Lecture 5: Evidence from the CAPM and the APT (and related models)

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Tests of the CAPM

- To test the validity of a model, we need to test its predictions
- CAPM predicts
  1. A linear relationship between expected excess returns and beta
  2. No other variable has marginal explanatory power/alpha is zero
  3. The risk premium for Beta is positive and equal to the market risk premium
- These predictions can be tested in various ways

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Time series tests

- Jensen's alpha: from the CAPM regression,

$$r_{i,t} - r_f = \alpha_i + \beta_i \left(r_{m,t} - r_f \right) + \epsilon_{i,t}$$

- Prediction: α_i should be jointly zero, i.e. $\alpha_1 = 0, \alpha_2 = 0, \ldots$
- Historical tests have not typically been favorable towards the CAPM
  - Reject that alphas are jointly zero (p=0.02) (Campbell, Lo, Mackinlay 1997)

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Time series tests

- Importance of time-series tests, however, is in their interpretation
- Can show joint tests of alpha are equivalent to the following:

1. Find the Sharpe ratio of the market portfolio and compare it to the portfolio with the highest realized Sharpe ratio over a given period

2. Time series tests provide statistical comparison of the predicted MVE (the market) and the actual MVE portfolio

Time series tests

- Said otherwise, non-zero alphas suggest the market portfolio lags the realized maximum Sharpe ratio portfolios by more than the CAPM would suggest
- Generally, it is useful to think about CAPM tests/factor model tests as figuring out if the market/factor portfolios are “efficient”

    - Big question: which stocks should we overweight in the market portfolio to make it more efficient?

Cross-sectional tests

- Cross-sectional tests have been somewhat more favorable
- Rather than regressing returns on returns, we now regress returns on betas
  - Hope to find that betas and returns line up as predicted by security market line
- Fama-Macbeth (1973) provides the standard framework for these tests

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Fama-Macbeth Cookbook

Two-step procedure:

1. Run time-series regressions to estimate beta for all stocks

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_{i,t}$$

2. Run cross-section regression of average excess returns $r_i - r_f$ on estimated betas

$$r_i - r_f = \lambda_0 + \lambda_1 \hat{\beta}_i + u_i$$

Prediction: $\lambda_0 = 0$ and $\lambda_1 = E(r_m - r_f)$

Fama-Macbeth Cookbook

- However, note that beta estimates are noisy
  - Regressing any variable on a noisy proxy will flatten the slope coefficient
- Why?
  - Imagine noise is added to x so that you observe x+e
  - True OLS coefficient is $\lambda_1 = \frac{Cov(x,y)}{Var(x)}$
  - Estimated OLS coefficient is $\lambda_1 = \frac{Cov(x+\epsilon,y)}{Var(x+\epsilon)} = \frac{Cov(x,y)}{Var(x)+Var(\epsilon)}$
- So, we expect any regression estimate of the security market line (SML) to be too flat.

Fama-Macbeth Cookbook

- In response, we form portfolios of stocks and hope that idiosyncratic noise in beta estimates disappears
  - Why might this help?
- Portfolios are formed based on firm betas
  - Why not random assignment?
- Actual procedure:
  1. Each year, calculate betas for all firms (past five years data – 60 months)
  2. Form 10 decile portfolios based on estimated betas
  3. Calculate realized portfolio returns and betas for 10 portfolios

