SYK Two-Point with using normal and hypergeometric distirbutions

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1. Construct a Majorana operator following the Jordan-Wigner Transformation:

$$\chi_i \to \begin{cases} \left(\prod_{j<k} \sigma_j^z\right) \sigma_k^x, \ i \text{ even} \\ \left(\prod_{j<k} \sigma_j^z\right) \sigma_k^y, \ i \text{ odd} \end{cases}$$

Easy does it. Now use these operators to construct the SYK Hamiltonian

$$H = (i^2) \sum_{i<j<k<l} \frac{J_{ijkl}}{4!} \chi_i \chi_j \chi_k \chi_l$$

Where

$$E[J_{ijkl}] = 0, \ E[J_{ijkl}^2] = \frac{6J^2}{N^3}$$

The equation for our two-point function:

$$F(\tau) = \frac{\mathrm{Tr}[e^{(-\beta+\tau)H} \chi_i(\tau) e^{-\tau H} \chi_i(0)]}{\mathrm{Tr}[e^{(-\beta+\tau)H} e^{-\tau H}]}$$

Notice we're working with χ_i. We make $\beta = 1$ for simplicity. Let us see what this looks like for N = 12 Majorana fermions.

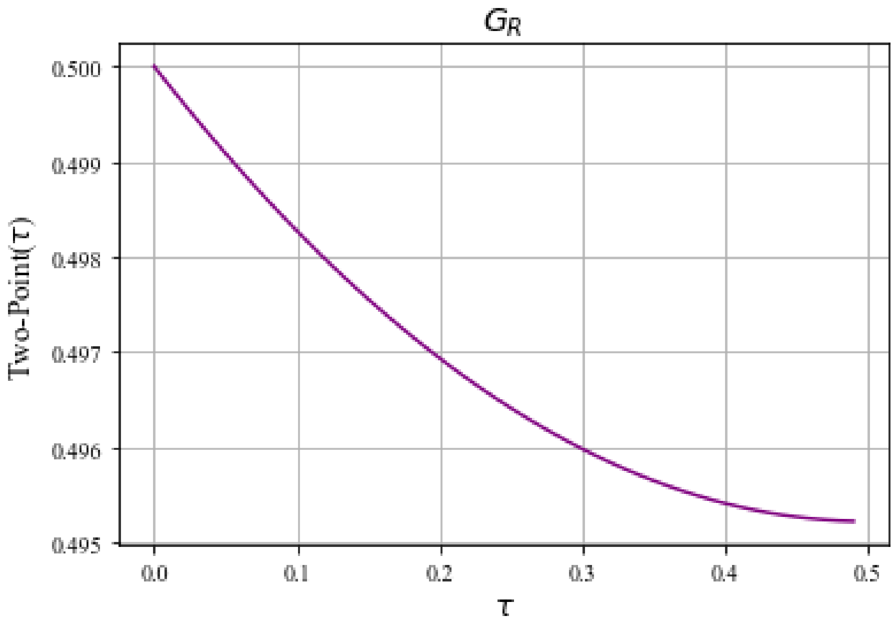

Figure 1: Normalized Two-Point Result. 12 Majorana operators with 4-interaction SYK Hamiltonian. The number of Majoranas will be increased. This is promising testament to a working script for small N. $\beta = 1$.

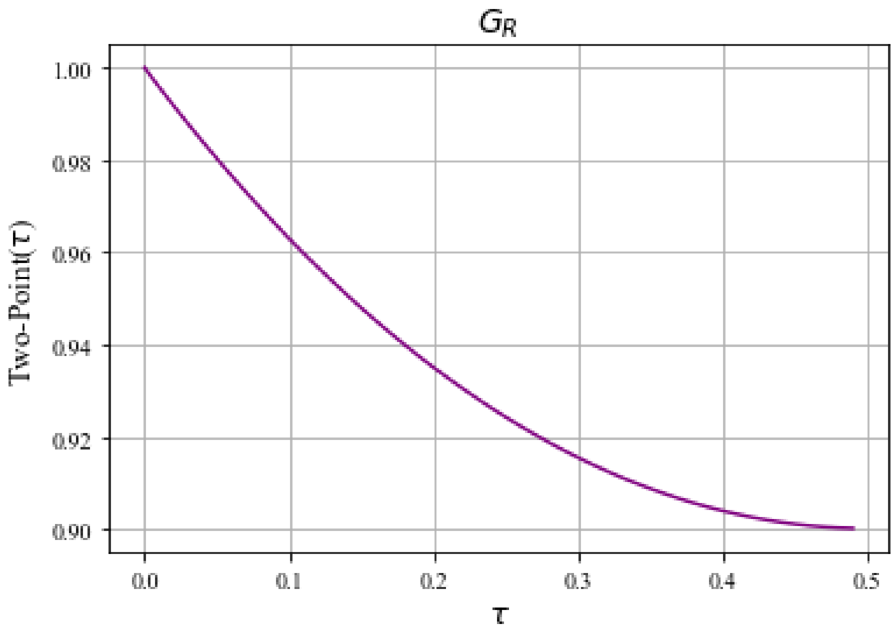

Figure 2: Normalized Two-Point Result. 12 Majorana operators with 4-interaction SYK Hamiltonian. The number of Majoranas will be increased. Notice here that a scaling factor $\frac{1}{\sqrt{2}}$ wasn't applied to the Majorana operators, making the Two-point's initial value 1 instead of .5. A nearly identical form is achieved. $\beta = 1$.