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The effect of demand variability on the adoption and design of a third party's pricing algorithm

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ABSTRACT

Consider a data analytics company supplying a pricing algorithm that adjusts price to a changing demand state. For this setting, I show the pricing algorithm is designed and priced so that higher demand variability results in more firms adopting the pricing algorithm. Furthermore, there is a critical threshold for demand variability whereby there is complete or near-complete adoption of the pricing algorithm. While widespread adoption of a third party's pricing algorithm among competitors has raised concerns of collusive conduct, it could instead reflect a strong efficiency delivered by a third party.

1. Introduction

There is a growing market where data analytics companies supply firms with pricing algorithms to more profitably set prices. A third-party developer is likely to offer a better pricing algorithm than would be created internally by a firm because it has more expertise and experience, access to more data, and stronger incentives to invest in their development (as the pricing algorithm can be licensed to many firms). Viewing a pricing algorithm as just another input, firms could well realize efficiencies from outsourcing this input. At the same time, competition authorities have expressed concern that this "make or buy" decision may have anticompetitive effect when it results in competitors adopting a pricing algorithm from the same third party.[2] Furthermore, there is some evidence, and claims of evidence, that these anticompetitive effects have occurred. In markets for apartment rentals and hotels in the United States, plaintiffs claim the use of a third-party's pricing algorithm resulted in supracompetitive prices.[3] A recent study found evidence of both anticompetitive and procompetitive effects in the market for apartment rentals (Calder-Wang and Kim, 2024). There is also evidence of anticompetitive effect in the supply of third-party pricing algorithms in the German retail gasoline market (Assad et al., 2024). In response to these concerns, the U.S. Senate has recently proposed legislation to constrain data analytics companies in how they use data.[4]

Towards better understanding the effects of a third party supplying a pricing algorithm to competitors, this study explores the determinants of the equilibrium adoption rate. The setting is one in which the third party is better at conditioning price on the demand state, where the state could vary across time (so the pricing algorithm allows price to respond to high-frequency demand shocks) or across markets (so it allows for third-degree price discrimination). The extent of this efficiency delivered by the third party is determined by the amount of demand variability. When adopting the pricing algorithm, a firm pays a fee to the third party (which is set by the third party to maximize its profit) and incurs a firm-specific adoption cost. The equilibrium adoption rate is shown to be increasing in the amount of demand variability and, furthermore, there is a critical threshold whereby there is complete or near-complete adoption of the pricing algorithm when demand variability exceeds that threshold. While widespread adoption of a third party's pricing algorithm among competitors may accentuate concerns of possible anticompetitive harm, this study shows the pro-competitive efficiency is strongest when there is widespread adoption. Consequently, policymakers should act with caution when restricting a third party in markets where its pricing algorithm is commonly used.

This paper is related to the literature on algorithmic pricing in the context of imperfectly competitive markets. Research that builds equilibrium theories in which firms choose and commit to a pricing algorithm includes Salcedo (2015), Brown and MacKay (2023), Johnson et al. (2023), Lamba and Zhuk (2024), Leisten (2024), and Musolff (2024). There is also a vast and rapidly growing body of research using numerical analysis to explore learning over algorithms with a focus

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[2] For statements by some competition authorities, see Harrington (2024a).

[3] The legal cases are cited in Harrington (2024a).

[4] "Preventing Algorithmic Collusion Act" https://www.congress.gov/bill/118th-congress/senate-bill/3686/text

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on when supracompetitive prices emerge and persist; many of those papers are referenced in Harrington (2024b). All of the just-mentioned studies assume a firm designs its own pricing algorithm. Analyses that model the design and supply of a pricing algorithm by a third party is limited to Harrington (2022, 2024b). The current paper directly builds on Harrington (2024b) by endogenizing the adoption rate and allowing for heterogeneity in a firm-specific adoption cost.

2. Model

Consider a market where firms have symmetrically differentiated products and a common and constant marginal cost c. Firm demand is linear, $a - bp_i + dP_{-i}$, where p_i is a firm's own price, P_{-i} is the average price of rival firms, and $b > d > 0$. For reasons of tractability, there is assumed to be a continuum of firms.[5]

A critical feature is demand variation with respect to a which has a continuously differentiable cdf $G : [\underline{a}, \overline{a}] \rightarrow [0,1]$ with mean μ and variance σ^2. The higher is a, the stronger is firm demand in the sense that the level of demand is larger and demand is more price-inelastic. The demand variable a has two interpretations. A firm could be facing a single demand curve and a is subject to a demand shock with distribution G, in which case price may condition on the current period's demand state. Alternatively, a firm faces a collection of market segments represented by G, in which case price may condition on the market segment a.

