

ONLINE THEORY APPENDIX TO

RENT SHARING TO CONTROL NON-CARTEL SUPPLY

IN THE GERMAN CEMENT MARKET

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Consider a market with a cartel that acts as a joint profit-maximizer and suppose there is a single intermediary and a single non-cartel supplier.¹ The cartel and the non-cartel supplier offer homogeneous products and the market demand curve is $D(p)$. The cartel produces at constant marginal cost c' . For the non-cartel firm to supply this market, it must operate through the intermediary. The non-cartel firm produces at constant marginal cost c'' , while the intermediary's services are provided at constant marginal cost g . Assume $c'' + g > c'$ – so the cartel is the more efficient supplier – and $D(c'' + g) > 0$ – so it is feasible for the non-cartel firm to price above cost and have positive demand.

Assume the demand curve is such that $(p - c)D(p)$ is strictly quasi-concave (when positive) and, therefore, the monopoly price $p^m(c) = \operatorname{argmax} (p - c)D(p)$ exists. It is also supposed that $p^m(c)$ is increasing in c so, in particular, $p^m(c'' + g) > p^m(c')$. Finally, assume that the cost differential between the cartel and the non-cartel supplier is not too large so that: $p^m(c') > c'' + g$.

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¹ For simplicity, we will specify a one-shot setting and assume a joint profit-maximizing cartel. However, all results can be derived as an equilibrium in an infinitely repeated game in which each firm maximizes the present value of its profit stream and firms' discount factors are sufficiently close to one. It is also assumed for simplicity that there is one intermediary and one non-cartel supplier. While we believe the resulting insight is robust to that assumption, extending the analysis to multiple intermediaries and multiple non-cartel suppliers would be a major complication as it would involve modelling multi-lateral bargaining.

A standard approach to analyzing this setting is to suppose that the cartel chooses a price to maximize its joint profit while taking into account how the non-cartel supplier will respond; that is, the extensive form is sequential move with the cartel acting as a price leader. In this case, if the cartel's price exceeds the cost of non-cartel supply then the partnership of the non-cartel firm and intermediary will price below it and leave the cartel with zero demand and zero profit. To avoid that outcome, the cartel will price just below $c'' + g$ in response to which the non-cartel firm prices at (or above) $c'' + g$. The cartel earns profit of approximately $(c'' + g - c')D(c'' + g)$. As long as the cost advantage of the cartel is not too great – so that $p^m(c') > c'' + g$ – the cartel is constrained to pricing no higher than the cost of alternative supply.²

Let us now enrich this setting by giving the cartel the opportunity to share rents with the intermediary for the purpose of preventing non-cartel supply from entering the market. With three players, there are a variety of possible extensive forms though we will argue for a particular one. As assumed above, the cartel commits to price and then those firms that are not part of the cartel select a price; thus, the cartel acts as a price leader.

Having set its price, the cartel is presumed to approach the intermediary about a possible sharing of profit if, in exchange, the intermediary does not offer its services to the non-cartel supplier. Rather than explicitly model the bargaining process between the cartel and the intermediary, we will take a reduced form approach by assuming that a per unit payment from the cartel to the intermediary is determined by the generalized Nash Bargaining Solution (NBS). If the two parties succeed in coming to an agreement then the game ends as the intermediary does not provide its services to the non-cartel firm and, as a result, there is no non-cartel supply. If the cartel and intermediary fail to come to an agreement, the non-cartel firm and the intermediary bargain over both the price to the consumer and how revenues are shared between the two parties. Again, the outcome of that negotiation is represented as a generalized NBS. If they fail to come to an agreement then there is no non-cartel supply and the cartel sells to the market at the price it set in the first stage. This sequence of moves is depicted in Figure A1.

² The sequential-move extensive form is used, for example, in Bos and Harrington (2010). If instead the two suppliers make simultaneous price decisions then there are many other Nash equilibria including the just described subgame perfect equilibrium for this sequential-move game. The latter outcome is generally thought to be the most reasonable Nash equilibrium for the simultaneous-move game.

