$$\frac{\partial \pi_1 \left( \phi_1(p_2^F), p_2^F \right)}{\partial p_1} + g = 0.$$

Note that

$$\phi_i'(p_j) = -\frac{\partial^2 \pi_i/\partial p_i \partial p_j}{\partial^2 \pi_i/\partial p_i^2} \in (0,1).$$

<sup>&</sup>lt;sup>24</sup>I assume the final price cannot exceed the initial price so that the model applies to settings in which the initial price is a posted price which, in practice, is an upper bound on the price that consumers face; the transaction price can be lowered through discounts and rebates but cannot be raised. At the end of this section, it is explained that our main result is robust to allowing the final price exceed the initial price.

 $<sup>^{25}</sup>$ At the end of this section, we argue that linearity is not necessary, though it does simplify the analysis.  $^{26}$ To relate this structure to the MFC clause literature, let  $p_1^I$  ( $p_1^F$ ) represent the period 1 (2) price. According to a MFC clause, if firm 1 lowers its price in period 2 then all those consumers who bought in period 1 are reimbursed the difference. Hence, the cost to firm 1 from lowering its price is  $D_1$  ( $p_1^I, p_2^I$ ) ( $p_1^I - p_1^F$ ) where  $D_1$  ( $p_1^I, p_2^I$ ) is firm 1's period 1 demand. In our setting, the cost is instead g ( $p_1^I - p_1^F$ ). The difference in functional form does require working out the paper's main results and there are some new findings related to the parameter g.

Let  $p^*$  denote the symmetric Nash equilibrium price when both firms' constraints are not binding,  $p^* = \phi_1(p^*)$ , or

$$\frac{\partial \pi_1 \left( p^*, p^* \right)}{\partial p_1} + g = 0. \tag{3}$$

Totally differentiating (3) with respect to the price adjustment parameter, we find

$$\frac{\partial p^*}{\partial g} = -\left(\frac{\partial^2 \pi_1\left(p^*, p^*\right)}{\partial p_1^2} + \frac{\partial^2 \pi_1\left(p^*, p^*\right)}{\partial p_1 \partial p_2}\right)^{-1} > 0.$$

Thus,  $p^*$  is higher when the cost of adjusting price is higher. Since g=0 implies  $p^*=p^N$ , it follows that  $p^*>p^N$  when g>0.

Figure 1 depicts the best response function without the price adjustment cost,  $\psi_i(p_j)$ , and when the price adjustment cost is present and operative,  $\phi_i(p_j)$ . The latter is just a shifting out of  $\psi_i(p_j)$ . Also depicted are the one-stage Nash equilibrium  $p^N$ , and the equilibrium price  $p^*$  when firms are incurring the price adjustment cost at a final price of  $p^*$ .

With that benchmark, we can solve the stage 2 game. The stage 2 best response function is defined by:

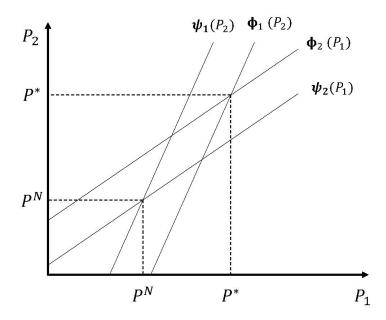
$$\phi_1^F \left( p_2^F, p_1^I \right) \equiv \arg \max_{p_1^F} \pi_1 \left( p_1^F, p_2^F \right) - g \left( p_1^I - p_1^F \right) \text{ subject to } p_1^F \leq p_1^I$$

which, by strict concavity of  $\pi_1\left(p_1^F,p_2^F\right)-g\left(p_1^I-p_1^F\right)$ , takes the form:

$$\phi_1^F\left(p_2^F,p_1^I\right) = \min\left\{\phi_1(p_2^F),p_1^I\right\}.$$

 $\left(\phi_1^F\left(p_2^F,p_1^I\right),\phi_1^F\left(p_1^F,p_2^I\right)\right):\left[0,p_1^I\right]\times\left[0,p_2^I\right]\to\left[0,p_1^I\right]\times\left[0,p_2^I\right]$  is a contraction mapping and thus has a unique fixed point.

Figure 1: Best Response Functions with and without Price Adjustment Cost



Lemma 1 shows that a property of subgame perfect equilibrium (SPE) is that a firm's initial and final prices are equal. This is not surprising given that a firm can anticipate what initial price a rival will charge.

**Lemma 1** A SPE outcome for the two-stage game satisfies:  $(p_1^F, p_2^F) = (p_1^I, p_2^I)$ .

The first result is that the two-stage game always has a SPE in which firms price at  $p^N$ . Thus, sharing prices need not affect firms' prices.

**Theorem 2**  $(p_1^I, p_2^I) = (p_1^F, p_2^F) = (p^N, p^N)$  is a SPE outcome for the two-stage game.

The next theorem is the main result. It provides sufficient conditions for it to be a SPE outcome for firms to price at  $p^*$ .<sup>27</sup>

#### Theorem 3 If

$$\frac{d^2\pi_1(p_1,\phi_2(p_1))}{dp_1^2} < 0 \ \forall (p_1,p_2)$$
 (4)

then  $\exists \overline{g} > 0$  such that if  $g \in (0, \overline{g}]$  then  $(p_1^I, p_2^I) = (p_1^F, p_2^F) = (p^*, p^*)$  is a SPE outcome for the two-stage game where  $p^* > p^N$ .

If there is strict concavity of  $\pi_1(p_1, \phi_2(p_1))$  (note that it takes into account how a firm's initial price affects the rival firm's final price) then sharing of prices can result in higher prices when it is not too costly to adjust prices (g is not too high). In Section 6, (4) is shown to hold for a version of the standard duopoly model with linear demand and cost functions.

