Vectors

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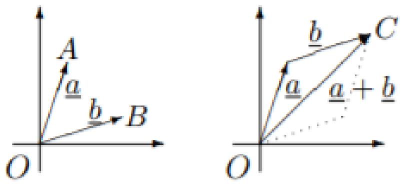
a + b = (a1 + b1, a2 + b2)
λa = (λa1, λa2)

a + b = (a1 + b1, a2 + b2, a3 + b3)
λa = (λa1, λa2, λa3)

Vector a + vector b = vector c where OABC is a parallelogram



Length and Distance

If $\underline{a} = (a_1, a_2) \in \mathbb{R}^2$ then we define the *length* $|\underline{a}|$ of $\underline{a}$ by

$$|\underline{a}| = \sqrt{a_1^2 + a_2^2}.$$

Similarly if $\underline{a} = (a_1, a_2, a_3) \in \mathbb{R}^3$, then we define the length of $\underline{a}$

$$|\underline{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

A vector is called a *unit vector* if its length is 1. The *distance* between $\underline{a}$ and $\underline{b}$ is defined to be $|\underline{b} - \underline{a}|$.

Normalisation

If we have a vector and need to find the unit vector then we carry out a process call normalisation
Take the modulus of the vector to get the length
Divide the vector by this value
e.g. |(2, -1)| = root 5
1/root5(2, -1) = 1
(2/root5, -1/root5) = 1

Scalar Product (Dot Product)

a.b = a1b1 + a2b2

Finding the Angle Between Vectors

$$\cos\theta = \cos(\beta - \alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha = \frac{a_1 b_1 + a_2 b_2}{|\underline{a}|\,|\underline{b}|}.$$

Two vectors are orthogonal (perpendicular) if their dot product is 0