Firms make simultaneous price decisions. Assume the demand state a is realized at a higher frequency than a firm's pricing decisions or, when G represents a collection of market segments, the firm cannot distinguish among those segments when pricing. Consequently, a firm is incapable of conditioning price on the demand state. Assume $\underline{a} - (b-d)c > 0$ so demand is positive for all demand states at the static Nash equilibrium. A symmetric Nash equilibrium price p^N is defined by:

$$p^N = \arg\max_{p_i \in \Re_+} \int (p_i - c)\left(a - bp_i + dp^N\right) G'(a)da \Leftrightarrow p^N = \frac{\mu + bc}{2b - d}. \quad (1)$$

Let us now introduce a third-party data analytics company whose business model is to design and sell a pricing algorithm. The efficiency it delivers is that its pricing algorithm can track the demand state a and condition price on it. An algorithm is denoted $\phi : [\underline{a}, \overline{a}] \rightarrow \Re_+$. The third party chooses a pricing algorithm ϕ and a fee f for the right to use it, while firms choose whether or not to adopt the pricing algorithm. An adopting firm's price is set by ϕ (which will condition on the demand state), while a non-adopting firm will choose a price which does not condition on the demand state. In addition to paying f, a firm incurs a cost k from adopting ϕ. This cost may be associated with integrating the pricing algorithm into its IT system, training personnel to implement and monitor the pricing algorithm, or simply capture managerial resistance to delegating pricing to an algorithm. To allow for heterogeneous adoption decisions (as is consistent with practice), k varies across the continuum of firms according to the cdf $H : [0, \overline{k}]$ where H is twice continuously differentiable. A lower bound of 0 is assumed to ensure the adoption rate is positive in equilibrium.

[5] This demand specification is the extension of a common specification to when there is a continuum of firms. When there is a finite number of firms, a firm's decision to adopt the third party's pricing algorithm affects the adoption rate which, as we will see, affects the third party's design of the pricing algorithm. That effect disappears with a continuum of firms as a single firm's adoption decision does not affect the adoption rate. In practice, for many market structures, the effect of a single firm's adoption decision on the design of the pricing algorithm is likely to be small in magnitude. Setting this small effect equal to zero, as occurs with a continuum of firms, greatly enhances analytical tractability. This assumption does mean the model is not suitable for market structures for which a firm is large enough that its adoption could be expected to affect how the third party designs the algorithm. Future research will need to explore that situation and assess the robustness of the findings of this paper.

3. Equilibrium

3.1. General description

There are four elements determined in equilibrium: pricing algorithm ϕ, fee f, (symmetric) non-adopter's price p^{NA}, and adoption rate θ (i.e., the share of firms that adopt ϕ). Anticipating θ, ϕ, and p^{NA}, a firm's gross willingness-to-pay (i.e., before netting out k) for adopting the pricing algorithm is

$$\begin{aligned} WTP\left(\phi, p^{NA}, \theta\right) \equiv & \int (\phi(a) - c)\left(a - b\phi(a) + d\left(\theta\phi(a) + (1-\theta)p^{NA}\right)\right) G'(a)da \quad (2) \\ & - \int (p^{NA} - c)\left(a - bp^{NA} + d\left(\theta\phi(a) + (1-\theta)p^{NA}\right)\right) G'(a)da. \end{aligned}$$

The first term is the average profit from adopting and the second term subtracted from it is the average profit from not adopting. A firm with adoption cost k adopts if and only if $WTP\left(\phi, p^{NA}, \theta\right) \geq k + f$. Assuming there is no marginal cost from selling the pricing algorithm and any fixed cost is the same for all designs, the third party will choose ϕ, f, and θ (i.e., how many licenses to sell) to maximize its revenue from selling the pricing algorithm, $f \times \theta$.

