The Effect of Outsourcing Pricing Algorithms on Market Competition*

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Abstract

A third party developer designs and sells a pricing algorithm that enhances a firm's ability to tailor prices to a source of demand variation, whether high-frequency demand shocks or market segmentation. The equilibrium pricing algorithm is characterized that maximizes the third party's profit given firms' optimal adoption decisions. Outsourcing the pricing algorithm does not reduce competition but does make prices more sensitive to the demand variation, and this is shown to decrease consumer welfare and increase industry profit. This effect is larger when products are more substitutable.

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1 Introduction

As a result of Big Data and algorithmic pricing, firms can condition prices on high frequency data, tailor prices to narrow submarkets, and engage in more effective learning to discover the most profitable pricing rules. While there are potential efficiency benefits from these advances, concerns have been raised about possible consumer harm. Enhanced price discrimination due to rich customer data may increase total welfare but could result in a transfer of surplus from consumers to firms. Automated pricing with high frequency data could make markets more efficient by increasing the speed of response to demand changes but it is unclear how it will affect price competition. Learning algorithms fueled by AI could deliver more profitable pricing rules but that could be because they facilitate collusion. An active competition policy debate has arisen regarding algorithmic pricing and whether legal and enforcement regimes are equipped to deal with the associated challenges.[1]

One of the primary implications of Big Data and algorithmic pricing is that it has become more attractive for a firm to outsource pricing. With prices determined more by data and less by the judgment of those employees in the firm with the best soft information, pricing can be delegated to a third party or to a pricing algorithm developed by a third party. A third party developer is likely to have better pricing algorithms than would be created internally because it has more expertise and experience, access to more data, and stronger incentives to invest in their development (as the pricing algorithm can be licensed to many firms). The use of a third party to provide pricing services is common on platforms such as Amazon Marketplace and Airbnb and more broadly in retail markets (e.g., Assad et al (2020) offer an analysis of the use of third party pricing algorithms in retail gasoline markets.)

While a firm may find it attractive to use a third party's pricing algorithm, the possibility of consumer harm has been voiced by various competition authorities. For example, the UK's Competition & Markets Authority (2018, pp. 26-27) expressed concern about the anticompetitive risk when "competitors decide ... that it is more effective to delegate their pricing decisions to a common intermediary which provides algorithmic pricing services" and noted that "[i]f a sufficiently large proportion of an industry uses a single algorithm to set prices, this could result in ... the ability and incentive to increase prices."

An open question in the area of competition policy is what the adoption of third party pricing algorithms means for consumers. Of particular relevance is that a third party's incentives when it comes to designing the pricing algorithm are likely to differ from those of a firm in a market. The latter is interested in selling more units at a higher price, while a third party developer is interested in selling more pricing algorithms at a higher fee. *What difference does it make if a pricing algorithm is designed by a firm interested in maximizing profit from the sale of the pricing algorithm rather than from the sale of the product which the algorithm is to price?*

To address this question, a stylized model is developed with a monopoly developer of pricing algorithms. While, in practice, there are many developers of pricing algorithms, it makes sense to first explore the monopoly case before examining the additional implications of competition among developers. The setting is one where the pricing algorithm's compar-

[1] Some of that discussion can be found in Mehra (2016), Ezrachi and Stucke (2017), Johnson (2017), Oxera (2017), Deng (2018), Harrington (2018), Gal (2019), Schwalbe (2019), and Calvano et al (2020a).

ative advantage is allowing price to condition on a source of demand variation such as high-frequency demand shocks (or what industry refers to as "dynamic pricing") or finely-grained market segments ("personalized pricing"). A third party designs the pricing algorithm and offers it to firms for a fee. Firms then decide whether or not to adopt it and, given those adoption decisions, choose prices.

The paper delivers new insight into pricing algorithms which we summarize here for when the demand variation is significant. A critical distinction between a pricing algorithm designed by a third party developer who intends to sell it and a firm who intends to use it is that the third party will take account of the possibility that the algorithm might compete against itself; that is, competitors might adopt the pricing algorithm. This could lead the third party to make the pricing algorithm less competitive in order to enhance the algorithm's performance and thus the demand for it. However, we do not find that to be the case in that outsourcing the pricing algorithm does not result in higher average prices. What the third party does instead is make price more sensitive to demand variation; thereby generating more profit when and where demand is strong. The problem with a higher average price is that it also enhances the attractiveness of *not* adopting the pricing algorithm when a firm's rival does, which would harm demand for the algorithm. By making price more sensitive to demand variation, the third party improves the profit from joint adoption without making it more attractive not to adopt. Though average price is not higher, outsourcing still harms consumers because of increased price variability. Furthermore, this harm is greater when products are more similar because prices remain highly sensitive to demand variation rather than becoming more closely tied to cost which is what would occur if there was no outsourcing. In sum, it does make a difference for the design of a pricing algorithm that it is done by a third party developer and, furthermore, consumers are harmed.[2]

This paper contributes to two literatures. Its primary contribution is to the emerging theoretical literature exploring the effects of algorithmic pricing.[3] A defining feature of these papers is how Big Data and algorithmic pricing are represented in the model. It can affect how much information firms have on demand (Miklós-Thal and Tucker, 2019; O'Connor and Wilson, 2021), how rapidly firms can respond to rivals' prices (Brown and MacKay, 2020), how firms simultaneously learn about their demand functions and optimal prices (Cooper, Homen-de-Mello, and Kleywegt, 2015; Hansen, Misra, and Pai, 2020), and how firms learn the best pricing algorithms (Salcedo, 2015; Klein, 2019; Calvano et al, 2020b). All of those studies assume the pricing algorithm is designed by the firm itself and thus do not consider the implications of it being designed by a third party with different incentives than that of the firm. For a more detailed literature review, the reader is referred to Appendix A.

The second literature to which this paper contributes is one that examines the welfare effects of third-degree price discrimination in oligopolistic markets.[4] Past research has focused on comparing welfare when a firm adopts a uniform price (so all markets are charged

[2] Outsourcing does result in higher average prices in Harrington (2020) where it is assumed adoption is exogneous and the third party designs the pricing algorithm to maximize the expected profit of an adopter. Here we show that higher prices do not emerge when adoption is endogenous and the third party designs the pricing algorithm to maximize its expected profit.

[3] There is a small empirical literature comprising Chen, Mislove, and Wilson (2016), Assad et al (2020), and Brown and MacKay (2020).

[4] For a survey, see Stole (2007).

the same price) and when it engages in third-degree price discrimination. The current paper considers third-degree price discrimination and compares welfare when outsourcing it to a third party (so it is designed to maximize the third party's profit) to when it is internally developed by the firm (so it is designed to maximize the firm's profit). As we show, the deleterious welfare effects of third-degree price discrimination are accentuated when the pricing algorithm is outsourced.

Section 2 provides the general model and characterizes equilibrium conditions both in the market for pricing algorithms and the market for firms' products. Under the assumption of linear demand, Section 3 derives a closed-form solution for the equilibrium pricing algorithm and Section 4 explores some implications of outsourcing the design of a firm's pricing rule. Section 5 summarizes and offers some directions for future research.

2 General Model and Equilibrium Conditions

2.1 General Model

Consider a collection of duopoly markets with differentiated products which differ in cost and demand conditions.[5] The set of market types is the finite set $\mathcal{H}$ and $\lambda(h)$ is the number of markets of type $h \in \mathcal{H}$. For a type h market, c^h is the common and constant marginal cost, and $D_i\left(p_1, p_2, a, h\right) : \Re_+^2 \times \Lambda \times \mathcal{H} \rightarrow \Re_+$ is the (symmetric) firm demand function which depends on firms' prices (p_1, p_2) and a demand variable $a \in \Lambda$ with cdf $G : \Lambda \times \mathcal{H} \rightarrow [0, 1]$. The demand variable a has two interpretations. A firm in market h could be facing a single demand curve and a is a demand shock with distribution G. In that case, price may condition on the current demand shock a. Alternatively, a firm in market h faces a collection of market segments represented by G. In that case, price may condition on the market segment a. We will generally use the "demand shock" interpretation in our exposition. Further demand assumptions will be made in Section 3. It is generally assumed $\Lambda = [\underline{a}, \overline{a}]$ and G is continuously differentiable though all analysis goes through if instead Λ is a finite set.[6]

Let us initially suppose the demand shock a occurs at a higher frequency than a firm's pricing decisions (or, when G represents a collection of market segments, the firm cannot distinguish among them). In that case, a firm is incapable of conditioning price on it and, therefore, its price depends only on G. A symmetric Nash equilibrium price $p^N(h)$ is defined by:

$$p^N(h) \equiv \arg\max_{p \in \Re_+} \int (p - c^h) D_1\left(p, p^N(h), a, h\right) G'(a, h) da.$$

As a convention, $G'(a, h) \equiv \partial G(a, h)/\partial a$.

The comparative advantage of the third party developer is that it can offer a pricing algorithm capable of tracking the high-frequency demand shock (or market segment) a so a firm's price can then condition on it. This algorithm is denoted $\phi : \Lambda \times \mathcal{H} \rightarrow \Re_+$. When a is a demand shock then ϕ assigns a price to each possible demand shock in Λ, and when a is a market segment then $\{\phi\}_{a \in \Lambda}$ is the vector of prices assigned to the set of market segments Λ. Once adopted by a firm, the algorithm is assumed to "learn" the firm's demand parameters,

[5] The duopoly case reduces the notational burden and is not essential to the paper's main insight.

[6] For the welfare analysis, Λ is a finite set so that some previous results in the literature can be used.

while a firm can program in its cost. As a result, ϕ conditions on h even though the third party may not know a particular market's type. Let Φ denote the space of mappings from $\Lambda \times \mathcal{H}$ into the price space $\Re_{+}$.

The third party chooses a fee f which a firm pays in order to adopt ϕ. As the fee is set ex ante, it is uniform across markets. For reasons of competition law, the fee is not tied to an adopting firm's profit. Given that both firms may adopt the algorithm, a third party that was compensated based on competitors' profits could effectively act as a cartel manager and coordinate firms' prices. It is also for this reason that the algorithm is not permitted to condition on the adoption decision of another firm in the market. If that were allowed, the algorithms could be programmed to "communicate" and coordinate their prices in the event that both adopted, which the third party may be inclined to do in order to generate more value for firms which would then allow it to charge a higher fee.[7,8] As we'll later show, in spite of these restrictions, outsourcing can cause consumer harm.

Next to be described is the sequence of moves and what agents know. In the first stage, the third party designs the pricing algorithm and sets the fee; that is, it chooses $(\phi(\cdot), f) \in \Phi \times \Re_{+}$. In the second stage, $(\phi(\cdot), f)$ is publicly revealed and firms make simultaneous adoption decisions. After adoption decisions are made and publicly observed, the third stage has the high-frequency demand shock realized and firms make simultaneous price decisions.[9] The solution concept is Subgame Perfect Equilibrium. In that final stage, if both firms did not adopt the pricing algorithm then equilibrium prices are $p^N(h)$. If both firms adopted then they price at $\phi(a, h)$. If one firm adopted and the other firm did not adopt then the former prices at $\phi(a, h)$ and the latter, which cannot condition its price on a, chooses a best response to $\phi(a, h)$ given its beliefs G on a. The assumption that $\phi(\cdot)$ is public information is clearly stylized but allows firms to form accurate beliefs on the profit associated with adoption and for a non-adopting firm to form accurate beliefs on an adopting firm's price.[10]

Before moving on, it is worth noting that an adopting firm is unable to modify the pricing algorithm. I am presuming the pricing algorithm is a "black box" to the firm so it cannot disentangle the demand state and start changing the price attached to it. While I believe there are situations where such an assumption is appropriate (as the firm lacks the

[7] This restriction is motivated by concerns expressed by various authorities. The German Monopolies Commission (2018, p. 23) has warned that a third party, in its design of a pricing algorithm, "could contribute to a collusive market outcome [and] it is even conceivable that [they] see such a contribution as an advantage, as it makes the algorithm more attractive for users interested in profit maximization." While the OECD (2017, p. 27) has warned: "concerns of coordination would arise if firms outsourced the creation of algorithms to the same IT companies and programmers. This might create a sort of 'hub and spoke' scenario where co-ordination is, willingly or not, caused by competitors using the same 'hub' for developing their pricing algorithms and end up relying on the same algorithms."

