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MODULE - 2

Maxwell's Equation & Travelling Waves

Modified Ampere's Circuital law

Ampere's Circuital law fails when the field varies (time varying situations) with time or the charges contained in a volume changes with time.

Ampere's circuital law, $\nabla \times H = J_c$

Let the modified form be, $\boxed{\nabla \times H = J_c + J_d}$

Take div on both sides,

$$\nabla \cdot (\nabla \times H) = \nabla \cdot (J_c + J_d)$$

$$0 = \nabla \cdot J_c + \nabla \cdot J_d$$

$$\nabla \cdot J_d = -\nabla \cdot J_c = \frac{\partial}{\partial t} \rho_v$$

$$\nabla \cdot J_d = + \frac{\partial}{\partial t} (\nabla \cdot D)$$

$\rho_v = \nabla \cdot D$

$D = \varepsilon E$

$$= \nabla \frac{\partial}{\partial t} (\varepsilon E)$$

from continuity eq^n

$$\nabla \cdot J_c + \frac{\partial \rho_v}{\partial t} = 0$$

$$\nabla \cdot J_c = -\frac{\partial}{\partial t} \rho_v$$

$$\nabla \cdot J_d = \nabla \cdot \varepsilon \frac{\partial E}{\partial t}$$

$$\boxed{J_d = \frac{\varepsilon \partial E}{\partial t} = \frac{\partial D}{\partial t}},$$

Total current density,

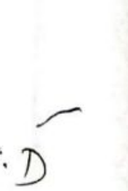
$$J = J_c + J_d$$

$$J = \sigma E + \frac{\partial D}{\partial t}$$

$$\left| \frac{J_c}{J_d} \right| = \left| \frac{\sigma E}{j\omega \varepsilon E} \right| = \frac{\sigma}{\omega \varepsilon}$$

J_D leads the total current density

$$J_T = J_c + J_D$$



$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$\sigma \rightarrow$ conductivity

Modified form of ampere's circuital law

$$\boxed{\nabla \times H = \sigma E + \frac{\partial D}{\partial t}}$$

Phasor form, $\frac{\partial}{\partial t} \rightarrow j\omega$

$\frac{\sigma}{\omega \varepsilon} << 1$, medium is insulator or dielectric.

$$\Rightarrow \boxed{\nabla \times H = \sigma E + j\omega D}$$

if $J_C = J_D$, then $\frac{\sigma}{\omega \varepsilon} = 1$

loss tangent, $\frac{\sigma}{\omega \varepsilon} >> 1$, the material is good conductor

1) Maxwell's equation derived from Guass law for <u>Electric fields</u>

~~1) Maxwell's equation derived~~ for

$\Psi = Q$

$\Psi = \oint_S D \cdot ds$

$\rho_v \Rightarrow$ volume charge density

$Q = \int \rho_v \cdot dV$

$$\boxed{\oint D \cdot ds = \int \rho_v \cdot dV}$$ Integral form

$\oint_S D \cdot ds = \int_V (\nabla \cdot D) dV \rightarrow$ Gauss divergence theorem

$\int_V (\nabla \cdot D) dv = \int_V \rho_v dv$

$$\boxed{\nabla \cdot D = \rho_v}$$ Point form / differential

2) Maxwell's equation derived from Guass's law for <u>magnetic field</u> -

For magnetic fields, the surface integral of B over a closed surface S is always zero

$\phi = 0$

$$\boxed{\oint B \cdot ds = 0}$$ integral form

Using divergence theorem,

$$\oint B \cdot ds = \iiint \nabla \cdot B \, dV = 0$$

$$\boxed{\nabla \cdot B = 0}$$ point form / differential form

iii) Maxwell's equation derived from Faraday's law.

$\nabla \times B = -\frac{\partial B}{\partial t}$

Faraday law

$$\oint E \cdot dl = -\iint_S \frac{\partial B}{\partial t} ds$$

$$\oint E \cdot dl = \iint_S \nabla \times E \, ds$$

stokes th

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

iv) Maxwell's eqn derived from Ampere's circuital law

$$\oint H \cdot dl = \int_S J \cdot ds = I$$

$$\oint H \cdot dl = \int_S (J_c + J_d) ds$$

$$\boxed{\oint_l H \cdot dl = \int_S \left(\sigma E + \frac{\partial D}{\partial t}\right) \cdot ds}$$ Integral form

Apply stoke's theorem on above eqn

$$\oint H \cdot dl = \int_S (\nabla \times H) ds$$

$$\therefore \int_S (\nabla \times H) ds = \int_S \left(\sigma E + \frac{\partial D}{\partial t}\right) ds$$

$$\boxed{\nabla \times H = \sigma E + \frac{\partial D}{\partial t}}$$

$\nabla \cdot A$ → divergence

$\nabla \times A$ → stokes

$$\left.\begin{aligned} \nabla \cdot D &= \rho \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= \sigma E + \frac{\partial D}{\partial t} \end{aligned}\right\}$$

