## Acknowledging and Incorporating Uncertainty in Model-Based Inferences

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Marc R. Gastonguay, Ph.D. Leonid V. Gibiansky, Ph.D.

metrumresearch group / metruminstitute

#### **Overview**

Acknowledging uncertainty in simulation parameters/models:

- Why?
  - Value of the approach
  - Some examples
- How?
  - Methods / Considerations
  - Useful features of simulation tool
  - Some available tools
- NMSUDs: new tool integrating R and NONMEM®

## **Uncertainty in Models & Parameters**

- CTS employ models and parameter values based on a variety of prior information sources and assumptions
- CTS often involve extrapolations to unobserved conditions

Problem: Substantial uncertainty can exist in the models and parameters used for CTS.

A Solution: Acknowledge the uncertainty by formal incorporation in the simulation process.

## Why Include Uncertainty in M&S?

- When uncertainty is not included, simulation results are only valid if the model and parameters are truth.
- Including uncertainty allows for a quantitative evaluation of the current state of knowledge
  - e.g. How confident are you in the simulation results?
- View simulation outcomes as a probability distribution; conditioned on current knowledge
- Results in Global Sensitivity Analysis of simulation outcome dependence on parameter (model) assumptions
  - e.g. What's the impact of model deficiencies?

# **Example 1: Optimal Design of a Pediatric Trial Using Simulations with Uncertainty**

In collaboration with John Mondick, Jeff Barrett
The Children's Hospital of Philadelphia

## Objectives of the Simulation Study

- To design a pediatric trial given the practical limitations
  - Sparse sampling
  - Time windows that patients are available for sampling
- To power the study to be able to estimate clearance for children < 1 year with sufficient precision and accuracy

## Range of Practical Limitations for Trial Design

- 100-200 patients
- Age: 0-18 years
- Dosing: combination of Drug 1 and Drug 2
- Sparse sampling: three samples no later than 6 hours post-dose; one sample at 24-30 or 48-96 hours post-dose.

## **Specific Aims**

- Select sampling times to characterize the population PK model
- Select number of patients sufficient to estimate the parameters with the desired precision

 Select proportion of patients with AGE < 1 year to sufficiently estimate age effect with the desired precision and accuracy

#### **Models**

- Drug 1: Three-compartment model parameterized in terms of CL, Q1, Q2, V1, V2 and V3. Characteristic half-lives: 10 minutes, 2 hours, 2 days
- Drug 2: Two-compartment model parameterized in terms of CL, Q, V1, and V2. Characteristic half-lives: 10 minutes, 2 hours

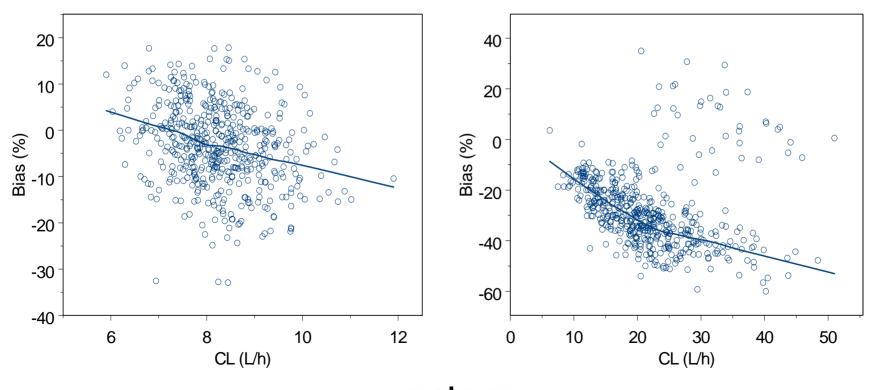
## **Initial Study Design**

- n=200
- Group 1: 4 samples at
  - 5 to 15 minutes
  - 0.75 to 1.5 hours
  - 3.5 to 4.5 hours
  - 48 96 hours (25% of patients)
- Group 2: 4 samples at
  - 15 to 30 minutes
  - 2 to 3 hours
  - 5 to 6 hours
  - 48 96 hours (25% of patients)