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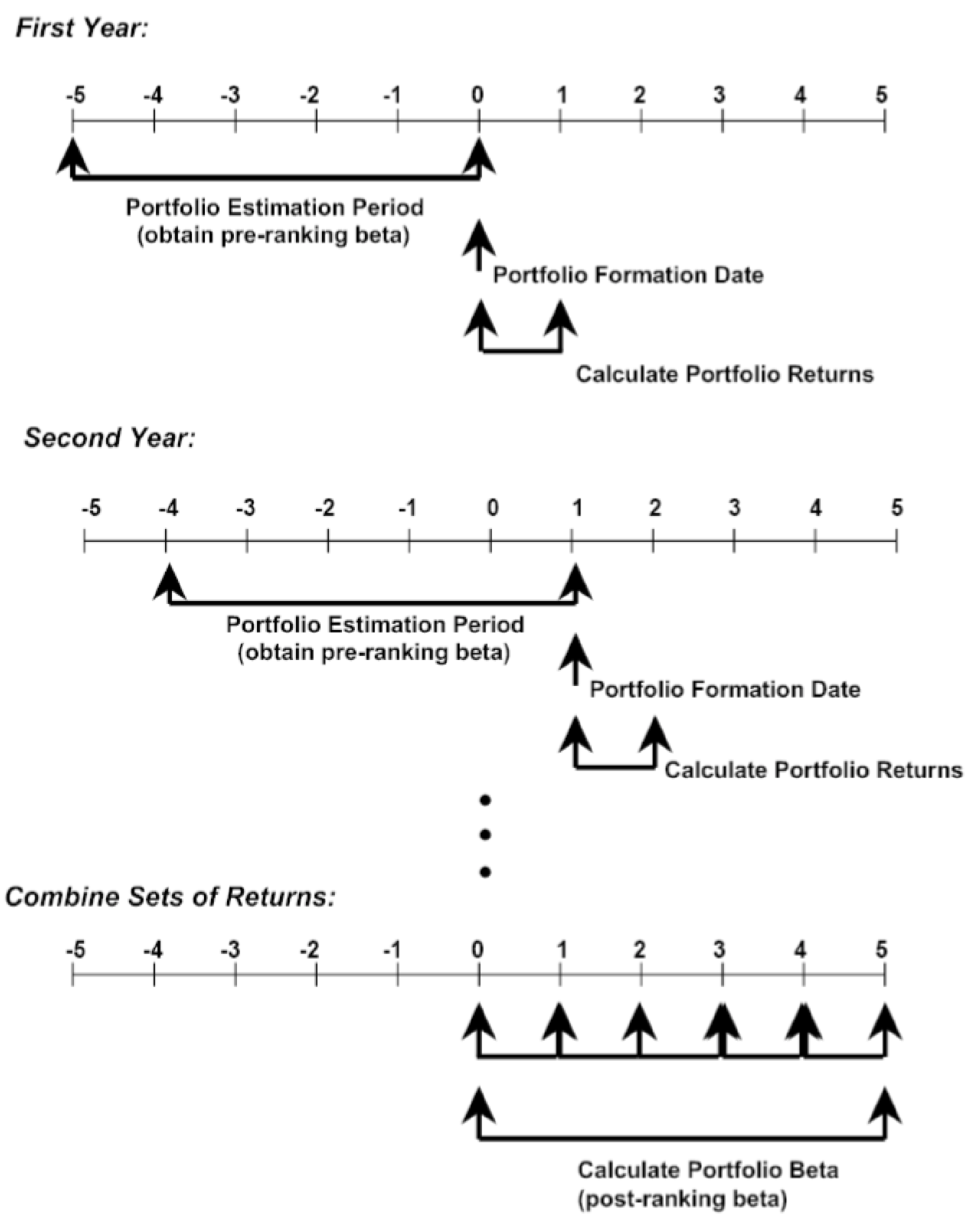

