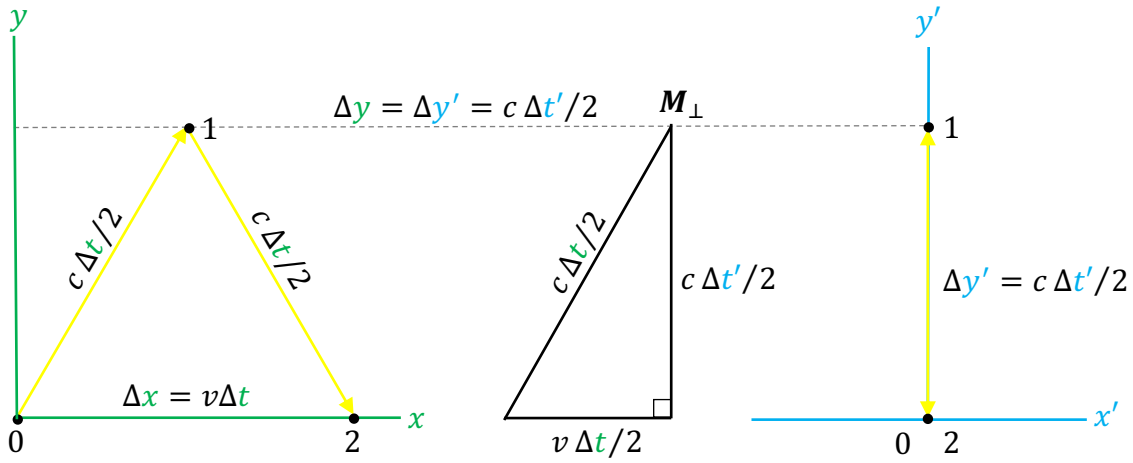
Time Dilation

Derivation of the time dilation formula using the light path reflected off mirror $\boldsymbol{M}_\perp$
which is offset from the light source perpendicular to the direction of motion
(example shown has $v = c/2$*)*

event 0: *send light pulse*
event 1: *bounce at mirror* $\boldsymbol{M}_\perp$
event 2: *receive light pulse (also send next light pulse)*



S' *moves at constant speed* v *in the* $+x$ *direction relative to* S
S *and* S' *perpendicular distances* Δy *and* $\Delta y'$ *are equal (no length contraction)*
let $\Delta t = S$ *time from event* 0 *to event* 2 $=$ *one* S *clock tick*
let $\Delta t' = S'$ *time from event* 0 *to event* 2 $=$ *one* S' *clock tick*
let $\Delta y'$ *be* S'*(proper) distance from event* 0 *to event* 1 $= c\,\Delta t'/2$
S *distance from event* 0 *to midway between events* 0 *and* 2 $= v\,\Delta t/2$
S *light path (diagonal) distance from event* 0 *to event* 1 $= c\,\Delta t/2$

use Pythagorean formula: $(c\,\Delta t/2)^2 = (v\,\Delta t/2)^2 + (c\,\Delta t'/2)^2$

$$c^2(\Delta t)^2 = v^2(\Delta t)^2 + c^2(\Delta t')^2$$
$$(c^2 - v^2)(\Delta t)^2 = c^2(\Delta t')^2$$
$$(\Delta t)^2 = \frac{c^2}{c^2 - v^2}(\Delta t')^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1}(\Delta t')^2$$
$$\Delta t = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\Delta t'$$

$$\boxed{\Delta t = \gamma \Delta t'}$$

$\gamma \geq 1 \Rightarrow$ *time dilation as seen by* S *frame*:
S *time duration* Δt *is greater (by a factor of* γ*) than* S' *time duration* $\Delta t'$
S *sees* S' *clocks tick slower than* S *clocks (by a factor of* γ*)*