553.633/433

Homework #5

Due Mon. 10/2/17

Three problems:

2.21 (You do not have to code or implement the inverse transform or A-R solution.)

A. Consider a Markov chain with state space {1, 2, 3} and with a stationary probability distribution of $\boldsymbol{\pi} = [3/11, 3/11, 5/11]$. Do the following:

(a) Determine the values of the entries in the transition matrix $\boldsymbol{P}$ under the constraint that the values in the upper-left 2×2 block are equal to each other (i.e., $p_{11} = p_{12} = p_{21} = p_{22}$) and that the sum of the entries in the first column is 13/15.

(b) Using the $\boldsymbol{P}$ in part (a), carry out a simulation of 5000 independent replicates of the Markov chain for two steps of the process. In particular, compute 5000 independent values of X_2 and compare the sample proportions of X_2 equaling each of {1, 2, 3} with the stationary probability values. Initialize each replicate at $X_0 = 1$. It is not necessary to carry out a statistical test to do the comparison.

(c) Repeat the analysis of part (b) with the exception of considering 20 steps of the Markov chain (producing values of X_{20}). Comment on whether the results are materially different from those in part (b) and, if so, comment on how the results differ.

B. (a) If $X \sim N(\mu, \sigma^2)$, derive $E[e^X]$.

(b) If $W(t)$ is the standard Wiener process and $U(W(t)) = \exp(t + \frac{W(t)}{2})$, derive $E[U(W(t))]$.