553.633/433

Homework #7

Due Wed. 10/18/17

Four problems:

A. We have talked a lot about simulation models of processes. Now you are going to write one! *Let's Make a Deal* is a television game show that originated in the U.S. in 1963 and has been broadcast in some form or another in the U.S. or overseas for most years since its introduction. For many years, the host of the show was a man named Monty Hall. Mr. Hall, who died on 30 September 2017 at the age of 96, is now associated with a famous, somewhat counterintuitive, problem in probability.

The "Monty Hall problem" is based on the following scenario that is similar to what occurs on *Let's Make a Deal*. The host (Monty) points to three doors and tells you (the player) that there are goats behind two of the doors, while behind the remaining door is a new car. If you choose the car, you get to keep it. Otherwise you are stuck with a goat. Monty asks you to choose a door. After you make your choice, Monty opens a different door and shows you a goat. Monty knows what lies behind all of the doors and chooses a door with a goat behind it in order to build suspense. After Monty opens the door, he gives you the option of changing doors. Should you accept the option?

The answer is that you *should* accept Monty's option. Many people (even well informed people!) are mystified by this choice and believe that there is no value in changing doors. In fact, the correct answer of changing doors is not hard to see by simple probability arguments.

Write *your own* computer simulation that simulates the Monty Hall problem. In the notation of the textbook, let ℓ be the probability of success and H be a 0 ("lose") or 1 ("win"). Run your simulation 1000 times for each of the following scenarios: the player always stands pat (i.e., does not change doors) and the player accepts Monty's offer and switches doors. From the simulation runs, form two corresponding values of $\hat{\ell}_{1000}$, together with 90 percent (approximate) confidence intervals around each of the estimates (for your confidence interval, you may use either the exact value of var(H), which is known in this fairly simple case, or the standard unbiased estimate of the variance.) Do your confidence intervals include the two true probabilities ℓ? (Approximately 1/10 of the students in the class should answer "no" to this question for each scenario.)

Submit the following: Turn in your code together with a summary of your numerical results (the two values of $\hat{\ell}_{1000}$, as well as the confidence intervals). All numerical results must be clearly labeled and not hidden in the code or code output. Include the brief derivation of the analytical solution.

3.3. (textbook)

4.2. (textbook)

4.3. (textbook)