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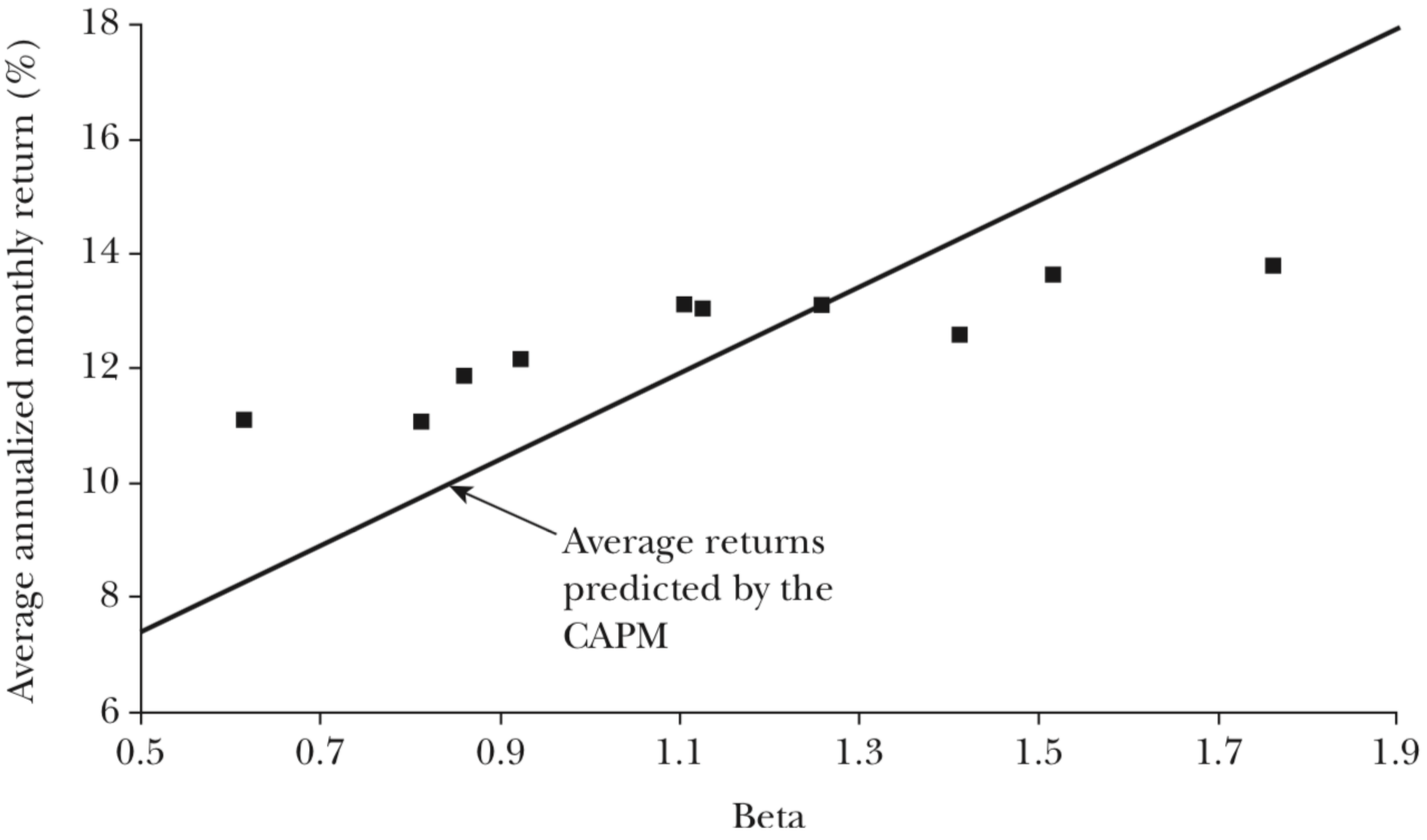
Results (Fama, French, JEP 2004)

- Positive relation between beta and portfolio returns, but fitted line too flat

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



Other predictions

- CAPM predicts no other measures of risk will predict cross-sectional returns
- In particular, CAPM says only covariance risk matters
  - What about idiosyncratic risk?
  - Fama-Macbeth (1973) control for idiosyncratic risk by including residual variances from firm time-series regressions in second-stage regression
  - Also beta squared

$$r_i - r_f = \lambda_0 + \lambda_1 \widehat{\beta}_i + \lambda_2 \widehat{\beta}_i^2 + \lambda_3 \widehat{\sigma}_e^2 + e_i$$

