Solutions to FIT5047 Tutorial on Knowledge Representation – First Order Logic

Exercise 1: Representation

Using the predicates COMPUTER-SYSTEM, INTELLIGENT, TASK, PERFORM, HUMAN, and REQUIRES-INTELLIGENCE, represent the following sentence in first-order logic:
"A computer system is intelligent, if it can perform a task which, if performed by a human, requires intelligence."

SOLUTION:
There are several interpretations to this sentence. The following predicate calculus representations depict two of these interpretations.

i. $\forall x$ { *COMPUTER-SYSTEM(x)* $\wedge$
  $\exists y$ *[TASK(y)* $\wedge$ *PERFORM(x,y)* $\wedge$
    $\exists z$ {*HUMAN(z)* $\wedge$ *[PERFORM(z,y)* $\Rightarrow$ *REQUIRES-INTELLIGENCE(z)]*}*]*
    $\Rightarrow$ *INTELLIGENT(x)* }

English translation: "For all x such that x is a COMPUTER SYSTEM, and for which conditions 1-3 are fulfilled, then x is INTELLIGENT." Where conditions 1-3 are:
1. There exists a y such that y is a TASK, and
2. x PERFORMs y, and
3. If a HUMAN performs y then the human REQUIRES INTELLIGENCE.

ii. $\forall x$ {*COMPUTER-SYSTEM(x)* $\wedge$
  $\exists y$ *[TASK(y)* $\wedge$ *PERFORM(x,y)* $\wedge$
    $\forall z$ { *HUMAN(z)* $\wedge$ *[PERFORM(z,y)* $\Rightarrow$
      REQUIRES-INTELLIGENCE(z)]} *]* $\Rightarrow$ *INTELLIGENT(x)* }

English translation: like # i, but now the sentence is interpreted as requiring *all* humans to require intelligence to perform TASK y.

Exercise 2: Equivalence

Are the following pairs of formulas equivalent?

(a) $\exists x \exists y P(x, y)$ and $\exists y \exists x P(x, y)$
SOLUTION: YES

(b) $\exists x\{P(x) \wedge Q(x)\}$ and $\exists y P(y) \wedge \exists x Q(x)$
SOLUTION:
NO: The left-hand-side demands that P and Q must be true for a single entity x, while the right-hand-side demands that P be true for one entity and Q be true for an entity. These may or may NOT be the same entity. The fact that the quantified variables are different is irrelevant (think of quantifiers as counters in a loop). The answer would be the same for $\exists x P(x) \wedge \exists x Q(x)$.

(c) $\forall x\{P(x) \vee Q(x)\}$ and $\forall y P(y) \vee \forall x Q(x)$
SOLUTION:
NO: The left-hand-side demands that P **or** Q be true for every entity in the world. This means that P may be true for some and Q for others, and both P and Q for others. In contrast, the right-hand-side demands that P be true for **all** the entities in the world, **or** Q be true for all the entities in the world. As above, the fact that the quantified variables are different is irrelevant.

Exercise 3: Unification

In the following pairs of expressions, w, x, y and z are variables, A and B are constants, and f and g are functions. Do these pairs of expressions unify? If they do, give the substitution that unifies them. If they don't, then indicate where the unification fails.

(a) $P(x, f(x))$ and $P(f(A), f(A))$
SOLUTION:
To unify the first term in both expressions, we need to substitute $\{x|f(A)\}$. This results in $P(f(A), f(f(A)))$ and $P(f(A), f(A))$. Now, the second term cannot unify, because $f(A)$ does not unify with A.
UNIFICATION FAILS.

(b) $P(x, f(x), f(x))$ and $P(y, z, f(y))$
SOLUTION:
Substituting $\{y|x\}$ we get $P(x, f(x), f(x))$ and $P(x, z, f(x))$. Now,
Substituting $\{z|f(x)\}$ we get $P(x, f(x), f(x))$ and $P(x, f(x), f(x))$. The mgu is $\{y|x, z|f(x)\}$.

Exercise 4: Resolution refutation

Consider the following statements:

1. John likes all food.
2. Anything that one eats and isn't killed by is food.
3. Bill eats peanuts.
4. Bill is still alive.

(a) Represent these statements as predicate calculus formulas using the predicates: LIKES, IS-FOOD, EATS and ALIVE.

SOLUTION:
1. $\forall x1$ *[FOOD($x1$) $\Rightarrow$ LIKES (John,$x1$)]*
2. $\forall x2 \forall y$ *[EATS($y, x2$) $\wedge$ ALIVE(y) $\Rightarrow$ FOOD($x2$)]*
3. *EATS(Bill,peanuts)*
4. *ALIVE(Bill)*

(b) Convert these statements to clauses.