Let us explain the proof of Theorem 3 and thereby convey the intuition behind why higher prices emerge in the two-stage structure. Begin by considering the situation faced by firms in stage 2 given their initial prices are  $p^*$ . Putting aside any price adjustment cost,  $p^*$  exceeds a firm's best response to its rival pricing at  $p^*$ :  $\psi_i(p^*) > p^*$  (see Figure 1). Hence, a firm may be inclined to set its final price below its initial price of  $p^*$ . By the construction of  $p^*$ , the marginal profit gain from setting its final price below  $p^*$ ,  $\partial \pi_1(p^*, p^*)/\partial p_1$ , equals the marginal cost of adjusting price, g; see (3). Thus, given the rival's final price is  $p^*$ , it is optimal for a firm to have its final price at its initial price of  $p^*$ . Now consider the situation faced in stage 1. Given the rival's initial price is  $p^*$ , the marginal profit from firm 1 changing its initial price is:

$$\frac{d\pi_1\left(p^*, p^*\right)}{dp_1^I} = \frac{\partial \pi_1\left(p^*, p^*\right)}{\partial p_1} + \left(\frac{\partial \pi_1\left(p^*, p^*\right)}{\partial p_2}\right) \left(\frac{\partial \phi_2\left(p^*\right)}{\partial p_1}\right). \tag{5}$$

The first term is negative (because lowering price raises profits holding fixed firm 2's price) and the second term is positive (because lowering the initial price causes firm 2 to lower its final price which then reduces firm 1's profit). The second term captures the ability of a rival firm to respond to a firm having set and shared a low price. Given that the first term

<sup>&</sup>lt;sup>27</sup>In the Online Appendix, it is shown that there are no other SPE outcomes.

equals -g by (3) and the second term is bounded above zero, (5) is negative as long g is sufficiently small. Hence, a firm's profit would decline by marginally lowering its initial price from  $p^*$  because of how its rival will react. (4) ensures that this local disincentive to lower price applies globally.

We conclude this section with four remarks. First, as argued in Section 4, forward induction and weak dominance can be used to select the SPE with price  $p^*$ . Second, Theorem 3 is robust to allowing the final price to exceed the initial price. With both firms having set the initial price above  $p^N$ , each firm's initial price exceeds its best response to the other firm's initial price (putting aside the cost of changing price). Thus, in stage 2, there is no incentive to raise price, only to possibly lower it. It follows that a firm will not want to set its final price above its initial price. What may not be robust is Theorem 2. If a firm can set its final price above its initial price, a firm may want its initial price to exceed  $p^N$ , given its rival's initial price is  $p^N$ , in order to induce its rival to raise its final price in stage 2 (according to the usual price leader logic). The key takeaway is that the anticompetitiveness of sharing prices is robust to allowing the executive to either raise or lower price after the information exchange.

Third, the linearity of the price adjustment cost function is not essential for Theorem 3. In equilibrium, price is not adjusted so what matters is the marginal cost of adjusting price at a price change of zero. Consider a general price cost adjustment function  $\rho\left(p_i^I - p_i^F\right)$ :  $\Re_+ \to \Re_+$  which is differentiable and increasing. As long as second-order conditions are satisfied, results are expected to go through by replacing g with  $\rho'(0)$ . The critical property is not linearity but rather that the marginal cost of adjusting price at a price change of zero is positive:  $\rho'(0) > 0$ .

The final remark explains the difference between Theorem 3 and the finding of Section 4. When a firm could only choose from two prices, it was shown there is an equilibrium with high prices if and only if the cost of adjusting price is neither too small nor too large. That result differs from Theorem 3 which only requires the adjustment cost is not too large. In the two-price model, if the adjustment cost was small then a firm finds it profitable to set a low final price after having set a high initial price, which undermines having high equilibrium prices. Given a fixed gain in profit from setting a final price below the initial price, it becomes profitable to do so when the adjustment cost is small enough. By comparison, in the many-price model of this section, the equilibrium initial price declines as the adjustment cost is reduced. As a result, the additional profit earned from lowering price in stage 2 is lessened, which compensates for the lower adjustment cost, and thus makes it unprofitable to set the final price below the initial price. Still, as in the two-price setting, the harm from sharing prices is greatest when the adjustment cost is moderate. When it is high, prices are unaffected. When it is low, prices are higher but the amount is small (as equilibrium prices are close to competitive prices when g is close to zero).

### 6 Analysis with List and Transaction Prices

In this section, we consider a simple model that presumes the initial and final prices are list prices, and specify how list prices are related to the prices paid by consumers. It is shown that the conditions associated with Theorem 3 are satisfied.

Suppose there are multiple submarkets which can vary in terms of their demand and the amount of discount off of the list price. Given list price  $p_i^L$  for firm  $i, \lambda^h\left(p_i^L\right)$  is the net price that firm i charges in submarket  $h \in \{1,...,H\}$ , so  $p_i^L - \lambda^h\left(p_i^L\right)$  is the discount. For example, one submarket could comprise small buyers who pay list price and another submarket comprise large buyers who receive a discount. For the executives who are choosing the list price, suppose they treat  $\lambda^h\left(\cdot\right)$  as exogenous because other individuals in the organization control discounts. Assume  $\lambda^h$  is twice continuously differentiable,  $\lambda^h\in(0,1]$ , and  $\partial\lambda^h/\partial p_i^L>0$ . Symmetry is maintained in that  $\lambda^h$  is common to both firms.