- They find that only β seems to matter – it's a linear relationship

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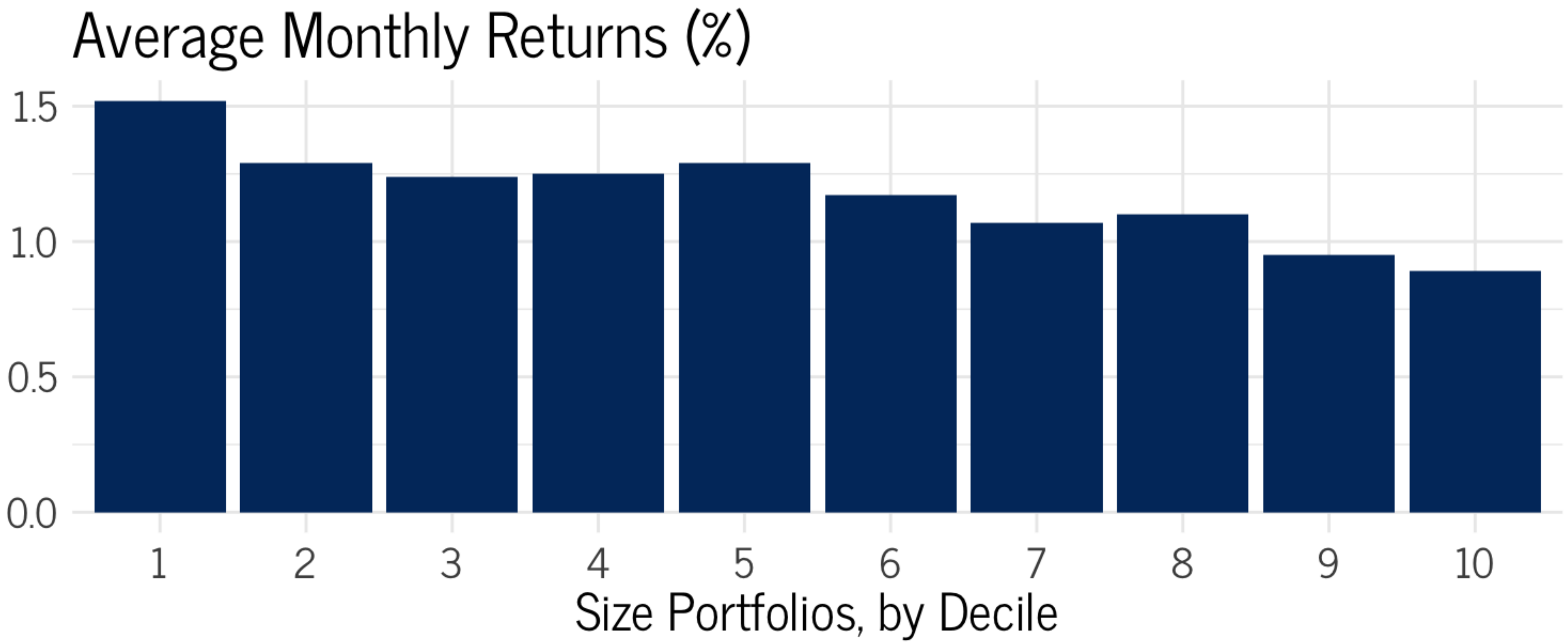
The search for anomalies begins

- However, we can go beyond beta-squared and residual variation to predict returns...
- For example:
  1. Earnings-to-price ratio $\longrightarrow$ high returns (Basu, 1977)
  2. Market cap $\longrightarrow$ low returns (Banz, 1981)
  3. Leverage $\longrightarrow$ high returns (Bhandari, 1988)
  4. Book-to-market $\longrightarrow$ high returns (Statman, 1980)
- All turn out to have predictive power over beta, in particular, size (market cap) and book-to-market
- So which wins in a race, beta or size?

The search for anomalies begins

- Fama-French (1992) test this by creating double sorted portfolios
  - First, sort firms on size; then, sort on beta (cov with market)
  - Can do the same with book-to-market
- Set up a horse-race between beta and the two other factors
- If beta is a good predictor, it should predict even with a bin of similarly sized firms

Average Returns, Post-Ranking Betas and Average Size on Portfolios (Fama-French 1992)