Optimality requires the third party to choose ϕ to maximize $WTP\left(\phi, p^{NA}, \theta\right)$.[6] At the same time, a non-adopting firm chooses price to maximize its profit. For a given adoption rate, the equilibrium values for ϕ and p^{NA} are the solution to:

$$\begin{aligned} \widehat{\phi}(\cdot, \theta) = \arg\max_{\phi(\cdot)} & \int (\phi(a) - c)\left(a - b\phi(a) + d\left(\theta\phi(a) + (1-\theta)\widehat{p}^{NA}(\theta)\right)\right) G'(a)da \quad (3) \\ & - \int (\widehat{p}^{NA}(\theta) - c)\left(a - b\widehat{p}^{NA}(\theta) + d\left(\theta\phi(a) + (1-\theta)\widehat{p}^{NA}(\theta)\right)\right) G'(a)da \end{aligned}$$

$$\widehat{p}^{NA}(\theta) = \arg\max_{p} \int (p - c)\left(a - bp + d\left(\theta\widehat{\phi}(a, \theta) + (1-\theta)\widehat{p}^{NA}(\theta)\right)\right) G'(a)da. \quad (4)$$

The associated gross willingness-to-pay is denoted

$$WTP^*(\theta) \equiv WTP(\widehat{\phi}(\cdot, \theta), \widehat{p}^{NA}(\theta), \theta).$$

To close the model, the third party chooses the fee and adoption rate to maximize its revenue:

$$\max_{f,\theta} f \times \theta \text{ s.t. } f \leq WTP^*(\theta) - k \;\forall k \leq H^{-1}(\theta). \quad (5)$$

The constraint ensures there is demand of at least θ given f, and it binds at the optimal solution as the fee is set equal to the net WTP: $f = WTP^*(\theta) - k$. Using the binding constraint to substitute for f in the objective in (5), the unconstrained problem is

$$\max_{\theta} \left(WTP^*(\theta) - H^{-1}(\theta)\right)\theta.$$

Equivalent to choosing the adoption rate is choosing the maximum adoption cost such that a firm adopts:

$$\max_{k} (WTP^*(H(k)) - k)H(k). \quad (6)$$

Defining

$$k^* \equiv \arg\max_{k} (WTP^*(H(k)) - k)H(k)$$

then, in equilibrium, all firms with adoption cost not exceeding k^* adopt the third party's pricing algorithm, the adoption rate is $H(k^*)$, the pricing algorithm is $\widehat{\phi}(\cdot, H(k^*))$, and the fee is $f^* = WTP^*(H(k^*)) - k^*$.

[6] If that were not true then the third party could choose another ϕ to raise $WTP\left(\theta, \phi, p^{NA}\right)$ which would generate more revenue by allowing it to increase the fee and/or the adoption rate.

3.2. Solution with an affine pricing algorithm

Given the linearity of demand, I will focus on an optimal pricing algorithm that is an affine function of the demand state: $\phi(a) = \alpha + \gamma a$ for some (α, γ). It is shown in Harrington (2024b) that the solution to (3)–(4) is

$$\hat{\phi}(a,\theta) = \frac{2bc(b-d\theta) + d\mu(1-2\theta)}{2(b-d\theta)(2b-d)} + \left(\frac{1}{2(b-d\theta)}\right) a \tag{7}$$

$$\hat{p}^{NA}(\theta) = \frac{\mu + bc}{2b-d}. \tag{8}$$

As the adoption rate increases, the pricing algorithm $\hat{\phi}(a,\theta)$ rotates around $a = \mu$ with a steeper slope, as shown in Fig. 1.[7] Thus, price is more sensitive to the demand state when more firms adopt. A notable property is that the average price,

$$E_a\left[\hat{\phi}(a,\theta)\right] = \frac{2bc(b-d\theta) + d\mu(1-2\theta)}{2(b-d\theta)(2b-d)} + \left(\frac{1}{2(b-d\theta)}\right)\mu = \frac{\mu + bc}{2b-d},$$

is independent of the adoption rate and the same as the uniform price for non-adopters. In addition, average price is the same as under competition without a third party; compare to (1). Here we provide a brief explanation for these properties and the reader is referred to Section 4 of Harrington (2024b) for more details.

The third party wants to design the pricing algorithm to maximize the WTP for it. It contributes to that objective when it raises the profit from adopting while not raising the profit from not adopting, given the WTP is the difference between them. If the pricing algorithm has a high average price, that would contribute to high adopter's profit but also to high non-adopter's profit as a non-adopting firm could best respond to adopting rival firms by undercutting their high average price with its uniform price. As a result, the third party does not impose a supracompetitive markup in the pricing algorithm – even when the adoption rate is high – because it would be exploited by non-adopters, thereby reducing the WTP and lowering the third party's revenue from selling the pricing algorithm.

The design challenge for the third party is to offer a pricing algorithm that delivers high profit for adopters without being exploitable by non-adopters. The solution is to have the pricing algorithm be supracompetitively *sensitive* to the demand state – raising price a lot when demand is strong and lowering it a lot when demand is weak – because a non-adopter cannot exploit that sensitivity due to not being able to condition price on the demand state; it can only exploit a high average price and not a highly sensitive price. In fact, the slope of the pricing algorithm is the same as if the third party was a cartel manager with θ firms in the cartel, though the average price is the same as under competition.