Initial design guided by D-optimality based on typical individual

## Initial Design Results: CL

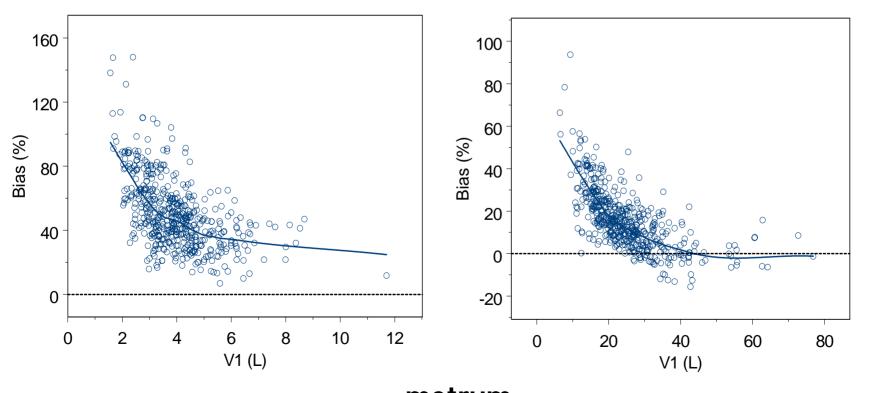
Drug 1 Drug 2



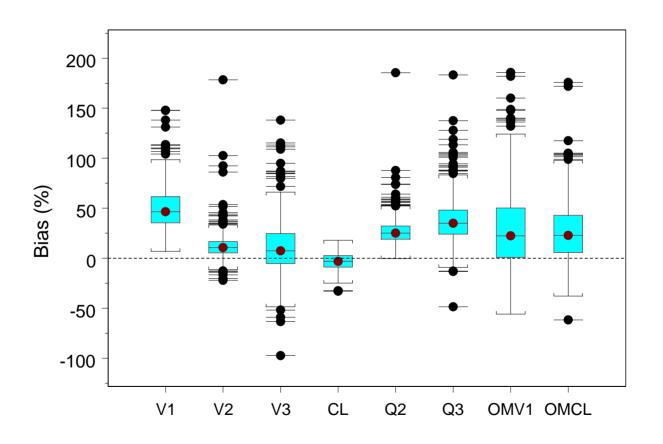
## Initial Design Results: V

Drug 1

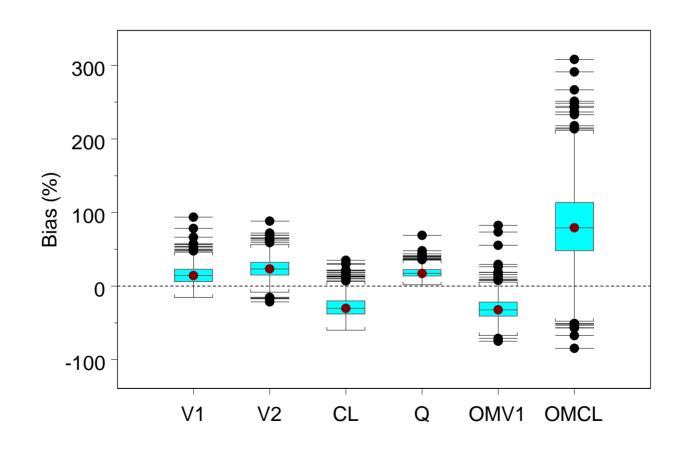
Drug 2



## **Initial Design Results: Drug 1 Bias**



## Initial Design Results: Drug 2 Bias



#### What to do?

 Improve our knowledge about population parameters (reduce uncertainty)

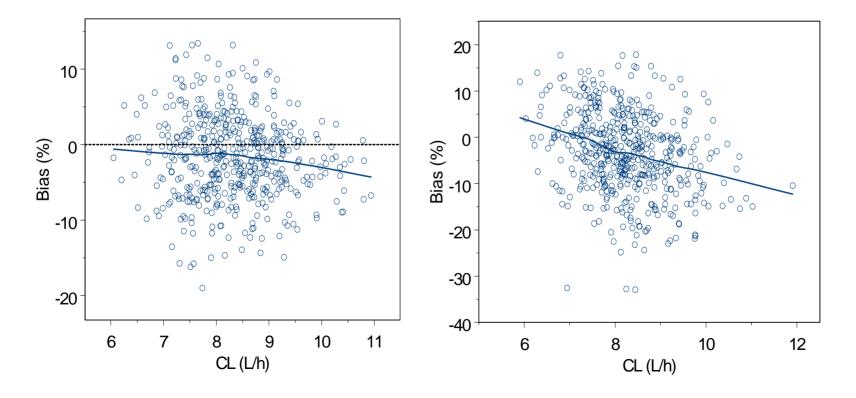
 Improve design to make it robust to the assumptions about the model parameters

## **Final Study Design**

- 24 hour sample added in 50% of patients
- Patients with a sample collected 48 96 hours increased to 50%
- Sample fixed at 5 minutes included for both schedules
- Sampling windows adjusted for remaining times

## Final Design Results: CL

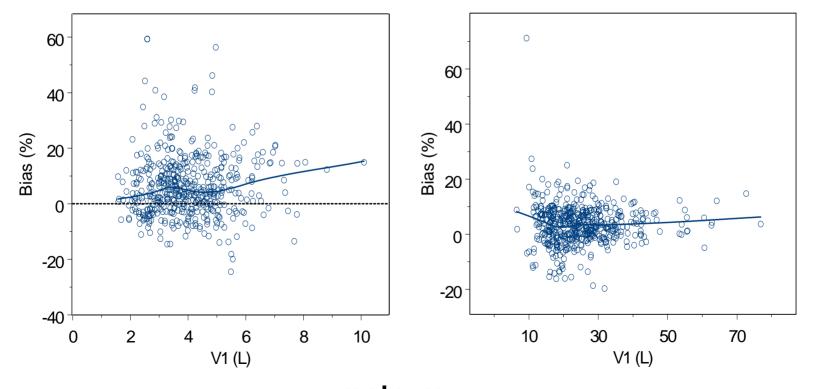
Drug 1 Drug 2



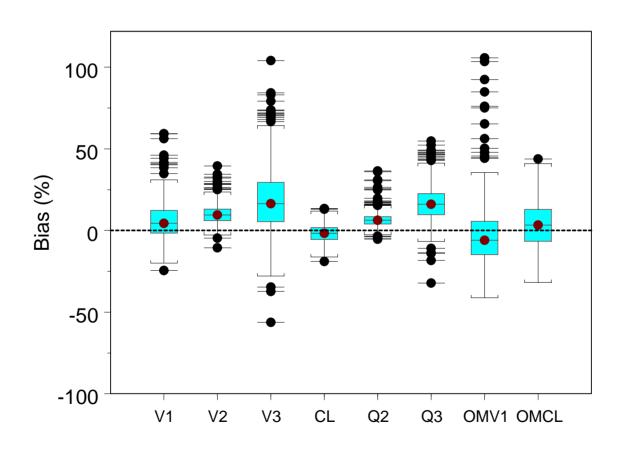
## Final Design Results: V

Drug 1

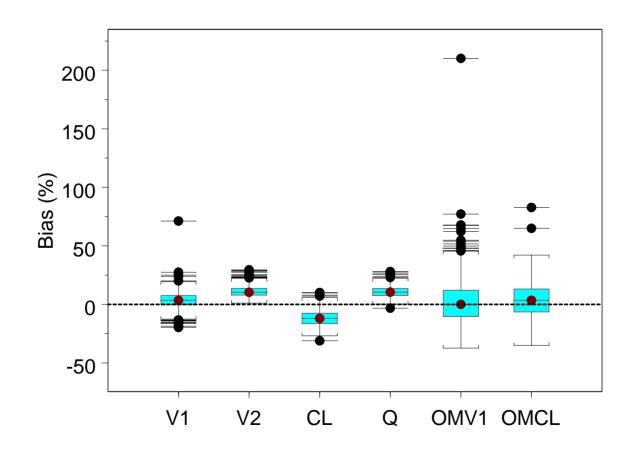
Drug 2



## Final Design Results: Drug 1 Bias



## Final Design Results: Drug 2 Bias



#### **Conclusions**

- Design was modified to make it robust to uncertainty across parameters
- Given the PK sample timing limitations, PK for both drugs could be accurately assessed
- 200 patients sufficient to characterize PK of both drugs
- 50 patients needed < 1 year old to characterize the suspected age effect on clearance

## Example 2: Evaluation of Trial Design and Dose/Regimen Selection

Hypothetical Example

## **Specific Aims**

#### Select dose

 To maximize % of patients with PD response at trough within a specific interval

Estimate (for a given design/dosing rule)

 % of patients with PD response at trough above and below the specified interval (goal = 90% of patients in target range)

