Intro to Financial Analytics

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Lecture I (7/10/2023)

Course Outline (Models):
Grading:

40% 4 class projects

25% Midterm

25% Final

10% Participation

Organization Forms:

- Sole Proprietorship - Stand alone business owned by one person. Principal only (partners). Single taxation - all business earning are taxed to individual (flow through). Unlimited liability - can take on debt (as the individual).
- Partnership - Principals only or principals and agents. Flow Through. Unlimited liability
- Corporation. Principals and agents. Double taxation. Separate legal entity. Limited liability. Retained earnings - reinvested in the company (not taxed twice) and earnings paid to shareholders. Called C-corporations (as opposed to S-corporations (small corporations)).
- LLC - Flow through taxation, but with limited liability.

Typical Corporation Structure:
C-Suite: Sit below the CEO. They are include CFO, COO, CIO, CTO, CSO, CMO. Under CFO there is the Treasurer/FP&A (financial planning and analysis) (great place to look for positions), Controller (accounting and auditing), and Corporate Development (M&A = mergers and acquisitions).

Investment Bank Structure:

Securities Division

- Equities

- Fixed income
- Commodities
- Derivatives

Investment Strategies (Called Strats):

- Quant
- Financial Engineering
- Portfolio Solutions

Investment Research:

- Economic Outlook
- Various Industries

I-Banking (investment banking)

- Industry groups
- ...

Finance – The economics of exchange
Finance involves valuing assets

- Accountants think of an assets as a sequence of future benefits
- in Finance we think about a sequence of future cash flows (CFs)

Time Value of Money (TVM):

- Follows from the idea of opportunity cost
- A dollar today will be

Conventions Side Note:

- Timeline: Thinking about a CF that acts on a single line, starting at time 0 and going until time T (time segment). We might have periods $1, 2, ...$ etc. The number of period refers to the end of that period (i.e. from 0 to 1 is "year one"). Might also have a time-ray (go off for forever, will have seperate formulas).
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Cash flow:
V = value, PV = present value. So P(Assets) = PV(All expected future CF)
FV = future value. $FV_n = PV_m(1+r)^{n-m}$. where the n period is greater than m, the present period.

$$PV_m = \frac{FV}{(1+r)^{n-m}} = FV_n(1+r)^{m-n}$$

PV (a stream of CFs). $CF \in \mathbb{R}$

$$PV = V_0 = \frac{CF_1}{(1=r)^1} + \frac{CF_2}{(1+r)^2} + ... + \frac{CF_T}{(1=r)^T} = V_0 = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}$$

Some shortcuts: If $CF_{t+1} = CF_t(1+g)$ (i.e. an annuity) then

$$V_0 = \frac{CF_0(1+g)}{(1+g)} + \frac{CF_0(1+g)^2}{(1+r)^2} + ... + \frac{CF_0(1+g)^T}{(1+r)^T} = CF_0 \sum_{t=1}^{T} \frac{(1+g)}{(1+r)}^t = ... = V_0 = CF_0 \frac{(1+r)}{(1+g)} \left(1 - \left(\frac{1+g}{1+r}\right)^T\right)$$

This leads to the **Growing Annuities Formula:**

$$V_0 = \frac{CF_n(1+g)}{r-g} \left(1 - (\frac{1+g}{1+r})^{T-n}\right)$$

Formula works for **growing annuities** such as bonds. A *level annuity* has $g = 0$ and CF constant. The (...) step is given by the trick

$$\sum_{t=1}^{T} x^t = \frac{x(1-x^T)}{1-x}$$

Perpetuities

We are looking at

$$\sum_{t=1}^{\infty} \frac{CF_0(1+g)^t}{(1+r)^t} =^* CF_0 = CF_0 \left(\frac{1+g}{r-g}\right)$$

The (*) denotes application of the geometric series formula. Special case: (level perp) $g = 0$ so $V_0 = \frac{CF}{r}$

APR/EAR Conventions:

- APR = annual percentage rate (not necessarily EAR). Stated on the agreement
- EAR = Equivalent annual rate (can be used as r).
- EAR =APR if no compounding
- See how many times APR is compounded to calculate the EAR.
- To convert, do $EAR = (1 + \frac{APR}{n})^n - 1$ where n is the number of time compounded. Continuous compounding: $FV = PE^{rt}$

HW Prob, number 10. Say you want to retire at age T_w and die at age T_r, have income I, EAR rate r, and start saving at age T_s so that you can withdraw W every year until you die. Then you portfolio is

$$V = \left(I \sum_{t=1}^{T_w - T_s} (1+r)^t\right) (1+r)^{T_r - T_w} - W \sum_{t=1}^{T_r - T_w} (1+r)^t$$

Solve for I to solve the problem.

