30 - Boolean Algebra

Thursday, November 29, 2018 9:43 AM

https://filestore2.aqa.org.uk/resources/computing/AQA-7516-7517-TG-BA.PDF

https://photos.google.com/share/AF1QipP3_VO6_GMJWDf4OIk23QIgznYzsN_5jNuDYNeyOAu3YFaohjFw-7rsPmcZo1JdxA?key=enFlQjNtckRwRjhvemNNM3h5S0JPRUJUV2dpTE9B

Remember from test that its safest to just put the dot in for multiplication anyway.

boolean_algebra_laws

Boolean Algebra Laws

Joe Iddon

January 21, 2019

In the following summary of the laws of boolean algebra $x, y, z \in \{true, false\}$; the conjunction *or* is denoted as $+$; the disjunction *and* is denoted as $\cdot$ and the negation *not* as $\overline{x}$.

Name of Law	OR Operation ($+$)	AND Operation ($\cdot$)
Identity	$x + 0 = x$	$x \cdot 1 = x$
Complementation	$x + \overline{x} = 1^\dagger$	$x \cdot \overline{x} = 0^\dagger$
Associativity	$x + (y + z) = (x + y) + z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
Commutativity	$x + y = y + x$	$x \cdot y = y \cdot x$
Distributivity	$x + (y \cdot z) = (x + y) \cdot (x + z)^\dagger$	$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
Annihilator	$x + 1 = 1^\dagger$	$x \cdot 0 = 0$
Idempotence	$x + x = x^\dagger$	$x \cdot x = x^\dagger$
Absorption	$x + (x \cdot y) = x^\dagger$	$x \cdot (x + y)^\dagger = x$
De Morgan's	$\overline{x + y} = \overline{x} \cdot \overline{y}$	$\overline{x \cdot y} = \overline{x} + \overline{y}$

Note that laws with † *do not hold in ordinary algebra.*

We also have the Double Negation Law, seen below.

$$\overline{(\overline{x})} = x$$

1

Questions on Boolean Algebra page 250 band book

1) $(\bar{A}+B)\cdot(A+\bar{B}) = \bar{A}\cdot A + \bar{A}\cdot\bar{B} + B\cdot A + B\cdot\bar{B}$

$= \bar{A}\cdot\bar{B} + B\cdot A.$ ✓

verify:

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+B$	$A+\bar{B}$	$(\bar{A}+B)\cdot(A+\bar{B})$	$\bar{A}\cdot\bar{B}$	$B\cdot A$	$\bar{A}\cdot\bar{B}+B\cdot A$
0	0	1	1	1	1	1	1	0	1
0	1	1	0	1	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0
1	1	0	0	1	1	1	0	1	1

✓ ✓

2) $A(\bar{A}+B)(\bar{B}+C) = (A\bar{A}+AB)(\bar{B}+C)$

$= AB(\bar{B}+C)$

$= AB\bar{B}+ABC$

$= ABC.$ ✓

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{A}+B$	$\bar{B}+C$	$A(\bar{A}+B)(\bar{B}+C)$	ABC
0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	1	0	1	0	1	0	0	0
0	1	1	1	0	1	1	0	0
1	0	0	0	1	0	1	0	0
1	0	1	0	1	0	1	0	0
1	1	0	0	0	1	0	0	0
1	1	1	0	0	1	1	1	1

✓ ✓

3) see above.