Assume submarket demand is linear, so firm i's demand in submarket h is

$$a^h - b^h \lambda^h \left( p_i^L \right) + e^h \lambda^h \left( p_j^L \right),$$

and  $0 < e^h < b^h$ . Firm i's profit function is:

$$\pi_i \left( p_1^L, p_2^L \right) = \sum_{h=1}^H \left( a^h - b^h \lambda^h \left( p_i^L \right) + e^h \lambda^h \left( p_j^L \right) \right) \left( \lambda^h \left( p_i^L \right) - c \right).$$

In the Online Appendix, it is shown that the assumptions imposed on the profit function in Section 4 hold when  $\partial^2 \lambda^h / \partial \left( p_i^L \right)^2$  is small enough. To further simplify the analysis, assume the discount schedules are linear:  $\lambda^h \left( p_i^L \right) = \lambda^h p_i^L$ ,  $i \in \{1,2\}$ . In the Online Appendix, it is shown that (4) in Theorem 3 holds. Thus, our main findings hold for this model.

With this additional structure, we can engage in some further analysis. It is straightforward to derive:

$$p^{N} = \frac{\sum_{h=1}^{H} (a^{h} + b^{h}c) \lambda^{h}}{\sum_{h=1}^{H} 2b^{h} (\lambda^{h})^{2} - \sum_{h=1}^{H} e^{h} (\lambda^{h})^{2}}, \quad p^{*} = \frac{g + \sum_{h=1}^{H} (a^{h} + b^{h}c) \lambda^{h}}{\sum_{h=1}^{H} 2b^{h} (\lambda^{h})^{2} - \sum_{h=1}^{H} e^{h} (\lambda^{h})^{2}}.$$

Assume the submarkets are identical,  $b^h = b, e^h = e \ \forall h$ . Note that  $e \in [0, b]$  where e = 0 when products are independent and e = b when products are homogeneous. Recalling that  $\overline{g}$  is the maximal value for the price adjustment cost parameter such that an equilibrium with  $p^*$  exists, it can be shown:

$$\bar{g} = \left(\sum_{h=1}^{H} \lambda^{h}\right) \left(\frac{e^{2}(a - (b - e)c)}{4b^{2} - 2be - e^{2}}\right).$$

Given  $p^*$  is increasing in g, an upper bound on the anticompetitive distortion in price due to the sharing of list prices is measured by  $p^*(g = \overline{g})/p^N$ .

$$\frac{p^*(g = \overline{g})}{p^N} = \frac{\overline{g} + (a + bc) \sum_{h=1}^{H} \lambda^h}{(a + bc) \sum_{h=1}^{H} \lambda^h} = \frac{(2b - e) (2cb^2 + 2ab - ce^2)}{(a + bc) (4b^2 - 2be - e^2)}.$$

The overcharge  $(p^*(g = \overline{g})/p^N)$  is increasing in e, which is shown in Figure 2 for when a = 100, b = 1, c = 10. As products become more substitutable, the distortion rises and at an increasing rate. When e = b,  $p^*(g = \overline{g})/p^N = 2$ . Thus, the price distortion is bounded above by 100%.

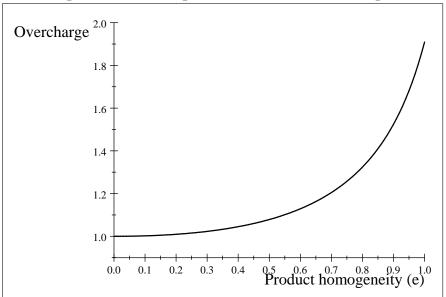


Figure 2: Overcharge from Information Exchange

### 7 Concluding Remarks

For courts to be convinced that a private information exchange of prices is anticompetitive, there needs to be a general and intuitive narrative. I believe such a narrative is offered here. The private sharing of prices by competitors gives each firm an opportunity to lower its price should it learn that its rival's price is relatively low. In anticipation of the information exchange and such a possible response by rival firms, a firm is incentivized to set and share a supracompetitive price. Notably, it is the information exchange agreement that creates harm for it is the anticipation of sharing prices that induces firms to initially set higher prices. While there is no agreement on prices, there is an agreement to share prices and there lies the unlawful agreement.

For economists to be convinced that a private information exchange of prices is anticompetitive, there needs to be a well-grounded rigorous theory which produces that narrative. The theory put forth rests on two key assumptions: 1) it is possible but costly for a colluding executive to change the firm's prices after prices have been shared with competitors; and 2) the increasing difference property of price games with differentiated products. The latter assumption delivers the property that a firm finds it more profitable to lower its price in order to be competitive with a rival's low price than it is to lower its price in order to undercut a rival's high price. The former assumption provides some level of commitment to the prices that are shared but not so much that a firm is locked into that price. As a result of these two assumptions, firms can independently set supracompetitive prices and, upon sharing them, each firm find it optimal to maintain that supracompetitive price as long as

rival firms have priced at a supracompetitive level but, should a rival firm have set a lower price, it is optimal to respond by decreasing price.

Though supracompetitive prices emerge without repeated interaction (as in the MFC clause literature), repetition may be required to induce truthful sharing of information, for there could be an incentive for a firm to misreport its price at the information exchange. However, as long as the true price is eventually revealed and firms interact sufficiently frequently, the usual argument of repeated games can be applied to incentivize firms to truthfully report their prices. While, by the theory of this paper, supracompetitive prices do not arise if the information exchange is cheap talk (in the sense that it is costless for an executive to change price), such an information exchange is still harmful and unlawful by the usual argument that firms are discussing prices which can allow them to coordinate their prices. The theory we offer is a complement to that standard argument for condemning such communications.