[8] Given the private information in the market, the third party might want to offer a menu of algorithms and fees so that firms from different market types could self select. That possibility is left to future research.

[9] Even if adoption decisions are not observed, a firm could eventually infer that the other firm adopted from its high-frequency price changes.

[10] The assumption that $\phi(\cdot)$ is observed should not be taken too literally and instead seen as a proxy for the steady-state information that firms would have. For example, a firm that adopts would eventually learn the profit from adoption. A firm that does not adopt would eventually learn the distribution of a rival firm's price which did adopt, which is what a non-adopting firm needs to know to set its optimal price and to know the profit from non-adoption.

necessary knowledge), there are also situations where some modification would be possible. In exploring the latter, one would want to consider the constraints on a firm's ability to modify the algorithm lest one trivializes the role of a third party developer. This extension of the model is left for future research.

2.2 Equilibrium Conditions

Towards specifying the conditions defining the equilibrium pricing algorithm, we will begin by characterizing the market demand for pricing algorithms. For that purpose, let $V(I_i, I_j, \phi, h)$ denote gross profit (before netting out the third party's fee) for a firm with adoption decision $I_i \in \{A, NA\}$ given the other firm's adoption decision $I_j \in \{A, NA\}$, where A refers to *adoption* and NA to *no adoption*. The explicit expressions for $V(I_i, I_j, \phi, h)$ are provided later. A firm's total cost of adoption is $f + \varepsilon$ where f is the fee charged by the third party and ε is a market-specific adoption cost which is observable to the firms but not to the third party. ε is introduced so the probability of adoption is a smooth function of ϕ and f. ε has a continuously differentiable cdf $K : \Re_+ \rightarrow [0,1]$ and K puts sufficient mass near zero so that, at the equilibrium design and fee, there is positive expected demand.[11]

Derivation of the demand for pricing algorithms requires characterizing equilibrium adoption decisions. It is an equilibrium for a market to have zero adoptions when:

$$\begin{aligned} V(NA, NA, \phi, h) &\geq V(A, NA, \phi, h) - (f + \varepsilon) \Leftrightarrow \\ f + \varepsilon &\geq V(A, NA, \phi, h) - V(NA, NA, \phi, h); \end{aligned}$$

that is, the incremental value of adoption conditional on the rival firm not adopting is less than the cost of adoption. It is an equilibrium for a market to have one adoption when:

$$V(A, NA, \phi, h) - (f + \varepsilon) \geq V(NA, NA, \phi, h) \text{ and } V(NA, A, \phi, h) \geq V(A, A, \phi, h) - (f + \varepsilon),$$

where the first inequality says it is optimal for a firm to adopt given the rival firm does not adopt and the second inequality says it is optimal for a firm not to adopt given the rival firm does adopt. Those conditions are equivalent to:

$$V(A, NA, \phi, h) - V(NA, NA, \phi, h) \geq f + \varepsilon \geq V(A, A, \phi, h) - V(NA, A, \phi, h).$$

Finally, it is an equilibrium for a market to have two adoptions when:

$$\begin{aligned} V(A, A, \phi, h) - (f + \varepsilon) &\geq V(NA, A, \phi, h) \\ &\Leftrightarrow V(A, A, \phi, h) - V(NA, A, \phi, h) \geq f + \varepsilon, \end{aligned}$$

so the incremental value of adoption conditional on the rival firm adopting exceeds the cost of adoption.

If

$$V(A, NA, \phi, h) - V(NA, NA, \phi, h) > V(A, A, \phi, h) - V(NA, A, \phi, h), \quad (1)$$

so adoptions are strategic substitutes, then, generically, there is a unique equilibrium number of adoptions. If

$$V(A, A, \phi, h) - V(NA, A, \phi, h) \geq f + \varepsilon \quad (2)$$

[11] Obviously, the third party will set f so that expected demand is positive.

then the market has 2 adoptions; if

$$V(A, NA, \phi, h) - V(NA, NA, \phi, h) \geq f + \varepsilon > V(A, A, \phi, h) - V(NA, A, \phi, h), \quad (3)$$

then the market has 1 adoption; and if $f + \varepsilon > V(A, NA, \phi, h) - V(NA, NA, \phi, h)$ then the market has 0 adoptions.

If instead

$$V(A, A, \phi, h) - V(NA, A, \phi, h) > V(A, NA, \phi, h) - V(NA, NA, \phi, h),$$

so adoptions are strategic complements, then the equilibrium number of adoptions is 0 or 2. If $V(A, NA, \phi, h) - V(NA, NA, \phi, h) > f + \varepsilon$ then the market has 2 adoptions, and if

$$V(A, A, \phi, h) - V(NA, A, \phi, h) \geq f + \varepsilon \geq V(A, NA, \phi, h) - V(NA, NA, \phi, h)$$

then the market has either 0 or 2 adoptions. When it is an equilibrium to have either 0 or 2 adoptions, an equilibrium selection is made that both adopt.[12]

The market demand for the third party's pricing algorithm is composed of those markets for which one firm adopts - (3) is satisfied - and those for which two firms adopt - so (2) is satisfied. The resulting demand is

$$\begin{aligned} &\sum_{h \in \mathcal{H}} [1 \times \max\{K(V(A, NA, \phi, h) - V(NA, NA, \phi, h) - f) \\ &-K(V(A, A, \phi, h) - V(NA, A, \phi, h) - f), 0\} \\ &+2 \times K(V(A, A, \phi, h) - V(NA, A, \phi, h) - f)]\lambda(h) \end{aligned} \quad (4)$$

where recall $\lambda(h)$ is the number of type h markets. The first term in brackets is demand coming from markets where one firm adopts the pricing algorithm. When (1) holds, it equals

$$K(V(A, NA, \phi, h) - V(NA, NA, \phi, h) - f) - K(V(A, A, \phi, h) - V(NA, A, \phi, h) - f),$$

and when (1) does not hold then it is zero. The second term in (4) is demand coming from markets where both firms adopt, and is the probability that (2) is satisfied.[13]

The third party chooses the design and fee to maximize its expected revenue.[14] The

[12] Our later analysis will show that this is part of the Pareto dominant equilibrium. It is generally in the interests of the third party to persuade firms to coordinate on the equilibrium with two adoptions as it prefers positive demand to zero demand. As shown in Sections 3 and 4 when there is linear product demand, a firm's gross equilibrium profit is higher when both adopt than when neither adopts. As a firm's net profit (gross profit less the equilibrium fee) must be at least as great as the profit from not adopting and that profit will be shown to exceed the Nash equilibrium profit when neither adopts then firms' net profit from adoption is higher compared to when both do not adopt. In sum, the third party and the two firms prefer the equilibrium when both firms adopt.

[13] Without ε, there could be many equilibrium designs only because there are many designs sufficient to result in the incremental profit from adoption exceeding f. Introducing ε rids the model of this indeterminancy.

[14] The design cost of the pricing algorithm is assumed to be independent of the design. The cost is also assumed to be sufficiently small so it is exceeded by the equilibrium revenue. Otherwise, the third party would not be in the market of supplying pricing algorithms.

equilibrium design and fee are the solution to:

$$\begin{aligned}(\phi^*, f^*) &= \arg \max_{(\phi,f)\in\Phi\times\Re_+} f \times \sum_{h\in\mathcal{H}}[1 \times \max\{K\left(V(A, NA, \phi, h) - V(NA, NA, \phi, h) - f\right) \\ &\quad -K\left(V(A, A, \phi, h) - V(NA, A, \phi, h) - f\right), 0\} \\ &\quad +2 \times K\left(V(A, A, \phi, h) - V(NA, A, \phi, h) - f\right)]\lambda(h)\end{aligned} \tag{5}$$

where

$$\begin{aligned}V(A, A, \phi, h) &= \int \left(\phi(a, h) - c^h\right) D_1\left(\phi(a, h), \phi(a, h), a, h\right) G'(a, h)da \\ V(A, NA, \phi, h) &= \int \left(\phi(a, h) - c^h\right) D_1\left(\phi(a, h), p^*(\phi, h), a, h\right) G'(a, h)da \\ V(NA, A, \phi, h) &= \int \left(p^*(\phi, h) - c^h\right) D_1\left(p^*(\phi, h), \phi(a, h), a, h\right) G'(a, h)da \\ V(NA, NA, \phi, h) &= \int \left(p^N(h) - c^h\right) D_1\left(p^N(h), p^N(h), a, h\right) G'(a, h)da\end{aligned} \tag{6}$$

$$p^*(\phi, h) = \arg\max_p \int \left(p - c^h\right) D_1\left(p, \phi(a, h), a, h\right) G'(a, h)da \tag{7}$$

Using the expression for market demand in (4), (5) is expected revenue. Given its rival adopts ϕ, the optimal price for a firm that does not adopt is $p^*(\phi, h)$ as defined in (7). $p^*(\phi, h)$ along with ϕ are used to define firms' values when one adopts and the other firm does not - $V(A, NA, \phi, h)$ and $V(NA, A, \phi, h)$ - as provided in (6). (6) also defines a firm's value when both adopt, $V(A, A, \phi, h)$, and when neither adopts, $V(NA, NA, \phi, h)$. We next turn to deriving a closed-form solution for (5).

3 Equilibrium under Linear Demand

For purposes of tractability, from hereon linear demand is assumed:

$$D_1(p_1, p_2, a, h) = a - bp_1 + dp_2,$$

where $b > d \geq 0$. Recall that $a \sim G$ and let a have mean μ and variance σ^2. Assume $a - (b - d)c > 0$ $\forall a \in \Lambda$ so demand is always positive. To save on extraneous notation, the market type h is dropped though it should be remembered that the pricing algorithm is designed for each market type.

3.1 Benchmark Equilibria

Suppose there is no third party and the firms are unable to condition price on the demand shock. In that case, the symmetric Nash equilibrium is $p^N = \frac{\mu+bc}{2b-d}$ with expected profit of

$$\pi^N \equiv \int \left(\frac{\mu + bc}{2b - d} - c\right)\left(a - (b - d)\left(\frac{\mu + bc}{2b - d}\right)\right) G'(a)da = \frac{b\left(\mu - (b - d)c\right)^2}{(2b - d)^2}.$$

While this is a relevant benchmark, it is not the appropriate one for assessing the effect of outsourcing because the use of a third party's pricing algorithm confounds outsourcing with engaging in third-degree price discrimination (with respect to the demand variable a). In order to separate these effects, the proper benchmark is when firms condition price on the demand shock but use a pricing algorithm that each firm internally develops in order to maximize its profit. This is the standard Nash equilibrium with third-degree price discrimination, $\phi^I(a) \equiv \frac{a+bc}{2b-d}$ where I refers to "internal development."

While expected price is the same as when firms do not condition on the high-frequency demand shock, $E\left[\phi^I(a)\right] = \frac{\mu+bc}{2b-d}$, expected profit with the pricing algorithm is higher,

$$\pi^I \equiv \frac{b\left(\mu-(b-d)c\right)^2}{\left(2b-d\right)^2} + \frac{b\sigma^2}{\left(2b-d\right)^2} > \pi^N,$$

because a firm is able to raise price when demand is stronger (and more price-inelastic) and lower price when demand is weaker (and more price-elastic). By comparing the third party's equilibrium pricing algorithm ϕ^* with ϕ^I, we will identify the difference due to outsourcing or, alternatively stated, to the pricing algorithm having been designed to maximize the profit from selling the algorithm (i.e., the developer's profit) rather than from using the algorithm (i.e., a firm's profit).

