$$\Rightarrow E[T_1 T_2^T] = E\left[\frac{(a^T\hat{\beta}_1 - a^T\beta_1)(b^T\hat{\beta}_2 - b^T\beta_2)}{MSE\sqrt{a^T(X^TX)^{-1}a\, b^T(X^TX)^{-1}b}}\right] =$$

$$= \frac{1}{\sqrt{a^T(X^TX)^{-1}a\, b^T(X^TX)^{-1}b}}\, E\left[\frac{1}{MSE}\left(a^T\hat{\beta}_1 b^T\hat{\beta}_2 - a^T\hat{\beta}_1 b^T\beta_2 - a^T\beta_1 b^T\hat{\beta}_2 + a^T\beta_1 b^T\beta_2\right)\right]$$

$$= \frac{1}{\sqrt{a^T(X^TX)^{-1}a\, b^T(X^TX)^{-1}b}}\, E\left[\frac{1}{MSE}\right] a^T b^T \left(E[\hat{\beta}_1]E[\hat{\beta}_2] - E[\hat{\beta}_1]E[\beta_2] - E[\beta_1]E[\hat{\beta}_2] + E[\beta_1]E[\beta_2]\right)$$

(since MSE, $\hat{\beta}_1$, $\hat{\beta}_2$, β_1, β_2 independent)

$$= \frac{a^T b^T}{\sigma^2\sqrt{a^T(X^TX)^{-1}a\, b^T(X^TX)^{-1}b}}\left(\beta_1\beta_2 - \beta_1\beta_2 - \beta_1\beta_2 + \beta_1\beta_2\right) = 0$$

$$\Rightarrow \operatorname{Cov}(T_1, T_2) = 0$$

Hence, the confidence intervals are independent.