Aerosonde Modeling and Control, Checkpoint1

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I. Introduction

Autonomous drones are used in many different fields to achieve many different goals, and today their operations include reconnaissance, transportation, and data collection. An early pioneer in autonomous drones was Aerosonde Robotic Aircraft Ltd of Australia (now part of Textron Systems), which devopled the Aerosonde fixed wing drone in the 1990s, as shown in Figure 1. In 1998 the Arosonde "Laima" made the first autonomous flight across the Atlantic, crossing in 26 hours and 45 minutes [1]. The Arosonde is designed as a remote sensing platform with GPS navigation that can measure temperature, humidity, pressure, and wind [2]. The Aerosonde's specifications and modeling parameters are well documented and publicly available. The specifications were provided in the AeroSim Blockset for Simulink. These values have also been used to validate the modeling parameters derived for other similar platforms [3].

Fig. 1. The Aerosonde Laima.

In this paper, the equations of motion for a general fixed wing drone will be described and then applied specifically to model the Aerosonde. Then the equations will be lineraized so that the control characteristcs of the system can be determined.

II. Assumptions

The drone will be modeled as a rigid body with a plane of mass symmetry. Looking from above the drone, a line can be drawn from the nose to the tail which will divide the drone symmetrically.

Three separate reference frames will be used to describe the motion of the drone. The earth frame will have its origin centered at ground level, with the x axis pointing north, the y axis pointing east, and the z axis pointing into the earth. This axis will remain fixed in place, so the position of the drone with respect to this frame will help determine if it has reached set waypoints. The earth frame is assumed to be an inertial frame, so it is assumed that the earth does not rotate or accelerate during flight. A flat earth is also assumed.

The body frame will have its origin at the center of gravity of the drone. The x axis points from the center to the nose of the drone along the plane of mass symmetry, the z axis points down along the plane of mass symmetry, and the y axis points out along the wing to complete the right handed coordinate frame. The body frame is useful for describing angular velocity since is is the only frame where the moments of inertia are constant.

Finally, the wind frame also has its origin at the center of gravity, but the x axis points along the velocity vector for the drone, the z axis normal to the x axis in the plane of mass symmetry, and the y axis completing the right hand coordinate frame. The wind frame is useful for describing the aerodynamic forces.

Twelve state variables will be used to describe the system. (x, y, z) will describe the location of the drone in the earth frame. (ϕ, θ, ϕ) are the Euler angles that represent the rotation from the earth frame to the body frame. (u, v, w) are the velocities of the drone with respect to the body frame. (p, q, r) are the angular velocities with respect to the body frame.

Four inputs will be used to control the system. $Thrust$ will represent the force applied by the motor. $\delta elevator$ will represent the angle of the elevator surface used to control pitch. $\delta aileron$ will represent the angle of the ailerons used to control roll. Finally, $\delta rudder$ will represent the angle of the rudder used to control yaw.

The aerodynamic forces of Lift (L) and Drag (D) will be applied in the wind frame. The drag force acts opposite to the velocity vector. The lift force acts normal to the velocity vector in the plane of mass symmetry. The force of gravity, g will act in the earth frame, and gravity is constant. F_y is a force that acts in the y axis of the body frame, which is influenced by p, r, $\delta aileron$, and $\delta rudder$.

The torques about the body x axis, y axis, and z axis are represented as (l, m, n), respectively.

It is assumed that the atmosphere is stationary with respect to the earth frame.

It is assumed that the thrust is a force produced along the body x axis. It is assumed that the thrust vector originates at the center of mass, and thus the thrust does not generate a pitching torque m. This is assumed because there is no Aerosonde parameter given to relate m as a function of $Thrust$. Just from pictures of the Aerosonde, this appears to be a reasonable assumption, since the motor seems very near to the center of mass.