### 8 Appendix

**Proof of Lemma 1.** Contrary to the lemma, suppose  $p_1^F < p_1^I$  and  $p_2^F \le p_2^I$ . As these are stage 2 NE prices, it follows that  $p_1^F = \phi_1\left(p_2^F\right)\left(< p_1^I\right)$ . Consider firm 1 choosing a slightly lower stage 1 price,  $p_1^I - \varepsilon$ , so that its stage 2 pricing constraint still does not bind:  $\phi_1\left(p_2^F\right) < p_1^I - \varepsilon$ . Hence, the stage 2 NE prices are unchanged: firm 1 prices at  $\phi_1\left(p_2^F\right)$  and firm 2 prices at  $p_2^F$ . Given that firm 1's payoff has increased by the reduction in adjustment costs,  $g\left(p_1^I - \phi_1\left(p_2^F\right)\right) - g\left(p_1^I - \varepsilon - \phi_1\left(p_2^F\right)\right) = g\varepsilon$ , we have a contradiction that the original stage 1 prices were equilibrium prices.

**Proof of Theorem 2.** Consider a symmetric strategy profile in which a firm prices at  $p^N$  in stage 1. If both firms price at  $p^N$  in stage 1, it is immediate that the unique stage 2 Nash equilibrium is for both to price at  $p^N$  in stage 2. That is because  $p^N$  is a firm's best response to  $p^N$  when it is unconstrained and faces no price adjustment costs. Thus, it is also the best response when the constraint is not binding  $(p^N \leq p_i^I)$  and there are price adjustment costs from setting a price below  $p^N$ . Thus, if  $(p_1^I, p_2^I) = (p^N, p^N)$  then firm 1 expects profit of  $\pi_1(p^N, p^N)$ . The only way in which firm 1's payoff could rise with  $p_1^I \neq p^N$  is if it caused firm 2 to raise its stage 2 price. However, that is not possible as firm 2 is pricing in stage 2 at the highest feasible level given  $p_2^I = p^N$ . Thus,  $p_1^I = p^N$  is the unique best reply to  $p_2^I = p^N$ .

**Proof of Theorem 3.** First note:

$$\frac{d^{2}\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{d\left(p_{1}^{I}\right)^{2}}$$

$$= \frac{\partial^{2}\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{1}^{2}} + 2\left(\frac{\partial^{2}\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{1}\partial p_{2}}\right)\left(\frac{\partial\phi_{2}\left(p_{1}^{I}\right)}{\partial p_{1}}\right)$$

$$+ \left(\frac{\partial^{2}\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{2}^{2}}\right)\left(\frac{\partial\phi_{2}\left(p_{1}^{I}\right)}{\partial p_{1}}\right)^{2} + \left(\frac{\partial\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{2}}\right)\left(\frac{\partial^{2}\phi_{2}\left(p_{1}^{I}\right)}{\partial p_{1}^{2}}\right)$$

Towards proving the theorem, we'll need the following partial characterization of Nash equilibria for the stage 2 game. First note that, when  $p^* \leq p_1^I$  and  $p^* \leq p_2^I$ , it is immediate that the unique stage 2 NE is  $\left(p_1^F, p_2^F\right) = \left(p^*, p^*\right)$ . Next we claim that if  $p_i^I < p^* \leq p_j^I$  then the unique stage 2 NE is  $\left(p_i^F, p_j^F\right) = \left(p_i^I, \phi_j\left(p_i^I\right)\right)$ . Given  $\phi_j\left(p^*\right) = p^* \leq p_j^I$  and  $p_i^I < p^*$ , it follows from  $\phi_j$  being an increasing function that  $\phi_j\left(p_i^I\right) < p_j^I$ . Thus,  $\phi_j^F\left(p_i^I\right) = \phi_j\left(p_i^I\right)$ . In stage 2,  $p_i^I$  is a best reply to firm j choosing  $\phi_j\left(p_i^I\right)$  iff  $p_i^I \leq \phi_i\left(\phi_j\left(p_i^I\right)\right)$ . Note that

$$\frac{\partial \phi_i \left( \phi_j \left( p_i \right) \right)}{\partial p_i} = \phi_i' \left( \phi_j \left( p_i \right) \right) \phi_j' \left( p_i \right) \in (0, 1)$$

because  $\phi_i'\left(p_j\right), \phi_j'\left(p_i\right) \in (0,1)$ . Hence,  $p_i^l - \phi_i\left(\phi_j\left(p_i^I\right)\right)$  is increasing in  $p_i^l$ . Given  $p^* - \phi_i\left(\phi_j\left(p^*\right)\right) = 0$ , it follows from  $p_i^I < p^*$  that  $p_i^l - \phi_i\left(\phi_j\left(p_i^I\right)\right) < 0$ .

Now let us prove the theorem. Suppose  $p_2^I = p^*$ . If  $p_1^I > p^*$  then stage 2 NE prices are still  $(p_1^F, p_2^F) = (p^*, p^*)$  - so product market profits are unchanged - but price adjustment

costs rise by  $g(p_1^I - p^*)$ . Hence, firm 1's payoff is lower with  $p_1^I > p^*$  compared to  $p_1^I = p^*$ . Thus,  $p_1^I = p^*$  is preferred to  $p_1^I > p^*$ .

Next consider  $p_1^I < p^*$ . Given  $p_2^I = p^*$ , it was previously shown (for  $p_2^I \ge p^*$ ) that the stage 2 NE is  $\left(p_1^F, p_2^F\right) = \left(p_1^I, \phi_2\left(p_1^I\right)\right)$ . Hence, firm 1's stage 1 payoff is  $\pi_1\left(p_1^I, \phi_2\left(p_1^I\right)\right)$ . Take the total derivative of it with respect to  $p_1^I$ :

$$\frac{d\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{dp_{1}^{I}} = \frac{\partial\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{1}} + \left(\frac{\partial\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{2}}\right)\left(\frac{\partial\phi_{2}\left(p_{1}^{I}\right)}{\partial p_{1}}\right),$$

and evaluate it at  $p_1^I = p_2^I = p^*$ :

$$\frac{d\pi_1(p^*, p^*)}{dp_1^I} = \frac{\partial \pi_1(p^*, p^*)}{\partial p_1} + \left(\frac{\partial \pi_1(p^*, p^*)}{\partial p_2}\right) \left(\frac{\partial \phi_2(p^*)}{\partial p_1}\right). \tag{6}$$

In order to show that a stage 1 price below  $p^*$  is not preferred to pricing at  $p^*$ , (6) must be non-negative so that firm 1's profit does not rise by reducing  $p_1^I$  below  $p^*$ .