3.2 Characterization Theorems

In solving (5), the focus is on affine pricing algorithms: $\phi(a) = \alpha + \gamma a$ for some (α, γ).[15] Prior to presenting our characterization theorems, it will prove useful to initially consider a constrained problem for the third party. Suppose the third party chose to optimize while ensuring that adoptions are (weakly) strategic complements, which means ϕ satisfies:

$$V\left(A, A, \phi\right) - V\left(NA, A, \phi\right) \geq V\left(A, NA, \phi\right) - V\left(NA, NA, \phi\right).$$

In that case, the expression in (4) for expected demand is $K\left(V(A, A, \phi) - V(NA, A, \phi) - f\right)$. As long as $K\left(V(A, A, \phi) - V(NA, A, \phi) - f\right) > 0$ for some ϕ then maximizing that expression is equivalent to maximizing the incremental value of adoption conditional on the rival firm adopting, $V(A, A, \phi) - V(NA, A, \phi)$. Consequently, the pricing algorithm is designed to solve:

$$\max_{\phi \in \Phi} V(A, A, \phi) - V(NA, A, \phi) \tag{8}$$
$$\text{s.t. } V\left(A, A, \phi\right) - V\left(NA, A, \phi\right) \geq V\left(A, NA, \phi\right) - V\left(NA, NA, \phi\right).$$

It is shown in Appendix B (Lemma 3) that the solution to (8) is:

$$\phi^{sc}(a) \equiv \frac{2(b-d)bc - d\mu}{2\left(b-d\right)\left(2b-d\right)} + \left(\frac{1}{2(b-d)}\right) a.$$

Now let us turn to solving (5). We'll initially consider the case when demand is relatively stable, which includes $\sigma^2 = 0$ so there is no demand variation. Theorem 1 shows that the

[15] While it has not been shown that a solution must be an affine function, it would be surprising if that were not the case.

third party designs the pricing algorithm to make one firm into a price leader (which is why the pricing algorithm is denoted ϕ^{pl}). By adopting the third party's pricing algorithm, a firm commits to a higher average price which induces its non-adopting rival to raise its price, and that is what makes it profitable for the firm to adopt. Proofs are in Appendix B.[16]

Theorem 1 *If σ^2 is sufficiently low then the unique equilibrium pricing algorithm is*

$$\phi^{pl}(a) \equiv \frac{(b+d)\left((2b-d)bc+d\mu\right)}{b\left(4b^2-2d^2\right)} + \left(\frac{1}{2b}\right) a,$$

and either no firm adopts or one firm adopts.

It is straightforward to show that the pricing algorithm commits the adopter to a higher average price than when pricing algorithms are not adopted:

$$\begin{aligned} E\left[\phi^{pl}(a)\right] - p^N &= \frac{(b+d)\left(2cb^2-cdb+d\mu\right)}{b\left(4b^2-2d^2\right)} + \left(\frac{1}{2b}\right)\mu - \frac{\mu+bc}{2b-d} \\ &= \frac{d^2(\mu-(b-d)c)}{4b(b-d)(2b+d)+2d^3} > 0. \end{aligned}$$

As a result, the non-adopting firm prices higher because its price is the best response to $E\left[\phi^{pl}(a)\right]$,

$$\frac{\mu+bc+dE\left[\phi^{pl}(a)\right]}{2b},$$

and

$$\frac{\mu+bc+d\left(\frac{(b+d)\left(2cb^2-cdb+d\mu\right)}{b(4b^2-2d^2)} + \left(\frac{1}{2b}\right)\mu\right)}{2b} - \frac{\mu+bc}{2b-d} = \frac{d^3(\mu-(b-d)c)}{4b(2b(b-d)(2b+d)+d^3)} > 0.$$

In general, two forces are at play when it comes to adoption of a third party's pricing algorithm. First, adoption means committing to a pricing rule, which has strategic value but only when the rival firm does not adopt and thus can respond to that commitment. Second, adoption means having price condition on the demand shock. When a market's demand variance is low, the second force is sufficiently weak that the third party designs the pricing algorithm so as to exploit commitment.[17]

To elaborate on this explanation, consider $\sigma^2 = 0$. Without any demand variation, commitment to a pricing algorithm is equivalent to commitment to a particular price. Thus, adoption of the third party's pricing algorithm creates a sequential-move price game where the adopter is the price leader and the non-adopter is the price follower. As we know that the follower's profit is higher than the leader's in such a game, it is then an equilibrium for

[16] For Theorems 1 and 2, the uniqueness of the solution assumes $\exists\phi$ such that expected demand is positive. That is always true when $f = 0$. Of course, in equilibrium, $f > 0$ in which case it is possible expected demand is zero $\forall\phi$ for some market types. In that case, any ϕ is a trivial solution. The proofs are for the case when $\exists\phi$ such that expected demand is positive.

[17] In another context, Brown and Mackay (2020) examine commitment with regards to the frequency with which price changes.

only one firm to adopt (assuming $f+\varepsilon$ is sufficiently low and, otherwise, no firm adopts). When $\sigma^2 > 0$, adoption no longer creates a sequential-move price game (as the adopting firm's price responds to the demand variable at the same time as the non-adopting firm is choosing its price) but there is still a benefit from committing to a higher *average* price since the non-adopting firm's price is a best response to its rival's average price. As σ^2 is increased from zero, the expected profit from adoption rises because a firm can adjust its price to demand shocks, while the expected profit from non-adoption is unchanged (as it is based on the adopting firm's expected price which is independent of σ^2). As long as σ^2 is not too high, it will still be an equilibrium for only one firm to adopt and accordingly the third party designs the pricing algorithm to take advantage of the commitment that adoption delivers.

While the preceding analysis is interesting, the more relevant setting is when σ^2 is not low so high-frequency demand shocks are a significant factor in the market. That is the setting that is made more common with Big Data and which makes it especially attractive for a firm to turn to a third party so that it can condition price on demand fluctuations. Our next result characterizes the equilibrium pricing algorithm when the demand variance is high. Now we find the pricing algorithm is designed to make adoptions into strategic complements and the third party sells its pricing algorithm to both firms.

Theorem 2 *If σ^2 is sufficiently high then the unique equilibrium pricing algorithm is*

$$\phi^{sc} = \frac{2(b-d)bc - d\mu}{2\,(b-d)\,(2b-d)} + \left(\frac{1}{2(b-d)}\right) a = \frac{\mu + bc}{2b-d} + \frac{a-\mu}{2b-2d},$$

and either no firm adopts or both firms adopt.

When demand significantly fluctuates, it is attractive to a firm to be able to condition price on those demand movements and, consequently, equilibrium has both firms adopting the pricing algorithm (assuming $f+\varepsilon$ is low enough). Anticipating both firms may adopt, the third party designs the pricing algorithm to maximize each firm's willingness-to-pay (WTP) when both adopt, as that will maximize the likelihood that it exceeds the adoption cost $f+\varepsilon$ and thereby result in a sale. Thus, the design is chosen to maximize the incremental value of adoption which is $V(A, A, \phi) - V(NA, A, \phi)$. This then creates a design challenge for the third party developer: make it profitable to adopt the pricing algorithm - which encourages designing ϕ so that $V(A, A, \phi)$ is high – while not making it exploitable by a non-adopting rival firm - which encourages designing ϕ so that $V(NA, A, \phi)$ is low.

In solving this challenge, note that the third party does *not* design the pricing algorithm to maximize firms' joint profit which would mean maximizing $V(A, A, \phi)$. For consider the joint profit-maximizing (or monopoly) price as a candidate pricing algorithm:

$$p^M(a) \equiv \arg\max_p (p-c)(a-(b-d)p) = \frac{a+(b-d)c}{2(b-d)}.$$

If the third party were to use this design, it would result in a high value of $V(A, A, \phi)$ but also cause $V(NA, A, \phi)$ to be relatively high as a non-adopting firm would be able to exploit the high average price of a rival firm that has adopted. By instead having the pricing algorithm price slightly less than $p^M(a)$, there is no first-order effect on $V(A, A, \phi)$ but there is a first-order decrease of $V(NA, A, \phi)$, which raises the incremental value of adoption. Hence, the third party's design will have the pricing algorithm price below that which maximizes joint profit.

While the price level is less than the monopoly price, the sensitivity of the pricing algorithm to the demand shock is the same as for the monopoly price,

$$\frac{\partial \phi^{sc}(a)}{\partial a} = \frac{1}{2(b-d)} = \frac{\partial p^M(a)}{\partial a}.$$

Thus, the third party's pricing algorithm shifts down the pricing rule which maximizes the profit of both firms adopting:

$$\phi^{sc}(a) = p^M(a) - \frac{d\left(\mu - (b-d)c\right)}{2(2b-d)(b-d)}.$$

Towards understanding the optimality of this rule, consider how changing the pricing algorithm affects the incremental value of adoption, $V(A, A, \phi) - V(NA, A, \phi)$. Given that a non-adopting firm does not condition on the high-frequency demand shock, the expected profit of a non-adopter $V(NA, A, \phi)$ depends only on the expected price of its adopting rival which is the expectation of ϕ. In contrast, given adoption means conditioning price on the realization of the demand shock, $V(A, A, \phi)$ depends on the entire distribution of price (based on ϕ). Making the responsiveness of ϕ to a closer to the responsiveness of the monopoly price, while keeping the expectation of ϕ fixed, raises the expected profit from adopting without affecting the expected profit from not adopting; hence, the incremental value from adoption increases. For reasons that are not clear, it proves optimal to raise the responsiveness of ϕ to the point that it is the same as under joint profit maximization, while maintaining the expectation of ϕ at the Nash equilibrium price p^N. This simple modification of the monopoly price yields high profit when both firms adopt without creating high profit for a firm foregoing adoption and pricing competitively against a rival firm which did adopt.

3.3 Market-specific Fee

When the demand variance is high, the third party optimally designs the pricing algorithm so that adoptions are strategic complements (Theorem 2). Thus, expected demand is $2 \times K\left(V(A, A, \phi) - V(NA, A, \phi) - f\right)$ and the equilibrium price algorithm is that which maximizes the incremental value of adoption given the rival firm adopts, $V(A, A, \phi) - V(NA, A, \phi)$. When the demand variance is low, the third party optimally designs the pricing algorithm so that adoptions are strategic substitutes (Theorem 1). Expected demand is

$$K\left(V(A, NA, \phi) - V(NA, NA, \phi) - f\right) + K\left(V(A, A, \phi) - V(NA, A, \phi) - f\right) \tag{9}$$

and, at the equilibrium pricing algorithm, $V(A, A, \phi) - V(NA, A, \phi) \leq 0$ which implies expected demand simplifies from (9) to $K\left(V(A, NA, \phi) - V(NA, NA, \phi) - f\right)$. Thus, when the demand variance is low, the equilibrium price algorithm maximizes the incremental value of adoption given the rival firm does not adopt, $V(A, NA, \phi) - V(NA, NA, \phi)$. When the demand variance is neither low nor high, it is possible the third party chooses the pricing algorithm so adoptions are strategic substitutes and $V(A,A,\phi) - V(NA,A,\phi) > 0$ which means the solution that maximizes (9) will depend on K. That is why characterization is less clear in this case.[18] However, if the fee (as well as the design) is tailored to the market type and the adoption cost shock ε is eliminated then the equilibrium pricing algorithm can be characterized for all demand variances. That is what we do in this section.

Without the adoption cost shock, the third party will know exactly how many firms will adopt depending on ϕ and f, where both are now set for a market type. Thus, we can think of the third party deciding to sell the pricing algorithm to one firm or two firms. If it decides to sell it to two firms them the optimal design is ϕ^{sc} as that maximizes the WTP and thus maximizes the fee that can be charged. In that case, the WTP is the incremental value of adoption conditional on the rival firm adopting which (as shown in the proof of Lemma 3) is $\frac{\sigma^2}{4(b-d)}$ so the optimal fee is $f = \frac{\sigma^2}{4(b-d)}$. The third party's revenue from selling ϕ^{sc} to two firms at that fee is $\frac{\sigma^2}{2(b-d)}$. If it decides to sell the pricing algorithm to only one firm then it will want the design to maximize the incremental value of adoption conditional on the rival firm not adopting, as again that will maximize the fee that can be charged. The solution to that problem is ϕ^{pl} and the WTP (and fee) can be shown to be $\frac{d^4(\mu-(b-d)c)^2}{8b(2b-d)^2(2b^2-d^2)} + \frac{\sigma^2}{4b}$. Hence, the third party's revenue is $\frac{d^4(\mu-(b-d)c)^2}{8b(2b-d)^2(2b^2-d^2)} + \frac{\sigma^2}{4b}$.