#### **Simulation Model**

#### Study design:

- Oral administration
- Steady-state BID dosing
- 1000 patients

#### PK model:

- 3-compartment model;
- Terminal half-life ~ 30 hours

#### PK/PD model:

direct Emax model

## Simulations: Dose Selection Step

 Assuming perfect knowledge of population parameters, simulate study and compute expected endpoint values

 Assuming dose linearity, select the best dose that maximizes % of patients in the desired exposure range

## Simulations: Sensitivity Analysis

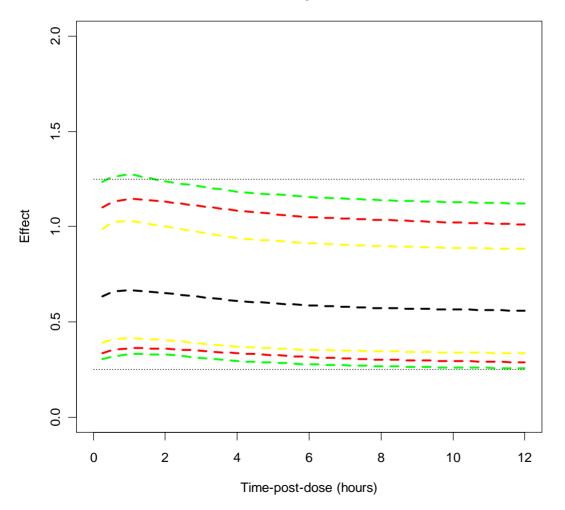
 Conduct simulations with uncertainty to estimate range of possible outcomes

Identify the most influential parameters

 Evaluate the effect of extra knowledge (decreased uncertainty)

## No Uncertainty in Model Parameters

#### Steady-state



#### **Effect-time course:**

Black: median

Yellow: 80% CI

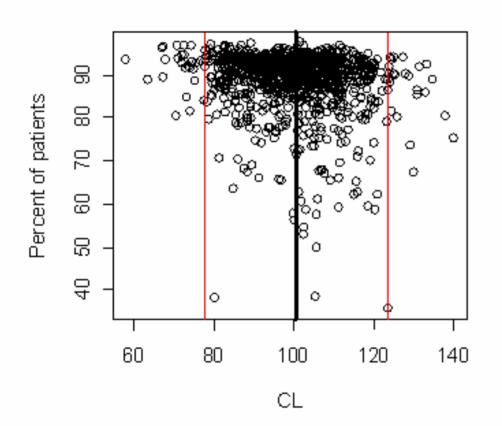
**Red:** 90% CI

Green: 95% CI

Dashed: desired range of trough effect (97% of patients were inside of this range)

## **Uncertainty in PK Parameters**

Effect within 0.25 - 1.25



Effect of uncertainty in CL: % of patients with trough effect within the desired range

**Simulated CL:** 

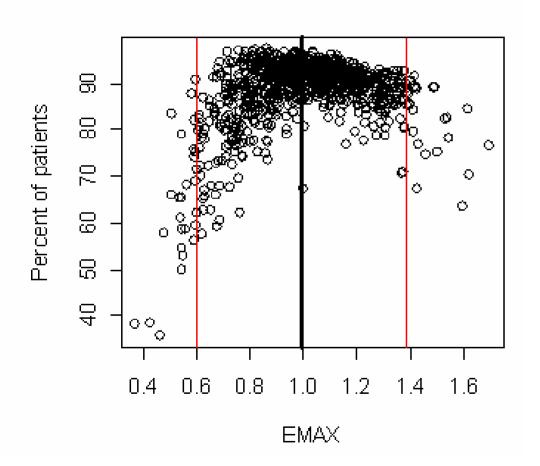
Black: median

**Red:** 95% CI

Conclusion: Uncertainty in CL is less important than uncertainty in PK/PD model parameters

## **Uncertainty in PD Parameters**

Effect within 0.25 - 1.25



Effect of uncertainty in EMAX: % of patients with trough effect within the desired range

**Simulated EMAX:** 

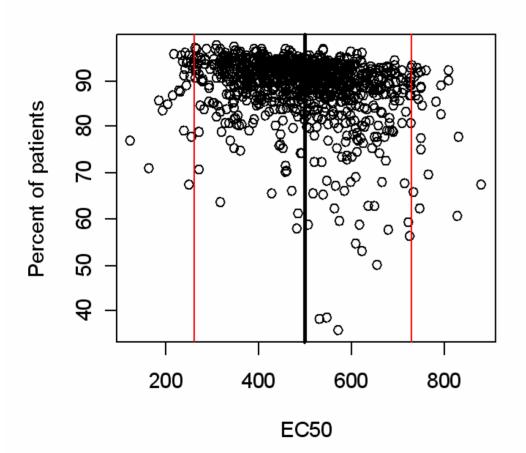
Black: median

**Red:** 95% CI

Conclusion: Precise knowledge of EMAX is very important

## **Uncertainty in PD Parameters**

Effect within 0.25 - 1.25



Effect of uncertainty in EC50: % of patients with trough effect within the desired range

**Simulated EC50:** 

Black: median

**Red:** 95% CI

Conclusion: Uncertainty in EC50 is less important than uncertainty in EMAX

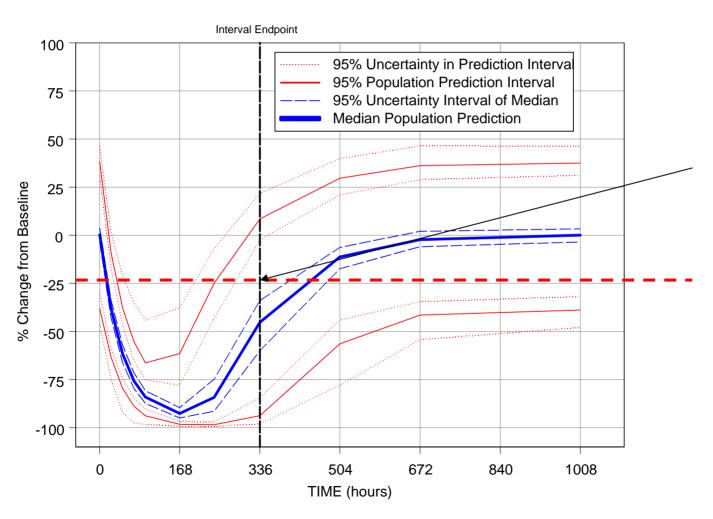
## **Conclusions for Example 2**

 When conditioned on current level of knowledge, this design results in:

P(>90% of patients in target range) = 0.67

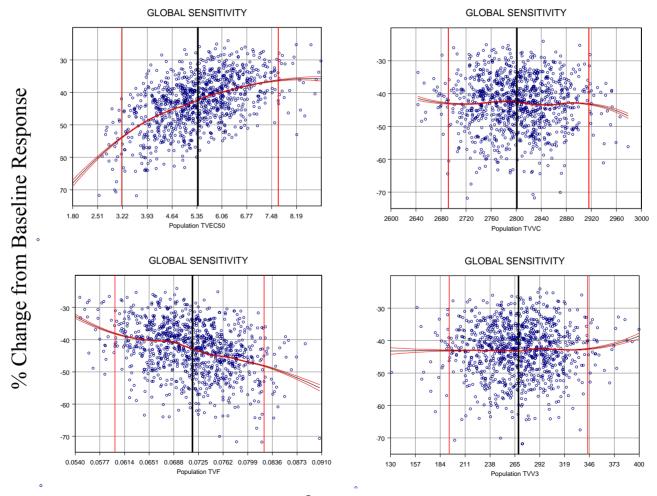
- Doses planned for the study are high enough so that exposure or EC50 are not as important as EMAX
- Improved estimates of EMAX may significantly improve precision of the simulation predictions of trial outcomes

# Example 3: View Population Variability and Uncertainty in Prediction For New Dose & Regimen



Competing therapy mean response at 2 weeks.

## Sensitivity of Simulation Endpoint to Parameter Assumptions



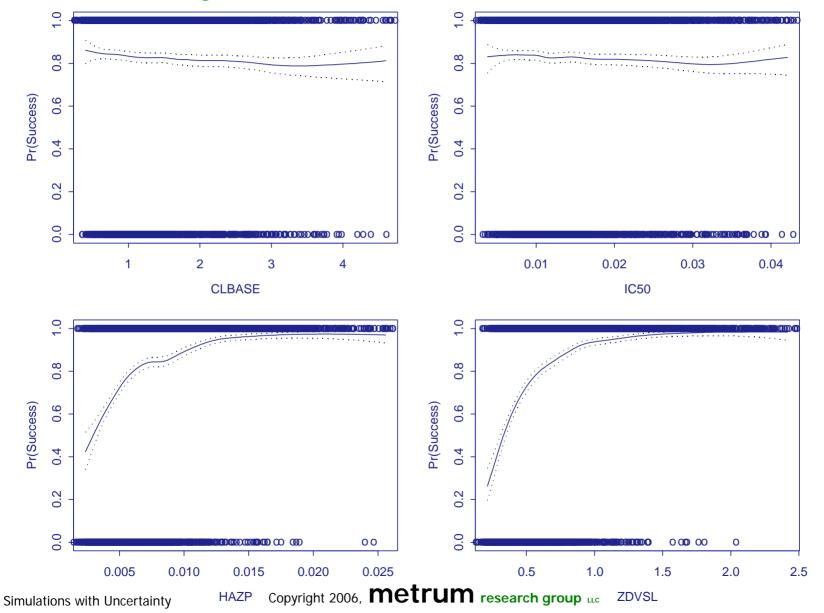
### **Example 4: Phase III Trial Simulation**

#### Results of Local Sensitivity Analysis

Fixed Value of ZDVSL	% Trials Successful <sup>a</sup>
0.25	30.6%
0.5	70.4%
0.735	93.0%
1.0	99.0%

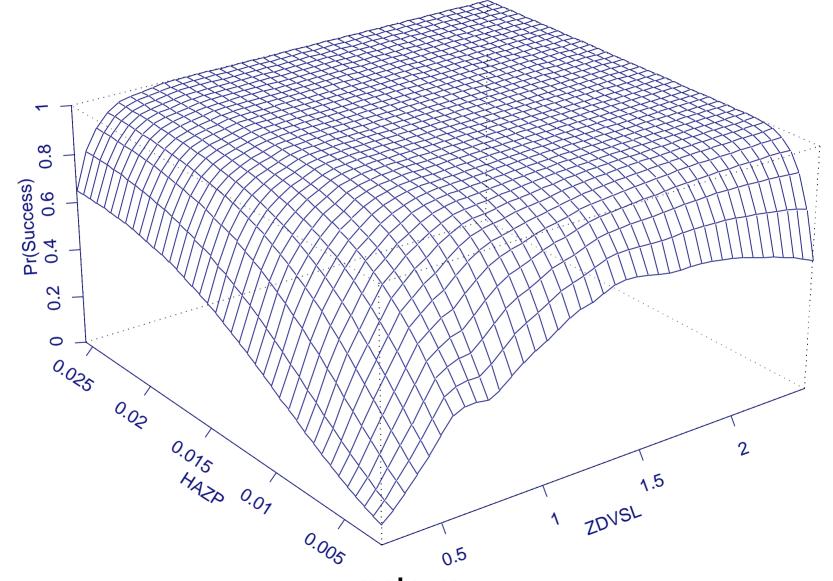
<sup>&</sup>lt;sup>a</sup>Results reflect 500 simulated trials of 2000 patients

# Global Sensitivity Analysis: Probability of Successful Trial vs. Uncertainty



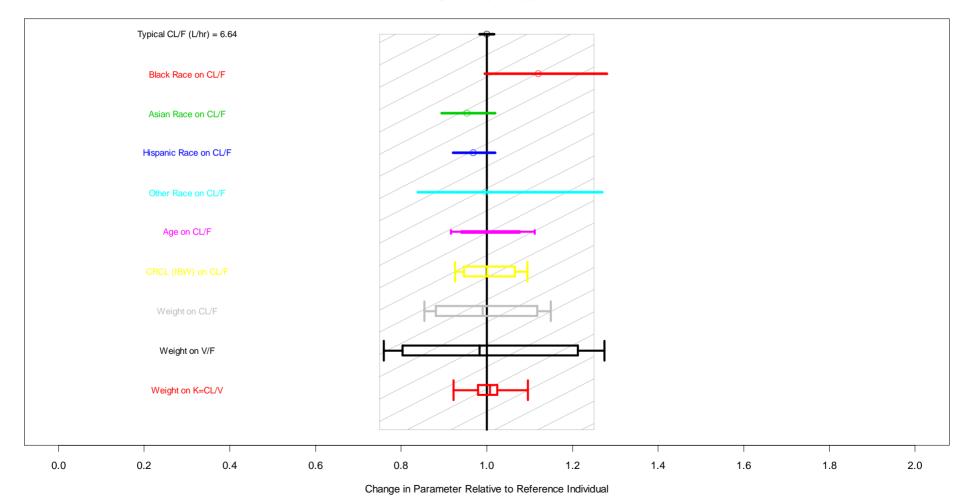
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# Sensitivity Analysis Surface: Most Influential Parameters