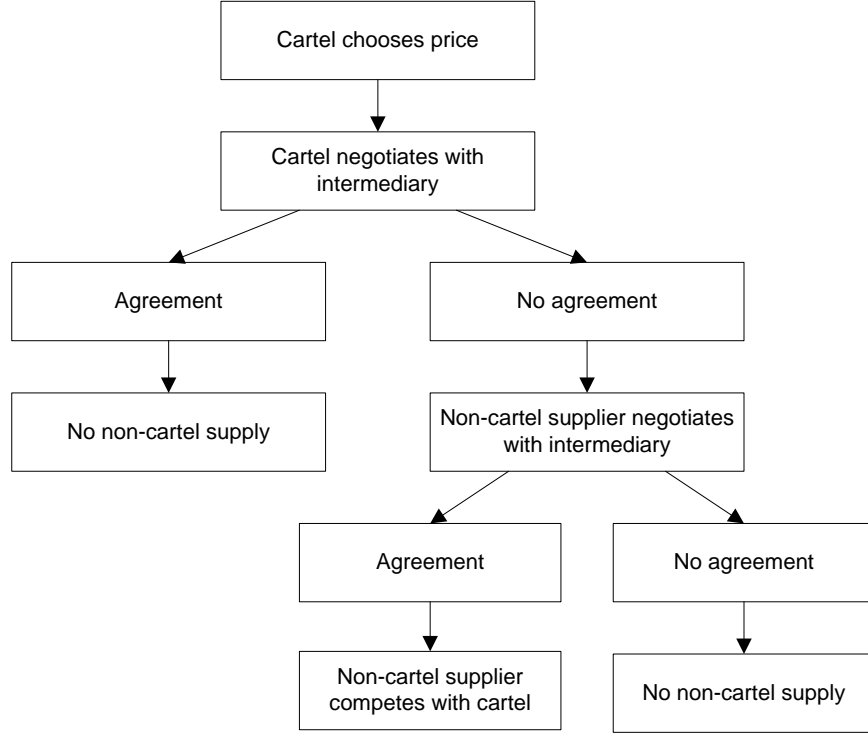


Figure A1: Sequence of Moves in Cartel-Intermediary-Non-Cartel Supplier Game

Note that we are assuming that the intermediary and the non-cartel firm bargain over both the product price and a sharing of revenues with the intermediary, while the intermediary and the cartel only bargain over the sharing of revenues as the cartel has already chosen its price. This asymmetric treatment is justified because the cartel can sell without the assistance of the intermediary, while the non-cartel firm cannot. Thus, the cartel can commit to a price and then approach the intermediary about making a deal. In contrast, the non-cartel firm and intermediary are both necessary for supplying the market so it is not meaningful for the non-cartel firm to have set the product price without having come to an agreement with the intermediary.

In sum, the game has three stages: 1) the cartel sets a price p^c at which it is willing to sell to customers; 2) the cartel and the intermediary negotiate over a per unit payment paid to the intermediary which represents a splitting of profit associated with the cartel selling at a price p^c ; and 3) if the cartel and the intermediary fail to reach an agreement then the intermediary and the non-cartel supplier bargain over the product price and how that revenue is allocated between the two parties. The presumption is that if the cartel fails to come to an agreement with the intermediary in stage 2 then the intermediary will cut a deal with the non-cartel supplier.

This three-stage game is solved using backward induction. Suppose bargaining between the cartel and the intermediary broke down so stage 3 is reached. With regards to the bargaining between the intermediary and the non-cartel firm, their threat points have them each earn zero profit because they do not supply the market. The intermediary and non-cartel supplier are assumed to choose the price p (charged to customers) and the payment received per unit by the intermediary. Denoting that per unit payment by r , the non-cartel supplier then receives $p - r$ per unit. Note that the NBS will have $p \in (c'' + g, p^c)$ for that yields positive total profit while any other price yields non-positive profit.

Letting β denote the bargaining power of the non-cartel supplier, p and r are chosen to solve the NBS objective:

$$\begin{aligned} \max_{p,r} [(p - c'' - r)D(p)]^\beta [(r - g)D(p)]^{1-\beta} \\ = \max_{p,r} (p - c'' - r)^\beta (r - g)^{1-\beta} D(p) \end{aligned} \quad (\text{A1})$$

