Joseph High

(c.) Let a be a $q \times 1$ vector and b a $(q-p) \times 1$ vector. Let (l_1, u_1) and (l_2, u_2) be the individual 95% confidence intervals for $a^T\beta_1$ and $b^T\beta_2$ based on $\hat{\beta}_1$ and $\hat{\beta}_2$, respectively. Is the confidence interval (l_1, u_1) independent of the confidence interval (l_2, u_2)? Justify your answer.

Answer: Testing whether the linear combination of the β_i for $1 \le i \le q$ is equal to $a^T\beta_1$ and whether the linear combination of the β_j for $q < j \le p$ is equal to $b^T\beta_2$.

Thus, our test statistics for our confidence intervals are:

$$T_1 = \frac{a^T\hat{\beta}_1 - a^T\beta_1}{\sqrt{MSE \times a^T(X^TX)^{-1}a}} \quad \text{and} \quad T_2 = \frac{b^T\hat{\beta}_2 - b^T\beta_2}{\sqrt{MSE \times b^T(X^TX)^{-1}b}}$$

for the individual 95% confidence intervals:

$$a^T\hat{\beta}_1 \pm qt(0.975; n-p)\sqrt{MSE \times a^T(X^TX)^{-1}a}$$

and

$$b^T\hat{\beta}_2 \pm qt(0.975; n-p)\sqrt{MSE \times b^T(X^TX)^{-1}b}$$

If it can be shown that the pairwise covariance of the test statistics, T_1 and T_2, are zero, then it implies that the overall confidence probability is still $1-\alpha = 0.95$.

$$Cov(T_1, T_2) = Cov\left[\frac{a^T\hat{\beta}_1 - a^T\beta_1}{\sqrt{MSE \times a^T(X^TX)^{-1}a}}, \frac{b^T\hat{\beta}_2 - b^T\beta_2}{\sqrt{MSE \times b^T(X^TX)^{-1}b}}\right]$$

$$= \frac{a^T}{\sqrt{a^T(X^TX)^{-1}a}} Cov\left[\frac{\hat{\beta}_1 - \beta_1}{\sqrt{MSE}}, \frac{\hat{\beta}_2 - \beta_2}{\sqrt{MSE}}\right]\frac{b}{\sqrt{b^T(X^TX)^{-1}b}}$$

$\longrightarrow$

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