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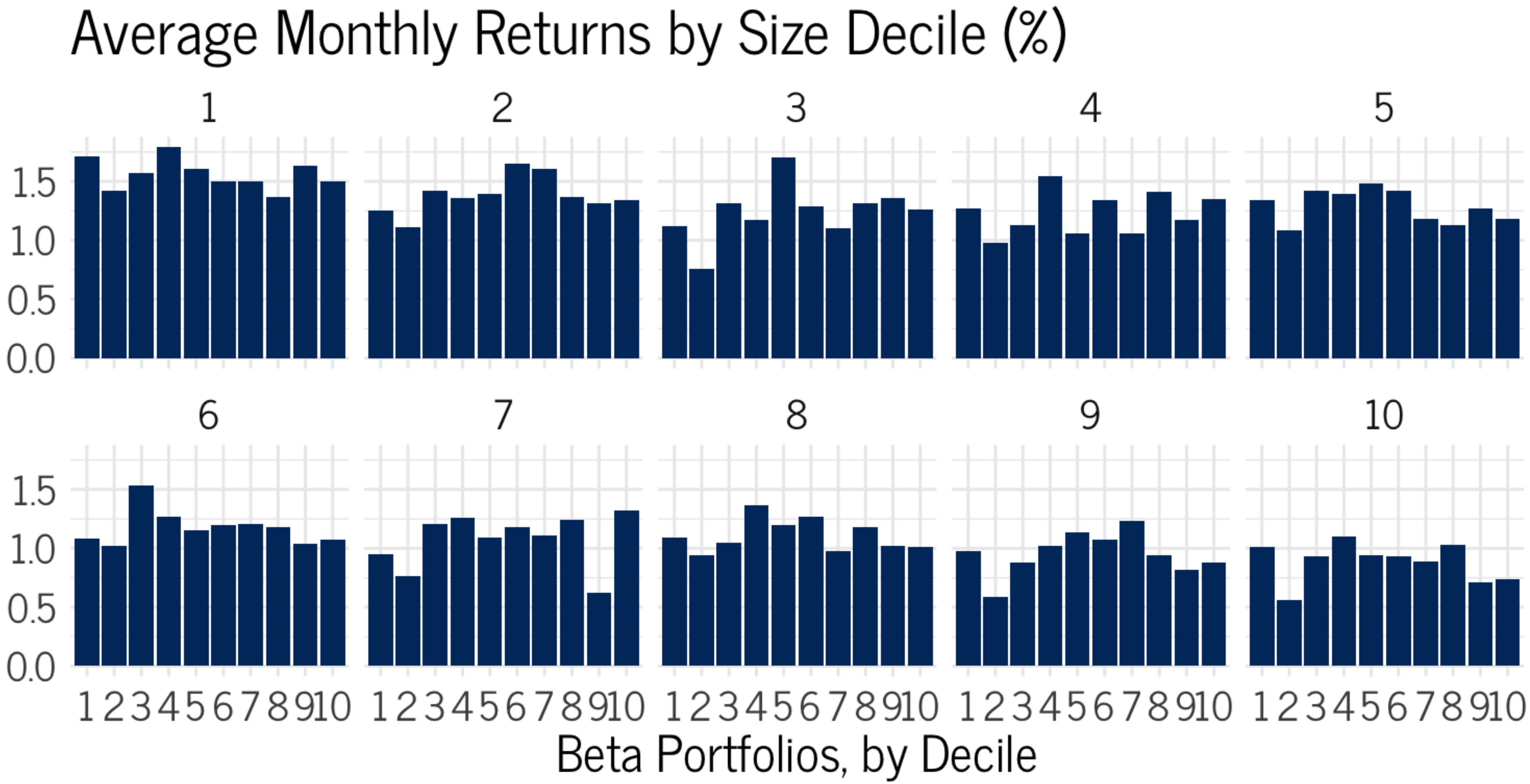
Average Returns, Post-Ranking Betas and Average Size on Portfolios (Fama-French 1992)



