# Solutions to FIT5047 Tutorial on Knowledge Representation – Propositional Logic

## Exercise 1: Propositional logic, models, validity and satisfiability

Consider a vocabulary with only four propositions $A, B, C$ and $D$. How many models are there in the following sentences? Are these sentences valid, satisfiable (but not valid) or unsatisfiable?

(a) $B \vee C$

SOLUTION:
The sentence is false only if B and C are false, which occurs in 4 cases for all the combinations of A, B, C and D, leaving 12 models.

Another way to look at this is:
A={T,F}, B=T, C=F, D={T,F}: 4 models
A={T,F}, B=F, C=T, D={T,F}: 4 models
A={T,F}, B=T, C=T, D={T,F}: 4 models
Total: 12 models.

It doesn't hold in four models: A={T,F}, B=F, C=F, D={T,F}

So, this sentence is satisfiable but not valid.

(b) $\neg A \vee \neg B \vee \neg C \vee \neg D$

SOLUTION:
The sentence is false only if A, B, C, and D are true, which occurs in 1 case, leaving 15 models.

So, this sentence is satisfiable but not valid (doesn't hold in one model).

## Exercise 2: Propositional logic, representation

According to some political pundits, a person who is radical ($R$) is electable ($E$) if s/he is conservative ($C$), but otherwise a radical person is not electable. Represent this assertion in propositional logic. Can you express it in Horn form?

SOLUTION:
This assertion may be represented as follows:
$[(R \wedge C) \Rightarrow E] \wedge [(R \wedge \neg C) \Rightarrow \neg E]$, which is equivalent to:
$[\neg(R \wedge C) \vee E] \wedge [\neg(R \wedge \neg C) \vee \neg E]$
$[\neg R \vee \neg C \vee E] \wedge [\neg R \vee C \vee \neg E]$

Each clause can be expressed in Horn form as it has at most one positive literal.

**Exercise 3: Propositional logic, validity and satisfiability**

Consider the following sentence:

$$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drink) \Rightarrow Party]$$

Convert the left-hand and right-hand sides of the main implication to CNF, and determine whether this sentence is valid, satisfiable (but not valid) or unsatisfiable.

SOLUTION:

$$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$$

$$\neg[(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \vee [\neg(Food \wedge Drinks) \vee Party]$$

$$\neg[\neg Food \vee \neg Drinks \vee Party] \vee [\neg Food \vee \neg Drinks \vee Party] \equiv True$$

So, the sentence is **valid**.

**Exercise 4: Horn clauses, forward and backward reasoning**

Consider the following statements:

1. If Susan gets high marks she is successful.
2. If Susan is bright and studies she will get high marks.
3. If Susan is not bright she won't pass the subject.
4. If Susan is diligent she will study.
5. If Susan is not diligent she will have fun.
6. If Susan has fun she will not study.
7. Susan is diligent.
8. Susan has passed the subject.

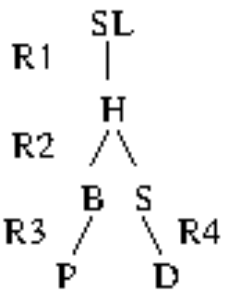
(a) Using the propositions H (high marks), SL (successful), B (bright), S (study), P (pass), D (diligent) and F (have fun), translate the above statements to propositional logic. Which of these clauses can be converted to Horn clauses?

SOLUTION:

| | |
|---|---|
| R1. $H \Rightarrow SL$ | $\neg H \vee SL$ |
| R2. $(B \wedge S) \Rightarrow H$ | $\neg B \vee \neg S \vee H$ |
| R3. $\neg B \Rightarrow \neg P$ | $B \vee \neg P$ – change to $P \Rightarrow B$ |
| R4. $D \Rightarrow S$ | $\neg D \vee S$ |
| R5. $\neg D \Rightarrow F$ | $D \vee F$ – NOT HORN CLAUSE |
| R6. $F \Rightarrow \neg S$ | $\neg F \vee \neg S$ |
| – $D$ | $D$ |
| – $P$ | $P$ |

(b) Apply backward reasoning to prove that Susan is successful.

SOLUTION:



(c) Apply forward reasoning to prove that Susan is successful.
SOLUTION:

| Agenda | Count | | | | | Inferred |
|---|---|---|---|---|---|---|
| | R1 | R2 | R3 | R4 | R6 | |
| $D, P$ | 1 | 2 | 1 | 1 | 1 | |
| $D, B$ | 1 | 2 | 0 | 1 | 1 | $P$ |
| $B, S$ | 1 | 2 | 0 | 0 | 1 | $P, D$ |
| $S$ | 1 | 1 | 0 | 0 | 1 | $P, D, B$ |
| $H$ | 1 | 0 | 0 | 0 | 1 | $P, D, B, S$ |
| $SL$ | 0 | 0 | 0 | 0 | 1 | $P, D, B, S, H$ |