$$= \frac{a^T}{(a^T(X^TX)^{-1}a)^{1/2}}\left[Cov\left(\frac{\hat{\beta}_1}{\sqrt{MSE}}, \frac{\hat{\beta}_2}{\sqrt{MSE}}\right) + Cov\left(\frac{\hat{\beta}_1}{\sqrt{MSE}}, \frac{-\beta_2}{\sqrt{MSE}}\right) + Cov\left(\frac{-\beta_1}{\sqrt{MSE}}, \frac{\hat{\beta}_2}{\sqrt{MSE}}\right) + Cov\left(\frac{-\beta_1}{\sqrt{MSE}}, \frac{-\beta_2}{\sqrt{MSE}}\right)\right]\frac{b}{(b^T(X^TX)^{-1}b)^{1/2}}$$

It has already been shown in part (b) that $\hat{\beta}_1$ and $\hat{\beta}_2$ are independent.
MSE is also independent of $\hat{\beta}_1$ and $\hat{\beta}_2$.
Also notice that $a^T\beta_1$ and $b^T\beta_2$ are fixed linear combinations, i.e. constant.
Thus we have that each of the above covariances are equal to 0.

$$= \frac{a^T}{(a^T(X^TX)^{-1}a)^{1/2}}(0+0+0+0)\frac{b}{(b^T(X^TX)^{-1}b)^{1/2}} = 0$$

Thus, $Cov(T_1, T_2) = 0$ and thus the individual confidence intervals are independent, since the t-variables are jointly distributed.

Another way to show this:

$$Cov(T_1, T_2) = E[(T_1 - E(T_1))(T_2 - E[T_2])^T] =$$

$$= E[T_1T_2^T - T_1E[T_2]^T - E[T_1]T_2^T + E[T_1]E[T_2]^T]$$

where

$$E[T_1] = E\left[\frac{a^T\hat{\beta}_1 - a^T\beta_1}{\sqrt{MSE \times a^T(X^TX)^{-1}a}}\right] = \frac{a^T}{(a^T(X^TX)^{-1}a)^{1/2}}E\left(\frac{1}{\sqrt{MSE}}\right)\left(E[\hat{\beta}_1] - E[\beta_1]\right)$$

$$= \frac{a^T}{(a^T(X^TX)^{-1}a)^{1/2}} \cdot \frac{1}{\sigma}(\beta_1 - \beta_1) = 0$$

$$E[T_2] = \frac{b^T}{(b^T(X^TX)^{-1}b)^{1/2}} \cdot \frac{1}{\sigma}\left(E[\hat{\beta}_2] - E[\beta_2]\right) = 0$$

Then the above reduces to $E[T_1T_2^T]$ $\Longrightarrow$