Direction of Motion Light Bounce *(used with time dilation to derive length contraction)*

Direction of motion of S' *wrt* S *at speed* v *is from source event to destination event*
Let mirror $M_\parallel$ *be ahead of source event in direction of motion*
Let $\Delta t (\Delta t')$ *be* $S (S')$ *time from source event to destination event (one tick)*
$M_\perp$ *and* $M_\parallel$ *are at the same* S' *(proper) distance from source event:* $\Delta y' = \Delta x' = c\Delta t'/2$
Let Δx *be* S *distance from source to* $M_\parallel$ *(at source event time)*
Let Δt_1 *be* S *time from source event to bounce event at* $M_\parallel$
Let Δt_2 *be* S *time from bounce at* $M_\parallel$ *to destination event*
Δt *is the total* S *time from source event to destination event:* $\Delta t = \gamma \Delta t' = \Delta t_1 + \Delta t_2$

$$S \text{ distance from source event to bounce at } M_\parallel = \Delta x + v\Delta t_1 = c\Delta t_1$$
$$\Delta x = \Delta t_1 (c - v)$$
$$\Delta t_1 = \Delta x/(c - v)$$

$$S \text{ distance from source event to destination event} = v\Delta t = c\Delta t_1 - c\Delta t_2$$
$$v\Delta t = c\Delta t_1 - c(\Delta t - \Delta t_1) = 2c\Delta t_1 - c\Delta t$$
$$\Delta t(c + v) = 2c\Delta t_1 = 2c\,\Delta x/(c - v)$$
$$\Delta t = \gamma\Delta t' = 2c\,\Delta x/(c^2 - v^2) = 2c^{-1}\,\Delta x\Big/\left(1 - \frac{v^2}{c^2}\right)$$
$$\Delta x = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\left(1 - \frac{v^2}{c^2}\right)c\Delta t'/2 = \left(1 - \frac{v^2}{c^2}\right)^{1/2}\Delta x'$$

$$\boxed{\Delta x = \gamma^{-1}\Delta x'}$$

$\gamma > 1$: S *length* Δx *is less (by a factor of* γ*) than* S' *length* $\Delta x'$
length contraction in direction of motion
S *sees* S' *measuring rods shorter (in direction of motion) than* S *measuring rods*