V is the magnitude of the velocity vector. α is the angle of attack, or the angle from the z axis of the wind frame to the z axis of the body frame. β is the sideslip angle, or the angle from the the y axis of the body frame to the y axis of the wind frame.

$$V = \sqrt{u^2 + v^2 + w^2} \tag{1}$$

$$\alpha = \arctan w/u \tag{2}$$

$$\beta = \arcsin v/V \tag{3}$$

III. Equations of Motion

The translational kinematics of the drone in Equation 4 relate the velocities in the body frame (u, v, w) to the velocities in the earth frame $(\dot{x}, \dot{y}, \dot{z})$. s is $sine$ and c is $cosine$.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} c\theta c\psi & -s\psi c\phi + c\psi s\theta s\phi & s\theta s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{4}$$

The rotational kinematics of the drone in Equation 5 relate the angular velocities in the body frame (p, q, r) to the angular velocities in the earth frame $(\dot{\phi}, \dot{\theta}, \dot{\psi})$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\phi tan\theta & c\phi tan\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi sec\theta & c\phi sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{5}$$

The translational dynamics of the drone in Equation 6 are obtained by using the transport theorem to obtain Newton's second law in a non inertial frame (the body frame). Then all the forces are represented in the body frame and summed together. M is the mass of the drone.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -gs\theta - \frac{Dc\alpha c\beta}{M} + \frac{Ls\alpha}{M} + \frac{Thrust}{M} - qw + rv \\ gc\theta s\phi - \frac{Ds\beta}{M} + F_y - ru + pw \\ gc\theta c\phi - \frac{Ds\alpha c\beta}{M} - \frac{Lc\alpha}{M} - pv + qu \end{bmatrix} \tag{6}$$

The rotational dynamics of the drone in Equation 7 are obtained by using the transport theorem for Newton's second law for angular accelerations. I_{xx} is the moment of inertia about the x axis, I_{yy} is the moment of inertia about the y axis, and I_{zz} is the moment of inertia about the z axis. I_{xz} is the product of inertia, which is nonzero because the drone does not have mass symmetry about all axes.

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \tau_1 pq - \tau_2 qr + \tau_3 l + \tau_4 n \\ \tau_5 pr - \tau_6 (p^2 - r^2) + \frac{m}{I_{yy}} \\ \tau_7 pq - \tau_1 qr + \tau_4 l + \tau_8 n \end{bmatrix} \tag{7}$$

where

$$\tau = I_{xx} I_{zz} - I_{xz}^2 \tag{8}$$

$$\tau_1 = I_{xz}(I_{xx} - I_{yy} + I_{zz})/\tau \tag{9}$$

$$\tau_2 = (I_{zz}(I_{zz} - I_{yy}) + Ixz^2)/\tau \tag{10}$$

$$\tau_3 = I_{zz}/\tau \tag{11}$$

$$\tau_4 = I_{xz}/\tau \tag{12}$$

$$\tau_5 = (I_{zz} - I_{xx})/I_{yy} \tag{13}$$

$$\tau_6 = I_{xz}/I_{yy} \tag{14}$$

$$\tau_7 = ((I_{xx} - I_{yy})I_{xx} + I_{xz}^2)/\tau \tag{15}$$

$$\tau_8 = I_{xx}/\tau \tag{16}$$

S is the surface area of the wings. b is the wingspan. c is the wing chord. ρ is the air density.

L, D, and m are primarily influenced by α, q, and $\delta elevator$.

$$L = \frac{1}{2}\rho V^2 S(C_{L0} + C_{L\alpha}\alpha + C_{Lq}\frac{c}{2V}q \\ + C_{L\delta elevator}\delta elevator \tag{17}$$

$$D = \frac{1}{2}\rho V^2 S(C_{D0} + C_{D\alpha}\alpha + C_{Dq}\frac{c}{2V}q \\ + C_{D\delta elevator}\delta elevator \tag{18}$$

$$m = \frac{1}{2}\rho V^2 Sc(C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{c}{2V}q \\ + C_{m\delta elevator}\delta elevator \tag{19}$$

F_y, l, and n are primarily influenced by β, p, r, $\delta aileron$, and $\delta rudder$.

$$F_y = \frac{1}{2}\rho V^2 S(C_{y0} + C_{y\beta}\beta + C_{yp}\frac{b}{2V}p + C_{yr}\frac{b}{2V}r + C_{y\delta aileron}\delta aileron + C_{y\delta rudder}\delta rudder) \quad (20)$$

$$l = \frac{1}{2}\rho V^2 Sb(C_{l0} + C_{l\beta}\beta + C_{lp}\frac{b}{2V}p + C_{lr}\frac{b}{2V}r + C_{l\delta aileron}\delta aileron + C_{l\delta rudder}\delta rudder) \quad (21)$$

$$n = \frac{1}{2}\rho V^2 Sb(C_{n0} + C_{n\beta}\beta + C_{np}\frac{b}{2V}p + C_{nr}\frac{b}{2V}r + C_{n\delta aileron}\delta aileron + C_{n\delta rudder}\delta rudder) \quad (22)$$

The coefficients are dimensionless quantities that represent how much one variable affects another variable. The coefficients for the Aerosonde are publicly available [4]. Because the coefficients are dimensionless quantities, p, q, and r must be nondimensionalized, which is done using either the expression $\frac{b}{2V}$ or $\frac{c}{2V}$.

