Coordinate Transformations

Lorentz-Einstein transformations between coordinate systems S and S' with S' moving in $+x$ direction at speed v relative to S

$$\text{Lorentz factor: } \gamma \equiv (1 - v^2/c^2)^{-1/2}$$

$$\begin{aligned} t &= \gamma(t' + vx'/c^2) \\ x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \end{aligned}$$

$$\begin{aligned} t' &= \gamma(t - vx/c^2) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} \Delta x &= \gamma(\Delta x' + v\Delta t') \\ \Delta t &= \gamma(\Delta t' + v\Delta x'/c^2) \\ \Delta x/\Delta t &= (\Delta x' + v\Delta t')/(\Delta t' + v\Delta x'/c^2) \\ \Delta x' = 0 &\Longrightarrow \Delta x/\Delta t = v \end{aligned}$$

$$\begin{aligned} \Delta x' &= \gamma(\Delta x - v\Delta t) \\ \Delta t' &= \gamma(\Delta t - v\Delta x/c^2) \\ \Delta x'/\Delta t' &= (\Delta x - v\Delta t)/(\Delta t - v\Delta x/c^2) \\ \Delta x = 0 &\Longrightarrow \Delta x'/\Delta t' = -v \end{aligned}$$

S measures the speed of S' to be v, and S' measures the speed of S to be $-v$

Relativity of Time Dilation

S *duration*: $\Delta x = 0 \Longrightarrow \Delta t = \Delta t'/\gamma$

S' *duration*: $\Delta x' = 0 \Longrightarrow \Delta t' = \Delta t/\gamma$

Relativity of Length Contraction

S *length*: $\Delta t = 0 \Longrightarrow \Delta x = \Delta x'/\gamma$

S' *length*: $\Delta t' = 0 \Longrightarrow \Delta x' = \Delta x/\gamma$

S' clock is stationary in the S' frame with $\Delta x' = 0$, and moves at speed v relative to S. That means $\Delta x = \gamma(\Delta x' + v\Delta t') = v\Delta t$, so $\Delta x = \gamma v\Delta t' = v\Delta t$ which leaves $\Delta t' = \Delta t/\gamma$. The time between two events $\Delta t'$ as measured by the S' clock is smaller than the time duration Δt measured by the S clock by a factor of γ :

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{v^2\Delta t}{c^2}\right) = \gamma\Delta t\left(1 - \frac{v^2}{c^2}\right) = \gamma\Delta t\frac{1}{\gamma^2} = \frac{\Delta t}{\gamma}$$

As seen by S, the S' clock runs slower than the S clock by a factor of γ. As seen by S, any physical process in S' takes longer to complete by a factor of γ than in S (time dilation). As seen by S', any physical process in S takes longer to complete by a factor of γ than in S' (time dilation is symmetric in SR).

S' ruler oriented parallel to the x-axis moves at speed v. The distance between the end points of the ruler as measured by S is Δx with $\Delta t = 0$ between the two end-point events:

$$\Delta x' = \gamma(\Delta x - v\Delta t) = \gamma\Delta x$$

S' measures its co-moving ruler length to be bigger than what S measures it to be by a factor of γ. As seen by S, the S' ruler is shorter than an identical S ruler by a factor of γ. Any object co-moving with S' is shortened in the direction of motion by a factor of γ relative to what its measured length is in S (Lorentz contraction). Any object co-moving with S is shortened in the direction of motion by a factor of γ relative to what its measured length is in S' (Lorentz contraction is symmetric in SR)