Let us first solve for r and then solve for p . The first-order condition with respect to the payment to the intermediary is:

$$(p - c'' - r)^{\beta-1} (r - g)^{-\beta} [-\beta(r - g) + (1 - \beta)(p - c'' - r)] D(p) = 0, \quad (\text{A2})$$

which is then solved for r : $r^*(p) = (1 - \beta)(p - c'') + \beta g$. Given customers pay a price p for non-cartel supply, the intermediary earns profit of

$$(r^*(p) - g)D(p) = (1 - \beta)(p - c'' - g)D(p) \quad (\text{A3})$$

and the non-cartel supplier receives:

$$(p - c'' - r^*(p))D(p) = \beta(p - c'' - g)D(p). \quad (\text{A4})$$

Next let us solve for the optimal price to charge for non-cartel supply. Assuming that the cartel's price is no higher than its monopoly price, it follows from our earlier assumptions that

$$\frac{\partial(p - c'' - g)D(p)}{\partial p} > 0 \text{ for all } p < p^c (\leq p^m(c') < p^m(c'' + g)) \quad (\text{A5})$$

in which case the payoffs for the intermediary and the non-cartel supplier are maximized by pricing just below p^c . In other words, they set the product price to maximize their total

profit $(p - c'' - g)D(p)$ – which requires just undercutting the cartel's price - and then allocate that profit according to the their bargaining power. In conclusion, the stage 3 payoffs to the intermediary and non-cartel supplier are, respectively,

$$(1 - \beta)(p^c - c'' - g)D(p^c) \text{ and } \beta(p^c - c'' - g)D(p^c). \quad (\text{A6})$$

Let us now move to stage 2 where the cartel and the intermediary bargain given the cartel has set a product price of p^c . s will denote the per unit payment received by the intermediary in which case the cartel receives $p^c - s$ per unit. Given that the cartel does not need the services of the intermediary, the cost g is not incurred and the payment to the intermediary is only to prevent it from offering its services to the non-cartel supplier.

Letting α denote the bargaining power of the cartel, s is chosen to solve the NBS objective:

$$\max_s [(p^c - c' - s)D(p^c)]^\alpha [sD(p^c) - (1 - \beta)(p^c - c'' - g)D(p^c)]^{1-\alpha}$$

or

$$\max_s (p^c - c' - s)^\alpha [s - (1 - \beta)(p^c - c'' - g)]^{1-\alpha} D(p^c). \quad (\text{A7})$$

Note that the threat point for the cartel is zero because failure to agree results in the non-cartel firm (with the assistance of the intermediary) undercutting the cartel's price. In contrast, the intermediary's threat point is $(1 - \beta)(p^c - c'' - g)$ which is its profit from working with the non-cartel firm to undercut the cartel's price and supply the market.

The first-order condition is

$$0 = \{-\alpha(p^c - c' - s)^{\alpha-1}[s - (1 - \beta)(p^c - c'' - g)]^{1-\alpha} + (1 - \alpha)(p^c - c' - s)^\alpha[s - (1 - \beta)(p^c - c'' - g)]^{1-\alpha}\}D(p^c). \quad (\text{A8})$$

Solving it for the NBS per unit payment to the intermediary yields

$$s^*(p^c) = (1 - \alpha\beta)p^c - (1 - \alpha)c' - \alpha(1 - \beta)(c'' + g). \quad (\text{A9})$$

The intermediary's payment is decreasing in the bargaining power of the cartel and the bargaining power of the non-cartel supplier:

$$\frac{\partial s^*(p^c)}{\partial \alpha} = -(p^c - c') + (1 - \beta)(p^c - c'' - g) < 0 \quad (\text{A10a})$$

$$\frac{\partial s^*(p^c)}{\partial \beta} = -\alpha(p^c - c'' - g) < 0. \quad (\text{A10b})$$

Because less bargaining power with respect to the non-cartel supplier lowers the stage 3 payoff for the intermediary, its threat point in bargaining with the cartel is smaller which results in a lower payment; hence, $s^*(p^c)$ is decreasing in β . The payoffs to the intermediary and the cartel, respectively, are

$$s^*(p^c)D(p^c) = [(1 - \alpha\beta)p^c - (1 - \alpha)c' - \alpha(1 - \beta)(c'' + g)]D(p^c) \quad (\text{A11})$$

$$[p^c - c' - s^*(p^c)]D(p^c) = \alpha[\beta p^c - c' + (1 - \beta)(c'' + g)]D(p^c) \quad (\text{A12})$$

Arriving at stage 1, the cartel chooses the product price to maximize its profit taking into account how it will influence its bargaining with the intermediary. If it prices above $c'' + g$ then the cartel will need to come to an agreement with the intermediary as failure would result in the intermediary and the non-cartel supplier undercutting the cartel's price. Of course, it can always price just below at $c'' + g$ in which case there is no need to share rents with the intermediary. In that case, the cartel earns profit of $(c'' + g - c')D(c'' + g)$. We will begin by solving the cartel's optimal pricing problem assuming it will then successfully negotiate with the intermediary and then compare the associated profit with that from pricing just below the total unit cost of non-cartel supply.

