Coordinate Transformations

Lorentz-Einstein transformations between coordinate systems S and S':
S' moving at constant speed $\Delta x/\Delta t = v$ relative to S
S moving at constant speed $\Delta x'/\Delta t' = -v$ relative to S'

Lorentz factor: $\gamma \equiv (1 - v^2/c^2)^{-1/2}$

$$t = \gamma(t' + vx'/c^2) \qquad t' = \gamma(t - vx/c^2)$$
$$x = \gamma(x' + vt') \qquad x' = \gamma(x - vt)$$
$$y = y' \qquad y' = y$$
$$z = z' \qquad z' = z$$

$$\Delta t = \gamma(\Delta t' + v\Delta x'/c^2) \qquad \Delta t' = \gamma(\Delta t - v\Delta x/c^2)$$
$$\Delta x = \gamma(\Delta x' + v\Delta t') \qquad \Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta x/\Delta t = (\Delta x' + v\Delta t')/(\Delta t' + v\Delta x'/c^2) \qquad \Delta x'/\Delta t' = (\Delta x - v\Delta t)/(\Delta t - v\Delta x/c^2)$$
$$\Delta x' = 0 \Longrightarrow \Delta x/\Delta t = v \checkmark \qquad \Delta x = 0 \Longrightarrow \Delta x'/\Delta t' = -v \checkmark$$

Each clock is stationary in its frame

S measure of S' clock with $\Delta x' = 0$ | *S' measure of S clock with $\Delta x = 0$*

$$\Delta t_{S'\,clock} = \gamma(\Delta t' + v\Delta x'/c^2) = \gamma\Delta t' \qquad \Delta t'_{S\,clock} = \gamma(\Delta t - v\Delta x/c^2) = \gamma\Delta t$$

End points of a moving ruler are measured simultaneously

S measure of S' ruler with $\Delta t = 0$ | *S' measure of S ruler with $\Delta t' = 0$*

$$\Delta x' = \gamma(\Delta x - v\Delta t) = \gamma\Delta x_{S'\,ruler} \qquad \Delta x = \gamma(\Delta x' + v\Delta t') = \gamma\Delta x'_{S\,ruler}$$
$$\Delta x_{S'\,ruler} = \Delta x'/\gamma \qquad \Delta x'_{S\,ruler} = \Delta x/\gamma$$

SR is symmetric between S and S'

As seen by S, the S' clock runs slower than an identical S clock by a factor of γ: any physical process in S' takes longer to complete by a factor of γ than in S (time dilation). Likewise, as seen by S', any physical process in S takes longer to complete by a factor of γ than in S' (time dilation is symmetric in SR).

As seen by S, the S' ruler is shorter than an identical S ruler by a factor of γ: any object co-moving with S' is shortened in the direction of motion by a factor of γ than in S (length contraction). Likewise, any object co-moving with S is shortened in the direction of motion by a factor of γ relative to its length as seen by S' (length contraction is symmetric in SR).

Symmetry of Time Dilation	Symmetry of Length Contraction
$\Delta x' = 0 \Longrightarrow \Delta t = \gamma\Delta t'$	$\Delta t = 0 \Longrightarrow \Delta x = \Delta x'/\gamma$
$\Delta x = 0 \Longrightarrow \Delta t' = \gamma\Delta t$	$\Delta t' = 0 \Longrightarrow \Delta x' = \Delta x/\gamma$