SOLUTION:
1. $\neg$*FOOD($x1$)* $\vee$ *LIKES (John,$x1$)*
2. $\neg$*EATS(y,$x2$)* $\vee$ $\neg$*ALIVE(y)* $\vee$ *FOOD($x2$)*
3. *EATS(Bill,peanuts)*
4. *ALIVE(Bill)*

(c) Use resolution-refutation to prove that John likes peanuts.

SOLUTION:

Goal: *LIKES (John,peanuts)*

Negated goal: 5. $\neg$*LIKES (John,peanuts)*

5 and 1: 5. $\underline{\neg \mathit{LIKES}\ (\mathit{John},\mathit{peanuts})}$ 1. $\neg \mathit{FOOD}(x1) \vee \underline{\mathit{LIKES}\ (\mathit{John},x1)}$

mgu: $\{x1|\text{peanuts}\}$

resolvent: 6. $\neg$FOOD(peanuts)

6. $\underline{\neg\text{FOOD(peanuts)}}$ 2. $\neg \mathit{EATS}(y,x2) \vee \neg \mathit{ALIVE}(y) \vee \underline{\mathit{FOOD}(x2)}$

mgu: $\{x2|\text{peanuts}\}$

resolvent: 7. $\neg$EATS(y,peanuts) $\vee$ $\neg$ALIVE(y)

7. $\underline{\neg\text{EATS}(y,\text{peanuts})} \vee \neg\text{ALIVE}(y)$ 3. *EATS(Bill,peanuts)*

mgu: $\{y|\text{Bill}\}$

resolvent: 8. $\neg$ALIVE(Bill)

8. $\neg$ALIVE(Bill) 4. *ALIVE(Bill)*

resolvent: NIL

Exercise 5: Resolution refutation

Consider the following sentences in first order logic, assuming that the variables range over the natural numbers $0, 1, 2, \ldots, \infty$, and that the predicate $GE(x, y)$ means that x is greater than or equal to y.

A. $\forall x \exists y \; GE(x, y)$

B. $\exists y \forall x \; GE(x, y)$

(a) Translate these predicates to English.
SOLUTION:

A. For every natural number, there is a natural number that is less than or equal to it.

B. There is a particular natural number that is less than or equal to every natural number.

(b) Is **A** true?
SOLUTION: YES.

(c) Is **B** true?
SOLUTION: YES.

(d) Does **A** logically entail **B**?
SOLUTION:
NO, if for every natural number x there is a natural number that is less than or equal to it, then this second natural number is not necessarily the same one for every x.

(e) Does **B** logically entail **A**?
SOLUTION:
YES, if there is a particular natural number that is less than or equal to every natural number, then there is a natural number that is less than or equal to every natural number.

(f) Try to use resolution to prove that **B** is true when **A** is true, and that **A** is true when **B** is true. Which proof works and which fails and why?

SOLUTION:

- Given **A**: $\forall x \exists y\ GE(x, y)$
  Goal **B**: $\exists y \forall x\ GE(x, y)$

  Negate the goal $\neg$**B**:
  $\neg \exists y \forall x\ GE(x, y)$
  $\forall y \neg \forall x\ GE(x, y)$
  $\forall y \exists x\ \neg GE(x, y)$

  Convert to clauses:
  A: $GE(x, g(x))$
  $\neg$**B**: $\neg GE(g(y), y)$

  Resolve **A** with $\neg$**B**:
  Substitute $\{x|g(y)\}$ yielding
  A$\{x|g(y)\}$: $GE(g(y), g(g(y)))$
  $\neg$**B**$\{x|g(y)\}$: $\neg GE(g(y), y)$
  Unification fails as y is in $g(g(y))$

- Given **B**: $\exists y \forall x\ GE(x, y)$.
  Goal **A**: $\forall x \exists y\ GE(x, y)$

  Negate the goal $\neg$**A**:
  $\neg \forall x \exists y\ GE(x, y)$
  $\exists x \neg \exists y\ GE(x, y)$
  $\exists x \forall y\ \neg GE(x, y)$

  Convert to clauses:
  B: $GE(x, A)$
  $\neg$**A**: $\neg GE(B, y)$

  Resolve **B** with $\neg$**A**:
  Substitute $\{x|B, y|A\}$ yielding
  B: $GE(B, A)$
  $\neg$**A**: $\neg GE(B, A)$
  which resolves to nil.