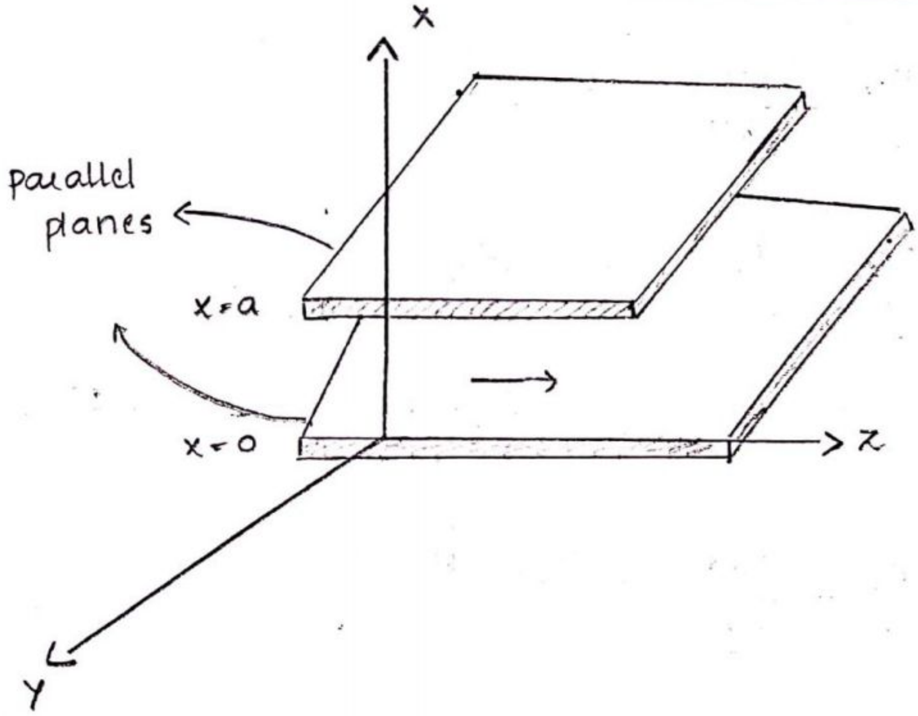
parallel plates]



16x2

From the Maxwell's equation

32

$$\nabla \times H = (\sigma + j\omega\varepsilon)E$$

44/76

$$\nabla \times E = -j\omega\mu H$$

wave equation is given by

$$\left.\begin{aligned} \nabla^2 E &= \gamma^2 E \\ \nabla^2 H &= \gamma^2 H \end{aligned}\right\} \qquad \gamma = \sqrt{(\sigma + j\omega\varepsilon)j\omega\mu}$$

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consider a rectangular coordinate system

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

~~The electric field~~ component in x-direction

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{dz} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned}\right\} \text{——} (1)$$

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\} \text{——} (2)$$

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\omega^2\mu\varepsilon H \end{aligned}\right\} \text{——} (3)$$

Assume the wave propagation in z-direction so the field is constant in the Y-direction.

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$\therefore$ in ①, the derivation wrto y is zero.

Here the wave is propagating in z-direction. so from ①

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

From ②

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$-\frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

— ④

Since the field components are propagating in the z-direction, it can be represented as

$$H_x = H_x^0 e^{-\gamma z}$$

$$H_y = H_y^0 e^{-\gamma z}$$

$$E_x = E_x^0 e^{-\gamma z}$$

$$E_y = E_y^0 e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y^0 e^{-\gamma z}$$

$$= -\gamma H_y$$

eqn ④ can be rewrite as

$$\left.\begin{aligned}
&\gamma H_y = j\omega\varepsilon E_x \\
&-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \\
&\cancel{\gamma E_y - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_z} \\
&\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \\
&\gamma E_y = -j\omega\mu H_x \\
&-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \\
&\frac{\partial E_y}{\partial x} = -j\omega\mu H_z
\end{aligned}\right\} \text{— (5)}$$

(3) can be rewrite as

$$\left.\begin{aligned}
&\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2\mu\varepsilon E \\
&\frac{\partial^2 H}{\partial x^2} + \gamma^2 H = -\omega^2\mu\varepsilon H
\end{aligned}\right\} \text{— (6)}$$