Differential form of Maxwell's Equation

1) $\nabla \cdot D = \rho_v$

2) $\nabla \cdot B = 0$

3) $\nabla \times E = \frac{-\partial B}{\partial t}$ $\quad B = \mu H$

4) $\nabla \times H = \sigma \dot{E} + \frac{\partial D}{\partial t}$ $\quad D = \varepsilon E$

Integral form

1) $\oint D \cdot ds = \int_V \rho_v \, dV$

2) $\oint B \cdot ds = 0$

3) $\oint_l E \cdot dl = -\iint_S \frac{\partial B}{\partial t} ds$

4) $\oint_l H \cdot dl = \int_S (\sigma E + \frac{\partial D}{\partial t}) ds$

Maxwell's eqn in phasor form

1) $\nabla \cdot D = \rho_v$

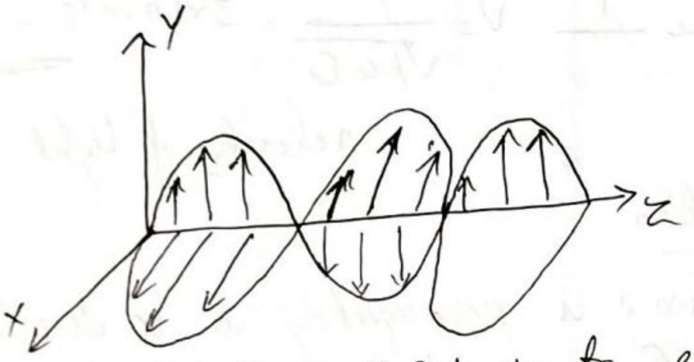
2) $\nabla \cdot B = 0$

~~$\nabla \times E = -\frac{\partial B}{\partial t}$~~

3) $\nabla \times E = -j\omega B \leftarrow \frac{\partial}{\partial t} = j\omega$

4) $\nabla \times H = \sigma E + j\omega D$

Plane wave



E_x, H_y, E & H $\perp$ to each other and also to the direction of propagation i.e. z axis

Uniform plane wave solution

Wave equation of uniform plane wave in free space

For free space ; $\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\varepsilon_r = \mu_r = 1$, $\rho_v = 0$, $J = 0$

$$\nabla \times E = -\mu_0 \mu_r \frac{\partial H}{\partial t}$$

$$\nabla \times H = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t} + \sigma E$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

$\Rightarrow$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \cdot D = 0$$

$$\nabla \cdot B = 0$$

$$\Rightarrow \nabla \cdot \varepsilon_0 E = 0, \quad \nabla \cdot E = 0 \longrightarrow ①$$

$$\nabla \cdot \mu H = 0, \quad \nabla \cdot H = 0 \longrightarrow ②$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \longrightarrow ③$$

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \longrightarrow ④$$

$D = \varepsilon E$
$J = \sigma E$
$B = \mu H$

Taking curl on both sides of eqn ③

(from either 3 or 4)

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t}\right)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\nabla \times H)$$

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t} \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\boxed{\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

or

$$\nabla^2 E = \frac{1}{V^2} \frac{\partial^2 E}{\partial t^2}$$

where $V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s = c$

velocity of light

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Plane wave solution

Assume the wave is propagating in x-direction. The variation of E field is independent of y and z directions. ie.,

$$\frac{\partial^2 E}{\partial y^2} = \frac{\partial^2 E}{\partial z^2} = 0 \qquad \nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \underbrace{\frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}}_{0}$$

$\Delta = \partial/\partial x$

$\Delta^2 = \partial^2/\partial x^2$

ie., $\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$ becomes

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

It is a 2nd order differential eqn and its solution can be written as

$$E = f_1(x - v_0 t) + f_2(x + v_0 t)$$

$\nabla \cdot D = 0$ in free space

$\nabla \cdot E = 0$

$\frac{\partial E}{\partial x} = 0$.

$\frac{\partial (\varepsilon E)}{\partial x} = 0$

$\varepsilon_0 \frac{\partial E}{\partial x} = 0$

This means $E = 0$ or E is a constant

If E is a constant, then electromagnetic field cannot be defined.

$\therefore$ E must be zero

i.e., $\boxed{E_x = 0}$

Characteristic impedance (intrinsic impedance) $\Rightarrow$ lossless dielectric

$\nabla \times H = \cancel{\sigma E} + \frac{\partial D}{\partial t}$

$$\Rightarrow \frac{\partial H_z}{\partial x} = -\varepsilon_0 \frac{\partial}{\partial t} E_y \longrightarrow (1)$$