IV. Linearizing the Model

The equations of motion can be linearized about a trajectory. Steady straight flight is the trajectory chosen here. In steady state flight, $u = u_0$, $w = w_0$, $\theta = \theta_0$, and $\psi = \psi_0$ where $(w_0, u_0, \theta_0, \psi_0)$ are constant. The drone will move in a straight line without accelerating, but the yaw angle and pitch angle do not have to be 0.

When the equations of motion are linearized, they will naturally separate into equations for lateral movement and equations for longitudinal movement. The lateral variable are (v, p, r, ϕ, ψ, y) with inputs $(\delta aileron, \delta rudder)$. The longitudinal variable are (u, w, q, θ, x, z) with inputs $(Thrust, \delta elevator)$.

The linearized longitudinal equation of motion is Equation 23.

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\\ \Delta\dot{x}\\ \Delta\dot{z}\end{bmatrix} = \begin{bmatrix} A_{uu} & A_{uw} & A_{uq} & A_{u\theta} & 0 & 0\\ A_{wu} & A_{ww} & A_{wq} & A_{w\theta} & 0 & 0\\ A_{qu} & A_{qw} & A_{qq} & A_{q\theta} & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ A_{xu} & A_{xw} & 0 & A_{x\theta} & 0 & 0\\ A_{zu} & A_{zw} & 0 & A_{z\theta} & 0 & 0\end{bmatrix} \begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\\ \Delta x\\ \Delta z\end{bmatrix} \begin{bmatrix}\frac{1}{M} & B_{u\delta elevator}\\ 0 & B_{u\delta elevator}\\ 0 & B_{u\delta elevator}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix} \begin{bmatrix}\Delta Thrust\\ \Delta\delta elevator\end{bmatrix} \quad (23)$$

The linearized lateral equation of motion is Equation 24.

$$\begin{bmatrix}\Delta\dot{v}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\phi}\\ \Delta\dot{\psi}\\ \Delta\dot{y}\end{bmatrix} = \begin{bmatrix} A_{vv} & A_{vp} & A_{vr} & A_{v\phi} & 0 & 0\\ A_{pv} & A_{pp} & A_{pr} & 0 & 0 & 0\\ A_{rv} & A_{rp} & A_{rr} & 0 & 0 & 0\\ 0 & 1 & A_{\phi r} & A_{\phi\phi} & 0 & 0\\ 0 & 0 & A_{\psi r} & A_{\psi\phi} & 0 & 0\\ A_{yv} & 0 & 0 & 0 & A_{t\psi} & 0\end{bmatrix} \begin{bmatrix}\Delta v\\ \Delta p\\ \Delta r\\ \Delta\phi\\ \Delta\psi\\ \Delta y\end{bmatrix} \begin{bmatrix} B_{v\delta aileron} & B_{v\delta rudder}\\ B_{p\delta aileron} & B_{p\delta rudder}\\ B_{r\delta aileron} & B_{r\delta rudder}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix} \begin{bmatrix}\Delta\delta aileron\\ \Delta\delta rudder\end{bmatrix} \quad (24)$$

The elements of the matrices for the longitudinal equations can be found in the appendix as an example. The rest of the elements can be found in Chapter 5 of *Small Unmanned Aircraft* [4].

V. Simulation

The simulation results can be seen in the included video. First, the equilibrium trajectory is shown. Next, the system response to longitudinal disturbances when θ is small is shown. Then, the system response to longitudinal disturbances when θ is large is shown. Finally, the system response to lateral disturbances is shown.

