Linear Comb & Subspaces

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https://warwick.ac.uk/fac/sci/dcs/teaching/material/cs131/part2

In R^2 the vector (5, 3) can be written in the form (5, 3) = 5(1, 0) + 3(0, 1) and also in the form (5, 3) = 1(2, 0) + 3(1, 1). In each case we say that (5, 3) is a linear combination of the two vectors on the right hand side.

If we wanted to express (6,6) as a linear combination of two other vectors:
(6, 6) = α(0, 3) + β(2, 1)

We would create two simultaneous equations
(6, 6) = (2β, 3α + β)
So we have (6, 6) = 1(0, 3) + 3(2, 1)

Span

Span. If $U = \{\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_m\}$ is a finite set of vectors in $\mathbb{R}^n$, then the *span* of U is the set of all linear combinations of $\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_m$, and is denoted by span U. Hence

$$\text{span } U = \{\alpha_1\underline{u}_1 + \alpha_2\underline{u}_2 + \cdots + \alpha_m\underline{u}_m | \alpha_1, \alpha_2, \ldots, \alpha_m \in \mathbb{R}\}$$

- (1) If U = {u} contains just a single vector, then span {u} = {αu | α ∈ R} is the set of all multiples of u.
- (2) In R^2 if U = {(1, 0),(0, 1)} then the span of U is R 2 . To see this note that we can write an arbitrary vector (x, y) in R 2 as a linear combination of (1, 0) and (0, 1) as follows: (x, y) = x(1, 0) + y(0, 1).
- (3) In R^3 the span of the set {(1, 0, 0),(0, 1, 0),(0, 0, 1)} is R^3
- (4) In R^3 let u = (1, 0, 1) and v = (2, 0, 3). Then αu + βv = α(1, 0, 1) + β(2, 0, 3) = (α + 2β, 0, α + 3β), so any linear combination of u and v has 0 for its middle component. In fact any vector with middle component 0 is a linear combination of u and v
- (5) In general, if u and v are not parallel, then the span of {u, v} is the plane determined by the three points u, v and 0.

Subspace

A Subspace is a Vector Space included in another larger Vector Space. Therefore, all properties of a Vector Space, such as being closed under addition and scalar multiplication still hold true when applied to the Subspace.

S subspace of R^n is a nonempty subset S of R^n with the following properties:
(1) u, v ∈ S ⇒ u + v ∈ S
(2) u ∈ S, λ ∈ R ⇒ λu ∈ S.

S = (x, y, 0) is a subspace of R^3 as any vector of S will also be in a the vector space R^3

To check if a set S is a subspace of some other vector space, simply check that the set is closed under addition and scalar multiplication

The set {0} consisting of just the zero vector is a subspace of R^n
The set R n itself is a subspace of R^n