The difference between the revenue from selling two units of ϕ^{sc} and one unit of ϕ^{pl} is

$$\begin{aligned} & \frac{\sigma^2}{2(b-d)} - \left(\frac{d^4 \left(\mu - (b-d)c\right)^2}{8b\left(2b-d\right)^2\left(2b^2-d^2\right)} + \frac{\sigma^2}{4b} \right) \\ = \; & \frac{(b+d)\,\sigma^2}{4b(b-d)} - \frac{d^4 \left(\mu - (b-d)c\right)^2}{8b\left(2b-d\right)^2\left(2b^2-d^2\right)}. \end{aligned}$$

It is then optimal for the third party to choose $\phi^{sc}(\phi^{pl})$ and sell to two firms (one firm) when

$$\sigma^2 > (<) \frac{(b-d)d^4\left(\mu-(b-d)c\right)^2}{2\left(2b-d\right)^2\left(2b^2-d^2\right)(b+d)}.$$

4 Effect of Outsourcing a Firm's Pricing Rule

In the remainder of the paper, we'll focus on the more interesting and relevant case when the demand variance is high enough that the third party's equilibrium pricing algorithm is ϕ^{sc} and both firms adopt. In order to assess the effect of outsourcing, I will compare the third party's pricing algorithm ϕ^{sc} with ϕ^I which is the pricing algorithm that conditions on the demand shock but is internally developed by the firm so as to maximize its expected profit. In that way, the effect of outsourcing is disentangled from the effect of third-degree price discrimination. Our primary goal is to shed light on the structure of the pricing algorithm which is attributable to it being developed by a third party.

[18] This is also the analytical impediment to allowing for firm-specific adoption costs as then either one or two firms may adopt in equilibrium which causes the characterization of the pricing algorithm to depend on K.

4.1 Sensitivity of Price to Demand Shocks

It will prove instructive to re-arrange the equilibrium pricing algorithm into the following expression:

$$\phi^{sc}(a) = \frac{a+bc}{2b-d} + \frac{d(a-\mu)}{2(b-d)(2b-d)}. \tag{10}$$

By comparison, the pricing algorithm that conditions on the demand shock which firms would develop on their own is

$$\phi^{I}(a) = \frac{a+bc}{2b-d}. \tag{11}$$

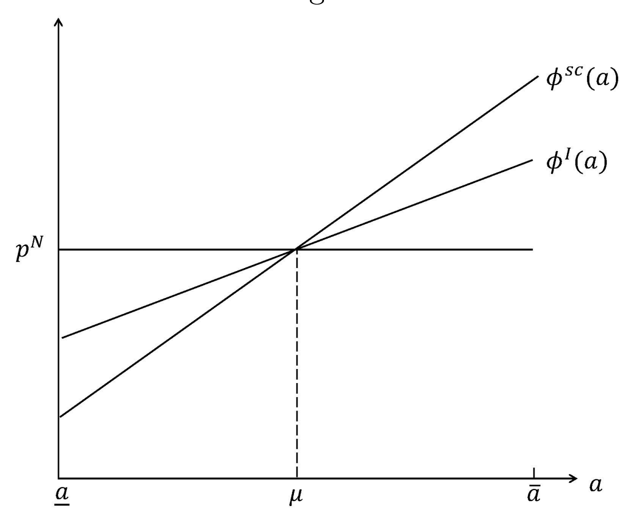
Recall that firms price at $p^N = \frac{\mu+bc}{2b-d}$ when they do not condition price on the high-frequency demand shock. The three pricing rules are depicted in Figure 1.

In comparing these pricing rules, the first point to note is that average price is $\frac{\mu+bc}{2b-d}$ whether or not firms condition price on the high-frequency demand shock and whether or not the pricing algorithm is developed internally or externally. The second point is that the pricing algorithm is more sensitive to the demand shock when it is developed by a third party:

$$\frac{\partial\phi^{sc}(a)}{\partial a} = \frac{1}{2(b-d)} > \frac{1}{2b-d} = \frac{\partial\phi^{I}(a)}{\partial a}.$$

In response to stronger demand, a firm raises its price but ϕ^I limits the amount of that price increase because of the prospect of losing demand to the other firm. However, the third party's pricing algorithm internalizes that effect - as it responds to demand variation as would a monopolist - and, consequently, price rises more in response to stronger demand. Outsourcing the pricing algorithm then results in greater price sensitivity to demand shocks and, therefore, greater price volatility.

Figure 1



A numerical example illustrates the effect of outsourcing on price variability. Assume $\mu = 100, b = 1, c = 10, d = 0.6$. In the proof of Theorem 2, it is shown that a sufficient (but not necessary) condition for ϕ^{sc} to be the equilibrium pricing algorithm is

$$\sigma^2 \geq \frac{d^3(b-d)\left(\mu-(b-d)c\right)^2}{2\left(2b-d\right)^2\left(2b^2-d^2\right)} = 123.86.$$

If a is uniformly distributed on $[80, 120]$ then $\sigma^2 = 133.33$ and the above condition is satisfied. The equilibrium pricing algorithm is $\phi^{sc}(a) = -46.\,43 + 1.25a$ and, consequently, price is uniformly distributed on $[53.57, 103.57]$. By comparison, $\phi^I(a) = 7.14 + 0.71a$ and price is uniformly distributed on $[63.94, 92.34]$. Outsourcing increases the range of prices by 76% from 28.40 to 50.00 and more than triples the variance from 67.21 to 208.33.

Before netting out the fee for using the pricing algorithm, expected profit is higher to firms when the pricing algorithm is developed by a third party. Expected profit under external development is

$$\frac{b\left(\mu-(b-d)c\right)^2}{\left(2b-d\right)^2} + \frac{\sigma^2}{4(b-d)}$$

and with internal development is

$$\frac{b\left(\mu-(b-d)c\right)^2}{\left(2b-d\right)^2} + \frac{(b+d)\sigma^2}{4b^2}.$$

The former is larger:

$$\left(\frac{b\left(\mu-(b-d)\right)^2}{\left(2b-d\right)^2} + \frac{\sigma^2}{4(b-d)}\right) - \left(\frac{b\left(\mu-(b-d)c\right)^2}{\left(2b-d\right)^2} + \frac{(b+d)\sigma^2}{4b^2}\right) = \frac{d^2\sigma^2}{4b^2(b-d)} > 0. \quad (12)$$

As explained above, the attractiveness of the third party's pricing algorithm is that it internalizes the effect of one's firm price on the other's demand and profit. Even if firms are capable of developing their own pricing algorithms at a comparable cost to the third party, it is collectively advantageous to have the the third party design them.[19,20]

4.2 Consumer Welfare

To analyze the consumer welfare effects of a third party's provision of a pricing algorithm, we will draw on some standard methods developed for third-degree price discrimination. The price discrimination literature has shown that a sufficient condition for third-degree price discrimination to lower consumer welfare, relative to a uniform price, is that total supply does not increase (Varian, 1989). Intuitively, for a given aggregate total supply, consumer

[19] This finding offers an interesting contrast from Corts (1998). In the setting of Corts (1998), price discrimination lowers firms' profits and that creates an incentive for firms to adopt practices - such as everyday low prices - so as make price *less* variable across demand states. We have a setting whereby price discrimination raises firms' profits and that creates an incentive for them to outsource their pricing algorithms so as to make price *more* variable across demand states.

[20] If a firm is given the option to develop its own pricing algorithm, it remains an open question whether a firm would do so or instead purchase the third party's. That question is left to future research. The maintained assumption of this paper is that the third party is the sole innovator of a pricing algorithm which can condition on this source of demand variation.

welfare is maximized by equating marginal utility across markets (or, in the current model, equating it across demand states) which can only be achieved with a uniform price. Thus, holding total supply constant across price regimes, price discrimination lowers consumer (and total) welfare compared to a uniform price. If total supply is lower under price discrimination then consumer (and total) welfare is even less. A corollary of that general finding is that, when comparing two price discrimination schemes which produce the same total supply, the one with more price dispersion across markets has lower consumer welfare.

Towards applying that insight, consider the three pricing rules: 1) uniform price (which does not condition on the demand shock); 2) internally developed pricing algorithm which conditions on the demand shock; and 3) externally developed pricing algorithm which conditions on the demand shock. Letting $p(a)$ represent a generic pricing rule, all three pricing rules have the same expected quantity (the analogue to "total supply") which follows from them having the same expected price:

$$\int (a-(b-d)p(a))\, G'(a)da = \mu-(b-d)E[p(a)] = \mu-(b-d)\left(\frac{\mu+bc}{2b-d}\right) = \frac{b\,(\mu-(b-d)c)}{2b-d}.$$

Of course, price dispersion is greater with the internally developed pricing algorithm than the uniform price, so consumer welfare is lower with internal development. Furthermore, price dispersion with an externally developed algorithm exceeds that when it is internally developed:

$$\phi^{sc}(a)-\phi^{I}(a) = \frac{d(a-\mu)}{2(2b-d)(b-d)} \gtreqless 0 as a \gtreqless \mu.$$

Therefore, outsourcing reduces consumer welfare as it exacerbates the harm from third-degree price discrimination.[21]

Recall from Section 2 that the design and fee structure were restricted in order not to facilitate coordinated conduct which would harm consumers. First, the fee was not allowed to be tied to an adopting firm's profit. Without that restriction, the third party could claim a share of firms' profit and thus be incentivized to have the pricing algorithm charge the monopoly price so as to maximize industry profit. Second, the pricing algorithm was not allowed to condition on how many firms adopted. Without that restriction, the third party could design the algorithm to price at the monopoly level but only when both firms adopted, and otherwise price competitively. In spite of those efforts to prevent outsourcing from causing consumer harm, we see that harm still occurs though it is not through higher prices but rather more volatile prices.