VI. Control Metrics

If x and z are removed from the linearized model and the model is linearized about a straight flight trajectory where θ is relatively small, the system has 2 pairs of complex eigenvalues that are stable, as seen in Figure 2 (If x and z are included, it only adds two eigenvalues that are 0). The first pair has a real part of -3.92, so the oscillations induced by these modes are relatively small. The second pair of eigenvalues has a real part of -0.05, so the oscillations induced by these modes are relatively large. These mean that u, w, q, and θ all oscillate and approach the equilibrium value after a disturbance, as seen in Figure 3

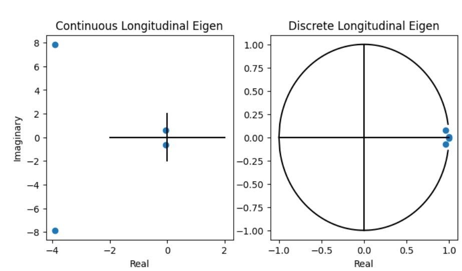

Fig. 2. The longitudinal eigenvalues when θ is small.

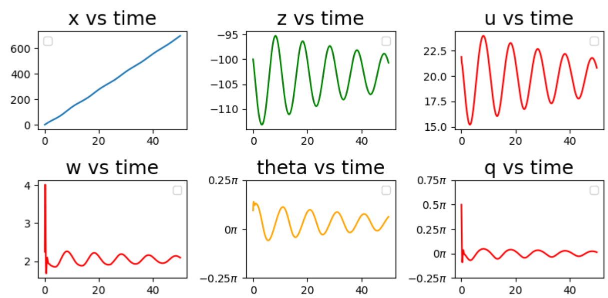

Fig. 3. The response to a longitudinal disturbance when θ is small.

When θ is large, one pair of eigenvalues for the longitudinal dynamics has a positive real part, meaning the pair is unstable, as seen in Figure 4. This means that the u, w, q, and θ will not return to the equilibrium trajectory, as shown in Figure 5

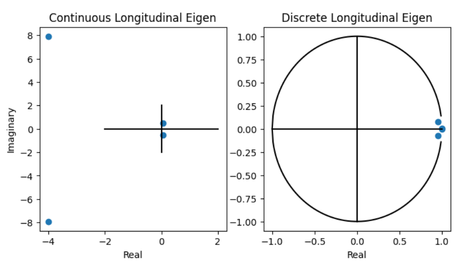

Fig. 4. The longitudinal eigenvalues when θ is large.

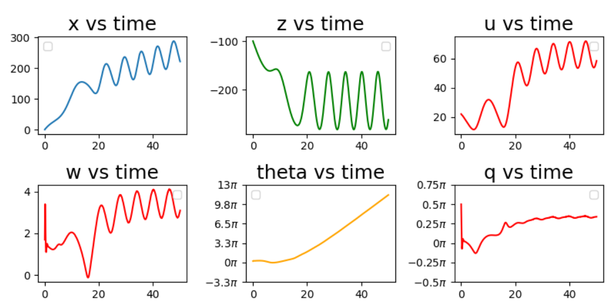

Fig. 5. The response to a longitudinal disturbance when θ is large.

If psi and y are removed from the lateral linearized model and the model is linearized about a straight flight trajectory, the system has a pair of complex eigenvalues that are stable, an additional eigenvalue that is stable, and a final eigenvalue that is unstable, as seen in Figure 6. When the system experiences lateral disturbances, it appears that v, p, and r die down to the equilibrium initially, but ϕ moves away from the equilibrium trajectory, as seen in Figure 7

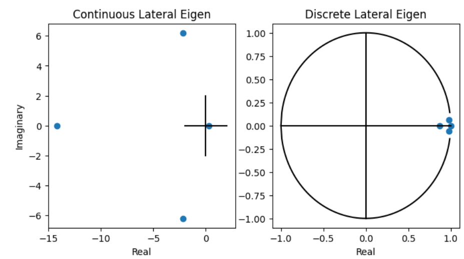

Fig. 6. The lateral eigenvalues.

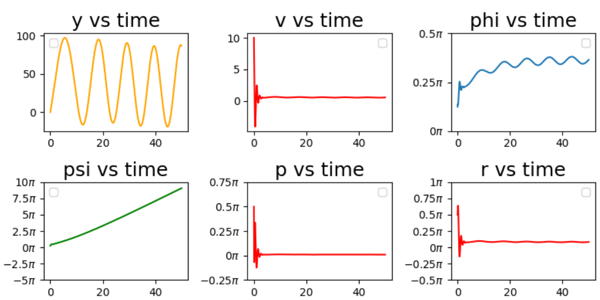

Fig. 7. The response to a lateral disturbance.

