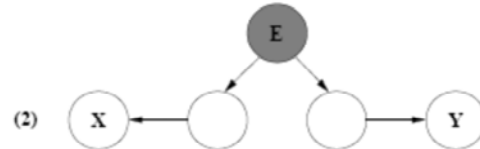


# Determining D-separation

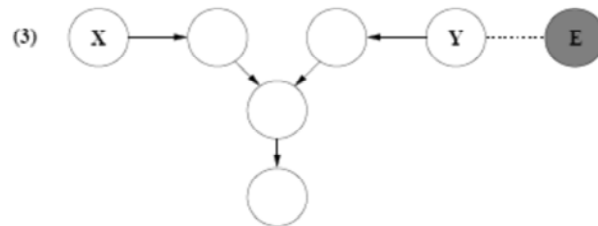
Chain



Common cause



Common effect

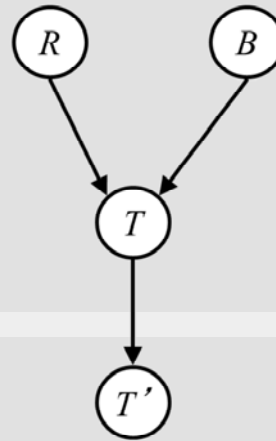


## D-separation – Example (I)

$$R \perp\!\!\!\perp B$$

$$R \perp\!\!\!\perp B | T$$

$$R \perp\!\!\!\perp B | T'$$



- R is independent of B, as the only path between them is through T, and there is no evidence for T or its descendant T' (common effect).
- R is NOT independent of B given T because of common effect, i.e., T is now in **E** (Evidence).
- R is NOT independent of B given T' because of common effect, i.e., T' is now in **E** (Evidence).

## D-separation – Example (II)

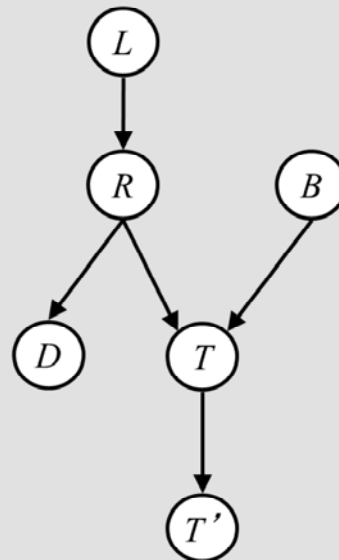
$$L \perp\!\!\!\perp T' | T$$

$$L \perp\!\!\!\perp B$$

$$L \perp\!\!\!\perp B | T$$

$$L \perp\!\!\!\perp B | T'$$

$$L \perp\!\!\!\perp B | T, R$$



- L is independent of T' given T because T is in **E**, blocking the path between L and T' (chain).
- L is independent of B because the only path between them is through T, and neither T nor its child T' are in **E** (common effect).
- L is NOT independent of B given T because of common effect (T is in **E**).
- L is NOT independent of B given T' because of common effect (T' is in **E**).
- L is independent of B given T and R because, even though T makes them dependent due to common effect (T is in **E**), R blocks the path between them (chain).

## D-separation – Example (III)

- **Variables:**

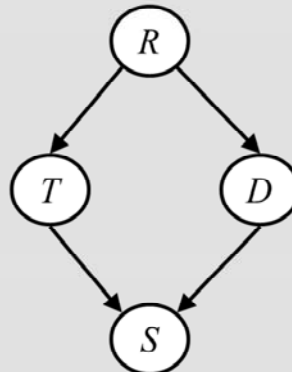
- R: Raining
- T: Traffic
- D: Roof drips
- S: I'm sad

- **Questions:**

$$T \perp\!\!\!\perp D$$

$$T \perp\!\!\!\perp D | R$$

$$T \perp\!\!\!\perp D | R, S$$



In this BN, we have two factors in play: common cause and common effect:

- Common cause causes independence between T and D when R is in **E**, **thereby** blocking the path between them.
- Common effect causes independence between T and D when S NOT in **E**, thereby blocking the path between them.

Now, let's examine each example:

- T is NOT independent of D – because there is one path between T and D
  - R is NOT in **E** (no common cause  $\rightarrow$  T and D are dependent through R)
  - S is NOT in **E** (no common effect  $\rightarrow$  T and D are independent through S), but this has no effect because there is another path.
- T is independent of D given R because the blockage through S remains, but now the path through R is also blocked because of common cause (R is in **E**).
- T is NOT independent of D given R and S – now the path through S has become unblocked (S is in **E**), so there is a path between T and D through S.