Given that

$$\frac{\partial \pi_1 \left( p^*, p^* \right)}{\partial p_1} + g = 0.$$

then (6) becomes:

$$\frac{d\pi_1\left(p^*, p^*\right)}{dp_1^l} = -g + \left(\frac{\partial \pi_1\left(p^*, p^*\right)}{\partial p_2}\right) \left(\frac{\partial \phi_2\left(p^*\right)}{\partial p_1}\right). \tag{7}$$

Since  $(\partial \pi_1/\partial p_2)(\partial \phi_2/\partial p_1)$  is bounded above zero, the following property holds for (7):

$$\lim_{g\to 0}\frac{d\pi_{1}\left(p^{*},p^{*}\right)}{dp_{1}^{I}}=\lim_{g\to 0}-g+\left(\frac{\partial\pi_{1}\left(p^{*},\phi_{2}\left(p^{*}\right)\right)}{\partial p_{2}}\right)\left(\frac{\partial\phi_{2}\left(p^{*}\right)}{\partial p_{1}}\right)>0.$$

Hence, if g is sufficiently small then (7) is positive, and firm 1's profit is reduced by marginally lowering its stage 1 price from  $p_1^l = p^*$ .

To complete the proof, we want to show that  $p_1^I = p^*$  is preferred to any  $p_1^I < p^*$ . For when g is sufficiently small, a sufficient condition is

$$\frac{d^2\pi_1\left(p_1^I,\phi_2\left(p_1^I\right)\right)}{d\left(p_1^I\right)^2} < 0. \tag{8}$$

If (8) holds then  $d\pi_1\left(p_1^I,\phi_2\left(p^*\right)\right)/dp_1^I>0$  implies  $d\pi_1\left(p_1^I,\phi_2\left(p_1^I\right)\right)/dp_1^I>0$  when evaluated at  $p_1^I< p^*$ .

### References

- [1] Andreu, Enrique, Damien Neven, and Salvatore Piccolo, "Delegated Sales, Agency Costs and the Competitive Effects of List Price," University of Naples, Centre for Studies in Economics and Finance, Working Paper No. 573, July 2020.
- [2] Cooper, Thomas E., "Most-Favored-Customer Pricing and Tacit Collusion," *RAND Journal of Economics*, 17 (1986), 377-388.
- [3] Dutta, Shantanu, Mark J. Zbaracki, and Mark Bergen, "Pricing Process of a Capability: A Resource-Based Perspective," *Strategic Management Journal*, 24 (2003), 615-630.
- [4] García Díaz, Antón, Roberto Hernán González, and Praveen Kujal, "List Pricing and Discounting in a Bertrand-Edgeworth Duopoly," *International Journal of Industrial Organization*, 27 (2009), 719-727.
- [5] Gill, David and John Thanassoulis, "Competition in Posted Prices with Stochastic Discounts," *Economic Journal*, 126 (2016) 1528-1570.
- [6] Hallberg, Niklas L., "The Micro-Foundations of Pricing Strategy in Industrial Markets: A Case Study in the European Packaging Industry," *Journal of Business Research*, 76 (2017), 179-188.
- [7] Hansen, Ann-Kristin, Kissan Joseph, and Manfred Krafft, "Price Delegation in Sales Organizations: An Empirical Investigation," *Business Research*, 1 (2008), 94-104.
- [8] Harrington, Joseph E., Jr., "The Anticompetitiveness of Sharing Prices," SSRN Working Paper 3621073, March 2020.
- [9] Harrington, Joseph E., Jr. and Lixin Ye, "Collusion through Coordination of Announcements," *Journal of Industrial Economics*, 67 (2019), 209-241.
- [10] Homburg, Christian, Ove Jensen, and Alexander Hahn, "How to Organize Pricing? Vertical Delegation and Horizontal Dispersion of Pricing Authority," *Journal of Marketing*, 76 (2012), 49-69.
- [11] Kaplow, Louis, Competition Policy and Price Fixing, Princeton: Princeton University Press, 2013.
- [12] Lubensky, Dmitry, "A Model of Recommended Retail Prices," RAND Journal of Economics, 48 (2017), 358-386.
- [13] Motta, Massimo, Competition Policy: Theory and Practice, Cambridge: Cambridge University Press, 2004.
- [14] Myatt, David P. and David Ronayne, "A Theory of Stable Price Dispersion," working paper, London Business School, June 2019.
- [15] Raskovich, Alexander, "Competition or Collusion? Negotiating Discounts off Posted Prices," *International Journal of Industrial Organization*, 25 (2007), 341-354.

- [16] Salop, Steven, C., "Practices that (Credibly) Facilitate Oligopoly Co-ordination," in New Developments in the Analysis of Market Structure, J. Stiglitz and G. Mathewson, eds., Basingstoke: Macmillan, 1986.
- [17] Simonetto, Mike, Larry Montan, Julie Meehan, and Junko Kaji, "Structuring and Managing an Effective Pricing Organization," in *The Oxford Handbook of Pricing Management*, Özalp Özer and Robert Phillips, eds. Oxford: Oxford University Press, 2012.
- [18] Vives, Xavier, Oligopoly Pricing: Old Ideas and New Tools, Cambridge, Mass.: The MIT Press, 1999.
- [19] Whish, Richard and David Bailey, *Competition Law*, Ninth Edition, Oxford: Oxford University Press, 2018.
- [20] Zbaracki, Mark J. and Mark Bergen, "When Truces Collapse: A Longitudinal Study of Price-Adjustment Routines," *Organization Science*, 21 (2010), 955-972.
- [21] Zbaracki, Mark J., Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen, "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets," *Review of Economics and Statistics*, 86 (2004), 514-533.