References

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[2] "Insitue aerosonde laima," in *The Muesume of Flight*, 2023.
[3] M. T. Burston, R. Sabatini, R. Clothier, A. Gardi, and S. Ramasamy, "Reverse engineering of a fixed wing unmanned aircraft 6-dof model for navigation and guidance applications," *Applied Mechanics and Materials*, vol. 629, pp. 164–169, 2014.
[4] R. W. Beard and T. W. McLain, *Small unmanned aircraft: Theory and practice*. Princeton university press, 2012.

VII. Appendices

The values for the elements of the A and B matrix for the linearized longitudinal equations.

$$A_{uu} = -\frac{u_0\rho S}{M}(C_{D0} + C_{D\delta elevator}\delta elevator_0 + C_{D\alpha}\alpha_0) + \frac{\rho S w_0 C_{D\alpha}}{2M} - \frac{\rho S c C_{Dq} q_0 u_0}{2mV_0} \tag{25}$$

$$A_{uw} = -\frac{w_0\rho S}{M}(C_{D0} + C_{D\delta elevator}\delta elevator_0 + C_{D\alpha}\alpha_0) - \frac{\rho S u_0 C_{D\alpha}}{2M} - \frac{\rho S c C_{Dq} q_0 w_0}{2mV_0} - q_0 \tag{26}$$

$$A_{uq} = -w_0 - \frac{\rho V_0 c S C_{Dq}}{4M} \tag{27}$$

$$A_{u\theta} = -g\cos\theta_0 \tag{28}$$

$$B_{u\delta elevator} = -\frac{\rho V_0^2 S C_{D\delta elevator}}{2M} \tag{29}$$

$$A_{wu} = -\frac{u_0\rho S}{M}(C_{L0} + C_{L\delta elevator}\delta elevator_0 + C_{L\alpha}\alpha_0) + \frac{\rho S w_0 C_{L\alpha}}{2M} - \frac{\rho S c C_{Lq} q_0 u_0}{2mV_0} + q_0 \tag{30}$$

$$A_{ww} = -\frac{w_0\rho S}{M}(C_{L0} + C_{L\delta elevator}\delta elevator_0 + C_{L\alpha}\alpha_0) - \frac{\rho S u_0 C_{L\alpha}}{2M} - \frac{\rho S c C_{Lq} q_0 w_0}{2mV_0} \tag{31}$$

$$A_{wq} = u_0 - \frac{\rho V_0 c S C_{Lq}}{4M} \tag{32}$$

$$A_{w\theta} = -g\sin\theta_0 \tag{33}$$

$$B_{w\delta elevator} = -\frac{\rho V_0^2 S C_{L\delta elevator}}{2M} \tag{34}$$

$$A_{qu} = \frac{u_0\rho S c}{I_{yy}}(C_{m0} + C_{m\delta elevator}\delta elevator_0 + C_{m\alpha}\alpha_0) - \frac{\rho S c w_0 C_{m\alpha}}{2I_{yy}} - \frac{\rho S c^2 C_{mq} q_0 u_0}{2I_{yy}V_0} \tag{35}$$

$$A_{qw} = \frac{w_0\rho S c}{I_{yy}}(C_{m0} + C_{m\delta elevator}\delta elevator_0 + C_{m\alpha}\alpha_0) - \frac{\rho S c u_0 C_{m\alpha}}{2I_{yy}} - \frac{\rho S c^2 C_{mq} q_0 w_0}{2I_{yy}V_0} \tag{36}$$

$$A_{qq} = \frac{\rho V_0 c^2 S C_{mq}}{4I_{yy}} \tag{37}$$

$$B_{q\delta elevator} = -\frac{\rho V_0^2 S c C_{m\delta elevator}}{2I_{yy}} \tag{38}$$

$$A_{xu} = \cos\theta_0 \tag{39}$$

$$A_{xw} = \sin\theta_0 \tag{40}$$

$$A_{x\theta} = w_0\cos\theta_0 - u_0\sin(\theta_0) \tag{41}$$

$$A_{zu} = -\sin\theta_0 \tag{42}$$

$$A_{zw} = \cos\theta_0 \tag{43}$$

$$A_{z\theta} = -u_0\cos\theta_0 - w_0\sin(\theta_0) \tag{44}$$