Substituting (7)–(8) into (2), it is shown in the Appendix that:

$$WTP^*(\theta) = \frac{\sigma^2}{4(b-d\theta)}. \tag{9}$$

The incremental value of using the pricing algorithm is increasing in the adoption rate:

$$\frac{\partial WTP^*(\theta)}{\partial\theta} = \frac{d\sigma^2}{4(b-d\theta)^2} > 0.$$

Thus, in designing the pricing algorithm, the third-party developer results in adoptions being strategic complements; the more firms who adopt the pricing algorithm, the more attractive it becomes to adopt it.

The final step is to characterize the equilibrium adoption rate. Using (9), (6) takes the form:

$$\max_k V(k) \equiv \left(\frac{\sigma^2}{4(b-dH(k))} - k\right) H(k).$$

[7] The parameters used in Fig. 1 are: $b = 1, d = 0.75, c = 10, \mu = 100$; and the adoption rate is increased from 0 to 0.25 to 0.50 to 0.75.

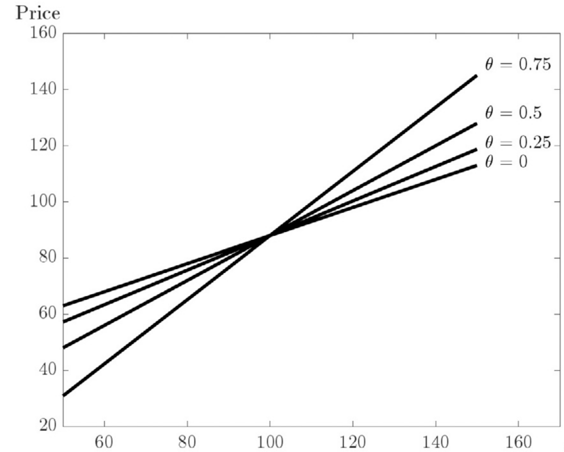

Fig. 1. Pricing algorithm rotates with a steeper slope as the adoption rate is increased.

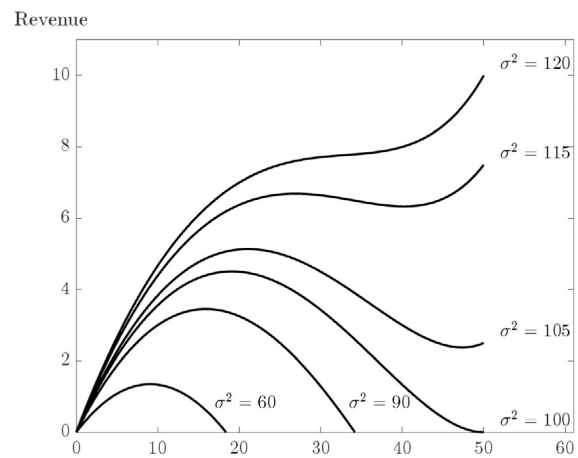

Fig. 2. Third party revenue function shifts up as demand variability is increased.

The third party chooses the adoption rate to maximizes its revenue taking into account the direct effect - a higher adoption rate means more sales of the pricing algorithm – and an indirect effect – a higher adoption rate affects the fee it can charge by influencing the WTP.

As the value added of the third party comes from allowing a firm to condition price on the demand state, this efficiency is greater when the demand state is more variable, as measured by σ^2. Under the assumption of a uniform distribution on firm-specific adoption costs, Theorem 1 shows the equilibrium adoption rate is higher when the third party's deliverable efficiency is greater. Furthermore, there is a threshold efficiency at which all firms adopt the pricing algorithm. The proof is in Appendix A.

Theorem 1. *Assume $H : \left[0, \overline{k}\right] \to [0,1]$ is uniform. $\exists \hat{\sigma}^2 > 0$ such that if $\sigma^2 \in \left(0, \hat{\sigma}^2\right)$ then $k^* \in \left(0, \overline{k}\right)$ and $\partial k^*/\partial\sigma^2 > 0$; and if $\sigma^2 \geq \hat{\sigma}^2$ then $k^* = \overline{k}$.*

The properties stated in Theorem 1 are illustrated in Fig. 2 which plots the third party's revenue $V(k, \sigma^2)$ for $\sigma^2 \in \{60, 90, 100, 105, 110, 115, 120\}$.[8] As σ^2 rises, $V(k, \sigma^2)$ shifts up – due to the greater value delivered by a third party when demand variability is greater – and the optimum shifts to the right. The increase in k^* is continuous in

[8] For Fig. 2, $b = 1, d = 0.5$, and H is uniform on $[0, 50]$.

