Complex Numbers

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$$
\begin{aligned}
a + ib = c + id &\Leftrightarrow a = c \text{ and } b = d, \\
(a + ib) + (c + id) &= (a + c) + i(b + d), \\
(a + ib)(c + id) &= (ac - bd) + i(bc + ad).
\end{aligned}
$$

A complex number a + ib can be represented by an ordered pair (a, b) of real numbers. The number a ∈ R is called the real part and b ∈ R is called the imaginary part. The set of all complex numbers will be denoted by C.

Complex Conjugate

The complex conjugate of an imaginary number a + ib is a – ib (simply a reflection in the real axis)

Polar Coordinates

The use of a distance and direction as a means of describing position is far more natural than using two distances on a grid. This means of location is used in polar coordinates and bearings.

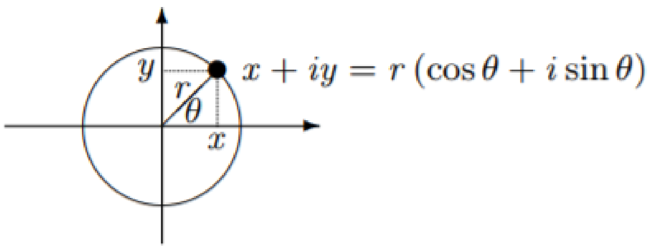
We say that (r,θ) are the polar coordinates of the point P, where r is the distance P is from the origin O and θ the angle between Ox and OP.

We can therefore express a complex number in polar coordinates

x = r cos θ, y = r sin θ

So x + iy = r(cosθ + isinθ)

r = sqrt(x^2 + y^2) , and θ satisfies tan θ = y/x

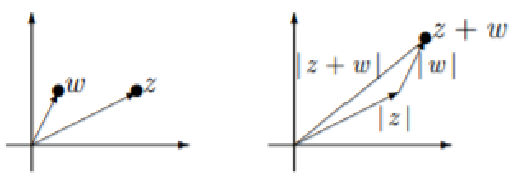


Modulus

The number $\sqrt{x^2 + y^2}$ is called the *modulus* of $x + iy$ and denoted by $|x + iy|$. Geometrically it represents the distance between $x + iy$ and the origin of the complex plane.

Properties of the modulus. *For any* $z, w \in \mathbb{C}$:   (1) $\mid z \mid = \mid \overline{z} \mid$,   (2) $\mid z \mid = \sqrt{z\overline{z}}$,   (3) $z\overline{z} = \mid z \mid^2$,   (4) $\mid zw \mid = \mid z \mid\mid w \mid$,   (5) $\mid z + w \mid \leq \mid z \mid + \mid w \mid$, *(the triangle inequality)*   (6) $\mid\mid z \mid - \mid w \mid\mid \leq \mid z - w \mid$.

Triangle inequality arises from representing the complex numbers as points in the plane as in the diagram:



De Moivres Theorem

Gives a formula for computing powers of complex numbers

$$\big(r(\cos\theta + i\sin\theta)\big)^n = r^n\big(\cos(n\theta) + i\sin(n\theta)\big).$$

Fundamental Theorem of Algebra

Every polynomial equation of degree n with complex coefficients has exactly n (not necessarily distinct) solutions in C