1.

$P(C) = 0.0001$ $\quad P(J \mid \neg C) = 0.6$

$P(J \mid C) = 0.64$ $\quad P(T \mid C) = 0.99 \rightarrow P(\neg T \mid C) = 0.01$

$P(T \mid \neg C) = 0.04 \rightarrow P(\neg T \mid \neg C) = 0.96$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$ Baye's Rule

(a) • Joint pain
• ve test

$$P(C \mid J \wedge T) = \frac{P(J \wedge T \mid C)\, P(C)}{P(J \wedge T)} = \frac{P(J \wedge T \mid C)\, P(C)}{(P(J \wedge T \mid C) \times P(C)) + (P(J \wedge T \mid \neg C) \times P(\neg C))}$$

$$P(J \wedge T \mid C) = P(J \mid C) \times P(T \mid C) = 0.64 \times 0.99 = 0.6336$$

$$P(J \wedge T \mid \neg C) = P(J \mid \neg C) \times P(T \mid \neg C) = 0.6 \times 0.04 = 0.024$$

$$P(C \mid J \wedge T) = \frac{0.6336 \times 0.0001}{(0.6336 \times 0.0001) + (0.024 \times 0.9999)} \approx 0.0026 < 0.5$$

$\therefore$ not enough evidence

(b) $$P(C \mid J \wedge T \wedge T) = \frac{P(J \wedge T \wedge T \mid C)\, P(C)}{(P(J \wedge T \wedge T \mid C)\, P(C)) + (P(J \wedge T \wedge T \mid \neg C) \times P(\neg C))}$$

$$P(J \wedge T \wedge T \mid C) = 0.6336 \times 0.99 = 0.627264$$

$$P(J \wedge T \wedge T \mid \neg C) = 0.024 \times 0.04 = 0.00096$$

$$P(C \mid J \wedge T \wedge T) = \frac{0.627264 \times 0.0001}{(0.627264 \times 0.0001) + (0.00096 \times 0.9999)} \approx 0.061 < 0.5$$

$\therefore$ still not enough evidence

to be confident

(c) $$P(C \mid J \wedge T^n) = \frac{P(J \wedge T^n \mid C) \times P(C)}{(P(J \wedge T^n \mid C) \times P(C)) + (P(J \wedge T^n \mid \neg C) \times P(\neg C))} > 0.5$$

$$\frac{P(J \mid C) \times P(T \mid C)^n \times P(C)}{(P(J \mid C) \times P(T \mid C)^n \times P(C)) + (P(J \mid \neg C) \times P(T \mid \neg C)^n \times P(\neg C))} > 0.5$$

$$\frac{0.64 \times 0.99^n \times 0.0001}{(0.64 \times 0.99^n \times 0.0001) + (0.6 \times 0.04^n \times 0.9999)} > 0.5$$

$$0.000064 \times 0.99^n > 0.000032 \times 0.99^n + 0.29997 \times 0.04^n$$

$$0.99^n > \tfrac{1}{2} 0.99^n + 4687 \times 0.04^n$$

$\boxed{\text{need 3 tests}}$ stick values in

2\. (a) $P(S \wedge I) = P(S) \wedge P(I)$

$= 0.132 + 0.011 = 0.143$

nine

(b) $P(C) = 0.132 + 0.21 + 0.098 + 0.184 = 0.624$

(c) $P(I \mid S) = \dfrac{P(I \wedge S)}{P(S)} = \dfrac{0.143}{0.463} \approx 0.309$

$P(S \mid I) = \dfrac{P(S \wedge I)}{P(I)} = \dfrac{0.143}{0.132 + 0.011 + 0.098 + 0.001} \approx 0.591$

$P(S \mid I) > P(I \mid S)$ $\therefore$ more likely someone who has insomnia also being a smoker

3. (a) f_1(Bad Speech)

f_2(Allies)

f_3(Bad Speech, Allies, Forms)

f_4(Forms, Traitor)

