Joseph High

Then when $\lambda < 1$, $\max_x \left\{ \frac{f(x)}{g(x)} \right\} = \frac{1}{\lambda(1-e^{-a})} = C_1$

When $\lambda > 1$, $\max_x \left\{ \frac{f(x)}{g(x)} \right\} = \frac{e^{(\lambda-1)a}}{\lambda(1-e^{-a})} = C_2$

When $\lambda = 1$, $\max_x \left\{ \frac{f(x)}{g(x)} \right\} = \frac{1}{(1-e^{-a})} = C_3$

Comparing the values of C above, it's easy to see that for any $\lambda < 1$ and all values of a, $C_3 < C_1$ since a $1/\lambda$ factor is the only difference between C_1 and C_3, where $\lambda < 1 \implies C_1 > C_3$.

For C_2 we have that $\lambda > 1$, so $e^{(\lambda-1)a} > 1$ but the $1/\lambda$ factor decreases C_2. However, $e^{(\lambda-1)a}$ increases at a faster rate than does λ as $\lambda \to \infty$. Thus, $C_3 < C_2$.

Thus, C_3 is the minimum C which occurs when $\lambda = 1$.

Thus, the minimum value of C for $f(x) \leq c\,g(x)$ to hold $\forall x \in [0, a]$ is $\boxed{C = \frac{1}{1-e^{-a}}}$

With this value of C, we can then proceed with the A-R method:

Generate $X \sim Exp(1)$ and $U \sim U[0,1]$.

If $U \leq \frac{f(x)}{c\,g(x)}$ Accept X

otherwise Return to beginning $//\square$