4) $\bar{A}+\bar{A}\cdot B = \bar{A}(1+B) = \bar{A}$ ✓

5) $A+\bar{A}\cdot B = (A+\bar{A})\cdot(A+B) = A+B$ ✓ $\because x+(Y\cdot Z) = (X+Y)\cdot(X+Z)$

6) $A+\bar{A}\cdot B+\bar{B}+C = (A+\bar{A})\cdot(A+B)+C = A+B+C$ ✓

7) $A+\bar{A}\cdot C+B+D(\bar{B}\bar{C}+A\bar{C})$

$= A+C+B+\bar{B}\bar{C}D+A\bar{C}D$

$= A+C+B+\bar{C}(\bar{B}D+AD)$

$= A+B+C+\bar{B}D+AD$

$= A+B+C+D+(A+A)\cdot(A+D)$

$= A+B+C+D+A+D$

$= A+B+C+D$ ✓

8) $A\bar{B}C + A\bar{B}\bar{C}$

$= A\bar{B}(C+\bar{C})$

$= A\bar{B}$ ✓

9) $A \cdot (\bar{A}+B) + A \cdot \bar{B}$

$= A \cdot (\bar{A} + B + \bar{B})$

$= A$ ✓

10) $(A \cdot \bar{B} + \bar{A} \cdot B) \cdot A \cdot B + A \cdot \bar{B}$

$= A \cdot B \cdot \bar{A} \cdot \bar{B} + A \cdot \bar{A} \cdot B \cdot B + A \cdot \bar{B}$

$= 0 + 0 + A \cdot \bar{B}$

$= A \cdot \bar{B}$ ✓

11) $(\bar{A} + B) \cdot (A + \bar{B})$

$= \bar{A} \cdot A + B \cdot A + B \cdot \bar{B} + \bar{A}\bar{B}$

$= 0 + B \cdot A + 0 + \bar{A}\bar{B}$

$= B \cdot A + \bar{A} \cdot \bar{B}$ as required. ✓

Summary of Laws:

		AND	OR
	Identity	$A \cdot 1 = A$	$A + 0 = A$
	Null	$A \cdot 0 = 0$	$A + 1 = 1$
doesn't change result →	Idempotence	$A \cdot A = A$	$A + A = A$
	Inverse	$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$
commute ↳ move around →	Commutative	$A \cdot B = B \cdot A$	$A + B = B + A$
associate ↳ group	Associative	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	$(A+B) + C = A + (B + C)$
	Distributive	$A + B \cdot C = (A+B) \cdot (A+C)$	$A \cdot (B + C) = A \cdot B + A \cdot C$
	Absorption	$A \cdot (A+B) = A$	$A + A \cdot B = A$
	De Morgan's	$\overline{(A \cdot B)} = \bar{A} + \bar{B}$	$\overline{(A+B)} = \bar{A} \cdot \bar{B}$

Double Complement $\bar{\bar{A}} = A$

1) $A \cdot A + B \cdot 1$ (idempotence + identity)

$= A + B$

1) $A \cdot A + B \cdot 1$ (idempotence + identity)

$= A + B$

2) $A \cdot (\bar{A} + B)$ (distributive)

$= A\bar{A} + AB$

$= AB$

3) $(\bar{A} + \bar{B}) \cdot (A + B)$ (3 x distributive)

$= \bar{A}A + \bar{A}B + \bar{B}A + \bar{B}B$

$= \bar{A}B + \bar{B}A$

4) $B + A\bar{B}$ (distributive)

$= (B + A) \cdot (B + \bar{B})$

$= B + A$

$= A + B$

5) $A + A\bar{B}$

$= A$ (absorption)

6) $A \cdot \bar{B} + \bar{A} \cdot B$ (De Morgan's)

$= \overline{(\bar{A} + B)} + \overline{(A + \bar{B})}$

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1) $(\bar{A}+B)\cdot(A+\bar{B})$

$= \bar{A}A + BA + \bar{A}\bar{B} + B\bar{B}$

$= \bar{A}\bar{B} + BA$

2) $A(\bar{A}+B)(\bar{B}+C)$

$= (A\bar{A} + AB)(\bar{B}+C)$

$= AB(\bar{B}+C)$

$= AB\bar{B} + ABC$

$= ABC$

3)

A	B	$\bar{A}+B$	$A+\bar{B}$	$(\bar{A}+B)\cdot(A+\bar{B})$	$\bar{A}\bar{B}+BA$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