In (5), from (a) ~~and (b)~~

$$H_y = \frac{j\omega\varepsilon E_x}{\gamma}$$

substituting in (c)

$$\frac{\partial\left(\frac{j\omega\varepsilon E_x}{\gamma}\right)}{\partial x} = j\omega\varepsilon E_z$$

$$\frac{j\omega\varepsilon}{\gamma}\frac{\partial E_x}{\partial x} = j\omega\varepsilon E_z$$

$\cancel{\frac{\partial E_x}{\partial y}}$ $\quad \frac{\partial E_x}{\partial x} = \cancel{\gamma E_z}$

$$\cancel{E_x = \int \gamma E_z \, dx}$$

from (5)

Solving

$$\left.\begin{aligned} H_x &= \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} \\ H_y &= \frac{-j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x} \\ E_x &= \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} \\ E_y &= \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \end{aligned}\right\} \quad -(7)$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

In (7), the electric and magnetic field components are expressed in terms of E_z and H_z. It is observed that there must be a z-component of either E or H otherwise all the components should be zero and there will be no wave propagation between the parallel plates.

[T_E] ~~Transfer~~ Transverse Electric Waves [$E_z = 0, H_z \neq 0$]

From equation (7) E_x and E_y also ~~seen~~ zero. Exist the value H_x and E_y

$$E_y = \frac{\partial^2 E_y}{\partial x} + \gamma^2 E_y = -\omega^2 \mu \varepsilon \cdot E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -(\omega^2\mu\varepsilon + \gamma^2) E_y$$

$$\frac{\partial^2 G_y}{\partial x^2} = -h^2 G_y \quad (8) \qquad \{ h = \omega^2 \mu \varepsilon + \gamma^2$$

$$E_y = G_y(x) e^{-\gamma z} \quad (9)$$

$$\frac{\partial^2 E_y^0}{\partial x^2} = -h^2 G_y$$

This is the differential equation equation of harmonic motion the solution can be written as;

$$E_y^0 = c_1 \sin hx + c_2 \cos hx$$

$$(9) \Rightarrow E_y = (c_1 \sin hx + c_2 \cos hx) e^{-\gamma z} \quad (10)$$

Applying the boundary conditions;

$E_y = 0 \quad x = 0$ for all z

$E_y = 0 \quad x = a$

Applying 1st boundary condition to (10)

$$0 = c_2 e^{-\gamma z} \Rightarrow \underline{\underline{c_2 = 0}}$$

Applying the second boundary condition

$$0 = c_2 e^{-\gamma z} \Rightarrow c_2 = 0$$

$$0 = c_1 \sin ha \, e^{-\gamma z}$$

$$\Rightarrow \sin ha = 0$$

$$\Rightarrow ha = m\pi$$

$$\Rightarrow h = \frac{m\pi}{a} \quad \text{where } m = 1, 2, 3, \ldots$$

The field components for transverse electric waves between parallel planes are

$$E_y = C_1 \sin \frac{m\pi}{a}x \, e^{-\gamma z}$$

$$H_x = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$$H_z = \frac{-m\pi}{j\omega\mu_0} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

The value of m specifies a particular field configuration or mode and wave associated with the integer m is designated as ~~TEm0~~ TEm0.

The lowest value of m is 1 so ~~and~~ the lowest mode that can exist in parallel plate wave guide is TE_{10}

<u>Transverse Magnetic Field</u> $[H_z = 0, \; E_z \neq 0]$

$H_z = 0 \Rightarrow H_x = E_y = 0$

Solving the wave equation by considering H_y component

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = -\omega^2 \mu \varepsilon H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \varepsilon) H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = -h^2 H_y \quad \text{---} \; (11)$$

This is a second order harmonic equation.

$$H_y = H_y^0(x) \, e^{-\gamma z} \quad \text{---} \; (12)$$

(11) → $\dfrac{\partial^2 H_y}{\partial x^2} = -h^2 H_y^0$ —— (13)

Solution can be expressed as

$$H_y^0 = C_1 \sin hx + C_2 \cos hx$$

$$H_y = (C_1 \sin hx + C_2 \cos hx)\, e^{-\gamma z} \quad \text{—(14)}$$

The tangential component of H is not zero at the surface of the conductor from ~~(4)~~ (5)

E_z can be obtained in terms of H_y

$$\frac{\partial H_y}{\partial x} = j\omega \varepsilon E_z$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}$$

From (14)

$$E_z = \frac{-1}{j\omega\varepsilon} \frac{\partial}{\partial x}\left[C_1 \sin hx + C_2 \cos hx\right] e^{-\gamma z} \quad \text{—(15)}$$

