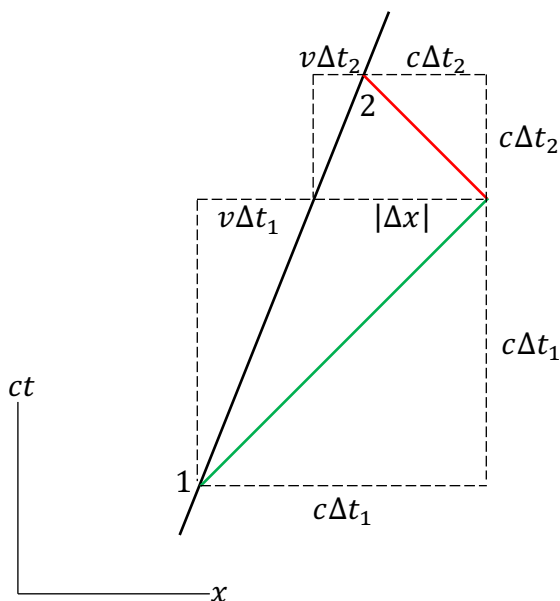


*Retarded wave* leaves particle when it is at 1 and arrives at  $(x, y)$  at time  $t$   
*Advanced wave* leaves particle when it is at 2 and arrives at  $(x, y)$  at time  $t$

$(x_p, y_p)$	particle position at time $t$
$(x, y)$	point on wave at time $t$
$\Delta x$	$x - x_p$
$\Delta y$	$y - y_p$
$t_1$	time when particle was at position 1
$t_2$	time when particle will be at position 2
$\Delta t_1$	$t - t_1$
$\Delta t_2$	$t_2 - t$
$x_1$	$x$ position of particle at time 1 $= x_p - v\Delta t_1$
$x_2$	$x$ position of particle at time 2 $= x_p + v\Delta t_2$

$\Delta y = 0$  (linear)



$$c\Delta t_2 = |\Delta x| - v\Delta t_2$$

$$(c + v)\Delta t_2 = |\Delta x|$$

$$\Delta t_2 = |\Delta x|/(c + v)$$

$$c\Delta t_1 = v\Delta t_1 + |\Delta x|$$

$$(c - v)\Delta t_1 = |\Delta x|$$

$$\Delta t_1 = |\Delta x|/(c - v)$$

$$\begin{aligned}
(c\Delta t_1)^2 &= (\Delta y)^2 + (v\Delta t_1 + \Delta x)^2 = (\Delta y)^2 + v^2(\Delta t_1)^2 + 2v\Delta t_1\Delta x + (\Delta x)^2 \\
c^2(\Delta t_1)^2 - v^2(\Delta t_1)^2 &= (\Delta y)^2 + 2v\Delta t_1\Delta x + (\Delta x)^2 \\
(c^2 - v^2)(\Delta t_1)^2 - 2v\Delta x\Delta t_1 - (\Delta y)^2 - (\Delta x)^2 &= 0 \\
(\Delta t_1)^2 - \frac{2v\Delta x}{(c^2 - v^2)}\Delta t_1 - \frac{(\Delta y)^2 + (\Delta x)^2}{(c^2 - v^2)} &= 0
\end{aligned}$$

$$\begin{aligned}
\Delta t_1 &= \left\{ \frac{2v\Delta x}{(c^2 - v^2)} \pm \sqrt{\left[ \frac{2v\Delta x}{(c^2 - v^2)} \right]^2 + 4 \frac{(\Delta y)^2 + (\Delta x)^2}{(c^2 - v^2)}} \right\} / 2 \\
&= \frac{1}{(c^2 - v^2)} \left[ v\Delta x \pm \sqrt{v^2(\Delta x)^2 + (c^2 - v^2)[(\Delta y)^2 + (\Delta x)^2]} \right] \\
&= \frac{1}{(c^2 - v^2)} \left[ v\Delta x \pm \sqrt{v^2(\Delta x)^2 + c^2(\Delta y)^2 - v^2(\Delta y)^2 + c^2(\Delta x)^2 - v^2(\Delta x)^2} \right] \\
&= \frac{1}{(c^2 - v^2)} \left[ v\Delta x \pm \sqrt{(c^2 - v^2)(\Delta y)^2 + c^2(\Delta x)^2} \right]
\end{aligned}$$

$$\pm \Rightarrow +$$

$$\boxed{\Delta t_1 = \frac{1}{(c^2 - v^2)} \left\{ v(x - x_p) + \sqrt{(c^2 - v^2)(y - y_p)^2 + c^2(x - x_p)^2} \right\}}$$