Lecture II (7/11/2023)

Side Note: We will denote inflation rate by π

Economic Policy Review:

Fiscal: Legislative and executive branches. Tools are 1. taxation, and 2. spending

Monetary: The FED (more specifically the FOMC). Some controls: 1. Open market Operation (buying and selling of securities), 2. Reserve Requirement (bank holdings requirement), 3. Federal Funds target rate (FF are what banks trade to other banks), 4. Discount Rate (what banks pay to borrow from the FF reserve directly).

- Aggregate Demand = C + I + G + NX
- Quantitative Easing - creating money back by the USFG

Asset Evaluation: Value(Asset) = PV(All Expected Future CFs)

$$V_0 = \sum_{t=1}^{T} \frac{CF_t}{(1+v)^T}$$

or

$$V_0 = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t}$$

Annuity (a finite series of related CFs). Assume $CF_{t+1} = CF_t(1+g)\ \ \forall t$. Recall that

$$V_n = \left(\frac{CF_n(1+g)}{r-g}\right)\left(1 - \left(\frac{1+g}{1+r}\right)^T\right) \quad r \neq g$$

Special case (level annuity): $g = 0$.

Perpetuity

$$V_n = \frac{CF_n(1+g)}{r-g}$$

Special case $g = 0$ so $V_n = \frac{CF}{r}$

Bonds

When an issuer borrows money, having a legal obligation to pay back.
Credit Ratings indicate the likelihood for the agent to pay back. (Investment Grade (IG) vs "High Yield", "Junk", "NonIG"
Investment grade bonds are BBB-/Baa3 and higher. SIP, Moody's, grade borrowers.

Bonds Terms

- Coupon Rate (CF)
- PAR Value (Principle), assume to be 1000

- Term to Maturity (how long does it last)
- Yield to Maturity (r)
- Compounding Frequency (# of periods in a year), and APR by convention. By convention in US this is twice a year

Example Bond: 5yr, AAA, 9% (coupon), XYZ Corp (issuer, can assume PAR=1000, semi-annual compounding), 6% ytm.

1000(9%/2) = 45 per period. So, $CF = 45$. Now, take $r = .03$, i.e. 6%/2. So, the Bond value is

$$P_{Bond} = \frac{1000}{(1+.03)^{10}} + 45\sum_{t=1}^{T}(1+.03)^{-t}$$

divide the total bond value by 10 to get the "percent of par"

All this math can be done with `price()` is excel.
Price - yield curve (concave up)

Computing Duration (interest rate sensitivity). Important for re-investment risk vs price-risk
O.I.D = zero coupon bonds (high-duration bond).

Lecture(7/13/2023)

Duration continued:

$$MacDur = \frac{1}{P_{bond}}\left(\sum_{t=1}^{T}\frac{CF}{(1+r)^t}\cdot t + \frac{T\cdot PAR}{(1+r)^T}\right)$$

How is this curve changing with r?

$$\frac{\partial P}{\partial r} = -\frac{MacDur \cdot P_{bond}}{1+r}$$

Note $ModDur = MacDur/(1+r)$, so

$$\frac{\partial P}{P} = -(ModDur)(\partial r)$$

This is a linearization of the price v. yield curve.

Interpretation of duration: This is the time at which a change in interests rate would balance the re-investment risk vs the holding risk

Asset-Liability mismatch: A liability with a shorter duration will not be discounted by a rise in interest rates as much as the money held up in a longer duration bond would be.

