Spectral Functions: NEUT and GENIE

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THE UNIVERSITY OF WARWICK

Notation

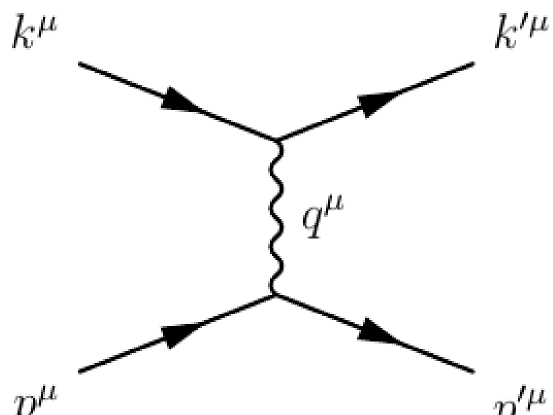


Figure 4: CCQE feynmann diagram

neutrino: $k^\mu = (E_\nu, \vec{k})$
muon: $k'^\mu = (E', \vec{k'})$
final state nucleon: $p'^\mu = (E_{\mathbf{p'}}, \vec{p'})$
four-momentum transfer: $q^\mu = k^\mu - k'^\mu$

$E_{\mathbf{p}}$ is the energy that a free nucleon with momentum $\vec{p}$ would have
$\tilde{E}$ is the removal energy, defined in the following way:

$$\tilde{E} = M + \omega - E_{\mathbf{p'}}$$

ω is the energy component of k^μ

SF nuclear model

- Total cross-section for a given neutrino

$$\sigma = \int d^3k' \int d\tilde{E} \int d^3p \frac{G_F^2 \cos^2\theta_C}{8\pi^2 E_\nu E' E_p E_{p'}} \delta(\omega + M - \tilde{E} - E_{p'}) P(\tilde{E}, \vec{p}) \times L_{\mu\nu} H^{\mu\nu}$$

- P(E, p) is the chance of finding a nucleon with a given momentum and energy

- $E^{\sim}$ is defined as: $M - E^{\sim} = E_{nu} - E_{lep} - E_{p'}$

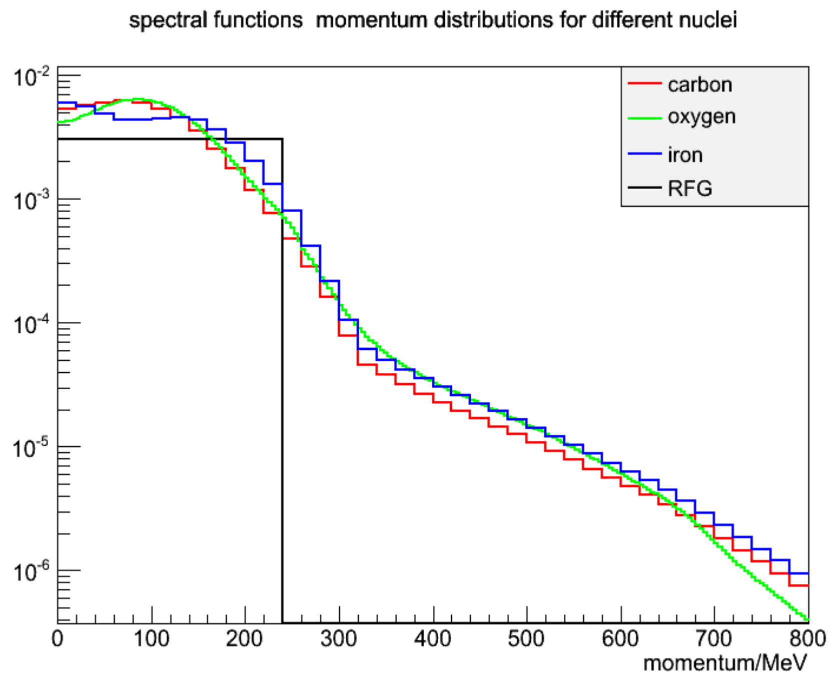
- RFG model has a flat momentum distribution, and a constant binding energy