<u>boundary conditions</u>

$E_z = 0 \quad x = 0$

$E_z = 0 \quad x = a$

applying 1^{st} boundary condition to ~~(14)~~ (15)

$$\underline{\underline{C_1 = 0}}$$

Applying 2^{nd} boundary condition to (15)

$$\sin ha = 0$$

$$\Rightarrow ha = \frac{m\pi}{a}$$

$$\Rightarrow \underline{\underline{h = \frac{m\pi}{a}}}$$

between parallel plane

be fiel
plate
~~plate~~

The field components ~~is~~ of TMmode waves in a parallel plate ~~plate~~ wave guide

$$E_x = \frac{\gamma E_z}{j\omega\varepsilon} \cos \frac{m\pi}{a} x \, e^{-\gamma z}$$

$$E_z = \frac{jm\pi}{\omega\varepsilon a} C_2 \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

$$H_y = C_2 \cos\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

Rectangular Wave guides

A rectangular wave guide is a hollow conducting tube with a rectangular cross section. In order to find the electromagnetic wave propagation through the wave guide solve ~~So~~ the maxwell's equation subjected to boundary conditions.

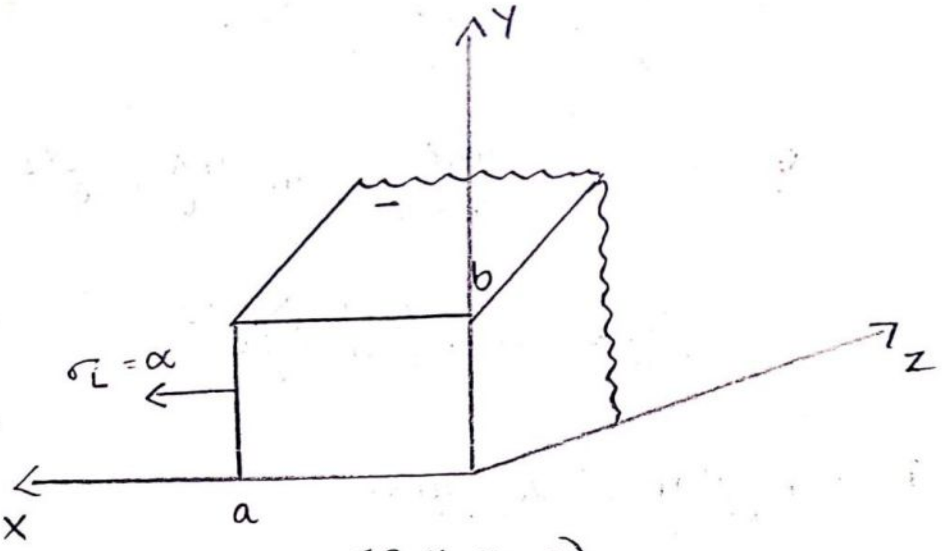
Electromagnetic wave of any frequency cannot propagate through the wave guide because of the boundaries and the related conditions. There ~~This~~ exist a minimum frequency below which the propagation is absent. Thus give rise to the concept of cutoff frequency. Wave guide permits propagation of electromagnetic wave of frequency greater than the cut off frequency ∴ we can consider the waveguide as a highpass filter.

(Transverse electric and magnetic wave)
TEM cannot exist in wave guides. The possible ~~no~~ different modes in waveguides are TM_{mn}

Attenuation is low in wave guides compared to the

Transmission Lines.

Consider a rectangular wave guide in X Y Z direction



electric field in varying field

Consider the rectangular wave guide. Assume that the wave guide is filled with source free $(\rho_v = 0, J = 0)$ lossless dielectric material $(\sigma = 0)$ and its walls are perfectly conducting $(\sigma_c = \infty)$

Using Maxwells equation,

$$\nabla^2 E_s + k^2 E_s = 0$$

$$\nabla^2 H_s + k^2 H_s = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

Let $E_s = (E_{xs}, E_{ys}, E_{zs})$

$H_s = (H_{xs}, H_{ys}, H_{zs})$

Consider the Z-component $E_z = \frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0$

The solution of E_z by assuming the wave propagation in z-direction

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) e^{-\gamma z}$$

This is for electric field component. For magnetic field component ①

$$H_{zs}(x,y,z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y) e^{-\gamma z}$$