Equities: DDM the Dividend Discount (A type of DCF model)

$$V_0 = \sum_{t=1}^{\infty}\frac{expectedDIV_t}{(1+r_E)^t}$$

Three cases:

1. Stock has a fixed dividend (no growth in DIV $= D$),
$$V_0 = \frac{D}{r}$$
2. Growing dividend
$$V_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$
3. Growing at rate one until it changes to rate 2 at period n. Need to account for delayed start of the perpetuity. This is, $PV(Annuity) + PV(PV(Perpetuity))$
$$V_0 = \frac{CF_0(1+g_1)}{r-g_1} \cdot \left(1 - \left(\frac{1+g_1}{1+r}\right)^n\right) + \left(\frac{CF_0(1+g_1)^n(1+g_T)}{r-g_T}\right)\left(\frac{1}{1+r}\right)^n$$

Note: $PV(Annuity)$ called "forecast horizon" or "high-growth phase". $PV(PV(Perpetuity))$ called the "Terminal value" **Calculating growth rate (Rudimentary)**: IF we know a company's total equity, dividends, and net income, we can calculate a growth rate g:
$$g = \frac{I - DIV_{total}}{E}$$
Note this is total dividends paid, not dividends per share (DPS).

Lecture (7/17/2023)

Some useful excel functions:

- `=price()`
- `=PV()`
- `=NPV()`
- `XNPV()`
- `=YEILD` or `=rate()` or `=IRR()` or `=XIRR()`

Begin hedged with regard to interest rates: When duration between assets and liabilities are matched.

Accounting vs Finance:

- Accounting: concerned with the "fairness" of the reporting of transactions.
- Accounting gives a distorted reality: expenses get time shifted towards their associated revenue
- The above is "Matching", following the rules of GAAP (Generally Accepted Accounting Principles) set by the FASB

Deprecation example: Straight-Line Depreciation (SLD). An original expense for capital used over n periods is going to be accounted over the expected life of the capital asset (i.e. expense/n). An "expenditure" (CAPX) is the original payment for the capital. The spread out accountings are called "expenses" (DEP).

Balance Sheeet–Looking at a certain point in time. On one side there Assests (A) and on the other there is Liabilities (L) and Equity (E). SO $A = L+E$. An income statement sees $REV - EXP = NI$ (net income).

Finance:

- Cares about the "actual cash flows"
- We need to convert the accounting language into time-relevant cash flows (proforma – i.e. projections)

Equity: CS + RE

Common Stock – Company issuing equity or buying back:

- Made up of PAR VAL and APIC
- PAR value is anitquated, minimum redeemable value for companies stock with company
- APIC is everything else (i.e. how much the market value has risen above PAR value).

Retained Earnings:

- Recall $NetIn = Rev - Expen$. Net Income splits into dividends and retained earnings
- $RE = \sum \Delta RE_t$ over previous time periods
- The dividend-payout ratio (DPR) is $= \frac{D}{NI}$
- The retention ratio (or plow-back ratio, or reinvestment ratio, or b) is $b = \frac{RE}{NI}$
- Why is RE not under cash in the BS? Remember NI might be something like credit (i.e. from customers), so computing $RE = NI - D$ will give an accounting number – i.e. it doesn't correspond to cash on hand
- $drp + b = 1$

Income statement:

(+) Revenues (sales) is the "topline"

(-) subtract COGS (Cost of Goods Sold) breaks down into FC and VC (fixed costs $\neq$ f(sales), and variable costs = f(sales)).

(=) Gross Profit

- Gross Profit / Revenues = Gross Margin

(-) Operating expenses (OPEX) (Selling, General, Administrative SGA)

(-) Depreciation expenses (DEPEX, spread out capital expenses)

(=) Operating Profit or Operating Income

(+) Non-operating profit (Non op income)

(=) EBIT or Earnings Before Interest and Taxes

(-) Interest expense = Debt*YTM

(=) EBT or NIBT - Net Income before taxes

(-) Tax expense (not tax expenditure (t*EBT)

(=) Net Income or Earnings, i.e. the Bottom Line

Some more terms:

- Net Income / Revenue = Net Profit Margin
- ROA = return on assets

(Lecture (7/18/2023)

Calculating Growth

- **SGR – Sustainable Growth Rate** = (ROE)*(b) = g or g = ROE(1-dpr)
- Recall: $b = \Delta RE/NI$ (change in retained earnings)
- Internal growth rate $IGR = ROA(b)$:
- ROE = NI / E (net income divided by equity)
- $g = \frac{\Delta RE}{Eq}$