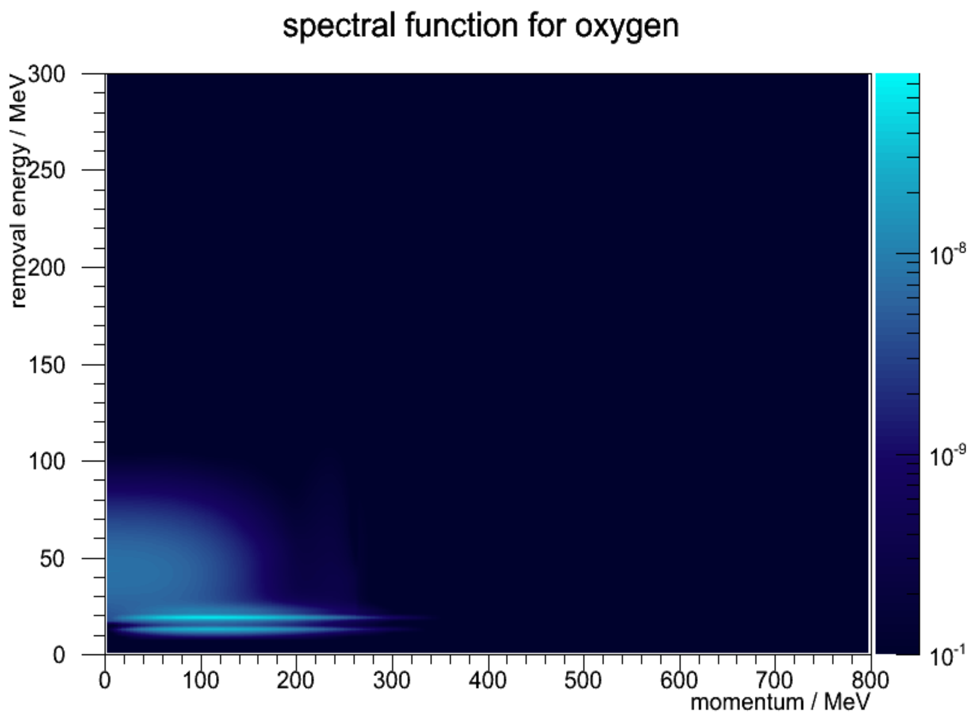
Spectral function

- O. Benhar's spectral function used in NEUT for oxygen, carbon, iron.

- Combination of nuclear theory and electron scattering fits (Nucl. Phys. A579 493)

- Features:
    - Clear structure from shell model
    - Long tail from correlated pairs of nucleons (also leads to 2-nucleon ejection)





Implementation in NEUT

- Implementation based on NuWro
    - Much from discussion with Jan Sobczyk
    - Currently no implementation of second nucleon ejection

- Basic idea:
    - Throw events evenly over the space that the xsec integral is over (d^3p, d^3k, $\tilde{E}$)
    - For each event, calculate the cross-section (differential in all variables)
        - given integrand on slide 3
    - Throw against a maximum differential cross-section (again, over all variables)

- Maximum xsec and total xsec pre-calculated
    - throw many events over phase space
    - save largest differential xsec found
    - total is average differential xsec over all events.

- Actually done in neutrino-nucleon centre-of-mass frame – integral over d^3k_{com}
- Requires Jacobian calculation – quite involved (in backups)

Final NEUT algorithm

1. Select a nucleon according to $P(\tilde{E}, \vec{p})$, using the rejection method
2. Calculate the centre-of-mass energy, this must be larger than the sum of the lepton and final hadron mass, if it isn't then reject the event
3. Shift into the neutrino-nucleon centre-of-mass frame
4. Create the outgoing lepton and nucleon in a random direction
5. Boost final state particles back into lab frame
6. Apply pauli blocking (if $p' < pF_{SF}$, reject the event)
7. Calculate $\vec{q}$ and $\tilde{\vec{q}} = (\tilde{\omega}, \vec{q})$, where $\tilde{\omega} = E_{p'} - E_p$
8. calculate weight from cross-section formula (where $L_{\mu\nu}H^{\mu\nu}$ is calculated using the reduced energy transfer, $\tilde{\omega}$)
9. Use rejection method again to decide whether event occurs or not. If not, goto 1.

