

# Peak to Average Power Reduction Analysis of Radar Spread Communication Signal

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**Abstract**—This paper is...

## I. INTRODUCTION

Introduction here.

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## II. RADAR SPREADING

Shared spectrum access between radar and communications (SSPARC) requires the mitigation of radar interference. One approach of this is to orthogonalize the communication and the radar signal. Spectral spreading the communication signal  $\mathbf{x} \in \mathbb{C}^{N_x}$  with the radar signal  $\mathbf{r} \in \mathbb{C}^{N_r}$  is one way to approximate this orthogonalization. The spread transmitted signal will be

$$\mathbf{y} = \mathbf{F}_r \mathbf{x} \quad (1)$$

where  $\mathbf{x}$  is the communication constellation symbols,  $\mathbf{F}_r$  is the circulant matrix of the radar signal. To maintain the proper size of the signal,  $N_x \geq N_r$ , and  $\mathbf{F}_r$  is a circular convolution with zero padding of  $N_x - N_r$  to be  $N_x \times N_x$  matrix.

The receiver will receive the signal

$$\mathbf{z} = \mathbf{F}_h \mathbf{F}_r \mathbf{x} + \beta[\mathbf{r}, 0, 0, \dots, 0, 0]^T + \mathbf{n} \quad (2)$$

where  $\mathbf{F}_h$  is the channel effect, The radar signal  $\mathbf{r}$  is zero padded to match the size of the communication signal.

Radar signals are generally composed of pulse compression waveform in order to decouple the range resolution and signal energy. This pulse compression waveform of the radar signal allows for the following proprieties:

$$\mathbf{F}_r^H \mathbf{r} \approx [1, 0, 0, \dots, 0, 0]^T \approx \mathbf{0} \quad (3)$$

$$\mathbf{F}_r^H \mathbf{F}_r \approx \mathbf{I} \quad (4)$$

$H$  denotes the Hermitian conjugate of matrix and  $T$  denote the Transpose of the matrix. The Hermitian conjugate of a circulant matrix is equal to the matched filter convolution matrix. (3)

The de-spreading of the signal is done by

$$\hat{\mathbf{x}} = \mathbf{F}_r^H \mathbf{z} \quad (5)$$

using the radar proprieties from (3) and (4) you get

$$= \mathbf{F}_h \mathbf{x} + \beta[h, 0, 0, \dots, 0, 0]^T + \mathbf{F}_r^H \mathbf{n} \quad (6)$$

This shows that radar signal  $\beta$  has compressed, mitigating the interference to just few samples. However there is a problem with this mitigation. The peak to average power (PAR) of the signal is very high.

The high PAR causes nonlinear distortion when the signals come close to or exceeds the saturation level of the power amplifier. To resolve this problem, several options were considered and analyzed.

## III. PAR REDUCTION

The PAR of the signal is defined as

$$PAR(x) = \frac{\|x\|_\infty^2}{\|x\|_2^2 / N_x} \quad (7)$$

where  $\|\cdot\|_l$  denotes the  $l$ -norm of the vector. Since radar spreading aggregate the signal to  $y_i = \sum_{j=0}^{N_x-1} F_{r,ij} x_j$ , and since  $x$  is i.i.d random variable with expectation  $E[x_j] = \mu$  and variance  $Var[x_i] = \sigma^2$ . This means that signal  $y$  has expectation  $E[y] \approx N_x \mu$  and variance  $Var[y] \approx N_x \sigma^2$ . As a result, the radar spread signal tends to have high PAR.

Having a high PAR means that the signal power inefficient and is prone to non-linear distortion from power amplifier. A general PAR reduction problem for aggregated signal is given by

$$\begin{aligned} &\text{Minimize} \quad \left\| \sum_{p=1}^P \alpha_p A_p \left( x_p^{(k)} + \epsilon_p^{(k)} \right) \right\|_\infty \\ &\text{Subject to} \quad k \in \mathcal{K} \\ &\quad \alpha_p \in \mathcal{A}_p \\ &\quad \epsilon_p^{(k)} \in \mathcal{E}_p \end{aligned} \quad (8)$$

where  $\mathcal{K}$  is a set of alternate signal,  $\mathcal{A}$  is a set of combination value, and  $\mathcal{E}$  is a set of allowed error values. Since the combination value have very limited effect on the PAR reduction, we will focus on the alternate signals and error values.

