$$\Rightarrow \begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} W^Ty \\ Z^Ty \end{bmatrix}$$

Since W, an $n \times q$ matrix, is of full column rank, $rk(W) = q$. Then W^T, a $q \times n$ matrix, is of full row rank, $rk(W^T) = q$. Since the rank of the product of 2 matrices, where at least one of the 2 matrices is of full rank, is equal to the smaller rank of the 2, we have $rk(W^TW) = q$ where W^TW is of size $q \times q$. Thus, W^TW is square and nonsingular. Thus, W^TW is invertible, i.e. $(W^TW)^{-1}$ is exists and is well-defined.
Similarly, Z has full column rank and Z^T full row rank, with $rk(Z) = rk(Z^T) = p-q$. By the same argument above $rk(Z^TZ) = p-q$ and Z^TZ is square of size $(p-q) \times (p-q)$, thus of full rank. Thus, $(Z^TZ)^{-1}$ exists and well-defined.
Since the inverse of a diagonal matrix is the diagonal matrix consisting of the inverse of its elements, we have that

$$\begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix}^{-1} = \begin{bmatrix} (W^TW)^{-1} & 0 \\ 0 & (Z^TZ)^{-1} \end{bmatrix}$$

Now we proceed as follows:

$$\begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix}^{-1} \begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix}^{-1} \begin{bmatrix} W^Ty \\ Z^Ty \end{bmatrix}$$

$$= \begin{bmatrix} (W^TW)^{-1} & 0 \\ 0 & (Z^TZ)^{-1} \end{bmatrix} \begin{bmatrix} W^TW & 0 \\ 0 & Z^TZ \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} (W^TW)^{-1} & 0 \\ 0 & (Z^TZ)^{-1} \end{bmatrix} \begin{bmatrix} W^Ty \\ Z^Ty \end{bmatrix}$$

$$= \begin{bmatrix} I_q & 0 \\ 0 & I_{(p-q)} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} (W^TW)^{-1}W^Ty \\ (Z^TZ)^{-1}Z^Ty \end{bmatrix} \Longrightarrow$$