4.3 Product Differentiation

In order to consider the effect of product differentiation, assume a representative agent's utility function:

$$\theta_1 q_1 + \theta_2 q_2 - \left(\frac{1}{2}\right)\left(\beta_1 q_1^2 + \beta_2 q_2^2 + 2\eta q_1 q_2\right)$$

[21] A more detailed proof of this result is provided in Appendix C. As we know from the price discrimination literature, welfare results can be sensitive to properties of the demand function; see, for example, Cowan (2016). Thus, it is an open question as to whether this finding extends to some non-linear demand functions.

where η is the degree of product similarity. Firms' products are independent when $\eta = 0$ and identical when $\eta = \beta$. Solving

$$\max_{(q_1,q_2)} \theta_1 q_1 + \theta_2 q_2 - \left(\frac{1}{2}\right)\left(\beta_1 q_1^2 + \beta_2 q_2^2 + 2\eta q_1 q_2\right) - p_1 q_1 - p_2 q_2$$

yields firm 1's demand function (with firm 2's demand function analogously defined):

$$D_1(p_1, p_2) = \left(\frac{1}{\beta^2 - \eta^2}\right)\left(\theta(\beta - \eta) - \beta p_1 + \eta p_2\right) = a - bp_1 + dp_2$$

where

$$a = \frac{\theta}{\beta + \eta}, b = \frac{\beta}{\beta^2 - \eta^2}, d = \frac{\eta}{\beta^2 - \eta^2}, \mu = \frac{\mu_\theta}{\beta + \eta}, \tag{13}$$

and μ_θ is the mean of θ. Substituting (13) into (10)-(11):

$$\phi^{sc}(a) = \frac{(2\beta - \eta)(\theta + c) - \eta(\mu_\theta - c)}{2(2\beta - \eta)} = \frac{\theta + c}{2} - \frac{\eta(\mu_\theta - c)}{2(2\beta - \eta)} \tag{14}$$

$$\phi^I(a) = \frac{(\beta - \eta)\theta + \beta c}{2\beta - \eta} = \frac{\theta + c}{2} - \frac{\eta(\theta - c)}{2(2\beta - \eta)} \tag{15}$$

Differentiating (14)-(15) with respect to the degree of product similarity parameter η yields:[22]

$$\frac{\partial \phi^{sc}(a)}{\partial \eta} = \frac{-\beta(\mu_\theta - c)}{(2\beta - \eta)^2} < 0 \ , \ \frac{\partial^2 \phi^{sc}(a)}{\partial \eta \partial \theta} = 0$$

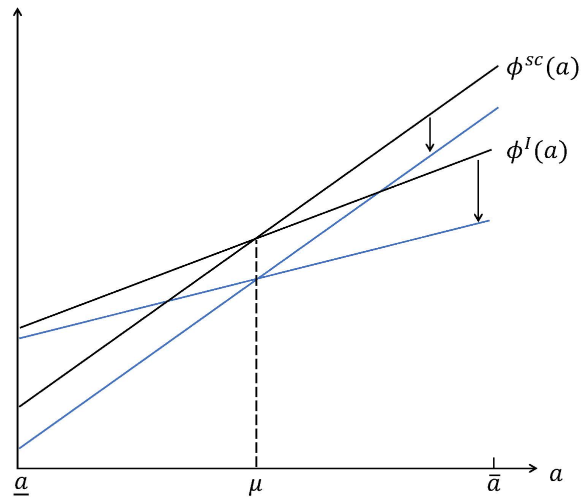
$$\frac{\partial \phi^I(a)}{\partial \eta} = -\frac{\beta(\theta - c)}{(2\beta - \eta)^2} < 0 \ , \ \frac{\partial^2 \phi^I(a)}{\partial \eta \partial \theta} = -\frac{\beta}{(2\beta - \eta)^2} < 0.$$

Figure 2 depicts the change in the two pricing algorithms when products are less differentiated. For all demand shocks, prices are lower for both pricing algorithms, per the usual explanation. As products are less differentiated, the internally developed pricing algorithm becomes less sensitive to the demand shock which is the consequence of more intense price competition. (Recall that, generally, equilibrium prices converge to cost when products become homogeneous; hence, they become independent of demand.) In contrast, the sensitivity to demand shocks of the externally developed pricing algorithm is unaffected by a change in the extent of product differentiation. This singular property is the result of the pricing algorithm's response to demand shocks being designed to maximize joint profit and that

[22] $\phi^{sc}(a)$ and $\phi^I(a)$ are derived assuming an interior solution, and that condition is violated when η is close enough to β. Thus, in reducing the degree of product differentiation, it is assumed products remain sufficiently differentiated.

response is independent of product differentiation.

Figure 2



In sum, outsourcing the pricing algorithm results in price being more responsive to demand shocks and, furthermore, this greater sensitivity is not diminished when firms' products are less differentiated. Using (12) and (13), the differential in expected profit between external and internal development is

$$\frac{\eta^2 \sigma_\theta^2}{4\beta^2 (\beta + \eta)}$$

which is increasing in η (where σ_θ^2 is the variance of θ). As products become more similar, price competition intensifies less when pricing algorithms are externally supplied and that enhances the profit from adoption attributable to outsourcing.

5 Concluding Remarks

The economics and management literature on pricing is based on the assumption that the firm selling the product is also the one that designed its pricing rule.[23] Contrary to that assumption, we are witnessing a growing role for third party developers, and little is understood about the implications of a firm using a pricing algorithm whose design was outsourced. With that motivation, this paper investigated how design incentives differ between a firm interested in selling the pricing algorithm and a firm interested in using the pricing algorithm, and what this means for consumers. In concluding, we summarize some general insight and offer some directions for future research.

Though the model of the paper has a specific and stylized structure, it delivers insight which seems intuitive and broadly relevant to the outsourcing of pricing algorithms. In maximizing its profit, a third party developer designs its pricing algorithm so as to increase

[23] Useful starting references for this immense literature are Waldman and Johnson (2007) and Özalp and Phillips (2012).

demand for it. In order to encourage multiple firms in a market to adopt the algorithm, one might think that a third party developer would make it less competitive. Indeed, a number of competition authorities have expressed exactly that concern. What is not appreciated is that, if the pricing algorithm sets higher prices, it will also make it more attractive for a firm *not* to adopt because it can profitably undercut the prices set by rival firms who did adopt. The challenge for the third party is to *design the pricing algorithm so that it prices cooperatively in a way that allows only a firm with the pricing algorithm to benefit.* In the context of our model for when there is substantive demand variation, this tactic manifests itself in making price highly sensitive to demand variation which a firm can condition on only if it has the pricing algorithm. Hence, a firm without the pricing algorithm cannot exploit a firm with the pricing algorithm. In this way, the third party's design raises the profit from adoption without raising the profit from not adopting, and that increases demand for the pricing algorithm.

As this is the first investigation of the outsourcing of pricing rules, there are many research directions. Building on the model of this paper, one could consider other demand structures. Our results are derived under linear demand with an additive source of demand variation, and we know from the price discrimination literature that welfare results can depend on the curvature of demand. Then there are other types of demand variation including those which affect the degree of product differentiation as well as firm-specific demand shocks.

A critical extension is to allow for multiple third party developers who compete in designing and selling their pricing algorithms. The introduction of competition raises many relevant policy questions. Does competition exacerbate or mitigate the consumer harm when there is a single developer? Does equilibrium involve firms adopting from the same developer? What is the effect of a policy that limits a third party to supplying at most one firm in a market? With such a policy, is there a trade-off between reducing competition in the market for pricing algorithms and increasing competition in the product market?

A more fundamental extension is to specify a different space of pricing algorithms. In this paper, a pricing algorithm conditions on market-specific cost and demand parameters, and that could be extended to condition on rival firms' prices, either prices from past periods (as in Calvano et al, 2020) or where there is a sequentiality to pricing (as in Brown and Mackay, 2020). Third party development is very likely to affect the design of pricing algorithms. For example, a firm might design its pricing algorithm to search for the lowest price among its competitors and charge a price just below that level.[24] However, a third party's pricing algorithm is unlikely to have that property for it would result in a downward spiral of prices should competitors adopt it. The question then is exactly how a third party developer's pricing algorithm will differ from that which a firm itself would design.

Within this broader class of pricing algorithms, one could also explore whether it is possible to prevent collusive pricing when competitors adopt pricing algorithms from the same third party. In the model of this paper, it was prevented by constraining the pricing algorithm so that it does not condition on another firm's adoption decision. One could impose an analogous constraint on this richer space of pricing algorithms by requiring, conditional on the history, the pricing algorithm to produce the same price regardless of rival firms'

[24] Such a rule was used by a poster seller on Amazon Marketplace (U.S. v. Topkins, U.S. Department of Justice, 2015). That there was collusion in that case is not relevant to the point being made.

adoption decisions. We already know from Calvano et al (2020) that collusion can emerge but there is the question of whether it becomes easier or more likely under third party development. A challenge to the prevention of collusion is that the price history will vary with rival firms' adoption decisions and that could provide a path to circumventing the constraint. For example, the algorithms could be programmed to have a pattern in the last two digits of a price and to shift to "collusive" mode when it observes that pattern in a rival's prices.

It seems clear there are many fascinating questions associated with the third party development of pricing algorithms and a considerable need for more research to address them.

6 Appendices

6.1 Appendix A: Literature Review

The theoretical literature examining the implications of Big Data and algorithmic pricing can be categorized along two dimensions: 1) the space of pricing algorithms; and 2) the criterion for selecting a pricing algorithm. The first dimension pertains to how Big Data and algorithmic pricing enrich the feasible set of pricing algorithms. One branch is behavior-based pricing which allows price to condition on a customer's history of purchases (or some other behavior such as clickstream activity). A second branch focuses on how Big Data and algorithmic pricing allows a firm to be more informed of demand when it sets price. This can mean using data to have a more accurate demand forecast or more finely segment the market or better tailor price to current market conditions. A third branch examines how pricing algorithms affect the way in which a firm's price responds to competitors' prices in terms of either the speed of response or committing to a particular response. This review focuses on the latter two branches, while behavior-based pricing is surveyed in Fudenberg and Villas Boas (2007, 2012).

The second dimension is how a firm selects a pricing algorithm. The conventional approach characterizes equilibrium pricing algorithms for a well-defined game. An alternative approach specifies a learning algorithm; that is, how past data (prices, sales, profits) is used to identify a better performing pricing algorithm.[25] Two classes of learning algorithms have been considered: estimation-optimization learning and reinforcement learning. The former embodies two distinct modules. The estimation module estimates the firm's environment and delivers predictions as to how the firm's price or quantity determines its profit or revenue. In particular, past prices and sales are used to estimate a firm's demand function (where various papers have used OLS, Maximum Likelihood, and an artificial neural network), and thereby have an estimate of how price affects a firm's profit (or revenue). With that estimated environment, the optimization module selects price to maximize profit (or revenue) using the estimated demand function, while adding some randomness to generate experimentation. An example discussed below is Cooper, Homen-de-Mello, and Kleywegt (2015), while den Boer (2015) provides an overview of this work.

An estimation-optimization learning algorithm separately estimates the environment and then optimizes in the selection of an action for the estimated environment. In comparison,

[25] Almost exclusively, the behavior-based pricing literature is based on the equilibrium approach.

reinforcement learning fuses estimation and optimization by learning directly over actions; it seeks to identify the best action for a particular state based on how various actions have performed in the past for that state. Its approach is model-free in that it operates without any prior knowledge of the environment. One common method of reinforcement learning is Q-learning. With this approach, there is a value assigned to each action-state pair (e.g., an action is a price and a state is a history of prices) and these values are updated based on realized profit. Given the current collection of values and the current state, the action is chosen which yields the highest value. Recent papers using Q-learning are Calvano et al (2020) and Klein (2019).[26] Hansen, Misra, and Pai (2020) uses the Upper Confidence Bound algorithm which, for each price, keeps track of the empirical average of the profit for that price and the number of times it was chosen. There is an index which is increasing in the empirical average profit and decreasing in the number of times a price was chosen. In any period, the price with the highest index is chosen, so a price is more likely to be selected when it has performed better and has been chosen less frequently.

Let me now turn to reviewing those papers that most directly examine how Big Data and algorithmic pricing affects market competition. The first four papers consider the impact of Big Data and algorithmic pricing on the propensity or extent of collusion. Firms interact in an infinitely repeated price game where pricing algorithms can arbitrarily condition on the history of past prices. Salcedo (2015) modifies the canonical perfect monitoring setting to allow for commitment to and observability of pricing algorithms. A pricing algorithm is a finite automaton which maps price histories into the set of feasible prices. A firm's pricing algorithm is a state variable in that it can be changed only during stochastic revision opportunities. At such an opportunity, a firm is assumed to know its rival's pricing algorithm. Thus, in selecting its pricing algorithm at a revision opportunity, a firm recognizes it will be committed to it until the next revision opportunity and, should its rival have a revision opportunity in the meantime, that rival will observe the firm's pricing algorithm and know it is committed to it. A striking result is derived: under certain conditions, all subgame perfect equilibria result in prices close to monopoly prices. However, a word of caution, for this result is erected on the untenable assumption that a firm observes a rival's pricing algorithm. The presumption is that past price data would allow a firm to "decode" its rival's pricing algorithm, though that cannot generally be possible (e.g., when the number of observations are fewer than the number of states in the finite automaton).