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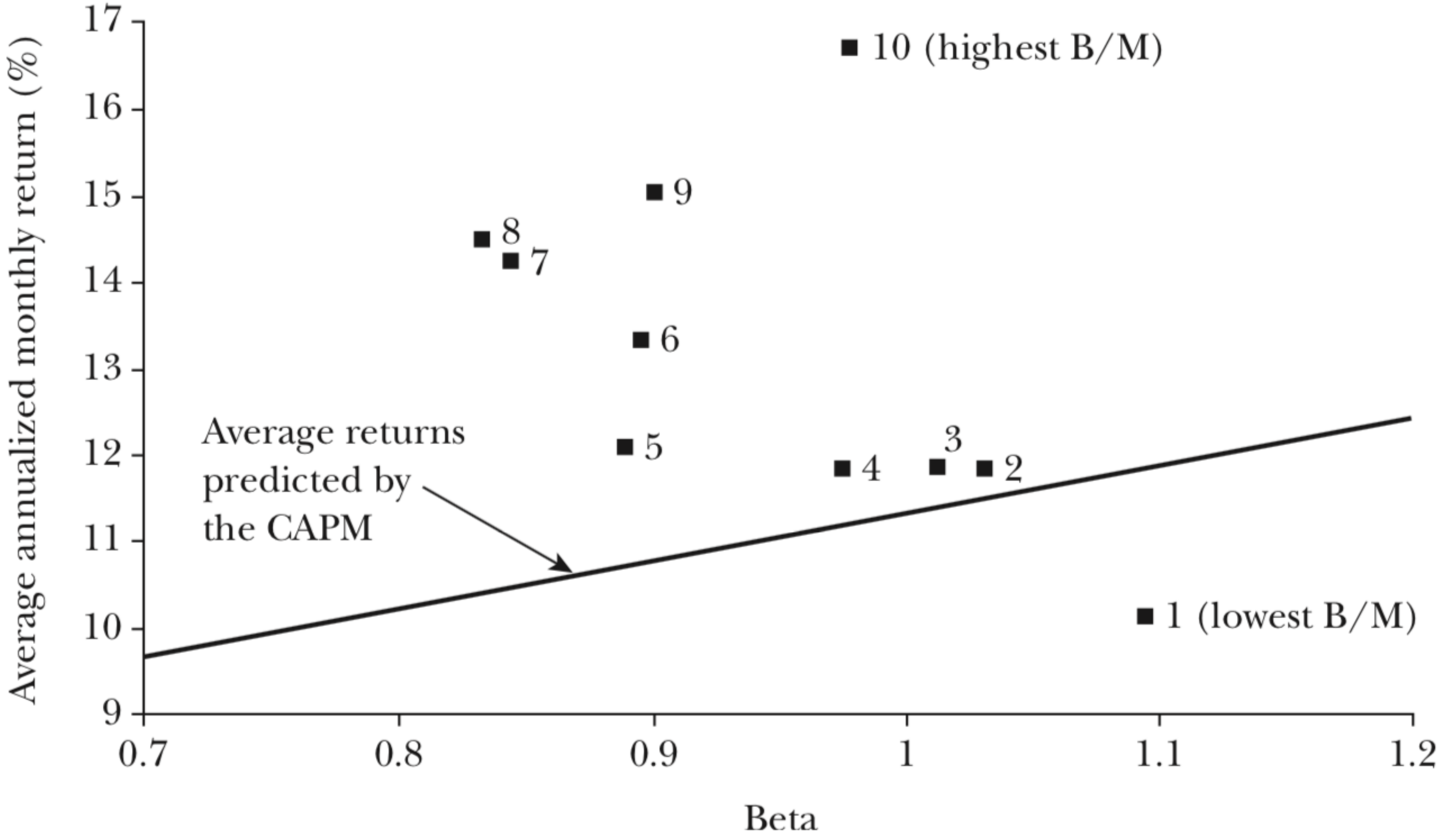
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



Is Beta is dead?

- 30 years after its birth, hard to say that CAPM isn't dead
- In reality, however, hard to say if CAPM or tests of the CAPM are flawed
- Roll's critique:
  - Tests of the CAPM are infeasible because the market portfolio is unobservable
  - Tests of CAPM are only tests of the efficiency of the market proxy used

Factor Models

- In spite of being largely credited with the temporary demise of the CAPM, Fama-French argue we need more flexible market proxies
- They advocate multiple factor models that capture the spirit of the CAPM
  - i.e. expected returns dictated by exposure to non- diversifiable risk
  - Size and book-to-market are not "characteristics" but proxies for economic risk factors

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Fama-French 3 factor model

- Create factor mimicking portfolios
  - HML (returns from high B/M stocks less returns from low B/M stocks)
  - SMB (returns from high market cap less returns from low market cap stocks)

  - Data available here: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
- Sort firms into portfolios based on size and value
  - Estimate the following regression for different portfolios

$$r_i - r_f = \alpha + b_i \times (r_m - r_f) + s_i \times SMB + h_i \times HML + \epsilon_i$$

- to test

$$E(r_i - r_f) = b_i \times E(r_m - r_f) + s_i \times E(SMB) + h_i \times E(HML)$$