### 9 Online Appendix

#### 9.1 No Other SPE Outcomes

It can be shown that there are no other (symmetric) SPE outcomes, and the proof can be tediously extended to show there are no asymmetric SPE outcomes.

**Theorem 4** If  $p_1^I = p_2^I \notin \{p^N, p^*\}$  then  $(p_1^I, p_2^I)$  is not a SPE outcome for the two-stage game.

**Proof.** Consider  $p_1^I = p_2^I = p'$ . Let us suppose p' is a SPE outcome and then derive a contradiction. Under that presumption, Lemma 1 implies  $p_1^F = p_2^F = p'$ .

Suppose  $p' < p^N$ . Given  $p' < \psi_1(p')$ , firm 1 would earn higher profits by raising its initial price to  $\psi_1(p')$  as then  $(p_1^F, p_2^F) = (\psi_1(p'), p')$ . This is a contradiction with equilibrium.

Suppose  $p' \in (p^N, p^*)$ .  $p' < p^*$  implies  $p' < \phi_i(p')$  and, therefore, a firm's stage 2 pricing constraint -  $p_i^F \le p_i^I$  - is strictly binding:  $p_i^F = p'$ . From that we can conclude that firm 1 will still set  $p_1^F = p'$  should firm 2 price a little lower in stage 2. Thus, consider firm 2 lowering  $p_2^I$  (and  $p_2^F$ ) by  $\varepsilon$  so that  $p' < \phi_1(p' - \varepsilon)$  and, therefore, firm 1's stage 2 pricing constraint is still binding. Firm 2's profit will be  $\pi_2(p', p' - \varepsilon)$ . Given  $p' > p^N$  then  $\psi_2(p') < p'$  and, therefore,  $\pi_2(p', p' - \varepsilon) > \pi_2(p', p')$ . Hence, firm 2's profit is higher by lowering its initial price from p' to  $p' - \varepsilon$ . This is a contradiction with equilibrium.

Suppose  $p' > p^*$ . Given  $\phi_1^F \left( p_2^F = p', p_1^I = p' \right) < p'$  then a firm would prefer to lower its final price which contradicts Lemma 1

In explaining why Theorem 4 is true, let us focus on  $p_1^I = p_2^I = p' \in (p^N, p^*)$ , which is the more subtle case. By Lemma 1, firm 1 will expect firm 2's final price to be p'. Given  $\psi_1(p') \in (p^N, p')$ , firm 1 would prefer a lower initial price so as to have a lower final price (and avoid the cost of adjusting price), holding fixed firm 2's final price. Of course, if firm 1 did set a lower initial price, and thus was expected to set a lower final price, firm 2 might respond with a lower final price; hence, firm 2's anticipated response could deter firm 1 from lowering its initial price from p'. However, when  $p' < p^*$ , firm 2's marginal profit gain from lowering price is less than its marginal cost of adjusting price. As long as firm 1 lowers its initial price just a little bit below p', firm 2's optimal final price will not change even though it anticipates firm 1 setting a lower final price. Hence, when  $p' \in (p^N, p^*)$ , a firm has an incentive to slightly lower its initial price from p' which means it is not an equilibrium initial price. In contrast, when  $p' = p^*$ , a firm would respond with a lower final price in response to its rival setting its initial price below  $p^*$  (at least when g is not too large) and that discourages a lower initial price.

#### 9.2 Derivations for Model with List and Transaction Prices

From Section 4, the assumptions imposed on the profit function are:

$$\frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i}^{2}} < 0 < \frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i} \partial p_{j}} < \left| \frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i}^{2}} \right|.$$

It is shown below that these conditions hold when  $\partial^2 \lambda^h / \partial \left( p_i^L \right)^2$  is small enough. Firm *i*'s profit function is:

$$\pi_i \left( p_1^L, p_2^L \right) = \sum_{h=1}^H \left( a^h - b^h \lambda_i^h \left( p_i^L \right) + e^h \lambda_j^h \left( p_j^L \right) \right) \left( \lambda_i^h \left( p_i^L \right) - c \right).$$

From Section 4, the assumptions we imposed on the profit function were that it is twice continuously differentiable, strictly concave in  $p_i^L$ , increasing in  $p_i^L$ , and

$$\frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i}^{2}} < 0 < \frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i} \partial p_{j}} < \left|\frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i}^{2}}\right|.$$