To find the field components E_{xs}, E_{ys}, H_{xs}, H_{ys} in terms of E_{zs} and H_{zs}, use the Maxwells equation

$$\nabla \times E_s = -j\omega\mu H_s$$

$$\nabla \times H_s = j\omega\varepsilon E_s$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_z & H_y & H_z \end{vmatrix}$$

$$\left.\begin{aligned}
\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} &= -j\omega\mu H_{xs} \\
\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} &= j\omega\varepsilon E_{xs} \\
\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} &= -j\omega\mu H_{ys} \\
\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} &= j\omega\varepsilon E_{ys} \\
\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} &= -j\omega\mu H_{zs} \\
\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} &= j\omega\varepsilon E_{zs}
\end{aligned}\right\} \quad \text{—— ③}$$

$$H_x = H_x^0 e^{-\gamma z}$$

$$H_y = H_y^0 e^{-\gamma z} \Rightarrow \frac{\partial H_y}{\partial z} = -\gamma H_y^0 e^{-\gamma z} = -\gamma H_y$$

$$E_x = E_x^0 e^{-\gamma z}$$

$$E_y = E_y^0 e^{-\gamma z}$$

Consider $E_{zs} \sim e^{-\gamma z}$

$$\frac{\partial E_{zs}}{\partial z} = -\gamma E_{zs}$$

$$\frac{\partial^2 E_{zs}}{\partial z^2} = \gamma^2 E_{zs}$$

By solving ③

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon = \gamma^2 + k^2$$

~~By solving~~

The expression of $E_{xs}, E_{ys}, H_{xs}, H_{ys}$ in terms of E_{zs} and H_{zs}.

$$\left.\begin{aligned}
E_{xs} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \\
H_{xs} &= \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} \\
H_{ys} &= \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y} \\
E_{ys} &= \frac{-\gamma}{h^2}\cdot\frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}
\end{aligned}\right\} \quad \text{— ④}$$

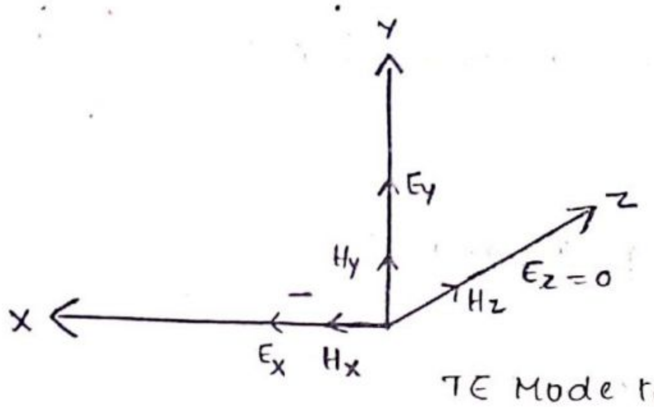
The different modes exist in a rectangular wave guide are

① $E_{zs} = H_{zs} = 0$ [Transverse Electric and Magnetic mode (TEM mode)]

From ④, all the field components vanish for this condition. The conclusion of this is that in a rectangular wave guide cannot support TEM mode.

② $E_{zs} = 0$, $H_{zs} \neq 0$ [Transverse electric mode (TE mode)]

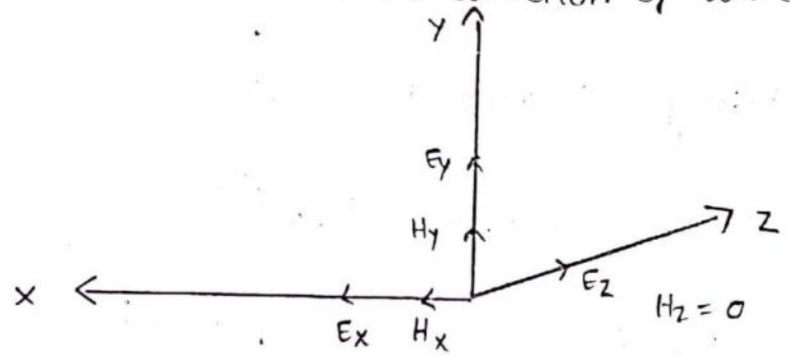
~~From ④~~ The components E_{xs} and E_{ys} of the electric field are transverse to the direction of propagation in the z-direction



TE Mode representation

③ $E_{zs} \neq 0$, $H_{zs} = 0$ [Transverse magnetic field]

The H field is transverse to the direction of wave propagation



TM Mode.

PT TEM wave cannot propagate through a rectangular wave guide.