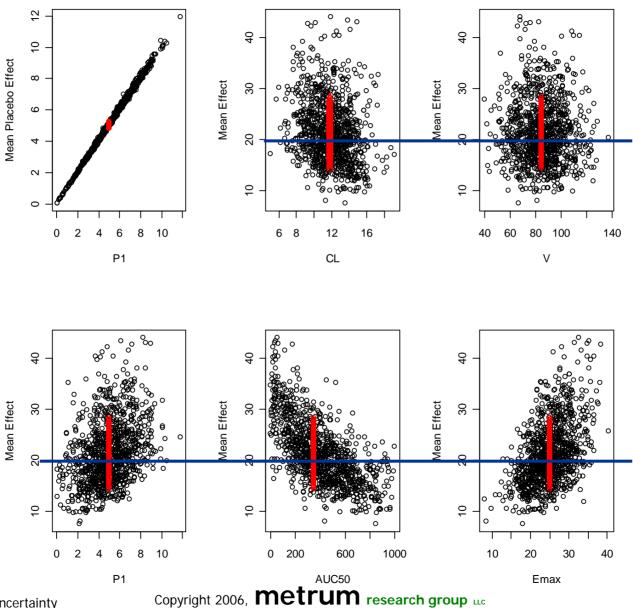


## **Example 5: Covariate Model for Population PK; Inferences in the Face of Uncertainty**

#### **Covariate Effects**



## **Example 6: Mean Response Across Parameter Uncertainty**



## **Summary of Examples**

- Acknowledge Uncertainty:
  - Predictions of expected responses are viewed in the context of the uncertainty in the simulation parameters (and/or model)
- Impact on Model-Based Inferences:
  - Sensitivity analysis allows for quantitative description of outcome dependencies on model assumptions
  - This approach leads to an informed application of simulation results in the decision making process
  - Implementation requires specific tools but not more CPUtime intensive than simulations without uncertainty

## How?

#### **Simulation Plan**

- Conventional CTS (without uncertainty):
  - Select model and model parameters
  - Simulate study 1000 times (with the same population parameters but different realizations of individual parameters)
  - Investigate range of possible outcomes (for fixed values of population parameters)
  - Repeat this process for different values of model parameters to investigate sensitivity of the results to assumptions (requires multiple repeats of simulations)

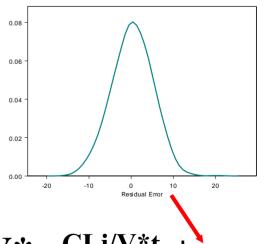
#### **Simulation Plan**

- CTS with uncertainty:
  - Select model and probability distribution of model / parameters (representing uncertainty)
  - Simulate study 1000 times (each time with different values of population parameters drawn from parameter distributions)
  - Investigate range of possible outcomes (given level of current knowledge)
  - Investigate sensitivity of the results to assumptions (does not require additional simulations)

## Hierarchy of Random Variability & Uncertainty in Simulation

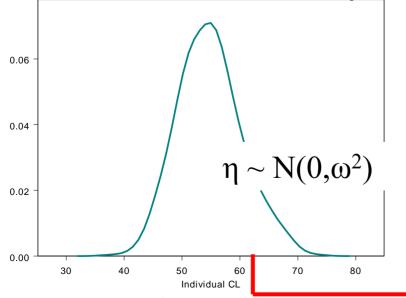
- Intra-individual, residual error (ε)
  - 1 draw from  $(0, \sigma^2)$  per observation, constant fixed-effect parameters  $(\theta)$
- Inter-individual error (η) in parameter
  - 1 draw from  $(0, \omega^2)$  per individual, constant fixed-effect parameters  $(\theta)$
- Uncertainty in models and parameters
  - 1 draw from prior distribution for  $\theta$ ,  $\Omega$ ,  $\Sigma$  per trial