$$\begin{aligned}
\Delta x = 0 &\Rightarrow \Delta t_1 = \frac{\sqrt{(c^2 - v^2)(\Delta y)^2}}{(c^2 - v^2)} = \frac{\Delta y}{\sqrt{c^2 - v^2}} \\
(\Delta t_1)^2 &= \frac{(\Delta y)^2}{c^2 - v^2} \\
c^2(\Delta t_1)^2 - v^2(\Delta t_1)^2 &= (\Delta y)^2 \\
(c\Delta t_1)^2 &= (v\Delta t_1)^2 + (\Delta y)^2 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
v = 0 &\Rightarrow \Delta t_1 = \frac{1}{c^2} \left[ \sqrt{c^2(\Delta y)^2 + c^2(\Delta x)^2} \right] \\
(\Delta t_1)^2 &= \frac{1}{c^4} [c^2(\Delta y)^2 + c^2(\Delta x)^2] \\
(c\Delta t_1)^2 &= (\Delta y)^2 + (\Delta x)^2 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
\Delta y = 0 &\Rightarrow \Delta t_1 = \frac{1}{(c^2 - v^2)} \left[ v\Delta x + \sqrt{c^2(\Delta x)^2} \right] \\
\Delta t_1 &= \frac{v|\Delta x| + c|\Delta x|}{(c + v)(c - v)} = \frac{\Delta x}{c - v} \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
(c\Delta t_2)^2 &= (\Delta y)^2 + (v\Delta t_2 - \Delta x)^2 = (\Delta y)^2 + v^2(\Delta t_2)^2 - 2v\Delta t_2\Delta x + (\Delta x)^2 \\
c^2(\Delta t_2)^2 - v^2(\Delta t_2)^2 &= (\Delta y)^2 - 2v\Delta t_2\Delta x + (\Delta x)^2 \\
(c^2 - v^2)(\Delta t_2)^2 + 2v\Delta x\Delta t_2 - (\Delta y)^2 - (\Delta x)^2 &= 0 \\
(\Delta t_2)^2 + \frac{2v\Delta x}{(c^2 - v^2)}\Delta t_2 - \frac{(\Delta y)^2 + (\Delta x)^2}{(c^2 - v^2)} &= 0
\end{aligned}$$

$$\begin{aligned}
\Delta t_2 &= \left\{ \frac{-2v\Delta x}{(c^2 - v^2)} \pm \sqrt{\left[ \frac{2v\Delta x}{(c^2 - v^2)} \right]^2 + 4 \frac{(\Delta y)^2 + (\Delta x)^2}{(c^2 - v^2)}} \right\} / 2 \\
&= \frac{1}{(c^2 - v^2)} \left[ -v\Delta x \pm \sqrt{v^2(\Delta x)^2 + (c^2 - v^2)[(\Delta y)^2 + (\Delta x)^2]} \right] \\
&= \frac{1}{(c^2 - v^2)} \left[ -v\Delta x \pm \sqrt{v^2(\Delta x)^2 + c^2(\Delta y)^2 - v^2(\Delta y)^2 + c^2(\Delta x)^2 - v^2(\Delta x)^2} \right] \\
&= \frac{1}{(c^2 - v^2)} \left[ -v\Delta x \pm \sqrt{(c^2 - v^2)(\Delta y)^2 + c^2(\Delta x)^2} \right]
\end{aligned}$$

$$\pm \Rightarrow +$$

$$\Delta t_2 = \frac{1}{(c^2 - v^2)} \left\{ -v(x - x_p) + \sqrt{(c^2 - v^2)(y - y_p)^2 + c^2(x - x_p)^2} \right\}$$

$$\begin{aligned}
\Delta x = 0 &\Rightarrow \Delta t_2 = \frac{\sqrt{(c^2 - v^2)(\Delta y)^2}}{(c^2 - v^2)} = \frac{\Delta y}{\sqrt{c^2 - v^2}} \\
(\Delta t_2)^2 &= \frac{(\Delta y)^2}{c^2 - v^2} \\
c^2(\Delta t_2)^2 - v^2(\Delta t_2)^2 &= (\Delta y)^2 \\
(c\Delta t_2)^2 &= (v\Delta t_2)^2 + (\Delta y)^2 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
v = 0 &\Rightarrow \Delta t_2 = \frac{1}{c^2} \left[ \sqrt{c^2(\Delta y)^2 + c^2(\Delta x)^2} \right] \\
(\Delta t_2)^2 &= \frac{1}{c^4} [c^2(\Delta y)^2 + c^2(\Delta x)^2] \\
(c\Delta t_2)^2 &= (\Delta y)^2 + (\Delta x)^2 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
\Delta y = 0 &\Rightarrow \Delta t_2 = \frac{1}{(c^2 - v^2)} \left[ -v\Delta x + \sqrt{c^2(\Delta x)^2} \right] \\
\Delta t_2 &= \frac{-v|\Delta x| + c|\Delta x|}{(c + v)(c - v)} = \frac{\Delta x}{c + v} \quad \checkmark
\end{aligned}$$