Transverse Magnetic (TM) Mode

Magnetic field components are transverse (normal/perpendicular) to the direction of wave propagation. In TM Mode set $H_z = 0$ and determine E_x, E_y, E_z, H_x and H_y using the boundary conditions

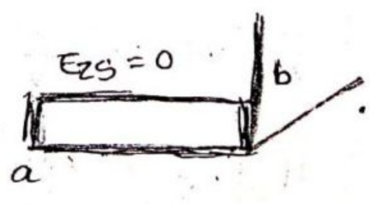
The boundary conditions are

$E_{zs} = 0$ at $y = 0$ — ⓐ

$E_{zs} = 0$ at $y = b$ — ⓑ

$E_{zs} = 0$ at $x = 0$ — ⓒ

$E_{zs} = 0$ at $x = a$ — ⓓ



From

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)$$

substituting ⓐ and ⓒ

$$A_1 = A_3 = 0$$

$$E_{zs} = A_2 A_4 \sin k_x x \sin k_y y \, e^{-\gamma z}$$

$$E_{zs} = E_0 \sin k_x x \sin k_y y \, e^{-\gamma z} \quad \text{—— (5)}$$

$$E_0 = A_2 A_4$$

Substituting (b) and ⓓ in (5)

$$0 = E_0 \sin k_x a \sin k_y b \, e^{-\gamma z}$$

$$\Rightarrow \sin k_x a = 0 \qquad \sin k_y b = 0$$

$$\Rightarrow k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b}$$

Substituting these values (5)

$$E_{zs} = E_0 \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \, e^{-\gamma z} \quad \text{—— (6)}$$

~~Fro~~ From ⓐ

(7)

$$\begin{cases} E_{xs} = \dfrac{-\gamma}{h^2}\left(\dfrac{m\pi}{a}\right) E_0 \cos \dfrac{m\pi}{a} x \sin \dfrac{n\pi}{b} y \, e^{-\gamma z} \\ E_{ys} = \dfrac{-\gamma}{h^2}\left(\dfrac{n\pi}{b}\right) E_0 \sin\left(\dfrac{m\pi}{a} x\right) \cos\left(\dfrac{n\pi}{b} y\right) e^{-\gamma z} \\ H_{xs} = \dfrac{j\omega\varepsilon}{h^2}\left(\dfrac{n\pi}{b}\right) E_0 \sin\left(\dfrac{m\pi}{a} x\right) \cos\left(\dfrac{n\pi}{b} y\right) e^{-\gamma z} \\ H_{ys} = \dfrac{-j\omega\varepsilon}{h^2}\left(\dfrac{m\pi}{a}\right) E_0 \cos\left(\dfrac{m\pi}{a} x\right) \sin\left(\dfrac{n\pi}{b} y\right) e^{-\gamma z} \end{cases}$$

where

$$h^2 = k_x^2 + k_y^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{--- (8)}$$

From (6), (7) and (8), each set of integers m and n gives a different ~~patte~~ field pattern or mode referred as TM_{mn}

- Integer m equals the number of half cycle variations in the x-direction and integer n is the number of half cycle variations in the y direction

From ~~(5)~~ (6) and (7)

if ~~mn~~ (m,n) is $(0,0)$, $(0,n)$ and $(m,0)$ all the field components vanish.

Thus neither m nor n can be zero.

So TM_{11} is the lowest order mode in ~~TM_{nm}~~ TM_{mn} mode

$$h^2 = k_x^2 + k_y^2$$

$$h^2 = \gamma^2 + k^2 \qquad k = \omega\sqrt{\mu\varepsilon}$$

$$\gamma^2 + k^2 = k_x^2 + k_y^2$$

Problem

$$\gamma = \sqrt{k_x^2 + k_y^2 - k^2}$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

Case-1

If $\gamma = 0$, $\alpha = \beta = 0$

$$k^2 - \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

This is the condition, the value of $\bar{\omega}$ for this condition is called the cut off angular frequency ω_c

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$= \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Case-2

$\gamma = \alpha$
, $\beta = 0$

$$k^2 = \omega^2 \mu \varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

In this case, there is no wave propagation. This not propagating or attenuating mode is called evanescent mode

Case-3

$\gamma = j\beta$, $\alpha = 0$

$$k^2 = \omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

This is the only condition for the wave propagation.