### A. Alternate signals

Lets say there are multiple signals to represent the same information or get the same radar knowledge, than the transmitted signal (1) can be expanded as

$$\begin{aligned} y^{(1)} &= F_r x_1^{(1)} + F_r x_2^{(1)} + F_r x_3^{(1)} + \dots \\ y^{(2)} &= F_r x_1^{(2)} + F_r x_2^{(2)} + F_r x_3^{(2)} + \dots \\ &\vdots \\ y^{(\mathcal{K})} &= F_r x_1^{(\mathcal{K})} + F_r x_2^{(\mathcal{K})} + F_r x_3^{(\mathcal{K})} + \dots \end{aligned}$$

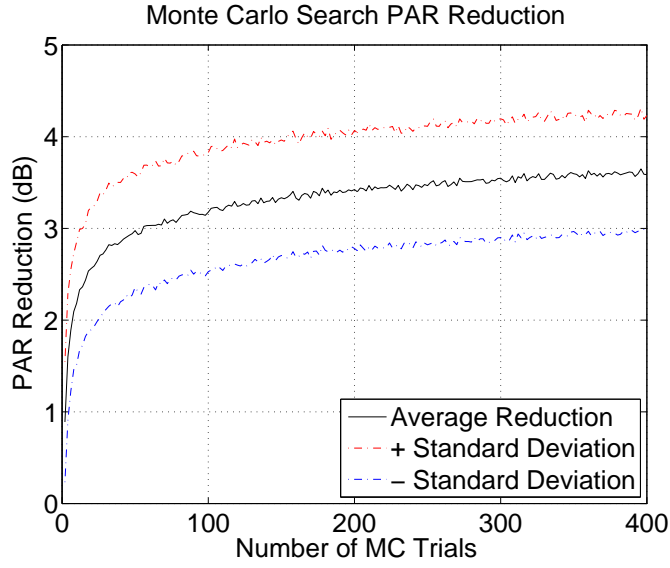


Fig. 1. Monte Carlo search of alternate signals

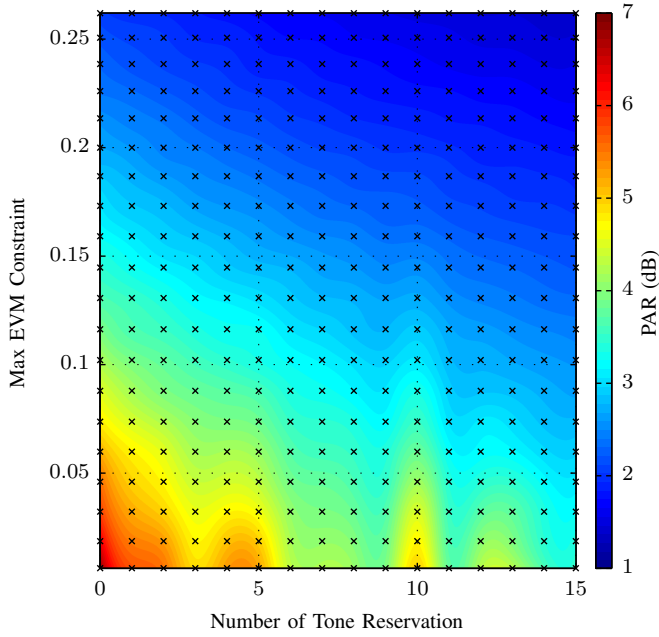


Fig. 2. enter caption here

where  $\mathcal{K}$  number of possible alternate signals. If the receiver is aware of all the alternative possibility and able to determine which signals was used. We may be able to reduce PAR by selecting a signal that gives the lowest PAR.

#### REFERENCES

- [1] H. Kopka and P. W. Daly, *A Guide to L<sup>A</sup>T<sub>E</sub>X*, 3rd ed. Harlow, England: Addison-Wesley, 1999.