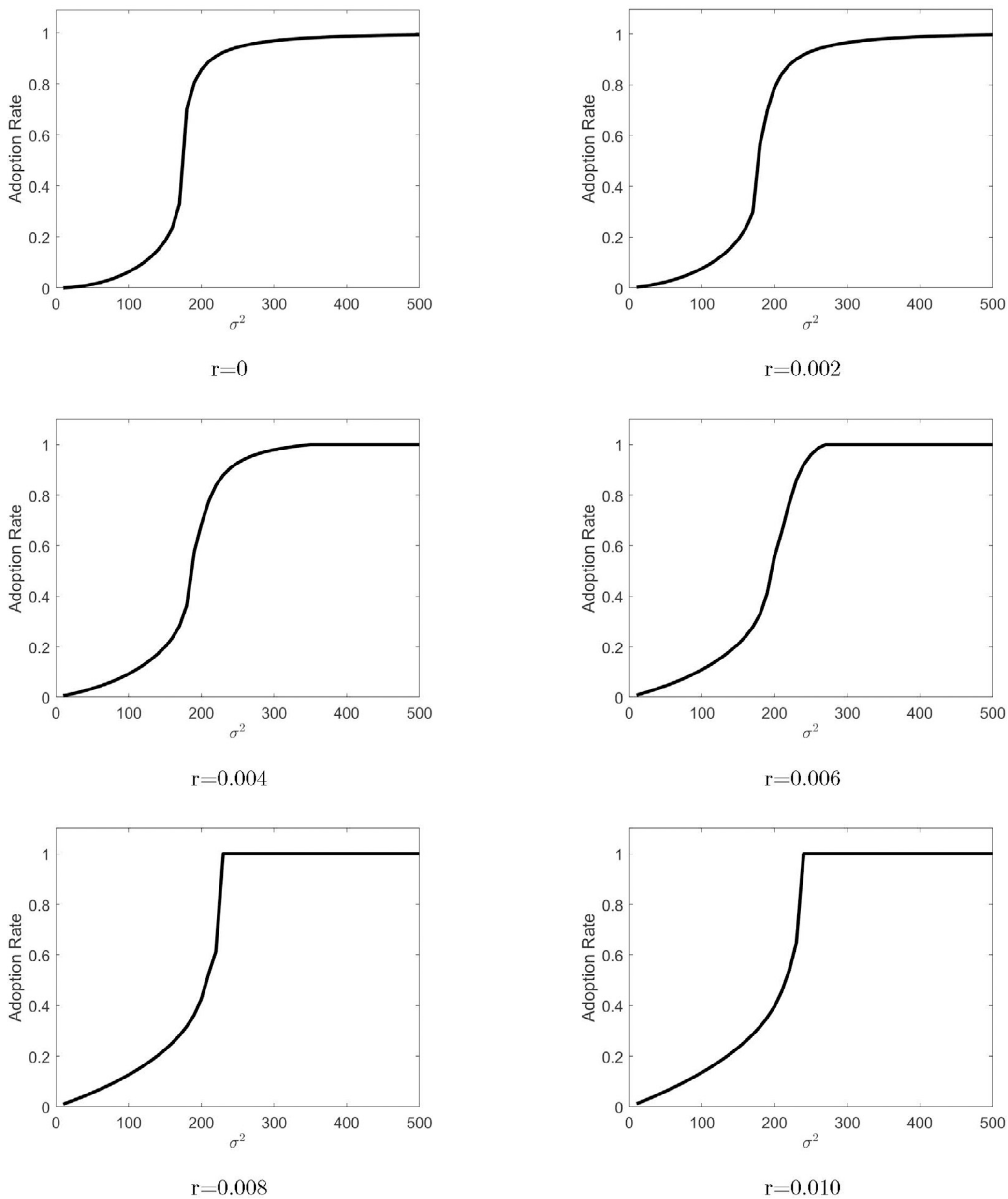

Fig. 3. Effect of the firm-specific adoption cost distribution on the relationship between demand variability and the adoption rate.

σ^2 until σ^2 is sufficiently high so there is full adoption. The optimum is $k^* = 23.49$ at $\sigma^2 = 110$ but then jumps to $k^* = 1$ at $\sigma^2 = 115$.

