

4.8 part a

Code:

```
%Exercise 4.8
%Part a

M=10000;
K=100;
N=30;
T=(M-K)/N;
mu=2;
lambda=1;
rho=lambda/mu;
n1=0;
nn1=n1;
ev_list=inf*ones(2,2);
t=0;
tot=0;
tt=0;
ev_list(1,:) = [-log(rand)/lambda, 1];
N_ev=1;
Tot = zeros(1, N);
while t < M
    t = ev_list(1,1);
    tt=[tt,t];
    ev_type = ev_list(1,2);
    switch ev_type
    case 1
        interarrival
    case 2
        service
    end
    N_ev = N_ev - 1;
    ev_list(1,:) = [inf,inf];
    ev_list = sortrows(ev_list, 1);
    nn1 = [nn1, n1];
    tot = tot + nn1(end-1)*(tt(end) - tt(end-1));
end
batches = zeros(1,N);
D = zeros(1,N);
for i = 1:N
    batches(i)=mean(nn1(101 + (i-1)*T:100 + T*i));
end
batchmean = mean(batches);
re = std(batches)/rho/sqrt(N)
res= tot/t;
fprintf('batches %g ; 0.95 CI (%g, %g) /n', batchmean, batchmean*(1-1.96*re),
batchmean*(1+1.96*re))
```

Definitions:

```
%interarrival.m
N_ev = N_ev + 1;
ev_list(N_ev, :) = [t - log(rand)/lambda, 1];
if n1 == 0
    N_ev = N_ev + 1;
    ev_list(N_ev, :) = [t - log(rand)/mu, 2];
end
n1 = n1+1;

% service.m
n1=n1-1;
if n1 ~= 0
    N_ev = N_ev + 1;
    ev_list(N_ev,:)=[t - log(rand)/mu, 2] ;
end
```

Output:

```
>> hw8prob8crap

re =

    0.0362

batches 1.01037 ; 0.95 CI (0.93872, 1.08202) /n>>
```

4.8 part b

Code

```
%Part B
M=10000;
K=100;
N=30;
T=(M-K)/N;
mu=2;
lambda=1;
rho=lambda/mu;
n1=0;
nn1=n1;
ev_list=inf*ones(2,2);
t=0;
tot=0;
tt=0;
ev_list(1,:) = [-log(rand)/lambda, 1];
N_ev=1;
R = zeros(1,N);
tau = zeros(1,N);
Rsum = 0;
regcount = 0;
```

```

lastregtime = 1;
for i = 1:numel(nnl)
while t < M
t = ev_list(1,1);
tt=[tt,t];
ev_type = ev_list(1,2);
switch ev_type
case 1
interarrival
case 2
service
end
Rsum = Rsum + nnl(i);
N_ev = N_ev - 1;
ev_list(1,:) = [inf,inf];
ev_list = sortrows(ev_list, 1);
nnl = [nnl, nl];
end
if nnl(i) == 0
regcount = regcount + 1;
R(regcount) = Rsum;
tau(regcount) = i - lastregtime;
Rsum = 0;
lastregtime = i;
end
end
regmean = mean(R)/mean(tau)
Covariance = cov(R, tau);
S = sqrt(Covariance(1,1)-2*regmean*Covariance(1,2) + regmean^2*Covariance(2,2))
RelError = S/mean(tau)/sqrt(M)
fprintf('regmean %g; 0.95 CI (%g, %g) \n', regmean, regmean*(1-1.96*RelError),
regmean*(1+1.96*RelError))

```

Output:

```

>> hw8prob8b

RelError =

    0.0589

regmean 1.06699; 0.95 CI (0.943813, 1.19016)

```

Part C

Relative Width = $1.96 \times 2 \times \text{Relative Error}$

For part a and part b, the relative width around the confidence intervals are 0.1176 for part a and 0.2309 for part b, this is for simulation time of $M = 10,000$. If we increase M to 70,000 we see that the relative error shrinks down to approximately 0.0145 for part a and if we increase to 170,000 for part b, we get an approximately relative error of 0.0145 as well. This gives a relative width of approximately 0.05 for each.

The output for each is below. Note: the code is not attached for this because it is exactly the same as the above just with $M = 70,000$ for part a and $M = 170,000$ for part b. (Note, I know the solution manual says 60,000 for both, but that did not work for my code).

Output for part a:

re =

0.0145

batches 0.992564 ; 0.95 CI (0.964332, 1.0208) /n>> hw8prob8b

Output for part b:

RelError =

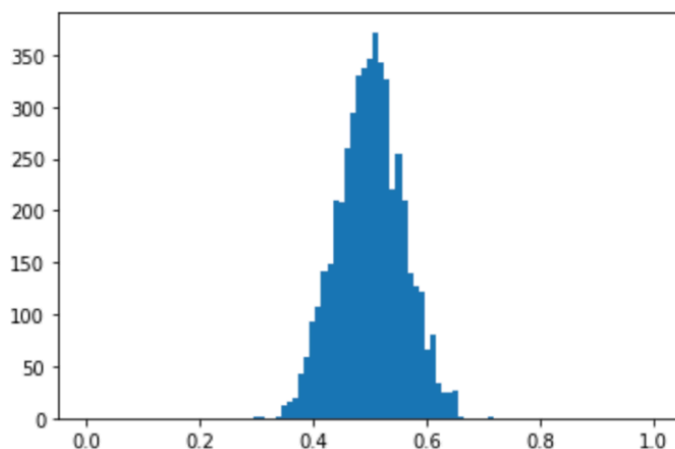
0.0145

regmean 0.989388; 0.95 CI (0.961363, 1.01741)

Exercise B

```
a np.random.seed(123)
   sample.size = 5000
   numpoints = 25
   d = np.random.uniform(0,1,(numpoints, sample.size))
   dbar = np.sort(np.mean(d,0))
   ugen = np.random.uniform(0,1,sample.size)
   unorm = sample.size*ugen
   unorm = unorm.astype(int)
   dbarbar = np.sort(dbar[unorm])
   plt.hist(dbarbar, bins = np.linspace(0,1,100))
   plt.show()
```

Output:



```
b  
updatedbar = np.mean(dbar)  
L = int(0.025*sample.size) - 1  
U = int(0.975*sample.size) - 1  
Lb = dbar[L] - updatedbar  
Ub = dbar[U] - updatedbar  
print("CI "str(Lb)" "+str(Ub)+')
```

```
Output:  
(-0.137899983311, 0.140012210078)
```