Validation

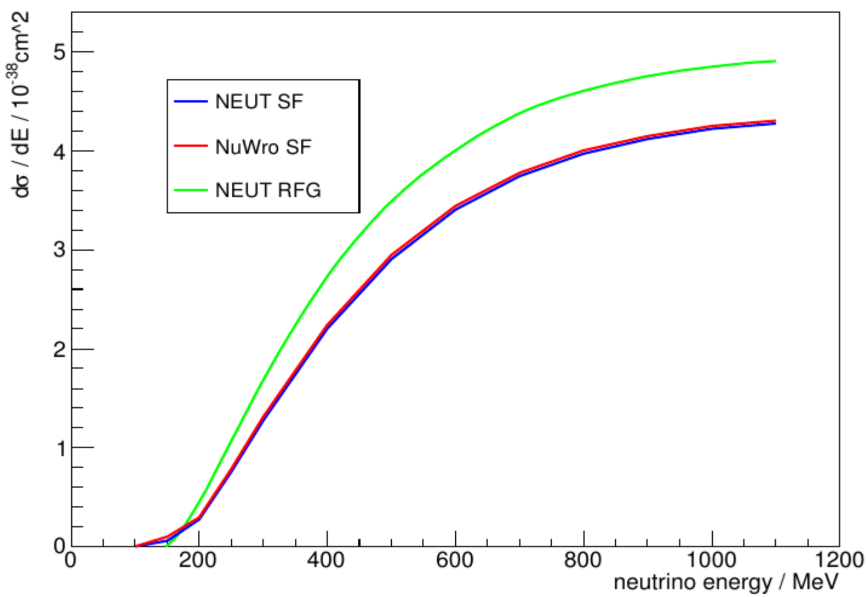
- Code was validated by comparing to NuWro output

- Comparisons done over a range of energies, neutrino types, and target nuclei

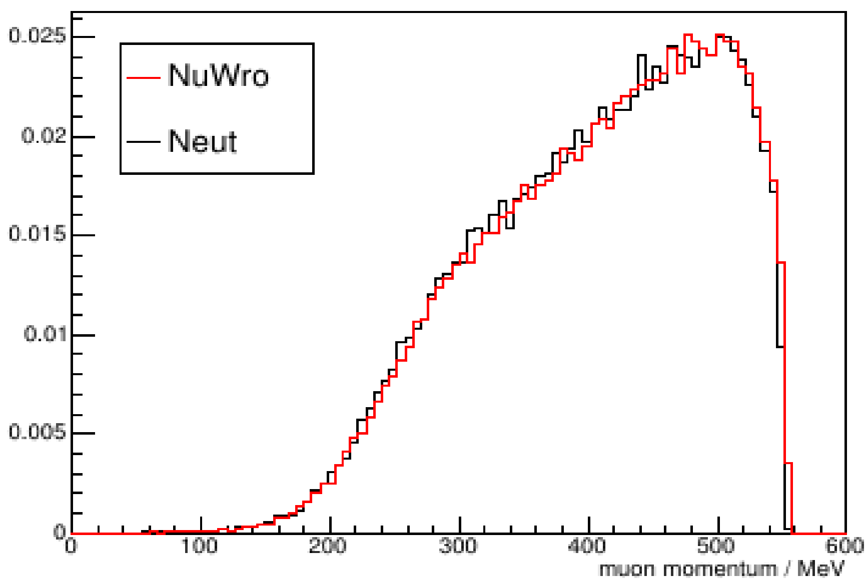
- Total cross-sections agree to < 1%
  - slight discrepancy at energy turn-on due to NuWro using an anlaytical fit to SF

