$\Rightarrow \mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_2) = 0$ and thus, $\hat{\beta}_1$ and $\hat{\beta}_2$ are independent. //

Another way to show this:

$\mathrm{Cov}[\hat{\beta}_1, \hat{\beta}_2] = \mathbb{E}[(\hat{\beta}_1 - \mathbb{E}[\hat{\beta}_1])(\hat{\beta}_2 - \mathbb{E}[\hat{\beta}_2])^T]$

where,

$\mathbb{E}[\hat{\beta}_1] = \mathbb{E}[(W^TW)^{-1}W^Ty] = \mathbb{E}[(W^TW)^{-1}W^T(X\beta + \varepsilon)]$

$= \mathbb{E}\left[(W^TW)^{-1}W^T\left([W\ Z]\begin{bmatrix}\beta_1 \\ \beta_2\end{bmatrix} + \varepsilon\right)\right] = \mathbb{E}\left[(W^TW)^{-1}W^T([W\beta_1 + Z\beta_2] + \varepsilon)\right]$

$= \mathbb{E}[(W^TW)^{-1}W^TW\beta_1 + (W^TW)^{-1}W^TZ\beta_2 + (W^TW)^{-1}W^T\varepsilon]$

$= \mathbb{E}[\beta_1 + (W^TW)^{-1}W^T\varepsilon]$ since $W^TZ = 0$

$= \mathbb{E}[\beta_1] + \mathbb{E}[(W^TW)^{-1}W^T\varepsilon] = \beta_1 + (W^TW)^{-1}W^T\mathbb{E}[\varepsilon]$

$= \beta_1$ since $\mathbb{E}[\varepsilon] = 0$

$\mathbb{E}[\hat{\beta}_2] = \mathbb{E}[(Z^TZ)^{-1}Z^T([W\beta_1 + Z\beta_2] + \varepsilon)]$

$= \mathbb{E}[(Z^TZ)^{-1}Z^TW\beta_1 + \beta_2 + (Z^TZ)^{-1}Z^T\varepsilon]$

$= \mathbb{E}[\beta_2 + (Z^TZ)^{-1}Z^T\varepsilon]$

$= \beta_2 + (Z^TZ)^{-1}Z^T\mathbb{E}[\varepsilon] = \beta_2.$

Thus, $\mathbb{E}[\hat{\beta}_1] = \beta_1$ and $\mathbb{E}[\hat{\beta}_2] = \beta_2$ (unbiased estimators when $W^TZ = Z^TW = 0$)

Then we have,

$\mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_2) = \mathbb{E}[(\hat{\beta}_1 - \beta_1)(\hat{\beta}_2 - \beta_2)^T] = \mathbb{E}[\hat{\beta}_1\hat{\beta}_2^T - \hat{\beta}_1\beta_2^T - \beta_1\hat{\beta}_2^T + \beta_1\beta_2^T]$

[From above we see that $\hat{\beta}_1 = \beta_1 + (W^TW)^{-1}W^T\varepsilon$ and
$\hat{\beta}_2 = \beta_2 + (Z^TZ)^{-1}Z^T\varepsilon$]

$= \mathbb{E}[(\beta_1 + (W^TW)^{-1}W^T\varepsilon)(\beta_2 + (Z^TZ)^{-1}Z^T\varepsilon)^T - (\beta_1 + (W^TW)^{-1}W^T\varepsilon)\beta_2^T -$
$- \beta_1(\beta_2 + (Z^TZ)^{-1}Z^T\varepsilon)^T + \beta_1\beta_2^T]$

$= \mathbb{E}[\beta_1\beta_2^T + \beta_1\varepsilon^TZ(Z^TZ)^{-1}$