#### **Residual Variability**

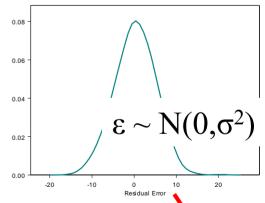


$$C_{t} = D/V * e^{-CLi/V*t} + \varepsilon_{t}$$





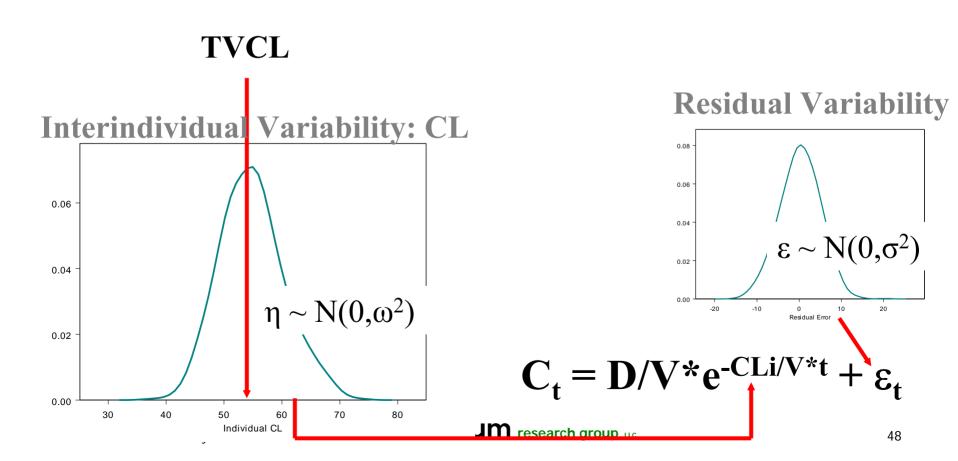
#### **Residual Variability**

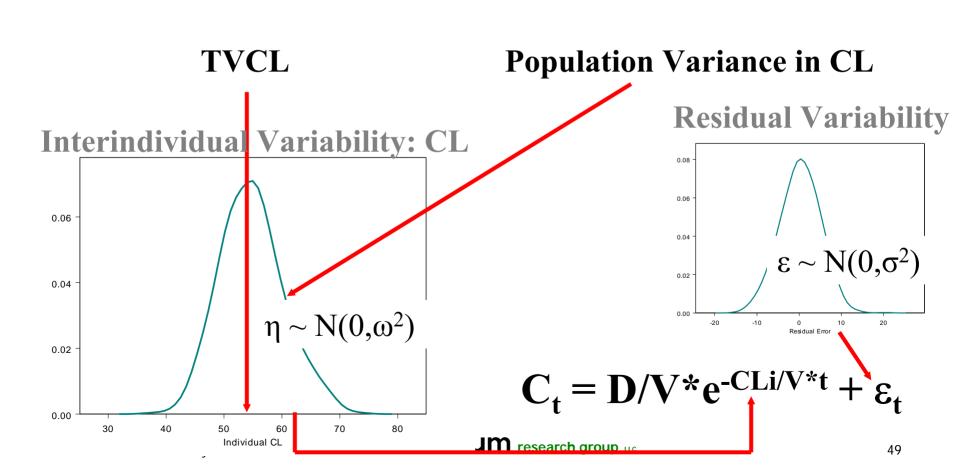


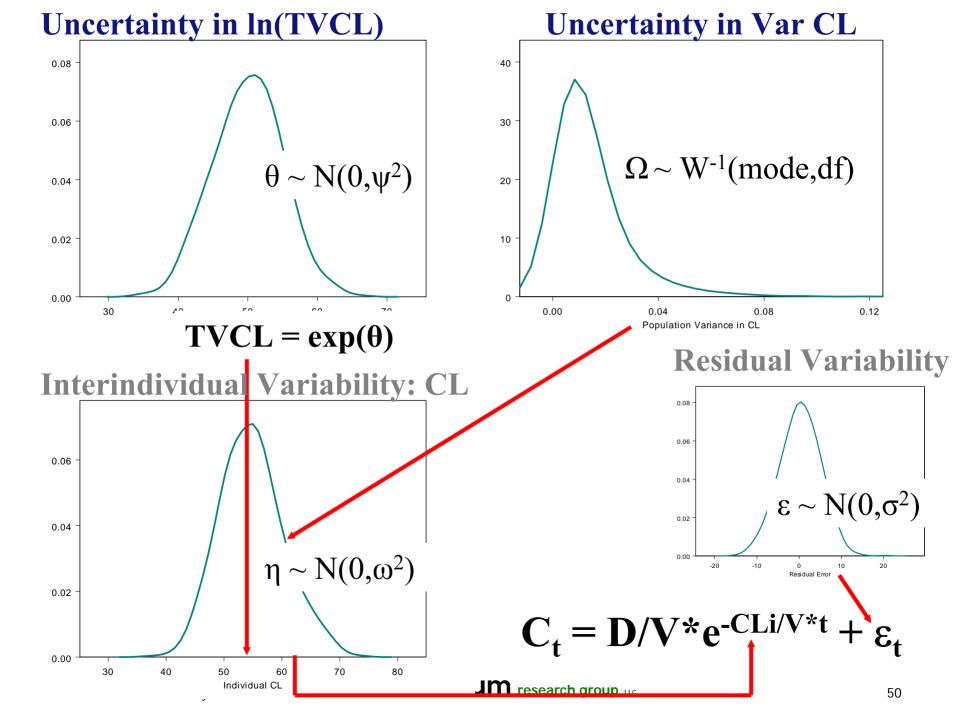
$$C_{t} = D/V * e^{-CLi/V * t} + \varepsilon_{t}$$

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### **Uncertainty in the Model (Competing Models)**

- Through Uncertainty in Parameters:
  - Combinations of some parameters approximate a different model structure.
  - e.g. High EC50 approximates linear model
- Simulate from Expected Probability of Each Model:
  - P(Model A) = 0.7 P(Model B) = 0.3
  - Draw random uniform variable (0-1), R
  - Model A if  $R \le 0.7$ ; Model B if R > 0.7

## **Obtaining Measures of Uncertainty**

- Results from prior modeling exercise
  - Variance-covariance matrix of estimates
  - Bootstrap parameter distributions
  - Bayesian posterior distributions
- Review of literature for ranges of plausible values
- Poll experts (everyone's view can be part of the simulation)

### **Assembling Simulation Model Components**

#### **Trial Simulation Model**

#### Pop PK Model

(NONMEM; NPBootstrap)

#### Clinical Outcome Model

(Meta-Analysis of Literature Data; Var-Cov Matrix of Estimates)

#### PD Models Efficacy/AEs

(BUGS; Posterior Distributions)

#### **Trial Conduct Model**

(Dropout Rate; Range of Expert Opinions)

## **Simulation Tool: Requirements**

- 1. Monte Carlo simulation hierarchy with multiple levels of nested random effects (at least 3)
- 2. Ability to incorporate joint uncertainty distributions from other methods (e.g. bootstrap, Bayesian)
- 3. Simulation and estimation (ML) for typical population PK and PD systems in same tool
- 4. Programmable/extensible language with data manipulation and graphics capability
- 5. Platform neutral (Win, Unix, Linux, Mac OS X)

#### **Current Simulation Tools**

Some programs with Monte Carlo simulation capabilities at parameter uncertainty level are available, but not all requirements are met:

- WinBugs
- NONMEM PRIOR subroutine
- Trial Simulator
- Others...

## NMSUDs R/NONMEM Package

 Generates draws from the uncertainty distributions at inter-trial level, maintaining joint distribution (covariance) of parameters

-OR-

- Samples from previously determined uncertainty distributions (e.g. Bootstrap, Bayesian Posteriors)
- Generates NONMEM control streams for simulation (estimation)
- 3. Runs NONMEM or R for simulation (and possibly estimation) of each trial
- Summarizes the results of each trial and across all trials