Miklós-Thal and Tucker (2019) considers a duopoly with homogeneous goods where there is one consumer type with fixed demand. A consumer's maximum willingness-to-pay (WTP) can take two possible values and is *iid* over time. In each period, firms receive a common signal of the WTP prior to choosing price. There are two possible signals and $\rho \geq 1/2$ is the probability that the signal is accurate. The influence of Big Data is captured by a higher value of ρ; hence, a firm has better demand information when it chooses price. The analysis focuses on grim trigger strategy equilibria under perfect monitoring. A higher value of ρ has two counteracting effects on the maximal collusive equilibrium price. More accurate demand information allows the cartel to better predict the joint profit-maximizing

[26] Earlier papers using Q-learning in an environment where multiple firms choose prices or quantities include Tesauro and Kephart (2002), Xie and Chen (2004), Waltman and Kaymak (2008), Dogan and Güner (2015), and Hilsen (2016).

price which increases the collusive value and thus makes collusion less difficult. However, more accurate demand information also increases the maximal deviation profit by better informing a prospective deviator when deviation profit is high, which makes collusion more difficult. When the discount factor is sufficiently high, more accurate demand forecasting harms consumers. When the discount factor is sufficiently low, it is possible for consumers to benefit from firms being better informed of demand.

Closely related in motivation is O'Connor and Wilson (2021) which also considers the implications of enhanced demand forecasting though under imperfect monitoring. Without Big Data, demand is affected by two unobservable demand shocks. With Big Data, one of those demand shocks is observed so price can condition on that shock. As with Miklós-Thal and Tucker (2019), the deviation payoff is higher because of the improved demand information which makes collusion harder, but monitoring is more effective which makes collusion easier. The net effect on prices is ambiguous.

The final paper that explores the implications of Big Data and algorithmic pricing (along with AI) for collusion is Calvano et al (2020b). This paper assumes each firm uses Q-learning to discover its pricing algorithm. The central question is whether collusive pricing rules can emerge under Q-learning and, if so, how robust a phenomenon it is. For the infinitely repeated price game with differentiated products, they find it is quite common for prices to converge to levels well above static Nash equilibrium levels. Furthermore, pricing algorithms evolve to having properties of collusive pricing rules.[27] For example, one pricing algorithm that emerged has firms settle on a supracompetitive price and, in response to a rival undercutting it, firms' prices significantly drop and then gradually climb back up to supracompetitive levels. The paper consider many variants of the basic model in concluding that collusion is a robust outcome of Q-learning. Firms whose pricing algorithms are determined by a general form of reinforcement learning can learn to collude.

The remaining papers show how Big Data and algorithmic pricing can result in supracompetitive prices under static optimization. In Brown and MacKay (2020), the profit function is fixed and known, and they focus on the implications of firms being able to respond more rapidly to rivals' prices. In the context of a duopoly game with differentiated products, firms can be heterogeneous in the frequency with which they can change price. For example, one firm may be able to change price every hour, while the other firm can only change price once a day. This heterogeneity introduces commitment in that the firm which is locked into its price over a longer period is effectively a price leader with respect to its rival. Allowing firms to choose their pricing technologies, firms are shown to select different frequencies because creating a leader-follower relationship yields higher prices and profits for both firms compared to when they simultaneously choose prices (which, by the model's timing structure, occurs when they choose the same frequency). So as to ensure itself of being the follower (which is more profitable than being a leader), one of the firms chooses the most rapid pricing technology. By allowing firms to commit to a pricing frequency, Big Data and algorithmic pricing produce higher prices.

Cooper, Homen-de-Mello, and Kleywegt (2015) and Hansen, Misra, and Pai (2020) con-

[27] "Collusion is when firms use history-dependent strategies to sustain supracompetitive outcomes through a reward-punishment scheme that rewards a firm for abiding by the supracompetitive outcome and punishes it for departing from it." Harrington (2017), p. 1.

sider a duopoly setting with differentiated products, where firms do not know their demand or profit functions and are endowed with a learning algorithm. The only available data to a firm are its own past prices and profits which means, in estimating the relationship between its price and profit, the firm has a misspecified model that does not take account of the other firm's price. With an omitted variable that is endogenous to what the pricing algorithm does, estimates will be biased. For example, if, when a firm raises its price, the other firm also happens to raise its price then the firm's demand will be estimated to be less price-elastic than it actually is. Underestimating the price elasticity of demand would cause firms to set higher prices than would be achieved for a full-information equilibrium. Both papers find that this misspecification results in supracompetitive prices. Cooper, Homen-de-Mello, and Kleywegt (2015) assumes prices are set optimally given an OLS-estimated demand curve. Hansen, Misra, and Pai (2020) view it as a multi-armed bandit problem where a pricing algorithm is chosen to minimize statistical regret (i.e., the difference between average profit achieved with the algorithm and ex-post optimal profit). They find that when the signal-to-noise ratio for sales is high (i.e., sales are relatively more responsive to price changes than to demand shocks), firms' prices are supracompetitive and positively correlated. It is when learning results in a high positive correlation that a firm finds a high price relatively profitable because the rival also tends to set a high price.

6.2 Appendix B: Proofs

Lemma 3: The unique affine solution to

$$\max_{\phi \in \Phi} V(A, A, \phi) - V(NA, A, \phi) \tag{16}$$

$$\text{s.t. } V(A, A, \phi) - V(NA, A, \phi) \geq V(A, NA, \phi) - V(NA, NA, \phi) \tag{17}$$

is

$$\phi^{sc} = \frac{2bc(b-d) - d\mu}{4b(b-d) - 2d(b-d)} + \frac{a}{2(b-d)}.$$

Proof: Our approach is to solve the unconstrained problem (16) and then show the solution satisfies the constraint (17). Given linear demand and cost and affine pricing algorithms, (16) takes the form:

$$\max_{(\alpha,\gamma)} \int (\alpha + \gamma a - c)\,(a - (b-d)(\alpha + \gamma a))G'(a)da$$
$$- \int \left(\frac{\mu + bc + d\,(\alpha + \gamma\mu)}{2b} - c \right) \left(a - b \left(\frac{\mu + bc + d\,(\alpha + \gamma\mu)}{2b} \right) + d(\alpha + \gamma a) \right) G'(a)da.$$

Taking the integral in the first term and simplifying the second term yields:

$$\max_{(\alpha,\gamma)} (\alpha - c)\mu - (\alpha - c)(b-d)(\alpha + \gamma\mu) - \gamma\mu(b-d)\alpha$$
$$+\gamma(1 - \gamma(b-d))\left(\mu^2 + \sigma^2\right) - \frac{(\mu - bc + d\,(\alpha + \gamma\mu))^2}{4b}. \tag{18}$$

Solving the first-order conditions to (18) yields:

$$\alpha = \frac{2bc(b-d) - d\mu}{4b(b-d) - 2d(b-d)}, \gamma = \frac{1}{2(b-d)}$$

which is ϕ^{sc}. Referring to the objective in (18) as W, second-order conditions are satisfied:

$$\frac{\partial^2 W}{\partial \alpha^2} = -\frac{(2b-d)^2}{2b} < 0, \quad \frac{\partial^2 W}{\partial \gamma^2} = -\frac{(4b(b-d)(\sigma^2+\mu^2) + d^2\mu^2)}{2b} < 0$$

$$\left(\frac{\partial^2 W}{\partial \alpha^2}\right)\left(\frac{\partial^2 W}{\partial \gamma^2}\right) - \left(\frac{\partial^2 W}{\partial \gamma \partial \alpha}\right)^2 = \frac{\sigma^2 (b-d)(2b-d)^2}{b} > 0.$$

The final step is to show that ϕ^{sc} satisfies (17). It is straightforward to show:

$$V(A, A, \phi^{sc}) = \frac{b(\mu - (b-d)c)^2}{(2b-d)^2} + \frac{\sigma^2}{4(b-d)}$$

$$V(NA, A, \phi^{sc}) = \frac{b(\mu - (b-d)c)^2}{(2b-d)^2},$$

so the incremental value of adoption when the other firm adopts is

$$V(A, A, \phi^{sc}) - V(NA, A, \phi^{sc}) = \frac{\sigma^2}{4(b-d)}. \tag{19}$$

Analogously, it can be shown the incremental value of adoption when the other firm does not adopt is

$$V(A, NA, \phi^{sc}) - V(NA, NA, \phi^{sc}) = \frac{(b-2d)\sigma^2}{4(b-d)^2}. \tag{20}$$

Inserting (19) and (20) into (17),

$$V(A, A, \phi^{sc}) - V(NA, A, \phi^{sc}) = \frac{\sigma^2}{4(b-d)} \geq \frac{(b-2d)\sigma^2}{4(b-d)^2} = V(A, NA, \phi^{sc}) - V(NA, NA, \phi^{sc})$$

which holds because

$$\frac{\sigma^2}{4(b-d)} - \frac{(b-2d)\sigma^2}{4(b-d)^2} = \frac{d\sigma^2}{4(b-d)^2} > 0.$$

■

Lemma 4: The unique affine solution to

$$\max_{\phi \in \Phi} V(A, NA, \phi) - V(NA, NA, \phi)$$

is

$$\phi^{pl} = \frac{(b+d)(2cb^2 - cdb + d\mu)}{b(4b^2 - 2d^2)} + \frac{a}{2b}.$$

Proof: First note that $\max_{\phi\in\Phi} V(A, NA, \phi) - V(NA, NA, \phi)$ is equivalent to $\max_{\phi\in\Phi} V(A, NA, \phi)$ given that ϕ does not appear in $V(NA, NA, \phi)$. With affine pricing algorithms, $\max_{\phi\in\Phi} V(A, NA, \phi)$ takes the form:

$$\max_{(\alpha,\gamma)} \int (\alpha + \gamma a - c) \left(a - b(\alpha + \gamma a) + d \left(\frac{\mu + bc + d(\alpha + \gamma\mu)}{2b} \right) \right) G'(a)da. \tag{21}$$

Taking the integral yields:

$$\begin{aligned} &(\alpha - c) \left(\mu - b(\alpha + \gamma\mu) + d \left(\frac{\mu + bc + d(\alpha + \gamma\mu)}{2b} \right) \right) \\ &+\gamma \left(\mu^2 + \sigma^2\right)(1 - b\gamma) + \gamma\mu \left(-b\alpha + d \left(\frac{\mu + bc + d(\alpha + \gamma\mu)}{2b} \right) \right). \end{aligned}$$

Solving the first-order conditions delivers:

$$\alpha = \frac{(b + d)(2cb^2 - cdb + d\mu)}{b(4b^2 - 2d^2)}, \ \gamma = \frac{1}{2b},$$

which is ϕ^{pl}. Referring to the objective in (21) as W, second-order conditions are satisfied:

$$\frac{\partial^2 W}{\partial \alpha^2} = -\frac{1}{b}\left(2b^2 - d^2\right) < 0, \ \frac{\partial^2 W}{\partial \gamma^2} = -\frac{1}{b}\left(2b^2\sigma^2 + (2b^2 - d^2)\mu^2\right) < 0$$

$$\left(\frac{\partial^2 W}{\partial \alpha^2}\right)\left(\frac{\partial^2 W}{\partial \gamma^2}\right) - \left(\frac{\partial^2 W}{\partial \alpha \partial \gamma}\right)^2 = 2\sigma^2\left(2b^2 - d^2\right) > 0.$$

■

Proof of Theorem 1: For the purpose of the analysis, the dependence of values on σ^2 is made explicit (where recall that σ^2 is an element of a market's type). Given f, the equilibrium algorithm maximizes expected demand (for a market type):

$$\begin{aligned} \max_{\phi} \quad & 1 \times \max\{K\left(V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2) - f\right) \\ & -K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right), 0\} \\ & +2 \times K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right). \end{aligned} \tag{22}$$

Consider $\sigma^2 = 0$. It is shown in the proof of Lemma 3 that

$$\max_{\phi} V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) = \frac{\sigma^2}{4(b - d)} \tag{23}$$

which implies $\max_\phi V(A, A, \phi, 0) - V(NA, A, \phi, 0) = 0$. Hence, at the solution to (22) for $\sigma^2 = 0$,

$$V(A, A, \phi, 0) - V(NA, A, \phi, 0) - f = -f \leq 0$$

which implies

$$K\left(V(A, A, \phi, 0) - V(NA, A, \phi, 0) - f\right) = 0.$$

Thus, the solution to (22) maximizes

$$K\left(V(A,NA,\phi,0)-V(NA,NA,\phi,0)-f\right)$$

or, equivalently, maximizes $V(A,NA,\phi,0)-V(NA,NA,\phi,0)$. It is shown in Lemma 4 that the solution to $\max_\phi V(A,NA,\phi,\sigma^2)-V(NA,NA,\phi,\sigma^2)$ is ϕ^{pl}. Hence, if $\sigma^2=0$ then the equilibrium pricing algorithm is ϕ^{pl}.

