Dimension

02 February 2020 15:37

The dimension of a subspace of Rn is the number of vectors in a basis for the subspace.

Since the standard basis for R n contains n vectors it follows that R n has dimension n

Show that the set S = {(x, y, z) | x + 2y − z = 0} is a subspace of R3. Find a basis for, and the dimension of, S

We can write: S = {(x, y, x + 2y) | x, y ∈ R} = {x(1, 0, 1) + y(0, 1, 2) | x, y ∈ R} = span {(1, 0, 1),(0, 1, 2)}
This shows that S is a subspace, **since the span of any nonempty finite subset of Rn is a subspace**
The spanning set {(1, 0, 1),(0, 1, 2)} is **linearly independent since neither vector is a multiple of the other**, and hence is a basis. Thus the dimension of S is 2

Theorem

Let {v1 , v2 , . . . , vm} be a set of nonzero vectors that spans a subspace S of Rn . Then removing each vi which is a linear combination of its predecessors will leave a basis for S

To see why this works, note that each vector removed is a linear combination of the remaining ones, so the span is not altered by the removal. Also the remaining vectors are linearly independent, since none is a linear combination of its predecessors.

Problem

Find a basis for and the dimension of the subspace S of R 4 spanned by the set {(2, 1, 0, −3),(−1, 0, −1, 2),(1, 2, −3, 0),(0, 0, 0, 1),(0, 1, −2, 0)}.

Solution

To find a basis we remove from the spanning set any vector which is a linear combination of its predecessors.

The remaining set of vectors: {(2, 1, 0, −3),(−1, 0, −1, 2),(0, 0, 0, 1)} is a basis for S, and hence the dimension of S is 3

Theorem

When the dimension of a subspace is known, then the task of deciding whether a given set is a basis can be simplified by using the following result. Instead of checking if the vectors form a spanning set we need only count them.

Let S be an m-dimensional subspace of Rn , then:
(1) any subset of S containing more than m vectors is linearly dependent
(2) a subset of S is a basis if and only if it is a linearly independent set containing exactly m vectors

When S = Rn

Any subset of Rn containing more than n vectors is linearly dependent. A subset of R n is a basis if and only if it is a linearly independent set containing exactly n vectors.

Subspaces of R2

(1) There is one 0-dimensional subspace {0}.
(2) A one-dimensional subspace is spanned by a single non-zero vector. Hence the one dimensional subspaces correspond to the straight lines through the origin. (3) There is only one two-dimensional subspace — R 2 itself.

Subspaces of R 3

(1) There is one 0-dimensional subspace {0}
(2) A one-dimensional subspace is spanned by a single non-zero vector. Hence the one dimensional subspaces correspond to the straight lines through the origin
(3) A two-dimensional subspace is spanned by two linearly independent vectors. Hence the two-dimensional subspaces correspond to planes which contain the origin.
(4) There is only one three-dimensional subspace — R 3 itself.