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Problem B

In[202]:=
```
sigma = 10;
mu = 20;
theta = 1;
t = 0.01;
x0 = 0;
n = 50;
proc = OrnsteinUhlenbeckProcess[mu, sigma, theta, x0];
```

In[238]:=
```
x = ItoProcess[proc];
```

In[246]:=
```
data = RandomFunction[x, {0, 5, t}, n]
```

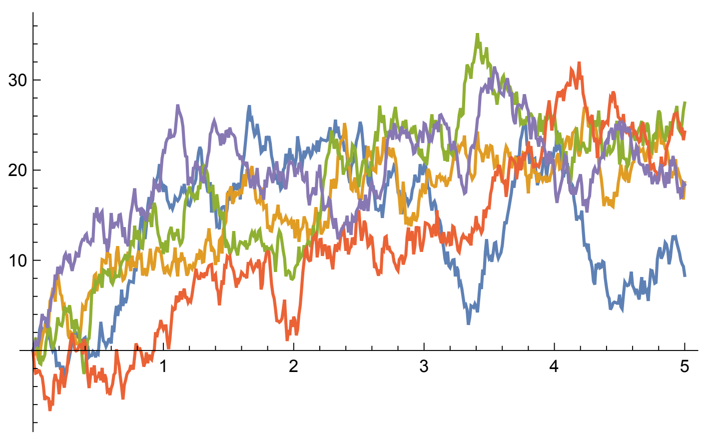
Out[246]= TemporalData[Time: 0. to 5. Data points: 25 050 Paths: 50]

In[247]:=
```
alldata = Table[data, n];
```

Part (a)

In[248]:=
```
ListLinePlot[data["Path", {1, 2, 3, 4, 5}], PlotRange → All]
```

Out[248]=

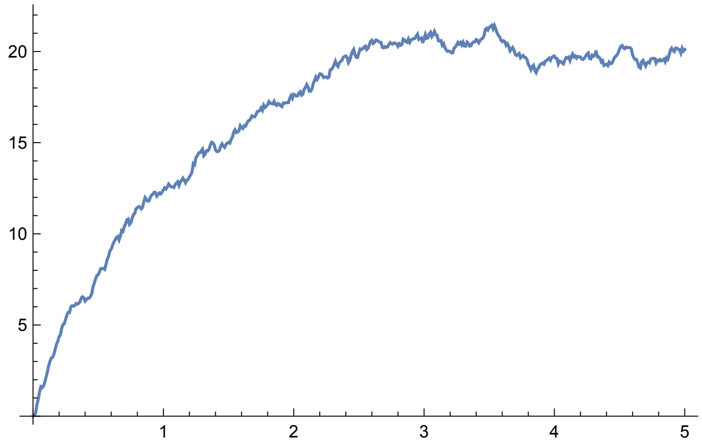


Part (b)

In[249]:=
```
meanpath = Mean[data["Path", {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
      17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33,
      34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50}]];
```

In[250]:=
```
ListLinePlot[meanpath, PlotRange → All]
```

Out[250]=



From Slide 10 “Example 2 of Ito SDE: Ornstein-Uhlenbeck Process” , E[X(t)] ⟶ μ = 20. It can be seen in the plot of the mean path above that the sample mean of the 50 simulated paths converge to 20 as t gets larger.

Part (c)

In[258]:=
```
meanx2 = Mean[data[2]];
```

In[259]:=
```
stdevx2 = StandardDeviation[data[2]];
```

Hypothesis Test: $H_0 : \mu = 20$, $H_1 : \mu \neq 20$

In[260]:=
```
tstat2 = ((meanx2 - mu) / (stdevx2)) * Sqrt[n]
```

Out[260]=
```
-2.59505
```

Thus, our t-statistics is -2.59505

In[271]:=
```
pvalue2 = CDF[StudentTDistribution[n - 1], tstat2] +
   (1 - CDF[StudentTDistribution[n - 1], -tstat2])
```

Out[271]=
```
0.0124431
```

Thus, our P-value is 0.0124431. At the 0.05 significance level, since our p-value < 0.05/2 = 0.025, then we reject the null hypothesis. That is, there is not enough evidence to claim that the mean of X(2) approaches the true mean.

```
meanx5 = Mean[data[5]];
stdevx5 = StandardDeviation[data[5]];
```

Hypothesis Test: $H_0 : \mu = 20$, $H_1 : \mu \neq 20$

In[263]:=
```
tstat5 = ((meanx5 - mu) / (stdevx5)) * Sqrt[n]
```

Out[263]=
```
0.101619
```

Thus, our t-statistics is 0.101619

In[272]:=
```
pvalue5 = CDF[StudentTDistribution[n - 1], -tstat5] +
   (1 - CDF[StudentTDistribution[n - 1], tstat5])
```

Out[272]=
```
0.919473
```

For X(5) we get a much larger P-Value = 0.919473. Then, at any significance level (say 0.05) since 0.919 < 0.05/2 = 0.025, we conclude that there is not enough evidence to reject the null hypothesis that the 50 simulations at X(5) approaches the true mean.