As the demand variability rises, it becomes more valuable to a firm to be able to condition price on the demand state. This raises the WTP for the pricing algorithm so a firm with a higher adoption cost is now willing to pay the fee. Consequently, the adoption rate is increasing in σ^2. As it was previously shown that adoptions are strategic complements, the higher adoption rate further raises the WTP. In sum, raising σ^2 causes more firms to adopt which leads yet more firms to adopt. Once σ^2 reaches a critical value, adoption becomes universal.

When a third party is supplying a pricing algorithm to firms, Theorem 1 implies that demand variability has a powerful effect on price variability. For a given adoption rate θ, the pricing algorithm in (7) results in an adopting firm's price variability equaling

$$\left(\frac{1}{2(b-d\theta)}\right)^2 \sigma^2,$$

which is naturally increasing in demand variability. That relationship is for a fixed pricing algorithm. But the pricing algorithm is not fixed, for higher demand variability increases the adoption rate which then changes the design of the pricing algorithm. In particular, the pricing algorithm is made more sensitive to the demand state when the adoption rate is higher,

$$\frac{\partial^2 \hat{\phi}(a,\theta)}{\partial a \partial \theta} = \frac{d}{2(b-d\theta)^2} > 0.$$

Thus, more demand variability directly increases price variability (as price depends on the demand state) and indirectly increases price variability through a higher adoption rate (which makes the price algorithm more sensitive to the demand state). Finally, the higher adoption rate from greater demand variability means more firms are using the pricing algorithm and thus having price respond to the demand state, rather than setting a uniform price. While one should generally expect more demand variability to result in more price variability, we find that this relationship is especially strong when firms are outsourcing its pricing algorithm to a third party.

3.3. Numerical analysis with triangular density function

Using numerical analysis, we show the robustness of the properties in Theorem 1. Consider a symmetric triangular density function on the firm-specific adoption cost k with domain $\left[\underline{k}, \overline{k}\right]$ and a peak at $\left(\underline{k}+\overline{k}\right)/2$.

$$H'(k) = \begin{cases} r + 4\left(\frac{1-r(\overline{k}-\underline{k})}{(\overline{k}-\underline{k})^2}\right)\left(k-\underline{k}\right) & if\, k \in \left[\underline{k}, \frac{\underline{k}+\overline{k}}{2}\right] \\ r + 4\left(\frac{1-r(\overline{k}-\underline{k})}{(\overline{k}-\underline{k})^2}\right)\left(\overline{k}-k\right) & if\, k \in \left[\frac{\underline{k}+\overline{k}}{2}, \overline{k}\right] \end{cases}.$$

r is the density at the extreme values: $H'(\underline{k}) = r = H'(\overline{k})$. Note that $r = 1/(\overline{k}-\underline{k})$ is the uniform case and the density function becomes more steep – so there is less mass in the tails – as r is reduced.

The analysis was conducted for parameter values $b = 1, d = 0.5, c = 20, \underline{k} = 0, \overline{k} = 100, \underline{a} = 75, \overline{a} = 125$ and a collection of density functions:

$$r \in \left\{0, \frac{.2}{\overline{k}-\underline{k}}, \dots, \frac{1}{\overline{k}-\underline{k}}\right\} = \{0, .002, .004, .006, .008, .010\}.$$

After setting a value for σ^2, k^* is numerically solved for where

$$k^* \equiv \arg\max_k H(k)\left(\frac{\sigma^2}{4(b-dH(k))} - k\right).$$

This exercise is performed for $\sigma^2 \in \{10, 20, \dots, 490, 500\}$.[9]

Fig. 3 reports the adoption rate $H(k^*)$ against σ^2 for different specifications for H. As the efficiency σ^2 increases, the adoption rate increases. Furthermore, when the efficiency is sufficiently great, the adoption rate rises drastically to a value near or at 1. The jump in the adoption rate is to full adoption when r is high so that the density function is close to the uniform case. We then find that the properties in Theorem 1 are robust in that the adoption rate is monotonically increasing in σ^2 and there is a threshold value for σ^2 after which the adoption rate rises sharply in σ^2.

4. Concluding remarks

Recent private litigation in the markets for apartment rentals and hotels claims there is an unlawful agreement between a third party and adopting firms which has resulted in supracompetitive prices. As coordinated conduct is easier to sustain and has a larger effect when it involves more competitors, a higher adoption rate gives more credibility to these claims. The analysis of this study offers a different interpretation: there may be no agreement and instead firms are independently deciding whether to adopt a third party's pricing algorithm with the third party designing it to maximize its profit, rather than acting like a cartel manager to maximize adopting firms' profits. From this perspective, the high adoption rate is a reflection of the efficiency delivered by the third party due to its pricing algorithm being able to condition price on a highly variable demand state.

