Matrix Algebra

02 February 2020 16:32

$$\begin{array}{ll}
(1) & A+(B+C)=(A+B)+C \\
(2) & A+O=A=O+A \\
(3) & A+(-A)=O=(-A)+A \\
(4) & A+B=B+A \\
(5) & (\lambda+\mu)A=\lambda A+\mu A \\
(6) & \lambda(A+B)=\lambda A+\lambda B \\
(7) & \lambda(\mu A)=(\lambda\mu)A.
\end{array}$$

Identity matrix

The identity matrix of order n is the n × n diagonal matrix whose diagonal elements are all 1. It is denoted by I or In.

A * A^-1 = I
AI = A

The identity matrix is like the number 1 in normal numbers.

Transpose.

The transpose AT of a matrix A is obtained by interchanging the rows and columns. Thus if A = [aij]m×n then AT = [a 0 ij]n×m where a 0 ij = aji. For example, if

$$\begin{array}{lll}
(1) & (A^T)^T = A & \\
(2) & (A+B)^T = A^T + B^T & \text{when } A+B \text{ exists} \\
(3) & (\lambda A)^T = \lambda A^T & \text{for any } \lambda \in \mathbb{R} \\
(4) & (AB)^T = B^T A^T & \text{when } AB \text{ exists.}
\end{array}$$

Matrix Inverse

If A and B are square matrices of the same order, then B is called the inverse of A if AB = I = BA
It can be shown that if A has an inverse, then that inverse is unique. It will be denoted by A^−1
The determinant of a 2 × 2 matrix A = |a b, c d |is defined to be the number ad − bc and is denoted by det(A), |A|

Now if a 2 × 2 matrix A has an inverse, then det(A) det(A−1) = det(AA−1) = det(I) = 1 which means that det(A) 6= 0. Conversely if det(A) 6= 0 then it is easy to verify that A has an inverse

A 2×2 matrix A is *invertible* if and only if its determinant is nonzero. If $\det(A) \neq 0$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$