- Shapes agree to within statistics

Total cross-section for numu on carbon



Cross-section against muon momentum for
600 MeV numu on carbon

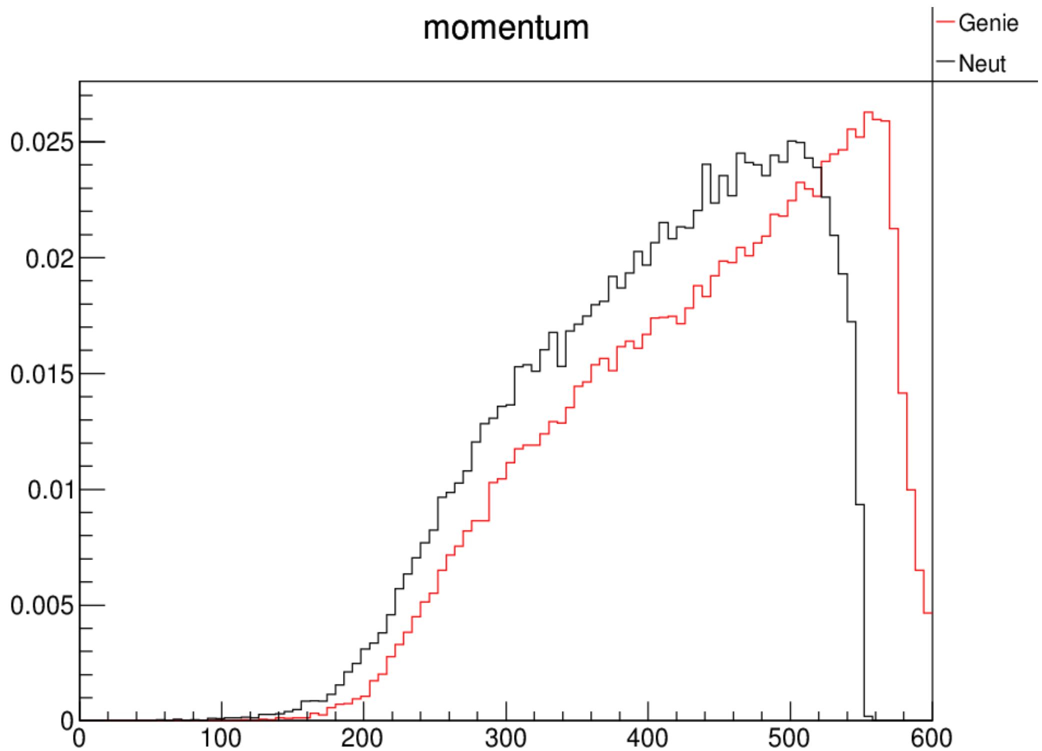


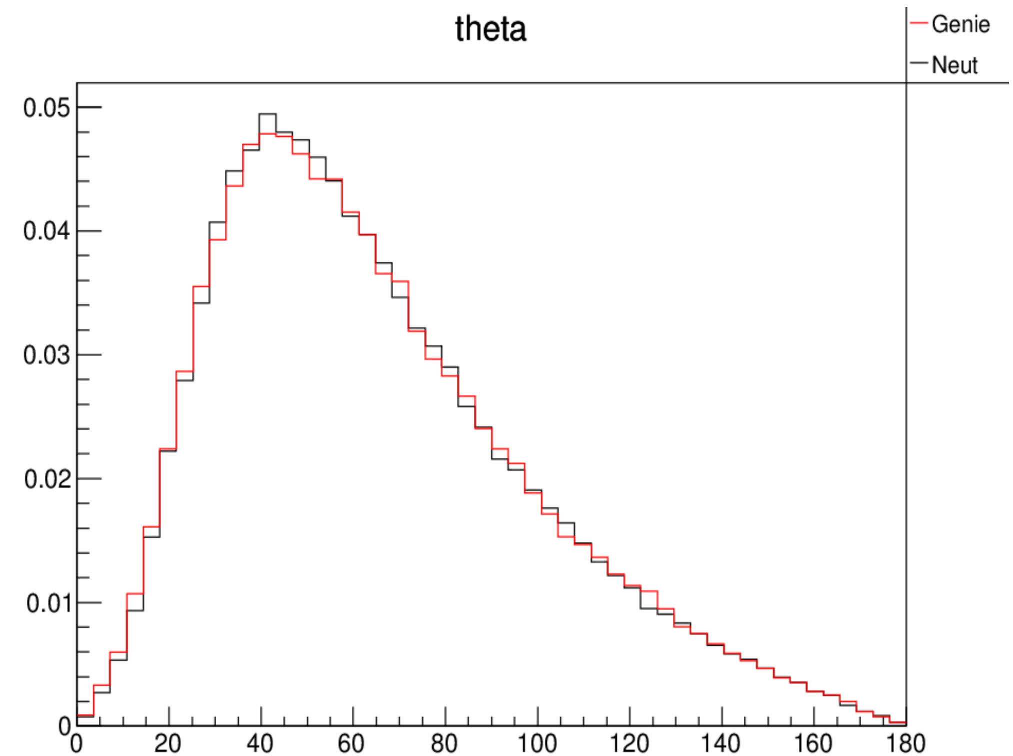
Old GENIE implementation

- Some time ago, Costas added the spectral function nuclear model into GENIE
  - Re-uses as much RFG code as possible (except selection of initial nucleon)