Parameter type (NONMEM name)	Distribution	Parameters of the distribution	Implementation	How to assign distribution parameters based on NONMEM run
Single uncorrelated population parameter (THETA)	Normal	Mean $\mu$ , variance $\sigma^2$	Standard R function rnorm(., $\mu$ , $\sigma$ )	μ: population parameter estimate; σ: standard error of the parameter estimate.
Set of correlated population parameters (THETA)	Multivariat Normal	Vector of mean values M, variance-covariance matrix Σ	Standard R function mvrnorm(., M, Σ)	<ul><li>M: vector of population parameter estimate;</li><li>Σ: variance-covariance matrix of the parameter estimates.</li></ul>
Variance of the random effect (OMEGA)	Scaled Inverse χ <sup>2</sup>	Number of degrees of freedom v, scale s <sup>2</sup> .	Standard R function v s²/rchisq(., v)	v: number of patients used to obtain the estimate; s²: estimated variance of the random effect.
Variance- covariance matrix of the random effects (OMEGA)	Inverse Wishart	Number of degrees of freedom v, scale matrix S. Implicit parameter is the S matrix dimension k.	Proprietary R function myriwish(k, v, vS) based on the standard riwish() function	v: number of patients used to obtain the estimate; vS: estimated variance- covariance matrix of the random effect.
Variance of the error term (SIGMA)	Scaled Inverse χ <sup>2</sup>	Number of degrees of freedom v, scale s <sup>2</sup> .	Standard R function vs <sup>2</sup> /rchisq(., v)	v: number between the number of patients and the number of observations used to obtain the estimate; s²: estimated variance of the error.

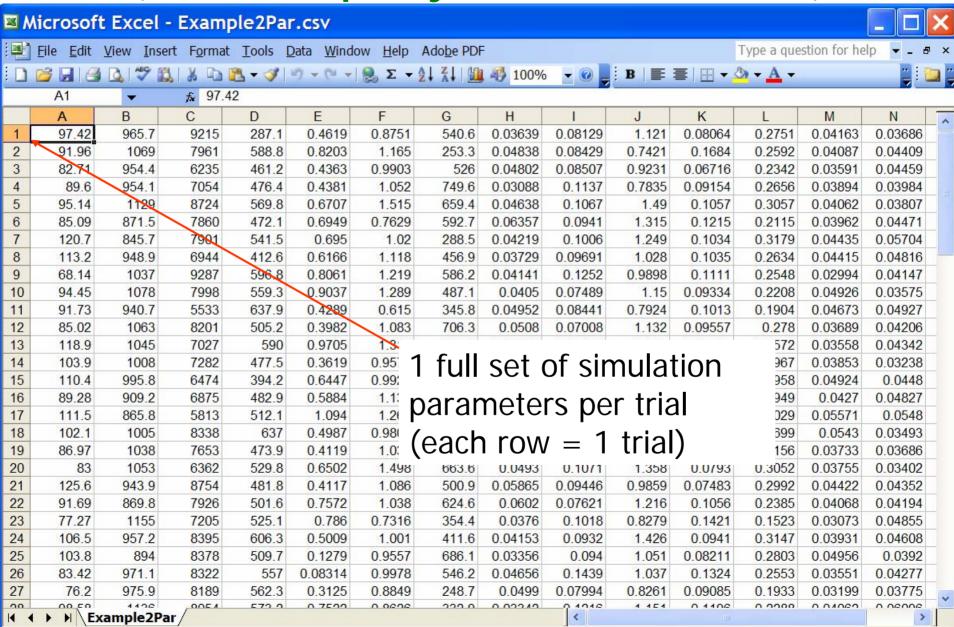
### **Sample from Uncertainty Distributions**

```
R C:\code\NMSUDSalpha1\Scripts\Example2Apr2006.R - R Editor
SourceDirName <- "c:/code/NMSUDSalpha1/Scripts/"
DirName <- "c:/code/NMSUDSalpha1/Example2/"
FigureDir <- DirName
source(paste(SourceDirName, "SimulationFromFileJan30.R", sep=""))
source(paste(SourceDirName,"CreateParametersOct24.R",sep=""))
source(paste(SourceDirName, "PostProcessingOct28.R", sep=""))
ThetaMean <-c(100,1000,7500,500,0.5,1,500)
ThetaCovar <- diag(c(150,15000,1000000,6400,0.1,0.04,15000))
OmegaModeList <-list(0.04,0.09,1,0.09,0.25)
OmegaDfList <-c(50,50,50,50,50)
SigmaModeList <- list(0.04,0.04)
SigmaDfList <- c(75,75)
# this part ensures reproducability:
set.seed(123)
rnorm(5)
# -0.56047565 -0.23017749 1.55870831 0.07050839 0.12928774
NsimPar <- 14
nsim <- 100
parameters <- CreateParametersForSimulation(nsim=1.5*nsim,
               ThetaMean=ThetaMean, ThetaCovar=ThetaCovar,
               OmegaModeList=OmegaModeList,OmegaDfList=OmegaDfList,
               SigmaModeList=SigmaModeList,SigmaDfList=SigmaDfList)
bounds <- data.frame(par =1:NsimPar,lower =rep(0,NsimPar),upper=rep(Inf,NsimPar))
parametersTruncated <- TruncateParametersForSimulation(parameters, bounds)</pre>
parametersTruncated <- parametersTruncated[1:nsim,]</pre>
write.table(parametersTruncated, file=paste(DirName,"Example2Par.csv",sep=""),
                            guote = F,sep=",",row.names = F,col.names = F)
```

Sample from Uncertainty Distributions

```
RGui - [C:\code\NMSUDSalpha1\Scripts\Example1Apr2006.R - R Editor]
File Edit Packages Windows Help
SourceDirName <- "c:/code/NMSUDSalpha1/Scripts/"
DirName <- "c:/code/NMSUDSalpha1/Example1/"
source(paste(SourceDirName, "SimulationFromFileApr2006.R", sep=""))
source(paste(SourceDirName,"CreateParametersApr2006.R",sep=""))
source(paste(SourceDirName, "PostProcessingOct28.R", sep=""))
ThetaMean <-c(11.8,85)
ThetaCovar \leftarrow matrix(c(0.232,0.449,0.449,12.8),2,2)
OmegaModeList <-matrix(c(0.0572,0.011,0.011,0.0615),2,2)
OmegaDfList <-20
SigmaModeList <- 0.0454
SigmaDfList <- 200
# this part ensures reproducability:
set.seed(123)
rnorm(5)
# -0.56047565 -0.23017749 1.55870831 0.07050839 0.12928774
parameters <- CreateParametersForSimulation(nsim=100,</pre>
               ThetaMean=ThetaMean, ThetaCovar=ThetaCovar,
               OmegaModeList=OmegaModeList,OmegaDfList=OmegaDfList,
               SigmaModeList=SigmaModeList,SigmaDfList=SigmaDfList)
bounds <- data.frame(par =c(1,2),lower =c(5,30),upper=c(20,150))
parametersTruncated <- TruncateParametersForSimulation(parameters, bounds)
write.table(parametersTruncated, file=paste(DirName,"Example1Par.csv",sep=""),
                            quote = F, sep=",", row.names = F, col.names = F)
```