Dupont Analysis:

- $g = f(profitability, efficiency, leverage, reinvestment\ policy)$

Calculating growth in Dividends / :

- $g_{DIV} = g_{FCFE} \approx ROE(b)$
- ROIC = return on invested capital (= Debt + Equity)
- $g_{FIRM\ or\ FCF} \approx (ROIC)(IR)$

Valuing a Firm:

- Intrinsic Value (IV) = Debt + Equity (financial term, not a Book Value)
- FCFF = Free Cash Flow to Firm (free as in 'free to distribute'

$$V_{ASSET} = \sum_{t=1}^{\infty} \frac{FCFF_t}{(1 + r_{FIRM})^t}$$

- CCF = Creditor Cash Flow (the money from the FCFF that goes to debt holders instead of equity holders)
- FCFE = Free Cash Flow to Equity (money given to equity holders)
- Clearly, FCFF = CCF + FCFE
- Each debt security is finite so

$$V_{DEBT} = \sum_{t=1}^{T} \frac{CCF_t}{(1 + r_{DEBT})^t}$$

Computing FCFF: A Free Cash Flow Worksheet (a mirror of the Accountant's Income Statement)

- When writing one of these statements, we do this every period in the future, if the project goes on forever we will need to make some assumptions and turn the values into perpetuities.
- So, the overall project value is

$$V_{project} = \sum_{t=1} \frac{FCF_t}{(1 + r_{proj})^t}$$

- g_{FCFF} = (ROIC)(IR) = $(\frac{NOPAT}{INVEST.CAP.})(\frac{REINVEST.}{NOPAT}) = (\frac{EBIT(1-t_{tax})}{LTD+E})(\frac{NCS+\Delta NWC}{EBIT(1-t_{tax})})$

Lecture (7/19/2023)

Capital Budgeting: Decisions

- NPV (>0 implies the project is a go). Net Present Value
- IRR ($\geq$ Hurdle Rate (Cost of Capital) implies project is a go)
- Payback Period ($\leq$ Target period (a subjective measure) implies project a go)
- Profitability index (>1 indicates a go).

Lecture (7/27/2023)

Father of modern portfolio theory (MPT): Harry Markowitz

- Wrote "Portfolio Selection" in the 1950s
- MVO = Mean Variance Optimization
- We use MPT (and eventually CAPM) to compute r.
- FF3F (Fama-French 3 factor model), FF5F, MFM are all more advanced theories

MVO:

- Complete (or Composite) contains: Risk Free portfolio (RF), and Risky (P)

- Risk Free portfolio has "no risk", (treasury bills, cash, etc.)
- Risky (bonds, stocks, Real Estate, commodities)
- Developed by Harry Markowitz in 1952
- Problems with MVO: tail risk (Kurtosis, skew not captured by the variance figure in a Gaussian distribution)

Define **risk** as variance or standard deviation.

Define **reward** as *expected return*.

Expected returns are usually positive (might be negative as a hedge)

$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$

We define the weights as $w_p = \frac{P}{Comp}$, similarly, $w_{RF} = \frac{P}{Comp}$ (note $w_p + w_{RF} = 1$

$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$

Note: for now, we are given a σ_i^2 and $E(r_i)$ for all our risky portfolio.

Covariance formula:

$$Cov(x, y) = \sum_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

Recall $\sigma_{ab} = \sigma_{ba}$

In Matrix notation: $w^T \mu$ where μ = vector of $E(r_i)$. So $E(r_p) = w \cdot \mu$.
In excel `= MMULT(TRANSPOSE(WTVEC), MUVEC)` then do Cntrl + Shift + Enter.

The covariance of the risky portfolio is:

$$\sigma_p^2 = w^T \Sigma w$$

were Σ is the covariance matrix. Note MMULT in excel only works with two inputs, so you'll have to do two MMULT function in excel.

An example covariance matrix:

$$COVMAT = \Sigma = \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{AB} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{AC} & \sigma_{BC} & \sigma_C^2 \end{pmatrix}$$

To easily generate a covariant matrix in excel, do $COVMAT = \frac{\epsilon^T \epsilon}{T-1}$ where ϵ is you vector of $x_i - \overline{x}$ (which is easy to compute from your column of returns)

Note: For the Risk Free scenario: $E(r_{RF}) = RF$ because risk free investment considered constant (not an expectation).

