

# Heuristics

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A **heuristic function**  $h(n)$ , takes a node  $n$  and returns a non-negative real number that is an estimate of the cost of the least-cost path from node  $n$  to a goal node. The function  $h(n)$  is an **admissible heuristic** if  $h(n)$  is always less than or equal to the actual cost of a lowest-cost path from node  $n$  to a goal.

How do we design our heuristics to make the best search algorithm possible?

## Admissible heuristics: Example

2	8	3
1	6	4
7		5

initial state

1	2	3
8		4
7	6	5

goal state

- 8-puzzle is just hard enough to be interesting
- Branching factor 3(ish), typical solution around 20 steps
- Exhaustive search:  $3^{20}$  states ( $= 3.5 \times 10^7$ )
- Eliminating repeating states:  $9! = 362880$  states
- Need a decent heuristic.

### Possible heuristics

$H(n)$  = number of misplaced tiles

$H(n)$  = Manhattan distance

## Characterising Heuristics

- If  $A^*$  tree-search expands  $N$  nodes and solution is depth  $d$ , then the effective branching factor  $b^*$  is the branching factor a uniform tree of depth  $d$  would have to contain  $N$  nodes.
- The closer the effective branching factor is to 1, then the larger the problem that can be solved.
- Can estimate  $b^*$  experimentally (usually fairly consistent over problem instances)
- $Q$  is  $h_2$  always better than  $h_1$ ? If  $h(n)$  is bigger than another  $h(n)$  for all  $n$  then that heuristic is said to dominate the other heuristic

## Deriving Heuristics

Can derive admissible heuristics of the exact solution cost of a relaxed version of the problem

A problem with less restrictions on operators is a relaxed problem

E.g. 8 puzzle, we can relax it and say that a tile can move from  $A$  to  $B$  if  $B$  is blank

If one dominates, use that. Else, calculate the  $h$  value for each state and use maximum value. If all options are admissible, then the chosen one will be admissible

## Subproblems

Derive admissible heuristics from solution cost of a subproblem of given problem  
Cost of subproblem = lower bound on cost of complete problem

We can store exact solution costs in a database which we can use to lookup values

We can combine pattern databases

### Disjoint pattern databases

If we can divide up the problem so moves only affect a single subproblem

### Statistical approach

Run search over training problems and gather statistics