Given it will achieve the NBS with the intermediary, the cartel's pricing problem is

$$\max_p (p - c' - s^*(p))D(p) = \alpha[\beta p - c' + (1 - \beta)(c'' + g)]D(p). \quad (\text{A13})$$

The first-order condition is

$$\alpha\beta D(p) + \alpha[\beta p - c' + (1 - \beta)(c'' + g)]D'(p) = 0. \quad (\text{A14})$$

So as to allow for the derivation of a closed-form solution, assume market demand is linear: $D(p) = a - bp$ where $a, b > 0$ and $a - b(c'' + g) > 0$. Solving the first-order condition for price yields

$$\begin{aligned} \hat{p} &= \left(\frac{1}{2b\alpha\beta} \right) [\alpha\beta a + b\alpha c' - b\alpha(1 - \beta)(c'' + g)] \\ &= \frac{a}{2b} + \frac{c'}{2\beta} - \left(\frac{1 - \beta}{2\beta} \right) (c'' + g). \end{aligned} \quad (\text{A15})$$

The intermediary's per unit payment is

$$\begin{aligned}
s^*(\hat{p}) &= (1 - \alpha\beta)\hat{p} - (1 - \alpha)c' - \alpha(1 - \beta)(c'' + g) \\
&= (1 - \alpha\beta)\left(\frac{a}{2b} + \frac{c'}{2\beta} - \left(\frac{1 - \beta}{2\beta}\right)(c'' + g)\right) - (1 - \alpha)c' - \alpha(1 - \beta)(c'' + g).
\end{aligned} \tag{A16}$$

Finally, the cartel's profit is

$$(\hat{p} - c' - s(\hat{p}))D(\hat{p}) = \left(\frac{\alpha}{4b\beta}\right)[\beta a - bc' + b(1 - \beta)(c'' + g)]^2. \tag{A17}$$

\hat{p} is the cartel's optimal price *when it anticipates coming to an agreement with the intermediary*. Interestingly, the optimal cartel price is *decreasing* in the cost of alternative supply. As $c'' + g$ rises, the payment to the intermediary $s^*(\hat{p})$ in (A16) falls which means the cartel's marginal cost (production cost plus payment to the intermediary) declines which then causes it to lower its price. The cartel's optimal price is also decreasing in the bargaining power of the non-cartel supplier, β . When the intermediary has less bargaining power vis a vis the non-cartel supplier (that is, β is higher) then its threat point in bargaining with the cartel is reduced which lowers the payment that the cartel makes to the intermediary which reduces the cartel's marginal cost and thus its optimal price.

To determine when the cartel prefers to price at \hat{p} (and share rents with the intermediary) rather than price at $c'' + g$ (and forego cooperation with the intermediary), define Φ as the difference in the profit from these two alternatives:

$$\begin{aligned}
\Phi &\equiv \\
&\left(\frac{\alpha}{4b\beta}\right)[\beta a - bc' + b(1 - \beta)(c'' + g)]^2 - (c'' + g - c')[a - b(c'' + g)].
\end{aligned} \tag{A18}$$

The cartel strictly prefers to price at \hat{p} if and only if $\Phi > 0$.

Relevant for deriving hypotheses for the ensuing empirical analysis, let us introduce a source of variation in firms' costs that will be a driver of the cartel's decision whether or not to share rents with the intermediary. For this purpose, let x denote some factor that influences the cost of the non-cartel supplier and may influence the cost of the cartel. Let us now denote the non-cartel's total unit cost (including the cost of the intermediary) to be $f(x)$ where $f: [0, \infty) \rightarrow [0, \infty)$ is an increasing continuously differentiable function, and the cartel's unit cost is denoted $h(x)$ where $h: [0, \infty) \rightarrow [0, \infty)$ is a continuously differentiable function. For example, suppose x captures the location of the buyer and a higher value means the buyer is closer to the cartel and farther away from the non-cartel firm. In that case, the cost of the non-cartel firm (cartel) is increasing (decreasing) in x : $f'(x) > 0 > h'(x)$. Consistent with this example, it is

assumed that the factor x has more of a positive impact on the non-cartel firm's cost than on the cartel's cost as described by the following condition: $(1 - \beta)f'(x) > h'(x)$. Finally, as the cartel has a cost advantage over the non-cartel supplier, $f(x) > h(x)$ for all $x > 0$ and, for purposes of the analysis, assume $f(0) = h(0)$ where $x \geq 0$.³