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

$$j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

squaring on both sides

$$-\beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2$$

$$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

For each mode characterized by a set of ~~m~~ intigers m and n, there is a corresponding cutoff frequency f_c. The cut off frequency is the operating frequency below which attenuation occurs and above which propagation takeplace.

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \longrightarrow \text{cutoff frequency}$$

where $u' = \frac{1}{\sqrt{\mu\varepsilon}}$

where $u' = \frac{1}{\sqrt{\mu\varepsilon}}$ is the phase velocity of uniform plane wave in a lossless dielectric medium

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cutoff wavelength, λ_c of the TM mode,

$$\lambda_c = \frac{u'}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

mode ∴ is the lowest

~~TM11 has~~ TM_{11} has the lowest cutoff frequency of all the TM modes. The phase constant β can be represented as

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\cancel{\beta' = \omega\sqrt{\mu\varepsilon}}$$

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$\beta' = \omega\sqrt{\mu\varepsilon}$ (expression for lossless ~~lossless~~ ~~uniform~~ dielectric medium ~~mode~~)

~~This is the expression for.~~

$$\gamma = \alpha = \beta'\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

$\gamma = \alpha + j\beta$

evanescent mode

Problems.

$$U_p = \frac{\omega}{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{U_p}{f}$$

Intrinsic wave impedence (η)

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-H_y}{E_x}$$

By substituting the values

$$\eta_{TM} = \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{1}{\omega\varepsilon}\beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{1}{\omega\varepsilon}\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

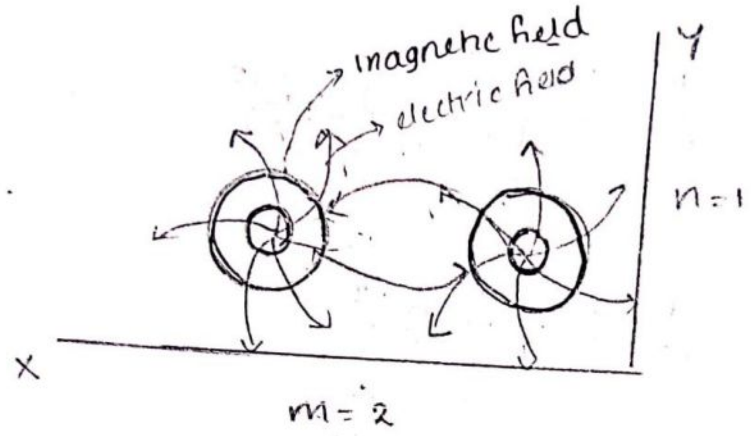
$$\therefore \eta_{TM} = \eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

where $\eta' = \sqrt{\frac{\mu}{\varepsilon}}$

η' is the intrinsic impedence of the uniform plane wave in the medium.

~~when v.~~ Note

The quantities ~~u a~~ u', β' and η' are the wave characteristics of the dielectric medium unbounded by the wave guide. i.e., ~~H~~ u' would be the velocity of the wave if the wave guide were removed and the entire space were filled with the dielectric. The quantities u, β and η are the wave characteristics of the medium bounded by the wave guide.



Transverse electric mode (TE mode)

The electric field is transverse to the direction of wave propagation.

- Rectangular wave guide
- fig
- different field component value and solution (upto (1) and (2))

~~After~~ If $E_z = 0$ and determine the field components E_x, E_y, H_x, H_y and H_z using boundary conditions.

$E_{xs} = 0$ at $Y = 0$ — (a)

$E_{xs} = 0$ at $Y = b$ — (b)

$E_{ys} = 0$ at $x = 0$ — (c)

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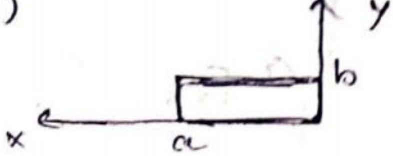
$E_{ys} = 0$ at $x = a$ — (d)



Substituting (a), (b), (c) and (d) in (4)

~~For y=0, y=b~~

$$\frac{\partial H_{zs}}{\partial y} = 0$$

$$\left.\begin{aligned} \frac{\partial H_{zs}}{\partial y} &= 0 \quad \text{at } y = 0 \\ \frac{\partial H_{zs}}{\partial y} &= 0 \quad \text{at } y = b \\ \frac{\partial H_{zs}}{\partial x} &= 0 \quad \text{at } x = 0 \\ \frac{\partial H_{zs}}{\partial x} &= 0 \quad \text{at } x = a \end{aligned}\right\} \text{— (9)}$$

