Lecture 8 : Performance Evaluation

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Why Performance Evaluation?

- Picking the manager of our money
  - Recall choice of portfolio manager plays big role in returns
- Compensation
  - Don't want to pay "too much" for given factor risk; or for passive management
  - Incentives matter – designing compensation on poor measurement may induce poor behavior
- Evaluating performance identifies the "value-add" of a manager or fund

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Risk adjustments

- Focus on three basic measures of risk-adjustment
  1. Sharpe ratio
  2. Information ratio
  3. Treynor ratio
- Each of these represent a rescaling / reinterpretation of alpha, so we begin there…

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Single Factor / CAPM alpha

We construct our portfolio alpha as

$$\alpha_p = \bar{r}_p - \left[\bar{r}_f + \beta_p(\bar{r}_m - \bar{r}_f)\right]$$

where

- $\bar{r}_p$: average return on the portfolio
- β_p: portfolio Beta (risk loading on the market factor)
- $\bar{r}_f$: average risk free rate
- $\bar{r}_m$: average return on market index portfolio

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Multifactor alpha

$$\alpha_i = \overline{r_i - r_f} - \beta_i(\overline{r_m - r_f}) - s_i\overline{SMB} - h_i\overline{HML}$$

- r_m: monthly return on market index portfolio
- r_f: monthly return on risk free asset portfolio
- SMB: monthly return on Fama-French small minus big portfolio
- HML: monthly return on Fama-French high minus low portfolio

Or, just run the regression and extract the α_i:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + s_i SMB_t + h_i HML_t + \epsilon_{it}$$

Running regression is doubly useful b/c also provides a way to identify factor weights that a portfolio is constructing – e.g. manager strategy.

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Risk Adjusted Performance: Sharpe

Sharpe Ratio

$$SR = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p}$$

- $\bar{r}_p$: average return on the portfolio
- $\bar{r}_f$: average risk-free rate
- σ_p : portfolio standard deviation

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Risk Adjusted Performance: M^2

- Sharpe ratio tells us the optimal risky portfolio
- M^2 provides an economic interpretation the Sharpe Ratio for portfolio p
  - Blend r_p with r_f to form the complete portfolio return (call it r_p^{blend}) with the same standard deviation as the market r_m
  - Given equivalent risk characteristics, r_m and r_p^{blend} can now be compared:

$$M^2 = r_p^{blend} - r_m$$

where $r_p^{blend} = (\sigma_m/\sigma_p)r_p + (1 - (\sigma_m/\sigma_p))r_f$

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Sharpe ratio of an active portfolio

- The Sharpe ratio of an optimally formed active+passive portfolio is

$$SR_{A+P} = \sqrt{SR_m^2 + \frac{\alpha_A^2}{\sigma_A^2(e)}}$$

where SR_m is the Sharpe ratio of the market, α_A and $\sigma_A^2(e)$ are the alpha and residual variance of the active portfolio

- Active funds should be evaluated on the basis of their contribution to the Sharpe ratio

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Risk Adjusted Performance: Information Ratio

$$IR = \alpha_p / \sigma(e_p)$$

α_p : portfolio alpha

$\sigma(e_p)$: Residual *standard deviation* from CAPM regression

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_{i,t}$$

- Note: idiosyncratic risk matters… assumes you will not fully diversify away

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Risk Adjusted Performance: Treynor

- Treynor Measure

$$T = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}$$

- $\bar{r}_p$ = Average return on the portfolio
- $\bar{r}_f$: Average risk free rate
- β_p portfolio beta

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Risk Adjusted Performance: Treynor

- Notice, adding the market variance to the denominator preserves the ranking of Treynor measures across portfolios/stocks

$$\frac{\bar{r}_p - \bar{r}_f}{\beta_p \sigma_m^2} = \frac{\bar{r}_p - \bar{r}_f}{Cov(r_m, r_p)}$$

- Recall, this is a portfolio's contribution to the "Reward/Risk" ratio of the market portfolio (see CAPM lecture notes)
- Big assumption: idiosyncratic risk is diversified away

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Performance Evaluation Summary

- Sharpe Ratio/ M^2: the optimal measure of our overall risky portfolio
- Information ratio give the contribution to overall Sharpe ratio of an active portfolio
  - Assumes active portfolio risk is not diversified away (and is significant)

- Treynor measure considers the contribution of a stock (or one of many funds) to portfolio Sharpe ratio
  - Assumes idiosyncratic risk will be diversified away

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Brimson's Decomposition

- Suppose we want to compare a fund/fund manager to a benchmark portfolio
- Allocate exceptional performance to three buckets
  - Asset Allocation
  - Stock Selection
  - Interaction

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Brinson's Decomposition

- If we know weights and returns in each asset class, difference in portfolio returns and benchmark can be written as

$$r_p - r_b = \sum_i \underbrace{(w_{i,p} - w_{i,b})r_{i,b}}_{\text{Asset Allocation}} + \sum_i \underbrace{(r_{i,p} - r_{i,b})w_{i,b}}_{\text{"Stock" Selection}} + \sum_i (w_{i,p} - w_{i,b})(r_{i,p} - r_{i,b})$$

- Interpretation for last term?

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