Basis

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Basis

Let S be a subspace of Rn . A set of vectors is called a basis of S if it is a linearly independent set which spans S.

Example

The set {e1 , e2 , e3} where e1 = (1, 0, 0), e2 = (0, 1, 0) and e3 = (0, 0, 1) is a basis for R3

To verify this we have two things to show:

First, the vectors are linearly independent since: α(1, 0, 0) + β(0, 1, 0) + γ(0, 0, 1) = 0 =⇒ (α, β, γ) = (0, 0, 0) =⇒ α = β = γ = 0

Second, the vectors are a spanning set since we can write any vector (x, y, z) in R3 as a linear combination thus: (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)

Thus the set of vectors {e1, e2, e3} is a basis for the subspace R3

To prove that a set of vectors is a basis, proof linear independence and the fact it spans the subspace (recall linear independence means all coefficients must be 0 for the sum to be 0 and spanning means any vector in the subspace can be formed as a product of some combination of the vectors)

Standard Basis

In Rn, the standard basis is the set {e1, e2,..., en} where er is the vector with r th component 1 and all other components 0.

For example, the standard basis for R5 is {e1, e2, e3, e4, e5} where

e1 = (1, 0, 0, 0, 0)

e2 = (0, 1, 0, 0, 0)

e3 = (0, 0, 1, 0, 0)

e4 = (0, 0, 0, 1, 0)

e5 = (0, 0, 0, 0, 1)

Theorem

Let S be a subspace of Rn . If the set {v1 , v2 , . . . , vm} spans S then any linearly independent subset of S contains at most m vectors.