- Unfortunately he couldn't get the distributions to line up with those from Omar

- Here is a comparison of the GENIE SF with NEUT SF





GENIE differences

- SF mode in GENIE does not agree with NEUT

1. Energy conservation (“off-shell-ness”) defined differently
   - initial nucleon energy now (M - E~)

2. Outgoing nucleon off-shell until after FSI
   - outgoing nucleon is now on-shell when created
   - Pauli-blocking and FSI applied to this on-shell nucleon

3. More complex – GENIE doesn't perform the selection over the whole phase space evenly.
   - initial nucleon selected, then Q2 selected from allowed region
   - Q2 allowed region depends on initial nucleon
   - not always selecting over the same Q2 region
   - events are weighted incorrectly

   - more detailed explanation on the next slides

GENIE algorithm

1. Select a nucleon from the nuclear model's momentum/binding energy distribution

2. move into nucleon rest frame

3. calculate physically allowed Q^2 range

4. select Q^2 from this region

5. calculate differential cross-section for this Q^2

6. throw against a maximum value
(calculated for the neutrino energy in nucleon rest frame)

7. if event is rejected, select a new Q^2 (step 4)

GENIE Q2 selection

- imagine a fake (very unrealistic) nuclear model for initial nucleons:
    - $p_x = p_y = 0$
    - p_z flat between -1 and +1 (+1 is same direction as neutrino beam)
    - no binding energy

- The cross section in the lab frame is now only dependent on p_z, k, θ_μ
    - azimuthal dependence removed
    - can be written in terms of p_z and Q^2

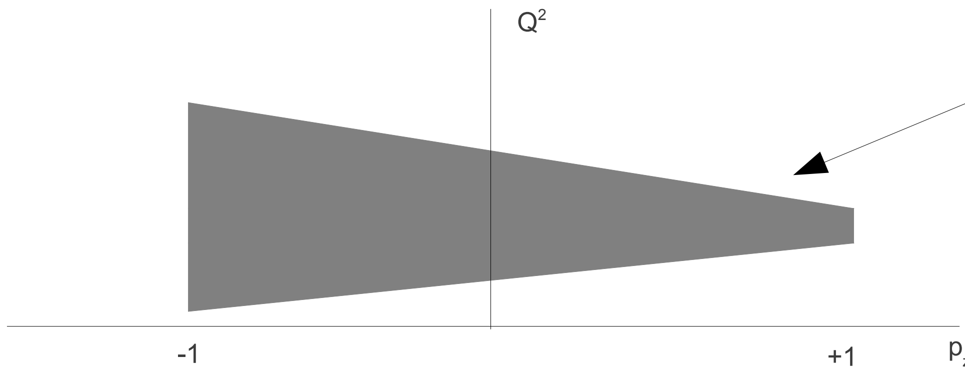
- we also have a fake differential cross-section is 1 for all physical events
    - 0 for unphysical events

- Differential cross-section can now be visualised in 2D easily
    - can sketch it based on what we know

- We will consider what the GENIE algorithm does for this model

GENIE Q2 selection - 2

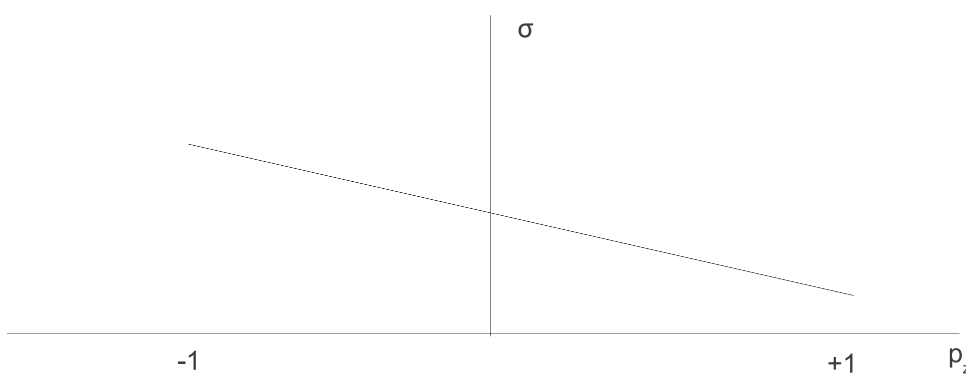
- Differential cross-section looks something like:



Less Q2 space available here – lower centre-of-mass energy

- Grey areas cross-section = 1

- If we integrate this over Q2, we find the cross-section as a function of pz looks like:



GENIE Q2 selection - 3

- What GENIE does:
  - select p_z at random
  - find the allowed region of Q^2 for that p_z
  - select a value of Q^2 within that region
  - calculate weight and throw against maximum weight (always selected here)
  - event is generated

  - this means, all p_z are equally likely
  - but I just showed the cross-section as a function of p_z
    - and it wasn't flat!