If  $\lambda_1^h$  and  $\lambda_2^h$  are twice continuously differentiable then so is  $\pi_i\left(p_1^L,p_2^L\right)$ .  $\pi_i\left(p_1^L,p_2^L\right)$  is strictly concave in  $p_i^L$  if

$$\frac{\partial^{2} \pi_{i} \left(p_{1}^{L}, p_{2}^{L}\right)}{\partial \left(p_{i}^{L}\right)^{2}} = -2 \sum_{h=1}^{H} b^{h} \left(\frac{\partial \lambda_{i}^{h} \left(p_{i}^{L}\right)}{\partial p_{I}^{L}}\right)^{2} + \sum_{h=1}^{H} \left(a^{h} + b^{h} c - 2b^{h} \lambda_{i}^{h} \left(p_{i}^{L}\right) + e^{h} \lambda_{j}^{h} \left(p_{j}^{L}\right)\right) \left(\frac{\partial^{2} \lambda_{i}^{h} \left(p_{i}^{L}\right)}{\partial \left(p_{i}^{L}\right)^{2}}\right) < 0$$

which is satisfied as long as  $\partial^2 \lambda_i^h / \partial \left( p_i^L \right)^2$  is small enough. Note that  $\pi_i \left( p_1^L, p_2^L \right)$  is increasing in  $p_j^L$ , and has increasing differences:

$$\frac{\partial^{2} \pi_{i}\left(p_{1}^{L}, p_{2}^{L}\right)}{\partial p_{i}^{L} \partial p_{j}^{L}} = \sum_{h=1}^{H} e^{h} \left(\frac{\partial \lambda_{i}^{h}\left(p_{i}^{L}\right)}{\partial p_{i}^{L}}\right) \left(\frac{\partial \lambda_{j}^{h}\left(p_{j}^{L}\right)}{\partial p_{j}^{L}}\right) > 0.$$

Finally, we need

$$\left|\frac{\partial^{2}\pi_{i}\left(p_{1}^{L},p_{2}^{L}\right)}{\partial\left(p_{i}^{L}\right)^{2}}\right|>\frac{\partial^{2}\pi_{i}\left(p_{1}^{L},p_{2}^{L}\right)}{\partial p_{i}^{L}\partial p_{i}^{L}}$$

which takes the form:

$$2\sum_{h=1}^{H} b^{h} \left( \frac{\partial \lambda_{i}^{h} \left( p_{i}^{L} \right)}{\partial p_{i}^{L}} \right)^{2} - \sum_{h=1}^{H} \left( a^{h} + b^{h} c - 2b^{h} \lambda_{i}^{h} \left( p_{i}^{L} \right) + e^{h} \lambda_{j}^{h} \left( p_{j}^{L} \right) \right) \left( \frac{\partial^{2} \lambda_{i}^{h} \left( p_{i}^{L} \right)}{\partial \left( p_{i}^{L} \right)^{2}} \right)$$

$$> \sum_{h=1}^{H} e^{h} \left( \frac{\partial \lambda_{i}^{h} \left( p_{i}^{L} \right)}{\partial p_{i}^{L}} \right) \left( \frac{\partial \lambda_{j}^{h} \left( p_{j}^{L} \right)}{\partial p_{j}^{L}} \right).$$

$$(9)$$

Given  $b^h > e^h \, \forall h$ , (9) holds if  $\partial^2 \lambda_i^h / \partial \left( p_i^L \right)^2$  is small enough. I conclude that the model of Section 5 is a special case of the model of Section 4.

Assuming linear discount schedules,  $\lambda^h(p_i^L) = \lambda^h p_i^L$ , firm i's profit function is

$$\pi_i \left( p_1^L, p_2^L \right) = \sum_{h=1}^H \left( a^h - b^h \lambda^h p_i^L + e^h \lambda^h p_j^L \right) \left( \lambda^h p_i^L - c \right).$$

The first-order condition is:

$$\frac{\partial \pi_i \left( p_1^L, p_2^L \right)}{\partial p_i^L} = \sum_{h=1}^H \left( a^h + b^h c - 2b^h \lambda^h p_i^L + e^h \lambda^h p_j^L \right) \lambda^h = 0$$

from which we can derive a firm's best response function:

$$\psi_i\left(p_j^L\right) = \frac{\sum\limits_{h=1}^{H} \left(a^h + b^h c\right) \lambda^h}{\sum\limits_{h=1}^{H} 2b^h \left(\lambda^h\right)^2} + \left(\frac{\sum\limits_{h=1}^{H} e^h \left(\lambda^h\right)^2}{\sum\limits_{h=1}^{H} 2b^h \left(\lambda^h\right)^2}\right) p_j^L.$$

We can solve for  $p^{N}=\psi_{1}\left( p^{N}\right) ,$ 

$$p^{N} = \frac{\sum_{h=1}^{H} (a^{h} + b^{h}c) \lambda^{h}}{\sum_{h=1}^{H} 2b^{h} (\lambda^{h})^{2} - \sum_{h=1}^{H} e^{h} (\lambda^{h})^{2}}.$$

Introducing the cost of adjusting price, the profit function is

$$\sum_{h=1}^{H} \left( a^h - b^h \lambda^h p_i^F + e^h \lambda^h p_j^F \right) \left( \lambda^h p_i^F - c \right) - g \left( p_i^I - p_i^F \right),$$

from which we can derive

$$\phi_i\left(p_j^F\right) = \frac{g + \sum_{h=1}^H \left(a^h + b^h c\right) \lambda^h + \sum_{h=1}^H e^h \lambda^h p_J^F}{\sum_{h=1}^H 2b^h \lambda^h}.$$

Solving  $p^* = \phi_i(p^*)$ , we have

$$p^* = \frac{g + \sum_{h=1}^{H} (a^h + b^h c) \lambda^h}{\sum_{h=1}^{H} 2b^h (\lambda^h)^2 - \sum_{h=1}^{H} e^h (\lambda^h)^2}.$$
 (10)