Now suppose σ^2 is close to zero. Define $\phi^o(a,\sigma^2)$ as the solution to (22) for a sequence of values for σ^2 that go to zero: $\{\phi^o(a,\sigma^2)\}_{\sigma^2\to 0}$. As $\sigma^2\to 0$, assume $\phi^o(a,\sigma^2)$ differs from $\phi^{pl}(a)$ for a set of positive measure of values for a. I will show $\exists\eta>0$ such that if $\sigma^2\in(0,\eta]$ then ϕ^{pl} yields a strictly higher expected demand than $\phi^o(\sigma^2)$. This contradiction will prove the result.

First suppose $\lim_{\sigma^2\to 0}\phi^o(a,\sigma^2)=\phi^{pl}(a)$ pointwise in a for all but a set of measure zero. Thus, $\phi^o(\sigma^2)$ is close to ϕ^{pl} when σ^2 is close to zero. Note that

$$0>V(A,A,\phi^{pl},0)-V(NA,A,\phi^{pl},0) \tag{24}$$

because, given the adopting rival firm chooses ϕ^{pl}, $V(NA,A,\phi^{pl},0)$ is the profit from choosing the unique best response to ϕ^{pl} and $V(A,A,\phi^{pl},0)$ is the profit from choosing ϕ^{pl} which is not the best response to ϕ^{pl}. By the continuity of $V(A,A,\phi,\sigma^2)-V(NA,A,\phi,\sigma^2)$ in σ^2 and ϕ and that, by supposition, $\lim_{\sigma^2\to 0}\phi^o(\sigma^2)=\phi^{pl}$, it follows from (24) that

$$0>V(A,A,\phi^o(\sigma^2),\sigma^2)-V(NA,A,\phi^o(\sigma^2),\sigma^2)$$

for σ^2 close to zero, which then implies

$$K\left(V(A,A,\phi^o(\sigma^2),\sigma^2)-V(NA,A,\phi^o(\sigma^2),\sigma^2)-f\right)=0$$

for σ^2 close to zero. Consequently, if $\phi^o(\sigma^2)$ is the solution to (22) then it maximizes

$$K\left(V(A,NA,\phi,\sigma^2)-V(NA,NA,\phi,\sigma^2)-f\right)$$

and, equivalently, maximizes $V(A,NA,\phi,\sigma^2)-V(NA,NA,\phi,\sigma^2)$.[28] However, $\phi^o(\sigma^2)\neq\phi^{pl}$ for a set of positive measure delivers a contradiction because ϕ^{pl} is the unique maximum of $V(A,NA,\phi,\sigma^2)-V(NA,NA,\phi,\sigma^2)$. This completes the proof that, for σ^2 close to zero, there cannot be a solution to (22) in a neighborhood of ϕ^{pl} that is different from ϕ^{pl}.

Now suppose $\lim_{\sigma^2\to 0}\phi^o(a,\sigma^2)\neq\phi^{pl}(a)$ for a set of positive measure of values for a so the claimed optimum is not in a neighborhood of ϕ^{pl}. We know that

$$\lim_{\sigma^2\to 0}V(A,NA,\phi^{pl},\sigma^2)-V(NA,NA,\phi^{pl},\sigma^2)>0,$$

while

$$\lim_{\sigma^2\to 0}V(A,A,\phi^{pl},\sigma^2)-V(NA,A,\phi^{pl},\sigma^2)<0 \tag{25}$$

[28]This equivalence does require $\exists\phi$ such that $K\left(V(A,A,\phi^o(\sigma^2),\sigma^2)-V(NA,A,\phi^o(\sigma^2),\sigma^2)-f\right)>0$ which is presumed to be true. Otherwise, expected demand is zero $\forall\phi$ in which case any ϕ is a solution.

which follows from (24) and continuity of $V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2)$ in σ^2. As $\sigma^2 \to 0$, we then have[29]

$$K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) > 0 = K\left(V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2) - f\right).$$

Given that $\phi^o(\sigma^2)$ is bounded away from ϕ^{pl} as $\sigma^2 \to 0$ then, given ϕ^{pl} is the unique maximum of $V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2)$,

$$\begin{aligned} & \lim_{\sigma^2 \to 0} V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) \\ > \; & \lim_{\sigma^2 \to 0} V(A, NA, \phi^o(\sigma^2), \sigma^2) - V(NA, NA, \phi^o(\sigma^2), \sigma^2). \end{aligned} \tag{26}$$

It then follows: as $\sigma^2 \to 0$,

$$\begin{aligned} & K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) \\ & -K\left(V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2) - f\right) \\ > \; & K\left(V(A, NA, \phi^o(\sigma^2), \sigma^2) - V(NA, NA, \phi^o(\sigma^2), \sigma^2) - f\right) \\ & -K\left(V(A, A, \phi^o(\sigma^2), \sigma^2) - V(NA, A, \phi^o(\sigma^2), \sigma^2) - f\right) \end{aligned} \tag{27}$$

because the first term on the LHS > first term on the RHS by (26) and the second term on the LHS $= 0 \leq$ second term on the RHS by (25). Given (27) and

$$K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) - K\left(V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2) - f\right) > 0$$

then the first term in (22) is higher with ϕ^{pl} than ϕ^o:

$$\begin{aligned} & \lim_{\sigma^2 \to 0} 1 \times \max\left\{ \begin{pmatrix} K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) \\ -K\left(V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2) - f\right) \end{pmatrix}, 0 \right\} \\ > \; & \lim_{\sigma^2 \to 0} 1 \times \max\left\{ \begin{pmatrix} K\left(V(A, NA, \phi^o(\sigma^2), \sigma^2) - V(NA, NA, \phi^o(\sigma^2), \sigma^2) - f\right) \\ -K\left(V(A, A, \phi^o(\sigma^2), \sigma^2) - V(NA, A, \phi^o(\sigma^2), \sigma^2) - f\right) \end{pmatrix}, 0 \right\}. \end{aligned}$$

Hence, a necessary condition for the optimality of ϕ^o is that the second term in (22) is higher with ϕ^o than ϕ^{pl}:

$$\begin{aligned} & \lim_{\sigma^2 \to 0} K\left(V(A, A, \phi^o(\sigma^2), \sigma^2) - V(NA, A, \phi^o(\sigma^2), \sigma^2) - f\right) \\ > \; & \lim_{\sigma^2 \to 0} K\left(V(A, A, \phi^{pl}, \sigma^2) - V(NA, A, \phi^{pl}, \sigma^2) - f\right). \end{aligned} \tag{28}$$

However, given (23) then

$$\lim_{\sigma^2 \to 0} K(V(A, A, \phi^o(\sigma^2), \sigma^2) - V(NA, A, \phi^o(\sigma^2), \sigma^2)) = 0$$

which implies (28) cannot be true. In sum, if $\phi^o(\sigma^2)$ is bounded away from ϕ^{pl} then, for σ^2 close to zero, ϕ^{pl} has higher expected demand. Again, we have a contradiction to the claim that $\phi^o(\sigma^2)$ is optimal.

[29] This does presume f is small enough that $K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) > 0$. Otherwise, there is a degenerate solution as expected demand is zero for all ϕ.

We conclude $\exists \eta > 0$ such that if $\sigma^2 \in [0, \eta]$ then the solution to (22) is ϕ^{pl}. ■

Proof of Theorem 2: For the ensuing analysis, we will break the third party's optimization problem into two sub-problems. Partition the set of pricing algorithms into those that result in adoptions being strategic complements - call that subset of pricing algorithms Φ^{sc} - and those for which adoptions are strategic substitutes, Φ^{ss}.

$$\begin{aligned}
\Phi^{sc} &\equiv \left\{\phi \in \Phi : V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) \geq V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2)\right\}. \\
\Phi^{ss} &\equiv \left\{\phi \in \Phi : V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) < V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2)\right\}.
\end{aligned}$$

We have already shown ϕ^{sc} is the solution when the choice set is Φ^{sc}. Note that the associated expected demand is

$$2K\left(V(A, A, \phi^{sc}, \sigma^2) - V(NA, A, \phi^{sc}, \sigma^2) - f\right) = 2K\left(\frac{\sigma^2}{4(b-d)} - f\right).$$

Consider the optimal algorithm subject to making adoptions strategic substitutes:

$$\begin{aligned}
\phi^{ss} &= \arg\max_{\phi \in \Phi^{ss}} 1 \times \left(K\left(V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2) - f\right)\right. \\
&\quad \left. -K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right)\right) \\
&\quad +2 \times K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right) \\
&= \arg\max_{\phi \in \Phi^{ss}} K\left(V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2) - f\right) \\
&\quad +K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right).
\end{aligned} \tag{29}$$

Strategic substitutes and $K' > 0$ imply

$$K\left(V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2) - f\right) > K\left(V(A, A, \phi, \sigma^2) - V(NA, A, \phi, \sigma^2) - f\right) \;\; \forall \phi \in \Phi^{ss}$$

and, therefore,

$$\begin{aligned}
& 2K\left(V(A, NA, \phi^{ss}, \sigma^2) - V(NA, NA, \phi^{ss}, \sigma^2) - f\right) \\
> \; & K\left(V(A, NA, \phi^{ss}, \sigma^2) - V(NA, NA, \phi^{ss}, \sigma^2) - f\right) \\
& +K\left(V(A, A, \phi^{ss}, \sigma^2) - V(NA, A, \phi^{ss}, \sigma^2) - f\right).
\end{aligned} \tag{30}$$

Note that the RHS of (30) is the maximal value of the objective in (29).

From Lemma 4,

$$\phi^{pl} = \arg\max_{\phi \in \Phi} V(A, NA, \phi, \sigma^2) - V(NA, NA, \phi, \sigma^2)$$

which means $2K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right)$ is an upper bound on the LHS of (30). Hence,

$$\begin{aligned}
& 2K\left(V(A, NA, \phi^{pl}, \sigma^2) - V(NA, NA, \phi^{pl}, \sigma^2) - f\right) \\
> \; & K\left(V(A, NA, \phi^{ss}, \sigma^2) - V(NA, NA, \phi^{ss}, \sigma^2) - f\right) \\
& +K\left(V(A, A, \phi^{ss}, \sigma^2) - V(NA, A, \phi^{ss}, \sigma^2) - f\right).
\end{aligned} \tag{31}$$

In sum, $2K\left(V(A,NA,\phi^{pl},\sigma^2)-V(NA,NA,\phi^{pl},\sigma^2)-f\right)$ is an upper bound on the expected demand from any $\phi\in\Phi^{ss}$.