Applying (9) ~~these conditions~~ to (2)

$$\text{(2)} \Rightarrow \quad H_{zs}(x,y,z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)\, e^{-\gamma z}$$

$$\frac{\partial H_z}{\partial y} = 0 \Rightarrow$$

$$\frac{\partial}{\partial y}\left[\underbrace{(B_1 \cos k_x x + B_2 \sin k_y y)}_{\text{constant}}(B_3 \cos k_y y + B_4 \sin k_y y)\, e^{-\gamma z}\right] = 0$$

From first two equations in (9)

Applying the conditions, we get

$$H_{zs} = H_0 \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} \text{ — (10)}, \quad H_0 = B_1 B_3$$

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad (11)$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

~~when m=n=0~~ [(0,0)]

All ~~no field components will vanish~~

when (0,1) and (1,0), ~~all the field component values will vanish~~.

field component exist.

For TE modes m, n may be (0,1) or (1,0) but not (0,0). when m and n becomes zero, then all the field components will vanish [From (11)]

So the lowest mode in TE mode can be TE_{10} or TE_{01}. Depending on the values of a and b normally a>b. so TE_{10} is the lowest mode because the cutoff frequency of TE_{10} mode is less than ~~cutoff fr~~ TE_{01}.

(Q1) cutoff frequency, $f_c = \frac{u'}{2a}$ and $f_c = \frac{u'}{2b}$ for (0,1) and (1,0)

$$\eta_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$\left(\sqrt{\frac{\mu}{\varepsilon}}\right) \cdot \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

where η' is the intrinsic impedence of the plane wave

$$\cancel{\eta_{TE}} \quad \eta_{TE} \cdot \eta_{TM} = (\eta')^2$$

1) In a rectangular wave guide. $a = 1.5$ cm, $b = 0.8$ cm,

$\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right) \cdot \sin\left(\pi \times 10^{11} t - \beta z\right) \text{ A/m}$$

a) Determine the mode of operation

b) cutoff frequency

c) phase constant (β)

d) Propagation constant (γ)

e) intrinsic wave impedence (η)

Ans.

Comparing H_x with equation of H_{xs}

given z, mode is E, because =0 in TM, similarly if z is given, it is TM

$$\frac{m\pi}{a} = \frac{\pi}{a} \Rightarrow m = 1$$

$$\frac{n\pi}{b} = \frac{3\pi}{b} \Rightarrow n = 3$$

From this

(a) The mode of operation is either TM_{13} or TE_{13}.

(b) cut off frequency of TM_{13}

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{4\mu_0\varepsilon_0}} = \frac{c}{\sqrt{4}} = \frac{c}{2}$$

$$= 1.5 \times 10^8$$

$$\cancel{f_c = \frac{1.5\times10^8}{2}\sqrt{\left(\frac{1}{1.5}\right)^2 +}}$$

$$f_c = \frac{1.5\times10^8}{2}\sqrt{\left(\frac{1}{1.5\times10^{-2}}\right)^2 + \left(\frac{3}{0.8\times10^{-2}}\right)^2}$$

$$\simeq 2.85\times10^{10}\ Hz$$

$$= 28.5\ GHz$$

cutoff frequency of $TE_{13} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 28.5$ GHz

(c) Phase constant $(\beta) = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$\beta' = \omega\sqrt{\mu\varepsilon}$$

given

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\underbrace{\pi\times10^{11}t - \beta z}_{}\right)\ A/m$$

$\omega t - \beta z$

$$\therefore \omega = \pi\times10^{11}$$

$$\beta' = \pi\times10^{11}\sqrt{4\mu_0\varepsilon_0} = \pi\times10^{11}\times\frac{1}{2c}$$

$$= \pi\times10^{11}\times\frac{1}{2\times3\times10^8}$$

$$= \cancel{523.598}\ 20\,9\,4.81$$

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 523.598\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

E_0 $\omega = 2\cancel{\pi}f = \cancel{\pi}\times10^{11}$

$$f = 5\times10^{10}$$

$$\beta = \frac{2094.81}{\cancel{528.598}} \sqrt{1 - \left(\frac{28.5 \times 10^9}{5 \times 10^{10}}\right)^2}$$

$$= \underline{1721.18}$$

phase constant for TM and TE = 1721.18

(d) $\gamma = j\beta = 1721.18j$

(e) wave impedence for $\overset{TM}{\cancel{TE}}$ mode.