To show that condition (4) in Theorem 3 holds, (4) is reproduced here:

$$\frac{\partial^{2} \pi_{1} \left(p_{i}^{I}, \phi_{2} \left(p_{1}^{I}\right)\right)}{\partial p_{1}^{2}} + 2 \left(\frac{\partial^{2} \pi_{1} \left(p_{1}^{I}, \phi_{2} \left(p_{1}^{I}\right)\right)}{\partial p_{1} \partial p_{2}}\right) \left(\frac{\partial \phi_{2} \left(p_{1}^{I}\right)}{\partial p_{1}}\right) + \left(\frac{\partial^{2} \pi_{1} \left(p_{1}^{I}, \phi_{2} \left(p_{1}^{I}\right)\right)}{\partial p_{2}^{2}}\right) \left(\frac{\partial \phi_{2} \left(p_{1}^{I}\right)}{\partial p_{1}}\right)^{2} + \left(\frac{\partial \pi_{1} \left(p_{1}^{I}, \phi_{2} \left(p_{1}^{I}\right)\right)}{\partial p_{2}}\right) \left(\frac{\partial^{2} \phi_{2} \left(p_{1}^{I}\right)}{\partial p_{1}^{2}}\right) .$$
(11)

Given

$$\pi_1 (p_1^I, p_2^I) = \sum_{h=1}^H (a^h - b^h \lambda^h p_1^I + e^h \lambda^h p_2^I) (\lambda^h p_1^I - c),$$

let us evaluate each term in (11):

$$\frac{\partial^{2}\pi_{1}\left(p_{1}^{I},\phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{1}^{2}}=-2\sum_{h=1}^{H}b^{h}\left(\lambda^{h}\right)^{2}<0$$

$$\left(\frac{\partial^{2} \pi_{1}\left(p_{1}^{I}, \phi_{2}\left(p_{1}^{I}\right)\right)}{\partial p_{1} \partial p_{2}}\right) \left(\frac{\partial \phi_{2}\left(p_{1}^{I}\right)}{\partial p_{1}}\right) = \left(\sum_{h=1}^{H} e^{h} \left(\lambda^{h}\right)^{2}\right) \left(\frac{\sum_{h=1}^{H} e^{h} \lambda^{h}}{\sum_{h=1}^{H} 2b^{h} \lambda^{h}}\right) > 0$$

$$\left(\frac{\partial^2 \pi_1\left(p_1^I, \phi_2\left(p_1^I\right)\right)}{\partial p_2^2}\right) \left(\frac{\partial \phi_2\left(p_1^I\right)}{\partial p_1}\right)^2 = 0 \times \left(\frac{\sum_{h=1}^H e^h \lambda^h}{\sum_{h=1}^H 2b^h \lambda^h}\right)^2 = 0$$

$$\left(\frac{\partial \pi_1\left(p_1^I,\phi_2\left(p_1^I\right)\right)}{\partial p_2}\right)\left(\frac{\partial^2 \phi_2\left(p_1^I\right)}{\partial p_1^2}\right) = \left(\sum_{h=1}^H e^h \lambda^h \left(\lambda^h p_1^I - c\right)\right) \times 0 = 0.$$

Hence, (4) is

$$-2\sum_{h=1}^{H}b^{h}\left(\lambda^{h}\right)^{2}+2\left(\sum_{h=1}^{H}e^{h}\left(\lambda^{h}\right)^{2}\right)\left(\frac{\sum_{h=1}^{H}e^{h}\lambda^{h}}{\sum_{h=1}^{H}2b^{h}\lambda^{h}}\right)<0.$$

Rearranging, we have:

$$2\sum_{h=1}^{H}b^{h}\left(\lambda^{h}\right)^{2}\sum_{h=1}^{H}b^{h}\lambda^{h} > \left(\sum_{h=1}^{H}e^{h}\left(\lambda^{h}\right)^{2}\right)\left(\sum_{h=1}^{H}e^{h}\lambda^{h}\right),$$

which holds because  $b^h > e^h \ \forall h$ . Thus, condition (4) in Theorem 3 holds.

Let us derive  $\overline{g}$ . From the proof of Theorem 3, g must be such that (7) is positive, which is reproduced here:

$$\frac{d\pi_1\left(p^*, p^*\right)}{dp_1^I} = -g + \left(\frac{\partial \pi_1\left(p^*, p^*\right)}{\partial p_2}\right) \left(\frac{\partial \phi_2\left(p^*\right)}{\partial p_1}\right) > 0. \tag{12}$$

Given the additional structure of Section 5 and re-arranging, (12) is:

$$g < \left(\sum_{h=1}^{H} e^h \lambda^h \left(\lambda^h p^* - c\right)\right) \left(\frac{\sum_{h=1}^{H} e^h \lambda^h}{\sum_{h=1}^{H} 2b^h \lambda^h}\right). \tag{13}$$

Substituting for  $p^*$  using (10) and re-arranging (13), we have an expression for  $\overline{g}$ :

$$\frac{\left(\sum_{h=1}^{H} e^{h} \lambda^{h}\right) \left(\sum_{h=1}^{H} a^{h} \lambda^{h} + c \sum_{h=1}^{H} b^{h} \lambda^{h}\right) \left(\sum_{h=1}^{H} e^{h} \left(\lambda^{h}\right)^{2}\right) - c \left(\sum_{h=1}^{H} 2b^{h} \left(\lambda^{h}\right)^{2} - \sum_{h=1}^{H} e^{h} \left(\lambda^{h}\right)^{2}\right) \left(\sum_{h=1}^{H} e^{h} \lambda^{h}\right)^{2}}{\left(\sum_{h=1}^{H} 2b^{h} \lambda^{h}\right) \left(\sum_{h=1}^{H} 2b^{h} \left(\lambda^{h}\right)^{2}\right) - \left(\sum_{h=1}^{H} (2b^{h} + e^{h}) \lambda^{h}\right) \left(\sum_{h=1}^{H} e^{h} \left(\lambda^{h}\right)^{2}\right)}.$$

Assuming the submarkets are identical,  $b^h = b, e^h = e \ \forall h$ , it simplifies to:

$$\bar{g} = \left(\sum_{h=1}^{H} \lambda^{h}\right) \left(\frac{e^{2}(a - (b - e)c)}{4b^{2} - 2be - e^{2}}\right).$$