In concluding, let me discuss the approach of this study in relation to other research evaluating the implications of algorithmic pricing for competition. The current approach starts with the algorithm having an efficiency in that it can better tailor price to the demand state. The question is whether an anticompetitive effect emerges due to a third party designing and supplying the pricing algorithm to competitors. Though concerns have been expressed that the third party may have an incentive to program in a supracompetitive markup, that is not found to be the case because a higher average markup lowers a firm's willingness-to-pay and thus the fee that the third party can charge. Rather than have a common third party design firms' pricing algorithms, most research has instead assumed firms independently design their pricing algorithms. One line of work models learning (e.g., Q-learning) and explores the implications of algorithms searching for optimal prices.[10] A second line of work models commitment (e.g., regarding the frequency with which prices are updated) and explores the implications of an algorithm locking a firm into a pricing rule.[11] The main takeaway from both research lines is that the use of algorithms produce an anticompetitive effect, even though firms are independently designing their pricing algorithms. This conclusion is based on using the competitive (static Nash) equilibrium price as the non-algorithmic benchmark which implicitly assumes the algorithm does not enhance efficiency. For example, the learning literature assumes that, in the absence of using algorithms, firms settle on the competitive equilibrium, though it is not specified how firms learn profit-maximizing prices nor whether algorithms are better or worse at doing so. That supracompetitive prices emerge is an interesting and important finding but a proper assessment of the net effect of algorithms will require taking account of the efficiencies that algorithms deliver.

Appendix A

A.1. Derivation of willingness-to-pay

Substituting $\alpha + \gamma a$ for $\phi(a)$ in (2),

$$\begin{aligned} WTP\left(\phi, p^{NA}, \theta\right) = & \int \left((\alpha+\gamma a) - c\right)\left(a - b(\alpha+\gamma a) + d\left(\theta(\alpha+\gamma a)\right.\right. \\ & \left.\left. +(1-\theta)p^{NA}\right)\right) G'(a)da \\ & - \int (p^{NA} - c)\left(a - bp^{NA} + d\left(\theta(\alpha+\gamma a)\right.\right. \\ & \left.\left. +(1-\theta)p^{NA}\right)\right) G'(a)da. \end{aligned}$$

Integrating yields:

$$\begin{aligned} & (\alpha - c)\left(\mu - b(\alpha+\gamma\mu) + d\left(\theta(\alpha+\gamma\mu) + (1-\theta)p^{NA}\right)\right) \\ & + \gamma\mu\left(-b\alpha + d\left(\theta\alpha + (1-\theta)p^{NA}\right)\right) + \gamma\left(1 - b\gamma + d\theta\gamma\right)\left(\mu^2+\sigma^2\right) \\ & - (p^{NA} - c)\left(a - bp^{NA} + d\left(\theta(\alpha+\gamma\mu) + (1-\theta)p^{NA}\right)\right). \end{aligned} \tag{10}$$

Using (7)–(8), substitute

$$\alpha = \frac{2bc(b-d\theta) + d\mu(1-2\theta)}{2(b-d\theta)(2b-d)},\ \gamma = \frac{1}{2(b-d\theta)},\ p^{NA} = \frac{\mu+bc}{2b-d}$$

into (10). Simplifying with Mathematica yields: $\frac{\sigma^2}{4(b-d\theta)}$.

A.2. Proof of Theorem 1

Given H is uniform, it is straightforward to show:

$$\frac{\partial^3 V}{\partial k^3} = \frac{3bd^2\sigma^2\left(H'(k)\right)^3}{2\left(b-dH(k)\right)^4} > 0,\ \forall k.$$

[9] All of these values for σ^2 are consistent with a having support $[75, 125]$. That is, there exists a cdf $G : [75, 125] \to [0, 1]$ with variance equal to that value.

[10] See, for example, Abada and Lambin (2023), Asker et al. (2023), Calvano et al. (2020), Epivent and Lambin (2024), Meylahn and den Boer (2022), and Waltman and Kaymak (2008).

[11] See Arunachaleswaran et al. (2024), Brown and MacKay (2023), Lamba and Zhuk (2024), Leisten (2024), Musolff (2024), and Salcedo (2015).

