Differential Transforms

$c = G = 1$

Schwarzschild (dt_{far}, dr_{far}) Global coordinates Far-Away Bookkeeper/Mapmaker		**Local Rain** (dt_{rain}, dr_{rain}) Raindrop coordinates Free-Falling Observer		**Local Shell** (dt_{shell}, dr_{shell}) Stationary coordinates Fiducial Observer
$\boldsymbol{dt_{far}}$	$=$	$\left(1-\frac{2M}{r}\right)^{-1}\left(dt_{rain} - \sqrt{2M/r}\,dr_{rain}\right)$	$=$	$\left(1-\frac{2M}{r}\right)^{-1/2} dt_{shell}$
$\boldsymbol{dr_{far}}$	$=$	$dr_{rain} - \sqrt{2M/r}\,dt_{rain}$	$=$	$\left(1-\frac{2M}{r}\right)^{1/2} dr_{shell}$
$dt_{far} + \left(1-\frac{2M}{r}\right)^{-1}\sqrt{2M/r}\,dr_{far}$	$=$	$\boldsymbol{dt_{rain}}$	$=$	$\left(1-\frac{2M}{r}\right)^{-1/2}\left(dt_{shell} + \sqrt{2M/r}\,dr_{shell}\right)$
$\sqrt{2M/r}\,dt_{far} + dr_{far}\left(1-\frac{2M}{r}\right)^{-1}$	$=$	$\boldsymbol{dr_{rain}}$	$=$	$\left(1-\frac{2M}{r}\right)^{-1/2}\left(\sqrt{2M/r}\,dt_{shell} + dr_{shell}\right)$
$\left(1-\frac{2M}{r}\right)^{1/2} dt_{far}$	$=$	$\left(1-\frac{2M}{r}\right)^{-1/2}\left(dt_{rain} - \sqrt{2M/r}\,dr_{rain}\right)$	$=$	$\boldsymbol{dt_{shell}}$
$\left(1-\frac{2M}{r}\right)^{-1/2} dr_{far}$	$=$	$\left(1-\frac{2M}{r}\right)^{-1/2}\left(dr_{rain} - \sqrt{2M/r}\,dt_{rain}\right)$	$=$	$\boldsymbol{dr_{shell}}$

$$\gamma \equiv \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-\frac{2M}{r}}} = \left(1-\frac{2M}{r}\right)^{-1/2} \qquad v = \sqrt{2M/r} = escape\ velocity\ at\ radius\ r$$

$\boldsymbol{dt_{far}}$	$=$	$\gamma^2(dt_{rain} - v dr_{rain})$	$=$	γdt_{shell}
$\boldsymbol{dr_{far}}$	$=$	$dr_{rain} - v dt_{rain}$	$=$	$\frac{dr_{shell}}{\gamma}$
$dt_{far} + \gamma^2 v dr_{far}$	$=$	$\boldsymbol{dt_{rain}}$	$=$	$\gamma(dt_{shell} + v dr_{shell})$
$v dt_{far} + \gamma^2 dr_{far}$	$=$	$\boldsymbol{dr_{rain}}$	$=$	$\gamma(v dt_{shell} + dr_{shell})$
$\frac{dt_{far}}{\gamma}$	$=$	$\gamma(dt_{rain} - v dr_{rain})$	$=$	$\boldsymbol{dt_{shell}}$
γdr_{far}	$=$	$\gamma(dr_{rain} - v dt_{rain})$	$=$	$\boldsymbol{dr_{shell}}$