(b) f_5(Forms) $\Rightarrow$ from f_4 setting Traitor to true =

Forms	¬Forms
0.85	0.2

f_6(Allies, Forms) $\Rightarrow$ from f_3 setting Bad Speech to true =

	Allies	¬ Allies
Forms	0.95	0.5
¬ Forms	0.05	0.5

f_7(Allies, Forms) $\Rightarrow f_5 \times f_6$ =

	Allies	¬ Allies
Forms	0.8075	0.425
¬ Forms	0.01	0.1

f_8(Allies) $\Rightarrow$ sum out Forms in f_7

Allies	¬ Allies
0.8175	0.525

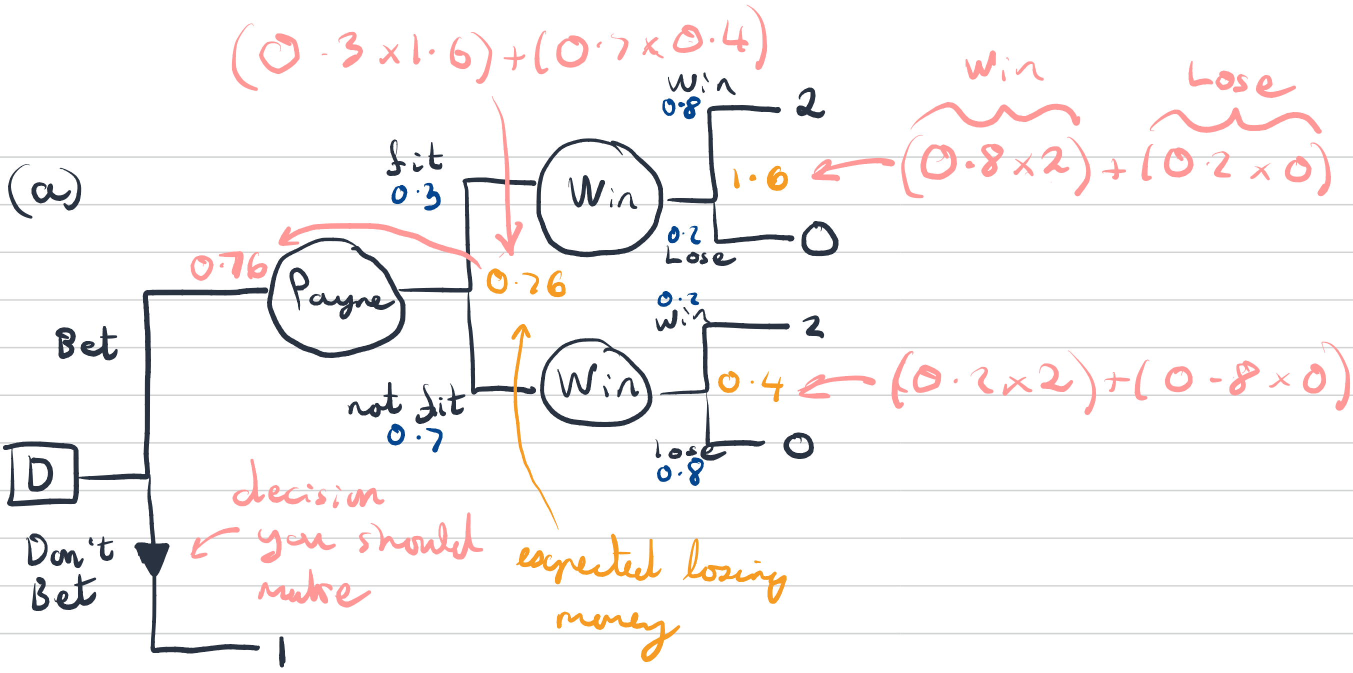
f_9(Allies) $\Rightarrow f_2 \times f_8$

Allies	¬ Allies
0.327	0.315

p(Allies | Bad Speech ∧ Traitor)

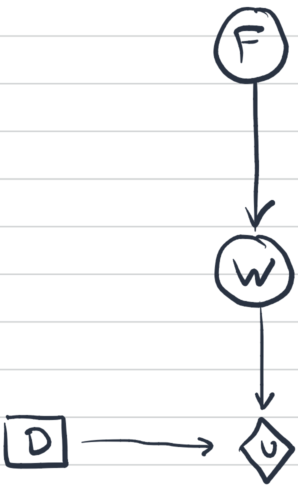
Allies	¬ Allies
$\frac{0.327}{0.642} \approx 0.5093$	$\frac{0.315}{0.642} \approx 0.4907$

4. (a)



(b) No, expected to receive back 76% of money bet
∴ lose money

(c)



fit

$P(F = f) = 0.3$

$P(F = u) = 0.7$

win

$P(W = w \mid F = f) = 0.8$

$P(W = L \mid F = f) = 0.2$

$P(W = w \mid F = u) = 0.2$

$P(W = L \mid F = u) = 0.8$

lose

$d_1 =$ Bet

$d_2 =$ No bet

$U(d_1, w) = 2$ ← double money

$U(d_1, L) = 0$ ← lose all

$U(d_2, w) = 1$ ← make no money

$U(d_2, L) = 1$