

553.633/433

Homework #9

Due Mon. 10/30/17

Two modest pencil-and-paper problems:

A. Suppose that a salesperson for a company is given a reward check (representing R) of \$1000 when he/she completes a specified number of sales. After receiving a check, the salesperson begins anew working toward the next reward check. Suppose that the probability density function $p(\tau)$ for τ , representing the time between reward checks, is given by $p(\tau) = \{\tau/3 \text{ for } 0 \leq \tau \leq 2; 2 - 2\tau/3 \text{ for } 2 < \tau \leq 3; 0 \text{ otherwise}\}$. Considering the above as a regenerative process, carry out parts (a) – (c) below:

- (a) Plot the function $p(\tau)$ (hand sketch is acceptable) and give the numerical value of ℓ .
- (b) Suppose that the standard regenerative estimate $\hat{\ell}$ is formed based on *one* sample (i.e., one regeneration period). What is the magnitude of the bias in $\hat{\ell}$ as an estimator of ℓ ?
- (c) Can the Kantorovich inequality (see HW #8) be used to provide an upper bound to the bias in part (b)? If so, compute the bound; if not, explain why not.

B. Suppose we have two *very* simple Monte Carlo “simulations” involving single random variables X and Y . Let U represent a $U(0,1)$ random number. Do the following:

- (a) Consider two cases for X and Y : (i) $X = U^2$ and $Y = U^3$ and (ii) $X = U^2$ and $Y = (1 - U)^3$. For each of the cases (i) and (ii), determine analytically whether CRNs (same U values in X and Y) lead to a decrease or increase in $\text{var}(X - Y)$ relative to independent sampling (independent U values) for X and Y . Provide brief comments on the reason for your conclusions.
- (b) Redo part (a) for case (ii) above, but with X and Y generated via the inverse transform method rather than the transforms given in part (a). (Note that for a random variable X with distribution function $F(x)$, the random variable $g(X)$ has distribution function $F(g^{-1}(x))$ if g is increasing and $1 - F(g^{-1}(x))$ if g is decreasing, where g^{-1} is the inverse function of g .) Why is the result different from the result in part (a)?