- What we have to do:
  - find the range of p_z and Q^2 allowed
  - select a pair
  - calculate the differential cross-section (either 0 or 1 in this case)
  - decide whether to accept that pair (accept if 1 for this model)
  - if rejected, select a new pair and start again

  - this way, we take into account the more limited phase-space of higher p_z

New GENIE algorithm

1. Throw lots of nucleons from the nuclear model, find the total Q^2 range allowed
   -also find the largest differential cross-section for the nucleus, save both

2. select a nucleon from the nuclear model

3. move into nucleon rest frame

4. select Q^2 from region found in step 1

5. calculate physically allowed Q^2 range

6. reject event if Q^2 is outside this range (if rejected – restart from step 2)

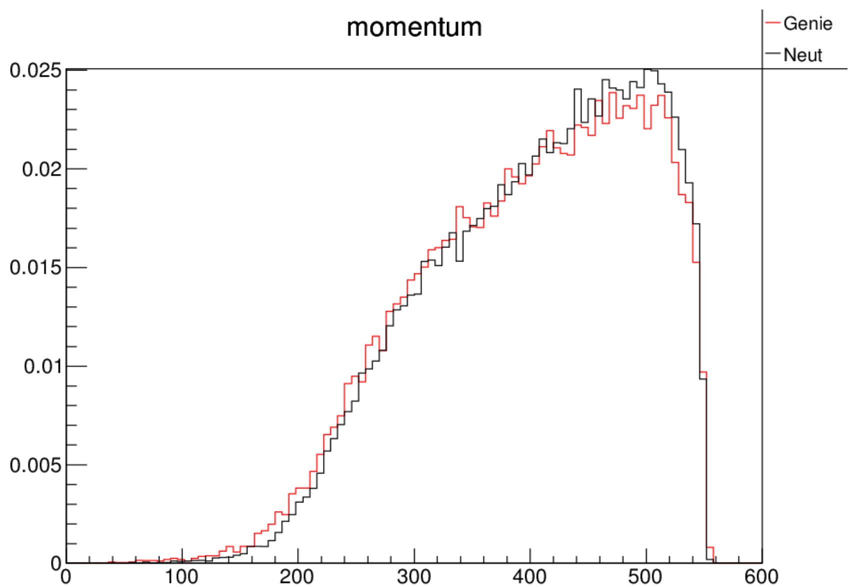
7. if allowed, calculate differential cross-section for this Q^2

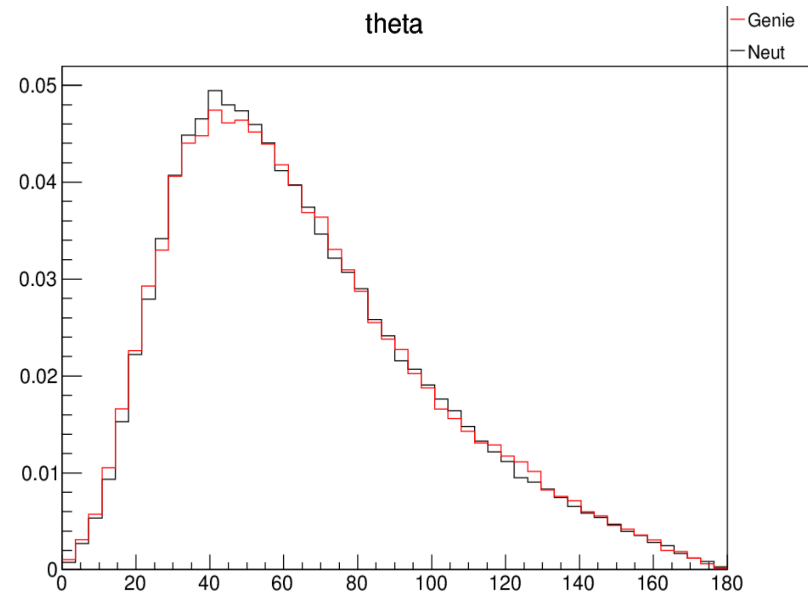
6. throw against a maximum value
(calculated for the neutrino energy in lab frame)

