553.633/433

Homework #10

Due Wed. 11/8/17

Two problems:

5.14 (textbook)

A. Let us consider importance sampling (IS) for the purpose of estimating tail probabilities. We will compare the standard method against a (non-optimal) IS approach. Suppose we wish to estimate $\ell \equiv P(|X| \geq 3.5)$ for $X \sim N(0, 1)$. Do the following:

(a) What is the true value of ℓ? (As needed, you may use numerical integral values or probability values that are determined via table look-ups or online.)

(b) Define the function $H(X)$ and calculate 10 estimates $\hat{\ell}$ in the standard way (as in Sect. 4.3 of the textbook), each using a sample of N = 100,000 independent values. List the 10 estimates. What is $\text{var}(\hat{\ell})$? (This should be an analytical calculation using the result of part (a); the calculation should not be based on the sample variance from the 100,000 samples.)

(c) Use IS with a sampling (proposal) density $N(3.5, 1)$. Calculate 10 estimates $\hat{\ell}$ using N = 100,000 independent values from the proposal density for each estimate. What is $\text{var}(\hat{\ell})$ under IS? (You may either estimate $\text{var}(\hat{\ell})$ from the sample or do an analytical calculation with or without table look-ups, as needed.)

(d) Comment on the relative accuracy of the standard method and the IS method above. Use the relative error in (4.6) (textbook) as the basis of the comparison.

(e) Extra credit (will not be used in the core grade calculation but will be considered favorably if your final course score is near the cutoff between a lower or higher grade, say a B+ or A-). Using whatever analytical or numerical method your wish, determine the optimal mean μ in an IS density $N(\mu, 1)$ in order to achieve minimum variance in the IS estimate $\hat{\ell}$. See pp. 152–154 of textbook for hints and relevant discussion. Calculate 10 values of $\hat{\ell}$ using N = 100,000 independent values from this proposal density and comment on how these compare with the results above.

Note: You should also be thinking about your paper review due on 11/4/17 (phase 1) and 11/17/17 (phase 2).