So, $E(r_{comp}) = w_{rf}RF + w_pE(r_p)$ and since $\sigma^2_{rf} = 0$ by definition, we have that $\sigma^2_{comp} = w_p^2\sigma_p^2$.

Constrained Optimization:

i) Goal 1: find

$$GMV_{portfolio} = \min_w \sigma_p^2 \text{ s.t. } \sum_i w_i = 1$$

ii) Goal 2: find efficient portfolios:

$$\min_w \sigma_p^2 \text{ s.t. } \sum w_i = 1 \text{ and } E(r_p) = \mu^*$$

1. GMV. Set up $\mathcal{L}(w_a, w_b, \lambda) = \sigma_p^2 + \lambda(w_a + w_b - 1)$. Take the derivatives:

$$\frac{\partial \mathcal{L}}{\partial w_A} = 2w_A\sigma_A^2 + 2w_B\sigma_{AB} + \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_B} = 2w_B\sigma_B^2 + 2w_A\sigma_{AB} + \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w_A + w_B - 1 = 0$$

in matrix notation we have

$$\frac{\partial \mathcal{L}}{\partial w} = 2\Sigma w + \lambda = \vec{0}$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w^T\vec{1}$$

which can be further condensed into

$$\begin{pmatrix} 2\Sigma & \vec{1} \\ \vec{1}^T & 0 \end{pmatrix} \begin{pmatrix} \vec{w} \\ \lambda \end{pmatrix} = \begin{pmatrix} \vec{0} \\ 1 \end{pmatrix}$$

Or simply $A\vec{x} = \vec{b}$ which has solution The solution, $x = A^{-1}b$ is the weights for a portfolio that is the GMV.

2. The Efficient portfolio.

$$\min_w w^T\Sigma w \text{ s.t. } w^T\mu = \mu^* \text{ and } w^T\vec{1} = 1$$

The Lagrangian:

$$\mathcal{L}(w, \lambda_1, \lambda_2) = w^T\Sigma w + \lambda_1(w^T\mu - \mu^*) + \lambda_2(w^T\vec{1} - 1)$$

so

$$\frac{\partial \mathcal{L}}{\partial w} = 2\Sigma w + \lambda_1\mu + \lambda_2$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = w^T\mu = \mu^*$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = w^T\vec{1} = \vec{1}$$

Which we simplify in matrix notation as

$$\begin{pmatrix} 2\Sigma & \vec{\mu} & \vec{1} \\ \mu^T & 0 & 0 \\ \vec{1}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{w} \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \vec{0} \\ \vec{\mu}^* \\ 1 \end{pmatrix}$$

Which we again solve in excel as the simplified $x = A^{-1}b$.

Lecture (8/1/2023)

Portfolio Optimization Continued

For the P.O. HW problem:

1. Download Returns (Adjusted Prices) – This would be good to programm and automize
2. Compute $E(r)$
3. Table of metrics: (Avg., σ, Skew, Kurt)
4. Build covariance matrix – (remember this is important in combining the variance of the stocks)
5. Evaluate the (periodic)current portfolio ($E(r)$, σ, Sharpe Ratio, annual $E(r)$, annual $\sigma_{ann} = \sqrt{12}\sigma_{monthly}$, annual Sharpe)
6. Optimize, note that the scatter plot of potential portfolios is the "feasible set"

a) GMV Portfolio

b) Efficient Portfolio

Avoid doing $x = A^{-1}b$ for the portfolio optimization by doing $w = \frac{\Sigma^{-1}\vec{1}}{\vec{1}^T\Sigma^{-1}\vec{1}}$

Covariance Shortcut: When computing the Covariance of two portfolios: $\sigma_{AB} = w_A^T \Sigma w_B$

Sharpe Ratio:

$$SR = \frac{w^T\mu - rfr}{\sqrt{w^T\Sigma w}}$$

where rfr is the Risk-Free rate

Say you had two portfolios from the effUsing the calculated expected returns in tandem with current MSFT share price and dividend payout allows us to calculate a growth raticient frontier, usually the GMV and (efficient) client portfolio. These form a basis and can be used to calculate the entire efficient frontier. Then you can generate the efficient frontier curve by computing an implicit problem:

$$E(r_{port}) = w_{GMV}E(r_{GMV}) + w_{EP}E(r_{EP})$$

$$\sigma_{port}^2 = w_{GMV}^2\sigma_{GMV}^2 + w_{EP}^2\sigma_{EP}^2 + 2w_{GMV}w_{EP}\sigma_{GMV,EP}$$