Substitute $f(x)$ for $c'' + g$ and $h(x)$ for c' in (A15). Assuming the cartel chooses to price above the non-cartel firm's cost and come to an agreement with the intermediary, the cartel's optimal price is

$$\hat{p}(x) = \frac{a}{2b} + \frac{h(x)}{2\beta} - \left(\frac{1 - \beta}{2\beta}\right)f(x), \quad (\text{A19})$$

and price is decreasing in x :

$$\hat{p}'(x) = -\left(\frac{1}{2\beta}\right)[(1 - \beta)f'(x) - h'(x)] < 0. \quad (\text{A20})$$

Next substitute $f(x)$ for $c'' + g$ and $h(x)$ for c' in (18) so that

$$\Phi(x) \equiv \left(\frac{\alpha}{4b\beta}\right)[\beta a - bh(x) + b(1 - \beta)f(x)]^2 - (f(x) - h(x))[a - bf(x)]. \quad (\text{A21})$$

Our objective is to learn when $\Phi(x)$ is positive (and the cartel bribes the intermediary) and when it is negative (and it avoids dealing with the intermediary by undercutting the cost of the non-cartel supplier).

First note that if the cartel has no cost advantage then it prefers to work with the intermediary:

$$\Phi(0) = \left(\frac{\alpha\beta}{4b}\right)(a - bh(0))^2 > 0. \quad (\text{A22})$$

In this situation, the cartel earns zero profit by pricing at the cost of alternative supply and does better by pricing higher and providing a payment to the intermediary. When $x = 0$, $f(0) = h(0)$ (or $c' = c'' + g$) and \hat{p} is the monopoly price:

$$\hat{p}(0) = \frac{a}{2b} + \frac{h(0)}{2\beta} - \left(\frac{1 - \beta}{2\beta}\right)f(0) = \frac{a + bc'}{2b} = p^M(c'). \quad (\text{A23})$$

³ For example, if x is the buyer's location then this just says that there is some buyer with location x' whose location is close enough to the non-cartel supplier that the cost of the latter is the same as the cost of the cartel. Normalizing, we have $x' = 0$.

When the cartel has no cost advantage, it prices at the monopoly level – which obviously exceeds the cost of non-cartel supply – and prevents non-cartel supply by bribing the intermediary with a per unit payment of

$$\begin{aligned} s^*(\hat{p}(0)) &= (1 - \alpha\beta)\hat{p}(0) - (1 - \alpha)h(0) - \alpha(1 - \beta)f(0) \\ &= (1 - \alpha\beta)\left(\frac{a + bc'}{2b}\right) - (1 - \alpha)c' - \alpha(1 - \beta)c' = (1 - \alpha\beta)\left(\frac{a - bc'}{2b}\right). \end{aligned} \quad (\text{A24})$$

When the cartel has no cost advantage, it prefers to share rents with the intermediary. At the end of this Appendix, it is shown that there exists a unique cost advantage defined by $x'' \in (0, x')$ such that the cartel sells through an intermediary (directly) if and only if $x < (>)x''$:

$$\Phi(x) \geq (\leq) 0 \text{ as } x \leq (\geq) x'', \text{ for } x \leq x'. \quad (\text{A25})$$

The cartel then optimally prices at $\hat{p}(x)$ when $x < x''$ and instead prices just below the cost of non-cartel supply when $x > x''$. This pricing strategy is depicted in Figure A2 for when $h'(x) = 0$, so that x only impacts the non-cartel firm's cost.⁴ If the cartel's cost advantage is sufficiently small (that is, $x < x''$) then the cartel prices at $\hat{p}(x)$ which exceeds the cost of non-cartel supply $f(x)$. In order to prevent that supply from coming onto the market, the cartel makes a per unit payment $s^*(\hat{p}(x))$ to the intermediary in exchange for it not providing its services to the non-cartel firm. The cartel then officially funnels the transaction through the intermediary with an invoice fee equal to $s^*(\hat{p}(x))$ per unit. As the cost advantage of the cartel rises (that is, x increases), the threat point of the intermediary declines and this translates into a lower per unit payment $s^*(\hat{p}(x))$. As a result, the cartel's optimal price declines. Due to bargaining, the cartel does not capture all of the possible gain associated with it having a bigger cost advantage. Thus, when the cost advantage is sufficiently large (that is, $x > x''$), the cartel chooses to forsake bribing the intermediary and instead prices below the cost of non-cartel supply. While this change in strategy involves a discrete drop in the collusive price (at $x = x''$), the cartel earns higher profit because it does not have to share any of the collusive rents with the intermediary. As the cartel does not involve the intermediary in the transaction, it engages in direct selling.