Given $\partial^3 V/\partial k^3 > 0$ then either: (i) $\partial^2 V/\partial k^2 < 0\ \forall k \in \left[0, \overline{k}\right]$; (ii) $\exists k' \in \left(0, \overline{k}\right)$ such that $\partial^2 V/\partial k^2 \lesseqqgtr 0$ as $k \lesseqqgtr k'$; or (iii) $\partial^2 V/\partial k^2 > 0\ \forall k \in \left[0, \overline{k}\right]$. This property implies there is at most one interior local optimum: k'' such that $\partial V(k'')/\partial k = 0$ and $\partial^2 V(k'')/\partial k^2 < 0$. Furthermore, the global optimum is either k'' (if it exists) or $\overline{k}$ (and could be both values but that would be non-generic).

If $\sigma^2 > 0$ then V is increasing in k at $k = \underline{k}(= 0)$:

$$\frac{\partial V(\underline{k})}{\partial k} = H'(\underline{k})\left(\frac{\sigma^2 - 4b\underline{k}}{4b}\right) > 0 \text{ if and only if } \sigma^2 > 4b\underline{k}(= 0),$$

which implies $k^* > 0$. Next note that if σ^2 is sufficiently small then V is increasing in k at $k = \overline{k}$:

$$\frac{\partial V(\overline{k})}{\partial k} = H'(\overline{k})\left(\frac{b\sigma^2 - 4(b-d)^2\overline{k}}{4(b-d)^2}\right) - 1 < 0 \text{ if } \sigma^2 \leq \frac{4(b-d)^2\overline{k}}{b},$$

which implies $k^* < \overline{k}$. In sum, if $\sigma^2 \in \left(0, \frac{4(b-d)^2\overline{k}}{b}\right)$ then $k^* \in \left(0, \overline{k}\right)$ and, in addition, $\partial^2 V(k^*)/\partial k^2 < 0$. Given that

$$\frac{\partial^2 V}{\partial k \partial \sigma^2} = \frac{bH'(k)}{4(b - dH(k))^2} > 0$$

then

$$\frac{\partial k^*}{\partial \sigma^2} = -\frac{\partial^2 V/\partial k \partial \sigma^2}{\partial^2 V/\partial k^2} > 0.$$

I conclude $\exists \widetilde{\sigma}^2 > 0$ such that if $\sigma^2 \in \left(0, \widetilde{\sigma}^2\right)$ then $k^* \in \left(0, \overline{k}\right)$ and $\partial k^*/\partial \sigma^2 > 0$.

Noting that

$$\frac{\partial^2 V}{\partial k^2} = \frac{bd\sigma^2\left(H'(k)\right)^2}{2(b - dH(k))^3} - 2H'(k)$$

and evaluating it at $k = \underline{k}$ reveals it is positive when σ^2 is sufficiently large:

$$\frac{\partial^2 V(\underline{k})}{\partial k^2} = \frac{bd\sigma^2\left(H'(\underline{k})\right)^2}{2(b - dH(\underline{k}))^3} - 2H'(\underline{k}) = \frac{bd\sigma^2\left(1/\overline{k}\right)^2}{2b^3} - (2/\overline{k}) > 0 \Leftrightarrow \sigma^2 > \frac{4b^2\overline{k}}{d}.$$

Given $\partial^3 V/\partial k^3 > 0$, it follows that if $\sigma^2 > 4b^2\overline{k}/d$ then $\partial^2 V/\partial k^2 > 0$ $\forall k$. Since $\sigma^2 > 4b^2\overline{k}/d(> 0)$ implies $\partial V(\underline{k})/\partial k > 0$ then $k^* = \overline{k}$. Thus, if $\sigma^2 > 4b^2\overline{k}/d$ then $k^* = \overline{k}$.

Define $\widehat{\sigma}^2 \equiv \inf\left\{\sigma^2 : k^* = \overline{k}\right\}$. Given $k^* = \overline{k}$ at $\sigma^2 = \widehat{\sigma}^2$ then $V(\overline{k}, \widehat{\sigma}^2) \geq V(k, \widehat{\sigma}^2)\ \forall k \in \left[0, \overline{k}\right)$. From $\partial^2 V/\partial k \partial \sigma^2 > 0$ it follows: if $\sigma^2 > \widehat{\sigma}^2$ then $V(\overline{k}, \sigma^2) > V(k, \sigma^2)\ \forall k \in \left[0, \overline{k}\right) \Rightarrow k^* = \overline{k}$. This completes the proof.

Data availability

No data was used for the research described in the article.

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