Note that $\sigma_{GMV,EP}$ can be more easily computed

Optimal Risky Portfolio (ORP):

- We can draw the line $E(r_{Total}) = w_p E(r_p) + (1 - w_p) rf$
- Note that $\sigma_{total} = w_p \sigma_p$
- Note that the slope is the Sharpe ratio
- We maximize the Sharpe Ratio, and so the line will meet the efficient frontier curve at one place, and this is where our ORP is

The Sharpe Ratio Optimization

$$\max_{w \in \mathbb{R}} \frac{w^T \mu - rfr}{\sqrt{w^T \Sigma w}} \text{ s.t. } \Sigma w = 1, \text{ and } w^T \vec{1} = 1$$

The answer is: $w = \frac{\Sigma^{-1}(\mu - rf \cdot \vec{1})}{\vec{1}^T \Sigma^{-1}(\mu - rf \cdot \vec{1})}$

Lecture (8/2/2023)

Sharpe ratio = $\frac{E(r_p) - rf}{\sigma_p} = \frac{Risk-premium}{risk}$

CAPM

- In the 1960's the Capital Asset Pricing Model was form (CAPM)
- Developed by Bill Sharpe (1964)
- John Linter and Jan Mossin developed variations on CAPM (1965, 1966)
- A model of expected returns $E(r_i)$ (expected rate of return on asset r_i
- A model for getting r in the DCF model

CAPM Assumptions

1. Investor are rational - Loophole: sentiment analysis, irrational behavior etc. Why do we assume it: on average, investors act in their own self interest
2. Investors are Mean-Variance optimizers: i.e. use the normal distribution (ignore tail-rsik, kurtosis, skew etc)
3. Homogeneous inputs to MVO (mean variance opt). So investors all have same $E(r)$, σ, and Covariance (ρ).
4. Closed system and no frictions

- From this, we get that $ORP = MKT$ (the Optimal risky portfolio is the portfolio of the market)

Two Studies

- Elton and Gruber published a study
- Statman (1987)
- Both wanted to study the effect of diversification
- How much diversification do you need to capture the market portfolio?
- Plot the average variance for all portfolios against the number of securities in the portfolio
- You get a diversification effect due to imperfect correlations: variance always decreases.
- That is, *idiosyncratic* risk is decreasing (idiosyncratic risk also known as *unique*, *non-systematic*, and *specific*)
- Takeaway: You 'get paid' for *systematic* risk exposure. You don't 'get paid' for idiosyncratic risk. Why? Because idiosyncratic risk can be diversified away.
- 'Paid' means paid in the form of expected return.

CAPM Continued

- The CAPM models is a two stage process:

1. Estimate exposure ot systematic risk β
2. Use β to estimate $E(r)$.

Estimate With β

- Do a linear regression
- (Dependent) $r_i - rf$ vs $r_m - rf$ (Independent) (m is the S&P500 for our purposes)
- We do a time series: collect data for each across the time period (say 2000 to 2022).
- If we plot this, we get a scattering around the origin
- Take a regression line, this is known as the **security characteristic line**. The intercept is α, the slope is β and ϵ_i is the distance between the line and the point.
- $r_{i_t} - rf = \alpha_i + \beta_i(r_{m_t} - rf) + \epsilon_{i_t}$
- Getting the variance of this equation: $\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_\epsilon^2$
- σ_i^2 is the Total risk of an asset i
- β_i^2 is the systematic risk
- σ_ϵ^2 is the non-systematic risk
- $\frac{syst-risk}{total-risk} = R^2 = \frac{\sigma_i^2\sigma_m^2}{\sigma_i^2}$
- Note $\sqrt{R^2}$ is correlation $\rho_{i,m}$
- SO $\rho_{i,m} = \beta_i(\frac{\sigma_m}{\sigma_i})$

- Solve for Beta: $\beta_i = \rho_{i,m}(\frac{\sigma_i}{\sigma_m}) = (\frac{\sigma_{i,m}}{\sigma_i \sigma_m})(\frac{\sigma_i}{\sigma_m})$ so

$$\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2} = \frac{Cov(i,m)}{Var(i,m)}$$

- As a statistician: it's still important to run the linear regression because you want to verify if the assumptions of CAPM hold.