A	B	C	$\bar{A}+B$	$\bar{B}+C$	$A(\bar{A}+B)(\bar{B}+C)$	ABC
0	0	0	1	1	0	0
0	0	1	1	1	0	0
0	1	0	1	0	0	0
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	0	1	0	1	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

4) $\bar{A} + \bar{A}\cdot B$

$= \bar{A}$

$\boxed{A + \bar{A}\cdot B = A+B}$

5) $A + \bar{A}\cdot B$

$= (A+\bar{A})\cdot(A+B)$

$= A+B$

6) $A + \bar{A}\cdot B + \bar{B}\cdot C$

$= (A+\bar{A})\cdot(A+B) + \bar{B}\cdot C$

$= A + B + \bar{B}C$

$= A + (B+\bar{B})\cdot(B+C)$

$= A+B+C$

7) $A+\bar{A}\cdot C + B + D\cdot(\bar{B}\bar{C} + A\bar{C})$

$= A\cdot C + B + D\bar{B}\bar{C} + DA\bar{C}$

$= AC + (B+D)(B+\bar{B})(B+\bar{C}) + DA\bar{C}$

$= AC + (B+D)(B+\bar{C}) + DA\bar{C}$

$= AC + B + B\bar{C} + BD + D\bar{C} + DA\bar{C}$

$= A+B+C$ (absorption laws)

$A + \bar{A}C + B + D(\bar{B}\bar{C} + A\bar{C})$

$= A + C + B + D\bar{B}\bar{C} + AD\bar{C}$

$= A + C + B + \bar{C}(D\bar{B} + AD)$

$= A + C + B + D\bar{B} + AD$

$= A + C + B + D + AD$

$= A + C + B + D$

8) $A\bar{B}C + A\bar{B}\bar{C}$

$= A\bar{B}(C+\bar{C})$

$= A\bar{B}$

9) $A(\bar{A}+B) + A\bar{B}$

$= A\bar{A} + AB + A\bar{B}$

$= AB + A\bar{B}$

$= A$

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10) $(\bar{A}\bar{B} + \bar{A}B)AB + A\bar{B}$

$\bar{A}\bar{B}AB + \bar{A}BAB + A\bar{B}$

$= A\bar{B}$

11) $(\bar{A} + B)(A + \bar{B})$

$= \bar{A}A + \bar{A}\bar{B} + BA + B\bar{B}$

$= \bar{A}\bar{B} + BA$

12) $\overline{\bar{A}\bar{B} + AB} = \overline{\overline{A+B} + \overline{\bar{A}+B}}$

13) $\overline{\bar{A}+B} + \overline{A+\bar{B}}$

$= A\bar{B} + \bar{A}B$

$= \overline{(\bar{A}+B)(A+\bar{B})}$

$= \overline{\bar{A}A + \bar{A}\bar{B} + BA + B\bar{B}}$

$= \overline{\bar{A}\bar{B} + BA}$

14) $\overline{AB} + A$

$= \bar{A} + \bar{B} + A$

$= 1$

15) $\overline{\overline{A\bar{B}}\,\overline{\bar{A}\bar{B}}}$

$= \overline{(\bar{A}+B)(A+B)}$

$= \overline{\bar{A}A + \bar{A}B + BA + BB}$

$= \bar{B}$

16) a) $\overline{\bar{A} + \bar{B} + A\bar{B}}$

$= AB(\bar{A} + B)$

$= AB\bar{A} + ABB$

$= AB$

b) $\bar{A}\overline{(\bar{A}+\bar{B})}$

$= \bar{A}AB$

$= 0$

c) $\bar{A}BC + \bar{A}B$

$= \bar{A}B$ (absorption)

d) $\overline{\bar{A} + \overline{\bar{A}B}}$

$= AB$

e) $\overline{AB} + \overline{A\bar{B}}$

$= \bar{A} + \bar{B} + \bar{A} + B$

$= 1$

17) $WX + WYZ$