In the standard theory of collusive pricing in the presence of non-cartel suppliers, if the cost differential is small enough that the cost of non-cartel supply is less than the unconstrained joint

⁴ The figure is qualitatively similar when $h(x)$ is either increasing or decreasing as long as $(1 - \beta)f'(x) > h'(x)$.

profit-maximizing price then the overcharge (which is the difference between the collusive price and the market price in the absence of collusion) equals the cost differential because the cartel is constrained to pricing just below the cost of alternative supply. As $f(x)$ is the cost of non-cartel supply then this means the cartel prices at (or just below) $f(x)$. However, when the cartel can share profits with the intermediary (and the intermediary is essential for non-cartel supply), we find instead that the overcharge is the monopoly overcharge when the cartel has no cost advantage! While increasing its cost advantage lowers the overcharge, it is still above the standard overcharge until the cost advantage is sufficiently large. The ability of the cartel to share rents with the intermediary can increase the damages created by the cartel.

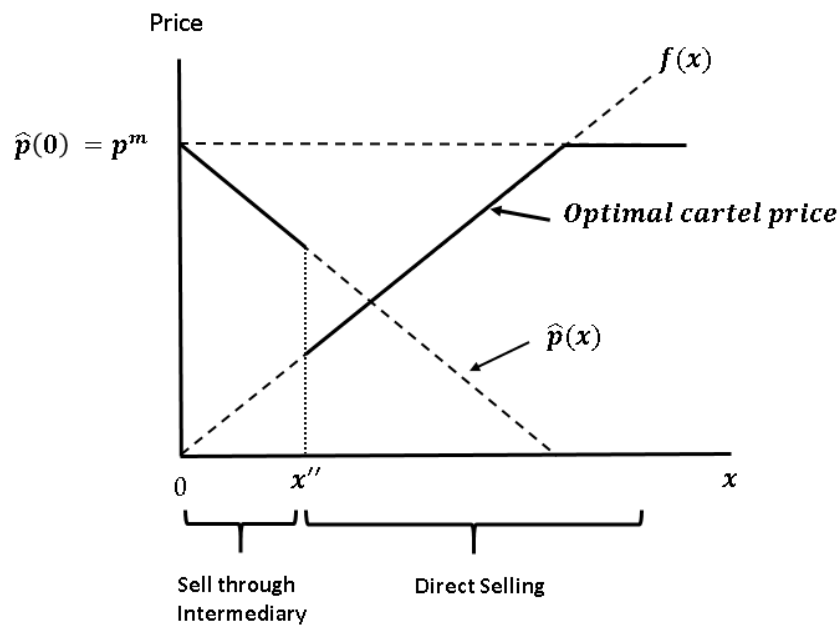


Figure A2: Cartel's Optimal Price

This section is concluded by deriving testable hypotheses from the theory. For this purpose, interpret x as the buyer's location where a higher value corresponds to the buyer being closer to a German cartel member and farther away from the nearest Eastern European non-cartel supplier. Thus, a higher value for x means higher transportation costs for the non-cartel firm and lower transportation costs for the cartel. As depicted in Figure A2, the theory predicts that when the buyer is sufficiently close to an Eastern European plant (that is, x is sufficiently low), the cartel member will sell through an intermediary and provide it with a payment. When instead the buyer is sufficiently distant from the Eastern European plant (that is, x is sufficiently high), the cartel member will sell directly to the buyer. Thus, the more distant is the buyer from

the nearest Eastern European plant, the more likely is it that the cartel member will engage in direct selling.

Proof that there exists a unique $x'' \in (0, x')$ such that (A25) is true.

