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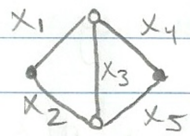
## EN.553.633 Homework #7

**Problem 4.2**: Prove that the structure function of the bridge system in Figure 4.1 is given by (4.11).

Pf: We want to show that

$$H(X) = 1-(1-x_1x_4)(1-x_2x_5)(1-x_1x_3x_5)(1-x_2x_3x_4)$$

for the bridge system:




We are told that the model system works if the black terminal nodes are connected by working links.

Now

$$X_i = \begin{cases} 1, & \text{component } i \text{ working} \\ 0, & \text{component } i \text{ NOT working} \end{cases} \quad \text{for } i=1,2,\dots,5$$

Going to prove this as an "if and only if" statement since we are proving an equality.

The black nodes are connected by 4 different paths.

$(x_1, x_4)$, $(x_2, x_5)$, $(x_1, x_3, x_5)$, $(x_2, x_3, x_4)$

The system works if and only if the black nodes connected by working links, if and only if at least 1 path contains all working links, if and only if at least one of

$(x_1=1$ and $x_4=1)$ OR $(x_2=1$ and $x_5=1)$ OR

OR $(x_1=1$ and $x_3=1$ and $x_5=1)$ OR $(x_2=1$ and $x_3=1$ and $x_4=1)$

holds if and only if

$H(X) = 1-(0) = 1 \iff H(X)=1$ if and only if the system works (is operational). / on the other hand, if each path contains at least one non-working link, then each of the products $x_1x_4 = x_2x_5 = x_1x_3x_5 = x_2x_3x_4 = 0$

$\iff H(X) = 1-1=0 \iff$ system NOT working $\Longrightarrow$

Thus, the given function $H(x)$ accurately depicts the operational state of the system. // $\square$

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Problem 4.3: Consider the bridge system in Figure 4.1. Suppose that all link reliabilities are $p$. Show that the reliability of the system is $p^2(2+2p-5p^2+2p^3)$.

Pf: The reliability of the entire system is given by

$$\ell = \mathbb{P}[Y=1] = \mathbb{P}[H(X)=1] = E[H(X)]$$

In 4.2 we showed that eq'n (4.11) is indeed the appropriate structure function for the model, so

$$H(X) = 1-(1-X_1X_4)(1-X_2X_5)(1-X_1X_3X_5)(1-X_2X_3X_4)$$

We are given that the reliability of each link is $p$.
Note: From pg. 110 (Example 4.2) "it is usually assumed that the $\{X_i\}$ are independent", so we assume here that $\{X_i\}_{i=1}^{5}$ are independent. Moreover recall $X_i \in \{0,1\}$, so that $X_i^2 = X_i$ $(1^2=1,\ 0^2=0)$. $\forall i=1,2,\dots,5$

Then,

$$E[H(X)] = E[1-(1-X_1X_4)(1-X_2X_5)(1-X_1X_3X_5)(1-X_2X_3X_4)]$$

$$= E[1-(1-X_2X_5-X_1X_4+X_1X_4X_2X_5)(1-X_1X_3X_5)(1-X_2X_3X_4)]$$

$$= \mathbb{E}[1-(1-X_1X_3X_5-X_2X_5+X_2X_1X_3X_5^2-X_1X_4+X_1^2X_4X_3X_5+ \\ +X_1X_4X_2X_5-X_1^2X_2X_3X_4X_5^2)(1-X_2X_3X_4)]$$

$$= \mathbb{E}[1-(1-X_2X_3X_4-X_1X_3X_5-X_2X_5+X_2X_1X_3X_5-X_1X_4 \\ +X_1X_4X_3X_5+X_1X_4X_2X_5\cancel{-X_1X_2X_3X_4X_5} \\ \cancel{+X_1X_2X_3X_4X_5}+X_2X_3X_4X_5-X_1X_2X_3X_4X_5+ \\ +X_1X_2X_3X_4-X_1X_2X_3X_4X_5\cancel{-X_1X_2X_3X_4X_5}+ \\ \cancel{+X_1X_2X_3X_4X_5}]$$

$\Longrightarrow$

Joseph [illegible]

$$= E[X_2X_5 + X_1X_4 + X_2X_3X_4 + X_1X_3X_5 - X_1X_2X_3X_5 - X_1X_3X_4X_5 - X_1X_2X_4X_5 - X_2X_3X_4X_5 - X_1X_2X_3X_4 + 2X_1X_2X_3X_4X_5]$$

$$= E[X_2X_5] + E[X_1X_4] + E[X_2X_3X_4] + E[X_1X_3X_5] - E[X_1X_2X_3X_5] - E[X_1X_3X_4X_5] - E[X_1X_2X_4X_5] - E[X_2X_3X_4X_5] - E[X_1X_2X_3X_4] + 2E[X_1X_2X_3X_4X_5]$$

and since the $\{X_i\}$ assumed independent

$$= E[X_2]E[X_5] + E[X_1]E[X_4] + E[X_2]E[X_3]E[X_4] + E[X_1]E[X_3]E[X_5] - E[X_1]E[X_2]E[X_3]E[X_5] - E[X_1]E[X_3]E[X_4]E[X_5] - E[X_1]E[X_2]E[X_4]E[X_5] - E[X_2]E[X_3]E[X_4]E[X_5] - E[X_1]E[X_2]E[X_3]E[X_4] + 2E[X_1]E[X_2]E[X_3]E[X_4]E[X_5]$$

$$= p^2 + p^2 + p^3 + p^3 - (p^4 + p^4 + p^4 + p^4 + p^4) + 2p^5$$ (since $E[X_i] = p$ for $i = 1, 2, \dots, 5$)

$$= 2p^2 + 2p^3 - 5p^4 + 2p^5 = p^2(2 + 2p - 5p^2 + 2p^3)$$

Thus, the reliability is

$$\ell = E[H(X)] = p^2(2 + 2p - 5p^2 + 2p^3)$$ // $\square$

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Problem 1 Monty Hall

To Compute 90% CI's

True values $l_{swap} = Prob(\text{win w/ Swap}) = 2/3$

$l_{stay} = Prob(\text{win w/ Stay}) = 1/3$

$\Rightarrow$ CI: for Swap: $\left(0.661 - 1.65\sqrt{\frac{2}{9000}},\ 0.661 + 1.65\sqrt{\frac{2}{9000}}\right)$

$= (0.636, 0.686)$

CI for Stay:

$= \left(0.339 - 1.65\sqrt{\frac{2}{9000}},\ 0.339 + 1.65\sqrt{\frac{2}{9000}}\right)$

$= (0.314, 0.364)$

Thus, $0.661 \in$ CI for Swap

and $0.339 \in$ CI for Stay