To complete the proof, we want to show: if σ^2 is sufficiently high then the expected demand from ϕ^{sc} exceeds the expected demand from ϕ^{ss},

$$\begin{aligned} & 2K\left(V(A,A,\phi^{sc},\sigma^2)-V(NA,A,\phi^{sc},\sigma^2)-f\right) \\ > \; & K\left(V(A,NA,\phi^{ss},\sigma^2)-V(NA,NA,\phi^{ss},\sigma^2)-f\right) \\ & +K\left(V(A,A,\phi^{ss},\sigma^2)-V(NA,A,\phi^{ss},\sigma^2)-f\right), \end{aligned} \tag{32}$$

so ϕ^{sc} is the equilibrium pricing algorithm. Given (31), a sufficient condition for (32) is

$$2K\left(V(A,A,\phi^{sc},\sigma^2)-V(NA,A,\phi^{sc},\sigma^2)-f\right)\geq 2K\left(V(A,NA,\phi^{pl},\sigma^2)-V(NA,NA,\phi^{pl},\sigma^2)-f\right)$$

or, equivalently,

$$V(A,A,\phi^{sc},\sigma^2)-V(NA,A,\phi^{sc},\sigma^2)\geq V(A,NA,\phi^{pl},\sigma^2)-V(NA,NA,\phi^{pl},\sigma^2). \tag{33}$$

One can show the RHS of (33) is

$$\frac{d^4\left(\mu-(b-d)c\right)^2}{8b\left(2b-d\right)^2\left(2b^2-d^2\right)}+\left(\frac{1}{4b}\right)\sigma^2.$$

Using (19), (33) takes the form:

$$\frac{\sigma^2}{4(b-d)}\geq\frac{d^4\left(\mu-(b-d)c\right)^2}{8b\left(2b-d\right)^2\left(2b^2-d^2\right)}+\frac{\sigma^2}{4b}$$

or

$$\sigma^2\geq\frac{d^3(b-d)\left(\mu-(b-d)c\right)^2}{2\left(2b-d\right)^2\left(2b^2-d^2\right)}. \tag{34}$$

Thus, a sufficient condition for ϕ^{sc} to be the equilibrium pricing algorithm is that σ^2 satisfies (34). ■

6.3 Appendix C: Consumer Welfare

Suppose there are two products and it'll be sufficient to focus on when a consumer consumes equal amounts of them. Let $U(q,a)$ be a consumer's utility function when consuming q units of each of the two products and the state is a. U is assumed to be concave in q. Assume $\Lambda=\{a_1,...,a_m\}$ where $a_1<\cdots<a_m$ and, for convenience, $\exists a^o\in\Lambda$ such that $\sum_{j=1}^m(1/m)a_j=a^o$. The m states are the analogue to m markets from the perspective of the price discrimination literature. Expected utility (or, equivalently, weighted aggregate utility over the m markets) is

$$\sum_{j=1}^m\rho\left(a_j\right)U(q(a),a)$$

where $\rho\left(a\right)$ is the probability of utility state a. Concavity of U implies concavity of its expectation.

Consider two price and quantity vectors: $(p'(a), q'(a))_{a\in\Lambda}$ and $(p''(a), q''(a))_{a\in\Lambda}$. Assume $p'(a)$ is non-decreasing in a and $p''(a)$ is increasing in a. Further suppose $p''(a)$ is greater (less) than $p'(a)$ as a is greater (less) than a^o, the quantities satisfy the relationship implied by these price vectors under decreasing demand, and the expected quantities are equal in the two configurations:

$$\begin{aligned} p''(a) &\gtreqqless p'(a) \text{ as } a \gtreqqless a^o \\ q''(a) &\lesseqqgtr q'(a) \text{ as } a \gtreqqless a^o \\ \sum_{j=1}^{m} \rho(a_j)\, q'(a_j) &= \sum_{j=1}^{m} \rho(a_j)\, q''(a_j). \end{aligned}$$

Assuming the quantities correspond to the associated demands, note that these three properties hold when: 1) p' is the uniform price p^N and p'' is the internally developed pricing algorithm ϕ^I; and 2) p' is the internally developed pricing algorithm and p'' is the externally developed pricing algorithm ϕ^{sc}.

By concavity,

$$\sum_{j=1}^{m} \rho(a_j)\, U(q''(a_j), a_j) \leq \sum_{j=1}^{m} \rho(a_j)\, U(q'(a_j), a_j) + \sum_{j=1}^{m} \rho(a_j)\, \frac{\partial U(q'(a_j), a_j)}{\partial q} \left(q''(a_j) - q'(a_j)\right). \tag{35}$$

Given that the quantity is chosen to maximize net surplus $U(q, a) - p(a)$ then $\partial U(q(a), a)/\partial q = p(a)$. As a result, (35) becomes:

$$\sum_{j=1}^{m} \rho(a_j)\, U(q''(a_j), a_j) \leq \sum_{j=1}^{m} \rho(a_j)\, U(q'(a_j), a_j) + \sum_{j=1}^{m} \rho(a_j)\, p'(a_j) \left(q''(a_j) - q'(a_j)\right),$$

which delivers an upper bound on the change in consumer welfare:

$$\sum_{j=1}^{m} \rho(a_j)\, U(q''(a_j), a_j) - \sum_{j=1}^{m} \rho(a_j)\, U(q'(a_j), a_j) \leq \sum_{j=1}^{m} \rho(a_j)\, p'(a_j) \left(q''(a_j) - q'(a_j)\right). \tag{36}$$

Given that, by construction,

$$p'(a^o) \sum_{j=1}^{m} \rho(a_j) \left(q''(a_j) - q'(a_j)\right) = 0$$

then

$$\begin{aligned} &\sum_{j=1}^{m} \rho(a_j)\, p'(a_j) \left(q''(a_j) - q'(a_j)\right) \qquad (37) \\ =\ &\sum_{j=1}^{m} \rho(a_j)\, p'(a_j) \left(q''(a_j) - q'(a_j)\right) + p'(a^o) \sum_{j=1}^{m} \rho(a_j) \left(q''(a_j) - q'(a_j)\right) \\ =\ &\sum_{j=1}^{m} \rho(a_j) \left(p'(a_j) - p'(a^o)\right) \left(q''(a_j) - q'(a_j)\right). \end{aligned}$$

Using (37), (36) becomes

$$\sum_{j=1}^{m} \rho(a_j) U(q''(a_j), a_j) - \sum_{j=1}^{m} \rho(a_j) U(q'(a_j), a_j) \leq \sum_{j=1}^{m} \rho(a_j) (p'(a_j) - p'(a^o)) (q''(a_j) - q'(a_j)) \tag{38}$$

The RHS of (38) is negative because $p'(a_j) - p'(a^o) < 0$ and $q''(a_j) - q'(a_j) > 0$ when $a_j < a^o$, and $p'(a_j) - p'(a^o) > 0$ and $q''(a_j) - q'(a_j) < 0$ when $a_j > a^o$.

It has then been shown that consumer welfare is highest with the uniform price, next highest with the internally developed pricing algorithm, and lowest with the externally developed pricing algorithm. Given constant and common marginal cost, the same ordering applies to total welfare (consumer welfare plus industry profit).

References

[1] Assad, Stephanie, Robert Clark, Daniel Ershov, and Lei Xu, "Algorithmic Pricing and Competition: Empirical Evidence from the German Retail Gasoline Market," working paper, December 2020.

[2] Brown, Zach Y. and Alexander MacKay, "Competition in Pricing Algorithms," working paper, December 2020.

[3] Calvano, Emilio, Giacomo Calzolari, Vincenzo Denicolò, Joseph E. Harrington, Jr., and Sergio Pastorello, "Protecting Consumers from Collusive Prices Due to AI," *Science*, 370 (27 November 2020a), 1040-1042.

[4] Calvano, Emilio, Giacomo Calzolari, Vincenzo Denicolò, and Sergio Pastorello, "Artificial Intelligence, Algorithmic Pricing and Collusion," *American Economic Review*, 110 (2020b), 3267-3297.

[5] Chen, Le, Alan Mislove, and Christo Wilson, "An Empirical Analysis of Algorithmic Pricing on Amazon Marketplace," *Proceedings of the 25th International Conference on World Wide Web*, April 2016, 1339-1349.

[6] Cooper, William L., Tito Homen-de-Mello, and Anton J. Kleywegt, "Learning and Pricing with Models That Do Not Explicitly Incorporate Competition," *Operations Research*, 63 (2015), 86-103.

[7] Corts, Kenneth S., "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment," *RAND Journal of Economics*, 29 (1998), 306-323.

[8] Cowan, Simon, "Welfare-increasing Third-degree Price Discrimination," *RAND Journal of Economics*, 47 (2016), 326-340.

[9] Deng, Ai, "What Do We Know About Algorithmic Tacit Collusion?," *Antitrust*, 33 (2018), 88-95.

[10] Ezrachi, Ariel and Maurice E. Stucke, "Artificial Intelligence and Collusion: When Computers Inhibit Competition," *University of Illinois Law Review*, 2017 (2017), 1775-1810.

[11] Fudenberg, Drew and J. Miguel Villas Boas, "Price Discrimination in the Digital Economy," *Oxford Handbook of the Digital Economy*, Martin Peitz and Joel Waldfogel, eds., Oxford and New York: Oxford University Press, 2012, 254-272.

[12] Fudenberg, Drew and J. Miguel Villas-Boas, "Behavior-Based Price Discrimination and Customer Recognition," in *Economics and Information Systems*, Terrence J. Hendershott, ed., Bingley, UK: Emerald, Elsevier, 2007, 377-436.

[13] Gal, Michal S., "Algorithms as Facilitating Practices," *Berkeley Technology Law Journal*, 34 (2019), 67-118.

[14] German Monopolies Commission, XXII. Biennial Report, Chapter on "Algorithms and Collusion," 2018.

[15] Hansen, Karsten, Kanishka Misra, and Mallesh Pai, "Algorithmic Collusion: Supra-Competitive Prices via Independent Algorithms," working paper, January 2020 (*Marketing Science*, forthcoming).

[16] Harrington, Joseph E., Jr., "Developing Competition Law for Collusion by Autonomous Artificial Agents," *Journal of Competition Law and Economics*, 14 (2018), 331-363.

[17] Harrington, Joseph E., Jr., "Third Party Pricing Algorithms and the Intensity of Competition," working paper, November 2020.

[18] Johnson, Paul A., "Should We Be Concerned That Data and Algorithms Will Soften Competition?," *CPI Antitrust Chronicle*, 2 (2017), 10-15.

[19] Klein, Timo, "Autonomous Algorithmic Collusion: Q-Learning Under Sequential Pricing," Amsterdam Center for Law & Economics Working Paper No. 2018-05, April 2019.

[20] Mehra, Salil K., "Antitrust and the Robo-Seller: Competition in the Time of Algorithms," *Minnesota Law Review*, 100 (2016), 1323-1375.

[21] Miklós-Thal, Jeanine and Catherine Tucker, "Collusion by Algorithm: Does Better Demand Prediction Facilitate Coordination Between Sellers?," *Management Science*, 4 (2019), 1552-1561.

[22] O'Connor, Jason and Nathan E. Wilson, "Reduced Demand Uncertainty and the Sustainability of Collusion: How AI Could Affect Collusion," *Information Economics and Policy*, 54 (2021), forthcoming.

[23] OECD, "Algorithms and Collusion - Background Note by the Secretariat," DAF/COMP(2017)4, 16 May 2017.

[24] Oxera, "When Algorithms Set Prices: Winners and Losers," Discussion Paper, 19 June 2017.

[25] Özer, Özalp and Robert Phillips, eds., *The Oxford Handbook of Pricing Management*, Oxford: Oxford University Press, 2012.

[26] Salcedo, Bruno, "Pricing Algorithms and Tacit Collusion," working paper, November 2015.

[27] Schwalbe, Ulrich, "Algorithms, Machine Learning, and Collusion," *Journal of Competition Law and Economics*, 14 (2019), 568-607.

[28] Stole, Lars, "Price Discrimination and Competition," *Handbook of Industrial Organization*, Volume 3, Mark Armstrong and Robert H. Porter, eds., Amsterdam: Elsevier, 2007, 2221-2299.

[29] United Kingdom Competition & Markets Authority, "Pricing Algorithms: Economic Working Paper on the Use of Algorithms to Facilitate Collusion and Personalised Pricing," 8 October 2018.

[30] Varian, Hal R., "Price Discrimination," in *Handbook of Industrial Organization*, Volume 1, Richard Schmalensee and Robert D. Willig, eds., Amsterdam: Elsevier, 1989, 597-654.

[31] Waldman, Michael and Justin Johnson, eds., *Pricing Tactics, Strategies, and Outcomes*, Cheltenham: Edward Elgar, 2007.