Transverse Light Bounce *(used to derive time dilation)*

Direction of motion of S' *wrt* S *at speed* v *is from source event to destination event*
Let mirror $M_\perp$ *be offset from source event perpendicular to direction of motion*
Let point P *be midway between source and destination events*
Let $\Delta t(\Delta t')$ *be* $S(S')$ *time from source event to destination event (one tick)*
S *distance in direction of motion from source to* P: $v\,\Delta t/2$
S' *(proper) distance from source to mirror* $M_\perp$: $\Delta y' = c\,\Delta t'/2$
S *and* S' *transverse distances* Δy *and* $\Delta y'$ *are equal (no length contraction), so*
S *transverse distance from* P *to mirror* $M_\perp$ *at* S *time of bounce*: $\Delta y = \Delta y' = c\,\Delta t'/2$
S *light path (diagonal) distance from source to mirror* $M_\perp$ *at time of bounce* $= c\,\Delta t/2$

$$\text{Pythagorean formula: } (c\,\Delta t/2)^2 = (v\,\Delta t/2)^2 + (c\,\Delta t'/2)^2$$

$$c^2\Delta t^2 = v^2\Delta t^2 + c^2\Delta t'^2$$
$$\Delta t^2(c^2 - v^2) = c^2\Delta t'^2$$
$$\Delta t^2 = \frac{c^2}{c^2 - v^2}\Delta t'^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1}\Delta t'^2$$
$$\Delta t = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\Delta t'$$

$$\boxed{\Delta t = \gamma\Delta t'}$$

$\gamma > 1$: S *time duration* Δt *is greater (by a factor of* γ*) than* S' *time duration* $\Delta t'$
time dilation: S *sees* S' *clock ticks slower than* S *clock*