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Fama-French 3 factor model

$$E(r_i - r_f) = b_i \times E(r_m - r_f) + s_i \times E(SMB) + h_i \times E(HML)$$

	BE/ME	Size	Ex Ret	a	b	s	h	t(a)	t(b)	t(s)	t(h)	R^2
						7/29-6/97						
S/L	0.55	22.39	0.61	-0.42	1.06	1.39	0.09	-4.34	30.78	19.23	1.73	0.91
S/M	1.11	22.15	1.05	-0.01	0.97	1.16	0.37	-0.18	53.55	19.49	9.96	0.96
S/H	2.83	19.05	1.24	-0.03	1.03	1.12	0.77	-0.73	67.32	39.21	26.97	0.98
M/L	0.53	55.85	0.70	-0.06	1.04	0.59	-0.12	-1.29	55.83	18.01	-4.30	0.96
M/M	1.07	55.06	0.95	-0.01	1.05	0.47	0.34	-0.15	32.98	17.50	9.50	0.96
M/H	2.18	53.21	1.13	-0.04	1.08	0.53	0.73	-0.90	47.85	8.99	11.12	0.97
B/L	0.43	94.65	0.58	0.02	1.02	-0.10	-0.23	0.88	148.09	-6.88	-13.52	0.98
B/M	1.04	92.06	0.72	-0.09	1.01	-0.14	0.34	-1.76	61.61	-4.96	13.66	0.95
B/H	1.87	89.53	1.00	-0.09	1.06	-0.07	0.84	-1.40	52.12	-0.86	21.02	0.93

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Fama-French 3 factor model

- Claim: Size and value premia reflect exposure to risk captured in SMB and HML
- High returns which are not associated with risk factors should be arbitraged away
- Alphas of size and book-to-market portfolios jointly zero, once we control for SMB and HML risk factors

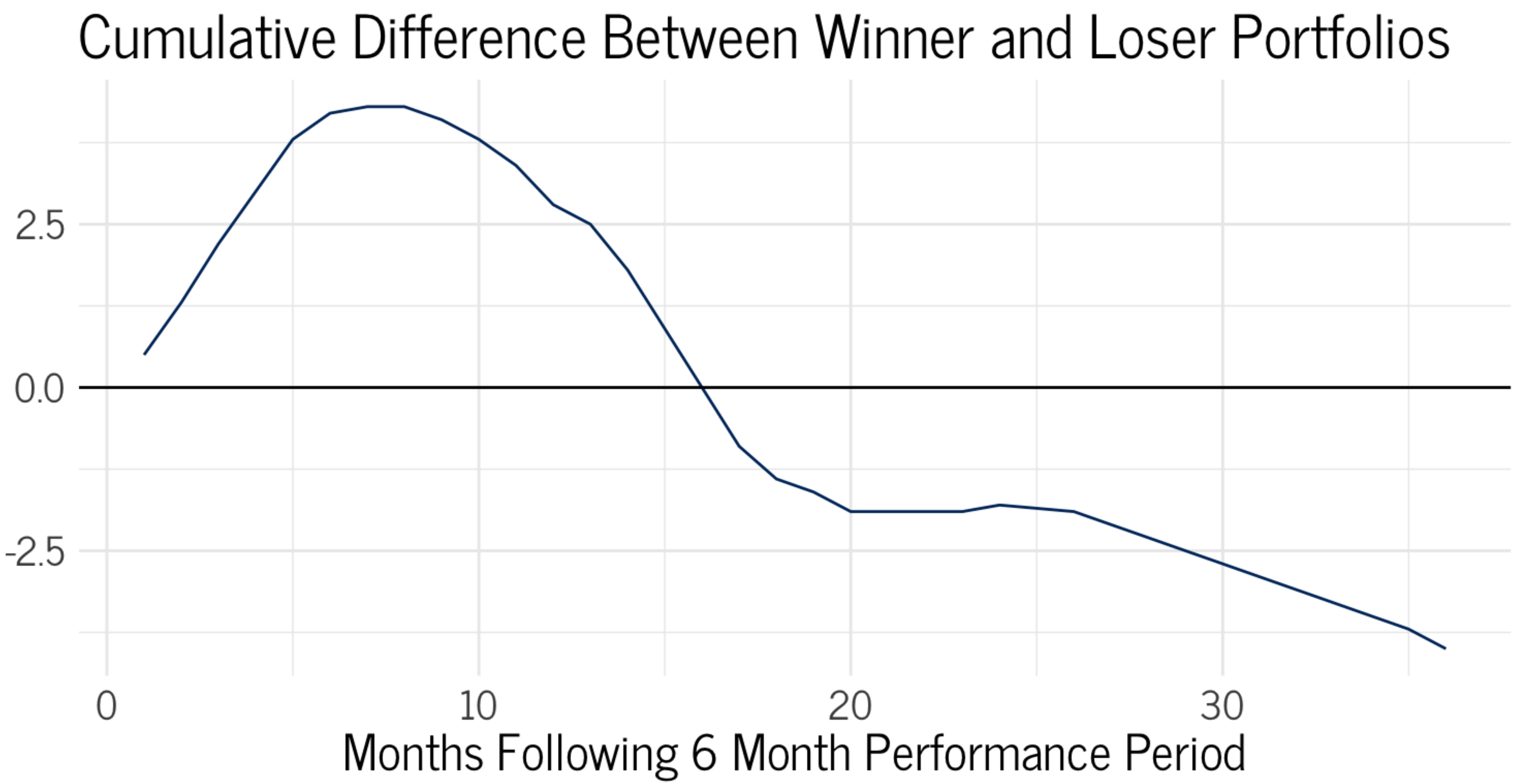
- This ensures the model is closer to a CAPM/APT story, but is source of some debate