Stage two: use β_i to estimate $E(r_i)$

Lecture (8/7/2023)

Review: Expected return and Variance for a scenario analysis

$$E(r) = \sum_{i=1}^{n} P(i) r_i$$

$$\sigma^2 = \sum_{i=1}^{n} P(i)[r_i - E(r)]^2$$

CAPM Continued

- ANOVA (Analysis of Variance) tables
- Has 3 rows: Regression, Residual, Total
- Has 3 columns: df (degrees of freedom) SS (sum of squares) and MS (mean of squares)
- Mean of squares is just variance $MS = \frac{SS}{df}$
- Note Residual MS is σ_ϵ^2
- This is the same from $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\epsilon^2$
- So $\beta_i^2 \sigma_m^2$ is systematic risk and σ_ϵ^2 is the idiosyncratic risk
- On the exame, σ_ϵ^2 and σ_i^2 will be blanked out from the ANOVA table so remember $MS_{Resid} = \frac{SS_{Res}}{df_{Res}}$ and $MS_{Total} = \frac{SS_{Total}}{df_{Total}}$

Stats review in context of CAPM:

- t-stat is $\frac{Estimated\ value - 0}{Standard\ Error}$
- For example

$$t_\alpha = \frac{\hat{\alpha} - 0}{s.e._\alpha}$$

- Usually, if $|t_\alpha| > 2$ then reject the null hypothesis
- Note that t_β is basically always going to be bigger than 2: the null hypothesis, is never satisfied (would imply a stock's risk is not at all associated with the market)
- p-value is probability of an observation being at the extremes of t-stat

- t-stat to confidence: 68% for 1, 95% for 2 (the assumed), 99% for 3.

Adjusted Beta: $Adj.\ \beta = (2/3)\beta_{computed} + (1/3)\beta_{market}$ and since $\beta_{market} = 1$, $Adj.\ \beta = (2/3)\beta_{computed} + 1/3$, a weighted average towards the market

Step 2 of CAPM: Use β_i to compute $E(r_i)$ (using SML)

- Plotting $E(r_i)$ vs β_i
- The result is
$$E(r_i) = rf + \beta_i(E(r_m) - rf)$$
- The post-facto plot of a portfolio on this plot may have a *Jensen's* α which is the difference between expected return and actual return. On average, Jensen's alpha should be 0. (It's not in practice, a reason to reject CAPM)

WACC or Weighted Average Cost of Capital

- Two ways of determining,
- From CAPM: $WACC = w_E k_E + w_D k_D(1 - t)$
- $WACC = f$(comp companies)
- Problem: To calculate WACC I need the value of the firm, but to calculate the value of the firm, I need the WACC
- Way around this problem, people will estimate future weights w_E and w_D

Comps (Comparable Companies) or Peer companies Analysis- To estimate β_i

- May be used to avoid problems with looking at historical data (potentially unreliable, what if company has changed, what if new firm)
- Do the CAPM steps to get β_E for each Peer company
- Can't just use $\overline{\beta}_E$, because different companies have different amounts of leverage
- Companies with more financial leverage will have higher βs

Comps Full steps

1. Collect Peer data $(t_E, \beta_E, D/E)$ - on test we'll ignore t and google D/E (the debt-to-equity ratio) and cite our source
2. Unlever each peer (this formula has problems, but widely used)
$$\beta_U = \frac{\beta_E}{1 + (1 - t)^{D/E}}$$
The above is known as "Hamada's Shortcut"
3. Take the average unlevered betas: $\overline{\beta}_U$
4. Relever $\overline{\beta}_U \to \beta_L$ according to the target firm's D/E ratio: $\beta_L = \overline{\beta}_U(1 + (1 - t)^{D/E})$
5. Finally, use β_L in SML to get $E(r_i)$
$$E(r_i) = rf + \beta_L(E(r_m) - rf)$$