# Parameters Generated from Uncertainty Distributions (or Bootstrap, Bayesian Posteriors, etc.)



#### **Typical NONMEM \$SIM Control Stream**

```
SPROB RUN# 001
SINPUT C ID AMT TIME EVID DV WT SS II
$DATA ../Example2Data.csv IGNORE=@
$SUBROUTINE ADVAN4 TRAN4
$PK
   TVCL = THETA(1)*(WT/70)**0.75
   TVV2 = THETA(2)*WT/70
     = THETA(3)*WT/70
      = THETA(4)*(WT/70)**0.75
   CL = TVCL*EXP(ETA(1))
   V2 = TVV2*EXP(ETA(2))
   F1 = 2
   S2 = V2/1000
   T1 = TVCL/TVV2
   T23 = O/TVV2
   T32 = Q/V3
   TL1 = ((T1+T23+T32)+SQRT((T1+T23+T32)**2-4*T1*T32))/2
   TVKA = THETA(5) + TL1
       = TVKA*EXP(ETA(3))
   EMAX = THETA(6)*EXP(ETA(4))
   EC50 = THETA(7)*EXP(ETA(5))
SERROR
   CONC=A(2)/S2
   EFF = EMAX*CONC/(EC50+CONC)
   Y=EFF*EXP(EPS(1))
```

```
STHETA
100
         ; 1 TVCL
1000
         ; 2 TVV2
7500
       ; 3 TVV3
500
        ; 4 TVO
0.5
         ; 5 TVKA
         ; 6 EMAX
500
         ; 7 EC50
SOMEGA
0.04
         ; 1 CT
     ; 2 V2
0.09
1.00
       ; 3 KA
0.09
     ; 4 EMAX
0.25
        ; 5 EC50
$SIGMA
0.01
         नन्त्र 1 ;
0.04
         ; 2 PK
$SIMULATION (12345) (6789 UNIFORM)
STABLE EVID TIME CONC IPRED EFF DV NOPRINT NOHEADER
       NOAPPEND FILE=../001.tab
```

IPRED=CONC\*EXP(EPS(2))

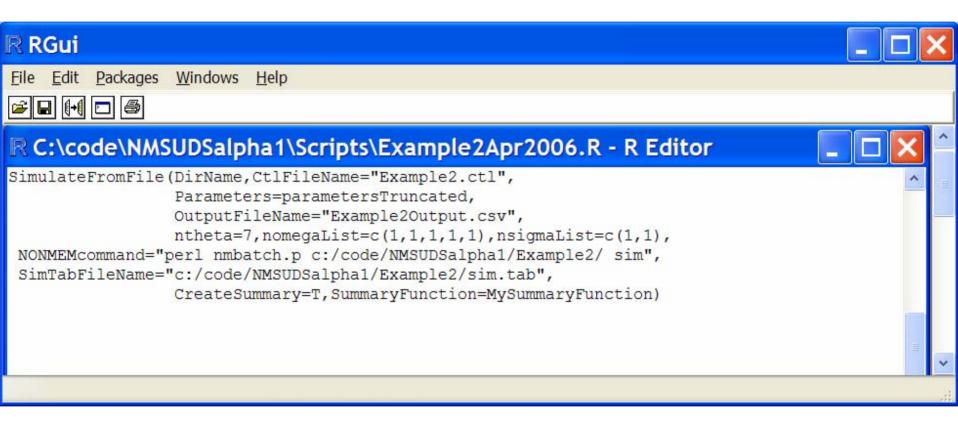
## **Constraining Simulated Parameters**

- When simulating from a multi-variate Normal covariance matrix, use caution about plausible values for population-level parameters.
- Constrain model so that plausible values are simulated, e.g.:

```
LNCL=THETA
CL=EXP(LNCL)
```

 Bootstrap distributions and Bayesian posteriors may already be constrained to plausible values

## Simulate from Uncertainty Distributions using NONMEM Model Control Stream

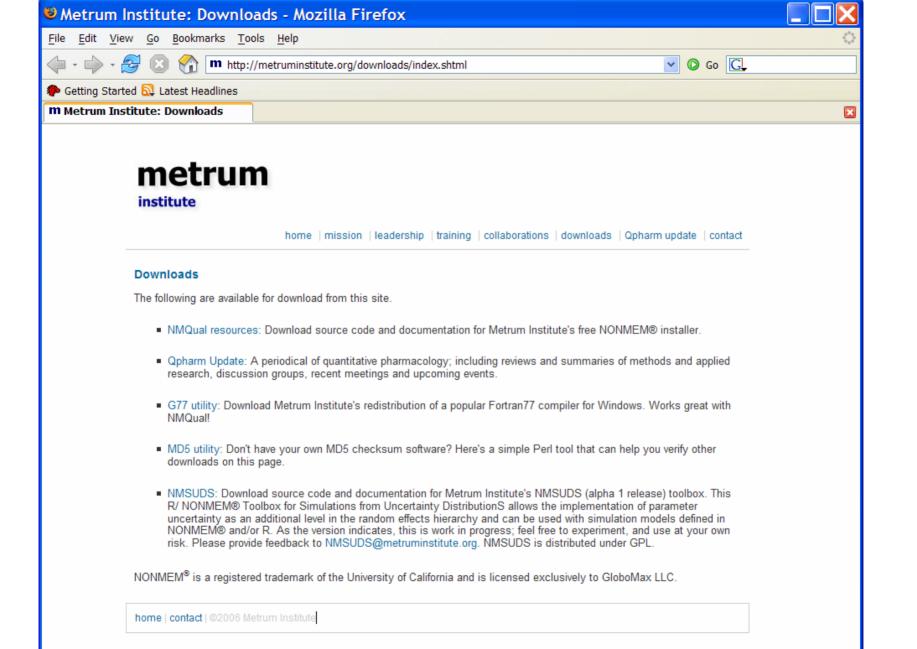


## NMSUDs R/NONMEM Package

Open-source tool, distributed under GPL.

Download alpha version of code from: www.metruminstitute.org/downloads

Forward questions/comments to: NMSUDs@metruminstitute.org



## **Acknowledgements**

Metrum Research Group Scientific Team

- John Mondick, Jeff Barrett (The Children's Hospital of Philadelphia)
- Industry collaborators (examples)

### **Questions or Comments?**