7. if event is rejected, select a new nucleon and Q^2 (restart from step 2)

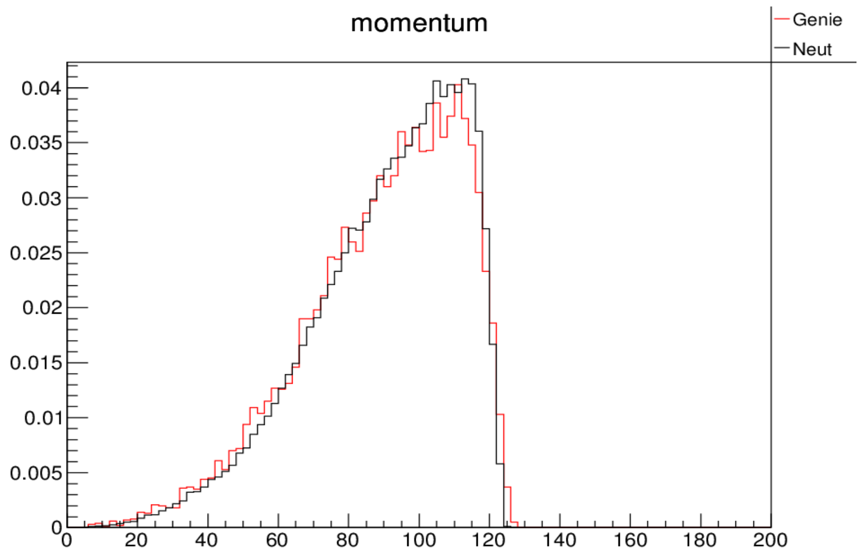
new GENIE comparisons

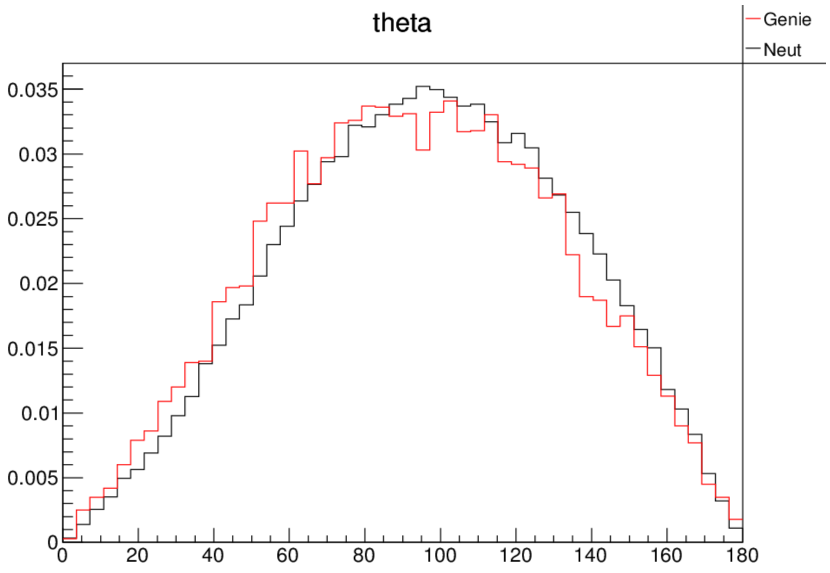
600MeV





200meV





Conclusions

- The GENIE algorithm for CCQE events is slightly incorrect

- This has a fairly large effect when using the SF nuclear model

- The algorithm can be edited fairly simply

- The SF development branch of GENIE contains these changes
    - (devel_branch_not_for_users_spectral_func_test_1)

- However:
    - Forcing the reselection of nucleons slows the algorithm down considerably
    - Generating electron scattering events is difficult due to inefficiency

Backup slides

(Taken from a talk by Cezary Juszczak about SF in NuWro)

Better algorithm – Jacobian computation

Notations

$$x = \frac{G_F^2 \cos^2\theta_C}{8\pi^2 E_{\mathbf{k}}}, \qquad y = E_{\mathbf{k}} + M - E, \qquad z = \frac{L^{\text{weak}}_{\mu\nu} H^{\mu\nu}_{\text{weak}}}{E_{\mathbf{p}} E_{\mathbf{k}'} E_{\mathbf{p}'}}$$

we can now write:

Cross section

$$d\sigma^{\text{weak}} = x \int dE\, d^3p\, P_{(n)}(\mathbf{p}, E) \int d^3k'\, \delta(E_{\mathbf{k}'} + E_{\mathbf{p}'} - y) z$$

Inside the integral over d^3k' the variable y is constant so we can proseed to solve the δ.