$$\eta = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad , \quad \eta' = \sqrt{\frac{\mu}{\varepsilon}}$$

$\frac{\mu_0}{\sqrt{\varepsilon_0}} \rightarrow$ intrinsic impedence of plane wave = 377

$$\eta' = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{1}{2} \times 377 = 188.5$$

$$\cancel{\eta_{TE13}} = \quad \eta = 188.5 \sqrt{1 - \left(\frac{28.5 \times 10^9}{5 \times 10^{10}}\right)^2}$$

$$= \underline{154.88}$$

wave impedence for $\overset{\cancel{TM}}{\underset{TE}{\cancel{TE}}}$ mode

$$\eta = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{188.5}{\sqrt{1 - \left(\frac{28.5 \times 10^9}{5 \times 10^{10}}\right)^2}}$$

$$= \underline{229.4}$$

Phas
It is a
phase

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Phase Velocity

It is defined as the rate at which the wave changes its phase in terms of gate wavelength or it is the velocity of propagation of equiphase surfaces along the guide.

$$u_p = \frac{\omega}{\beta}$$

$$u_p = \frac{\omega}{\sqrt{\omega^2 \mu \varepsilon - \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)}}$$

$$= \frac{\omega}{\sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon}}$$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1 - (f_c/f)^2}}$$

$$u_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Guide Wavelength (λ_g)

It is defined as the distance travelled by the wave inorder to undergo a phase shift of 2π radians.

$$\lambda_g = \frac{u_p}{f}$$

$$= \frac{\omega/\beta}{f}$$

$$= \frac{\omega}{\beta f}$$

$$= \frac{2\pi f}{\beta f}$$

$$= \frac{2\pi}{\beta}$$

$$\beta = \frac{2\pi}{\sqrt{\omega^2 \mu \varepsilon - \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)}}$$

$$= \frac{2\pi}{\sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon}}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{1}{f\sqrt{\mu\varepsilon}\sqrt{1 - (f_c/f)^2}}$$

$$= \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Group Velocity

It is defined as the rate at which the wave propagates through the wave guide

$$V_g = \frac{d\omega}{d\beta}$$

$$V_p V_g = c^2$$

$$V_p = \frac{c}{\sqrt{1-(f_c/f)^2}}$$

$$V_g = c\sqrt{1-(f_c/f)^2}$$

2) A rectangular wave guide with dimension 3 x 2 cm operates at 10 GHz (f). Find

a) f_c

b) λ_c

c) λ

d) λ_g

e) β_g

f) v_p for TE_{10} mode

{ operating frequency = f }

Ans

Dimension 3 x 2 cm

given data

$a = 3\text{ cm} = 0.03\text{ m}$

$b = 2\text{ cm} = 0.02\text{ m}$

$f = 10\ GHz$

TE_{10}

a) f_c (cut-off frequency)

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u' = \frac{1}{\sqrt{\mu \varepsilon}}$$

μ, ε are not given. Consider the free space condition

$$u' = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \times 10^8\ m/s$$

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.03}\right)^2 + 0}$$

$$\Rightarrow f_c = 5\times10^9 = \underline{\underline{5\ GHz}}$$

b) $\lambda_c =$ (cut off wavelength)

$$\lambda_c = \frac{u'}{f_c}$$

$$= \frac{3\times10^8}{5\times10^9}$$

$$= \underline{\underline{0.06\ m}}$$

c) λ (wavelength)

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = \underline{0.03\ m}$$

d) $$\lambda_g = \frac{c}{f\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \frac{c}{f} = \lambda$$

$$= \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{0.03}{\sqrt{1-\left(\frac{5\times10^9}{10\times10^9}\right)^2}}$$

$$= \frac{0.03}{\sqrt{1-0.25}}$$

$$= \underline{0.034\,m}$$

e) $\underline{\beta_g}$

$$\lambda_g = \frac{2\pi}{\beta_g} \rightarrow \beta_g = \frac{2\pi}{\lambda_g}$$

$$= \frac{2\pi}{0.034}$$

$$= \underline{\underline{184.79}}\ \text{rad/m}$$

f) $\underline{v_p}$ (phase velocity)

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^8}{\sqrt{1-0.25}} = \underline{\underline{3.4\times10^8\ m/s}}$$

V_g (group velocity) $\lambda, v_p, \lambda_c, f_c$
λ_g (guide wavelength)

find TE/TM

TE_{mn} / TM_{mn}