Consider the effect of raising the factor x (so that the cartel has a cost advantage):

$$\begin{aligned} \Phi'(x) = & \left(\frac{\alpha}{2\beta}\right) [\beta a - bh(x) + b(1 - \beta)f(x)][(1 - \beta)f'(x) - h'(x)] \\ & - (f'(x) - h'(x))[a - bf(x)] + b(f(x) - h(x))f'(x) \end{aligned} \quad (\text{A26})$$

Evaluate this derivative at $x = 0$,

$$\begin{aligned} \Phi'(0) = & \left(\frac{\alpha}{2\beta}\right) [\beta a - b\beta f(0)][(1 - \beta)f'(0) - h'(0)] \\ & - (f'(0) - h'(0))[a - bf(0)] \\ = & -[a - bf(0)] \left[\left(\frac{\alpha\beta}{2}\right) f'(0) + \left(\frac{2-\alpha}{2}\right) (f'(0) - h'(0)) \right] < 0 \end{aligned} \quad (\text{A27})$$

because $f'(0) - h'(0) \geq 0$ follows from assuming $(1 - \beta)f'(x) > h'(x)$. In sum, when the cartel does not have a cost advantage, $\Phi(0) > 0$ (so the cartel prefers to price above the non-cartel firm's cost at \hat{p} and provide a payment to the intermediary) and $\Phi'(0) < 0$ (so that the incremental profit from the option of sharing rents with the intermediary is decreasing in x where recall that a higher x corresponds to a greater cost advantage for the cartel).

Next let us derive sufficient conditions for there to exist $x' > 0$ such that $\Phi(x') < 0$ so the cartel prefers to undercut the non-cartel firm's cost. Given $\hat{p}(0) > f(0)$, $\hat{p}(x)$ is decreasing in x , and $f(x)$ is increasing in x , there exists $x' > 0$ such that $\hat{p}(x') = f(x')$:

$$\frac{a}{2b} + \frac{h(x')}{2\beta} - \left(\frac{1 - \beta}{2\beta}\right) f(x') = f(x') \rightarrow f(x') = \frac{\beta a + bh(x')}{b(1 + \beta)}. \quad (\text{A28})$$

We then have:

$$\hat{p}(x) > f(x) \text{ for all } x \in [0, x'). \quad (\text{A29})$$

Evaluate Φ at $x = x'$:

$$\begin{aligned} \Phi(x') = & \left(\frac{\alpha}{4b\beta}\right) [\beta a - bh(x') + b(1 - \beta)f(x')]^2 - (f(x') - h(x'))[a \\ & - bf(x')] \end{aligned} \quad (\text{A30})$$

$$\begin{aligned}
&= \left(\frac{\alpha}{4b\beta} \right) \left[\beta a - bh(x') + b(1 - \beta) \left(\frac{\beta a + bh(x')}{b(1 + \beta)} \right) \right]^2 \\
&\quad - \left(\frac{\beta a + bh(x')}{b(1 + \beta)} - h(x') \right) \left[a - b \left(\frac{\beta a + bh(x')}{b(1 + \beta)} \right) \right] \\
&\quad \Phi(x') = - \left[\frac{(1 - \alpha)\beta}{b(1 + \beta)^2} \right] (a - bh(x'))^2 < 0.
\end{aligned}$$

Next take the second derivative of Φ :

$$\begin{aligned}
\Phi''(x) &= \left(\frac{\alpha}{2\beta} \right) [\beta a - bh(x) + b(1 - \beta)f(x)] [(1 - \beta)f''(x) - h''(x)] \\
&\quad + \left(\frac{\alpha b}{2\beta} \right) [(1 - \beta)f'(x) - h'(x)]^2 \\
&\quad - (f''(x) - h''(x)) [a - bf(x)] + b(f'(x) - h'(x))f'(x) \\
&\quad + b(f'(x) - h'(x))f'(x) + b(f(x) - h(x))f''(x).
\end{aligned} \tag{A31}$$

We will assume $\Phi'' > 0$. Sufficient conditions for that to be the case are f and h are linear:

$$\Phi''(x) = \left(\frac{\alpha b}{2\beta} \right) [(1 - \beta)f'(x) - h'(x)]^2 + 2b(f'(x) - h'(x))f'(x) > 0, \tag{A32}$$

noting that previous assumptions make these two terms positive. As $\Phi(0) > 0 > \Phi(x')$ then, given Φ is convex, it follows that there exists a unique $x'' \in (0, x')$ such that (A25) is true.