Better algorithm – Jacobian computation

velocity of CMS frame

Since $\mathbf{k} + \mathbf{p} = \mathbf{k}' + \mathbf{p}'$ so

$$\mathbf{v} = \frac{\mathbf{p}' + \mathbf{k}'}{E_{\mathbf{k}'} + E_{\mathbf{p}'}} = \frac{\mathbf{p} + \mathbf{k}}{y}, \qquad \gamma \equiv \frac{1}{\sqrt{1 - \mathbf{v}^2}}.$$

Lets $E_{\mathbf{p}_0'}$, $E_{\mathbf{k}_0'}$, $\mathbf{p}_0'$, $\mathbf{k}_0'$ – energies and momenta in CMS.

δ in CMS frame depends only on $|\mathbf{k}_0'|$

Since $\mathbf{p}_0' + \mathbf{k}_0' = 0$ so

$$\delta(E_{\mathbf{k}'} + E_{\mathbf{p}'} - y) = \delta(\gamma(E_{\mathbf{p}_0'} + E_{\mathbf{k}_0'}) - y) =$$

$$= \delta(\gamma(\sqrt{M^2 + {\mathbf{k}_0'}^2} + \sqrt{m^2 + {\mathbf{k}_0'}^2}) - y)$$

Better algorithm – Jacobian computation

The radius of the sphere

$$|\mathbf{k}_0'| = \sqrt{\frac{(s + m^2 - M^2)^2}{4s} - m^2}, \qquad \text{where} \quad s = \left(\frac{y}{\gamma}\right)^2$$

Back in the LAB frame it is stretched in the **v** direction

$$\begin{cases} k_x' = \gamma k_{0x}' + \gamma v E_{\mathbf{k}_0'}, \\ k_y' = k_{0y}', \\ k_z' = k_{0z}', \end{cases}$$

Better algorithm – Jacobian computation

To use the formula

$$\int \delta(f(x))g(x)d^3x = \oint_{f(x)=0} \frac{g(x)}{|\nabla f(x)|}dS$$

we calculate

Gradient

$$|\nabla_{\mathbf{k}'}(E_{\mathbf{k}'} + E_{\mathbf{p}'} - y)| = \left|\frac{\mathbf{k}'}{E_{\mathbf{k}'}} - \frac{\mathbf{p}'}{E_{\mathbf{p}'}}\right| = |\mathbf{v}_{\mathbf{k}'} - \mathbf{v}_{\mathbf{p}'}|$$

and

Surface element

$$dS = \sqrt{\sin^2\theta_0 + \gamma\cos^2\theta_0}dS_0 \quad where \quad dS_0 = |\mathbf{k}'_0|^2 d\phi_0 d\cos\theta_0$$

Better algorithm – Jacobian computation

Cross section formula

$$d\sigma^{\mathrm{weak}} = \frac{G_F^2 \cos^2\theta_C}{8\pi^2 E_{\mathbf{k}}} \int dE\, d^3p\, P_{(n)}(\mathbf{p}, E) \int \frac{L_{\mu\nu}^{\mathrm{weak}} \widetilde{H}^{\mu\nu}_{\mathrm{weak}}}{E_{\mathbf{p}} E_{\mathbf{k}'} E_{\mathbf{p}'}} \frac{\sqrt{\sin^2\theta_0 + \gamma\cos^2\theta_0}}{|\mathbf{v}_{\mathbf{k}'} - \mathbf{v}_{\mathbf{p}'}|} |\mathbf{k}_0'|^2 d\phi_0 d\cos\theta_0$$

where:

θ_0 – angle between $\mathbf{k}_0'$ and $\mathbf{v}$.

ϕ_0 – angle of $\mathbf{k}_0'$ projected on the plane normal to $\mathbf{v}$.