More factors (?!): Momentum

- Often times a fourth factor – momentum – is added to the portfolio
- Based on results that suggest that a strategy of buying winners and selling losers can earn a significant premium over a buy-and-hold strategy

- Note: again, we have taken a firm characteristic (recent success), made a portfolio out of it, and called it a "risk-factor"
  - Is this reasonable?

Momentum Returns



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More factors – Liquidity

- Illiquid stocks tend to offer higher returns
  - Can be measured based on bid-ask spreads
  - CAPM assumes away transaction costs
- Alternatively, we can characterize liquidity as a risk factor
  - Illiquidity of stocks is correlated – “systemic” liqudity
  - Systemic liquidity varies over time
  - Stocks exposed to liuqidity risk need to compensate investors with additional risk premia

More factors—Liquidity

- Pastor and Stambaugh (2002) create a liquidity factor, LIQ
  - LIQ_t is low when order flows have a large impact on prices
- We can add this factor to our 3 factor model:

$$E(r_i - r_f) = \beta_i E(r_m - r_f) + s_i E(SMB) + h_i E(HML) + l_i E(RP_{liq})$$

- Hedge funds sell exposure to liquidity risk
  - As long as you don't need liquitiy when everyone else does, might as well get paid for it!

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Factors: risks or opportunities?

- Note the theme here
  - No shares are over/underpriced (almost)
  - Risk-premia paid on assets represent exposure to risk factors
  - Otherwise, “arbitrageurs” will quickly drive prices to equilibrium “correct” values
  - They need deep pockets!
- No free lunch → high returns = high risk exposure
- Different **kinds** of risk
  - Some institutions/investors prefer certain types of exposures

The Factor Zoo

- We discussed many potential factors
  - Fama French 3 Factors, Momentum, Liquidity
- Why stop at 5?
  - Can continue to capture as many risks as possible!
- Why such a small number of factors?

The Factor Zoo

- The estimation of systemic risk exposure relies on a limited amount of historical data
- Dumping in many historical risk factors that are correlated will give:
  - marginal gains
  - noisy estimates
- When you predict based on those estimates, you will get noisy output
  - Garbage in, garbage out!
- Many approaches to fix this

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Taming the Factor Zoo (Feng, Giglio and Xiu)

- A new approach: use machine learning to identify the "best" factors
- Feng Giglio and Xiu use "double-selection Lasso", which will identify factors which capture the most important loadings in the cross-section

- They test 15 new contributed factors in the literature, and 4 out of 15 predictive beyond what was already studied in the literature.