$\therefore \frac{\partial E}{\partial x} = 0$, then the soln can be written as

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = \frac{\partial}{\partial t}(\varepsilon E)$$

$$E_y = f(x - V_0 t)$$

Differentiating this expression w.r.t time

$\frac{\varepsilon_0 \partial}{\partial t}(E_x a_x + E_y a_y + E_z a_z)$

$$\frac{\partial}{\partial t} E_y = f'(x - V_0 t)(-V_0) \longrightarrow (2)$$

(2) in (1)

$$\frac{\partial H_z}{\partial x} = -\varepsilon_0 f'(x - V_0 t)(-V_0)$$

Integrating

$$H_z = -\varepsilon_0 f(x - V_0 t)(-V_0)$$

$$H_z = \varepsilon_0 V_0 f(x - V_0 t)$$

$$= \varepsilon_0 V_0 E_y$$

$$\frac{E_y}{H_z} = \frac{1}{\varepsilon_0 V_0} \longrightarrow (3)$$

wave velocity

$$\boxed{V_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

$$= \frac{1}{\varepsilon_0 \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

$$\boxed{\frac{E_y}{H_z} = \frac{\sqrt{\varepsilon_0 \mu_0}}{\varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

Similarly $\frac{E_z}{H_y} = -\sqrt{\mu_0 / \varepsilon_0}$

$$|E| = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$E_x^2 = H_x^2 = 0$$

$$|H| = \sqrt{H_x^2 + H_y^2 + H_z^2}$$

$$\frac{|E|}{|H|} = \frac{\sqrt{E_y^2 + E_z^2}}{\sqrt{H_y^2 + H_z^2}}$$

$$= \frac{\sqrt{H_z^2 \frac{\mu_0}{\varepsilon_0} + H_y^2 \frac{\mu_0}{\varepsilon_0}}}{\sqrt{H_y^2 + H_z^2}}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\sqrt{H_z^2 + H_y^2}}{\sqrt{H_y^2 + H_z^2}}$$

$$\boxed{\frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

Generally
Characteristic impedance, $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 377\,\Omega$ in free space

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Wave equation for a perfect dielectric (lossless) medium

For a perfect dielectric (lossless) medium,

$\sigma = 0$, $\rho_v = 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_r \mu_0$, $\sigma << \omega\varepsilon$

& Maxwell's eqn become

$$\nabla \cdot E = 0 \longrightarrow ①$$

$$\nabla \cdot H = 0 \longrightarrow ②$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \longrightarrow ③$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \longrightarrow ④$$

$$\nabla \cdot B = 0$$
$$\nabla \cdot D = \rho_v$$
$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad \left(-\frac{\partial B}{\partial t}\right)^{B=\mu H}$$
$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t}$$

Taking curl on both sides of eqn ③

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu \frac{\partial H}{\partial t}\right) = -\mu \frac{\partial}{\partial t}(\nabla \times H)$$

$$\nabla(\nabla\cdot E) - \nabla^2 E = -\mu\varepsilon \frac{\partial^2}{\partial t^2} E$$ from ④ & $\nabla\cdot E = 0$ from ①

$$\boxed{\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}}$$

Similarly

$$\boxed{\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}}$$

Wave equation for conducting medium

$\sigma = \infty$, $\varepsilon = \varepsilon_0\varepsilon_r$, $\mu = \mu_0\mu_r$, $\rho_v = 0$ (inside conductor), $\sigma \gg \omega\varepsilon$

(Maxwell genrl eqn)

Maxwell's equation becomes

$$\nabla\cdot E = 0 \longrightarrow ①$$

$$\nabla\cdot H = 0 \longrightarrow ②$$

$$\nabla\times E = -\mu\frac{\partial H}{\partial t} \longrightarrow ③$$

$$\nabla\times H = \sigma E + \varepsilon\frac{\partial E}{\partial t} \longrightarrow ④$$

Taking curl on both sides of eqn ③

$$\nabla\times(\nabla\times E) = \nabla\times\left(-\mu\frac{\partial H}{\partial t}\right) = -\mu\frac{\partial}{\partial t}(\nabla\times H)$$

$$\nabla(\nabla\cdot E) - \nabla^2 E = -\mu\frac{\partial}{\partial t}\left(\sigma E + \varepsilon\frac{\partial E}{\partial t}\right)$$

$$-\nabla^2 E = -\sigma\mu\frac{\partial E}{\partial t} - \varepsilon\mu\frac{\partial^2}{\partial t^2}E$$

$$\boxed{\nabla^2 E = \sigma\mu\frac{\partial E}{\partial t} + \varepsilon\mu\frac{\partial^2 E}{\partial t^2}}$$

Similarly

$$\boxed{\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \varepsilon\mu\frac{\partial^2 H}{\partial t^2}}$$

These are the wave eqn for a conducting medium. Here the electromagnetic wave give rise to a conduction current in the medium. This leads to attenuation of the wave and also dissipation of power. The terms responsible for the loss of energy are $\sigma\mu\frac{\partial E}{\partial t}$ & $\mu\sigma\frac{\partial H}{\partial t}$.

Derivation of α & β for a conducting medium

Phasor form of Maxwell's equation

$\nabla\cdot D=0$ $\quad \nabla\cdot E=0 \longrightarrow (1)$

$\nabla\cdot \varepsilon E=0$ $\quad \nabla\cdot H=0 \longrightarrow (2)$

$\nabla\cdot B=0$ $\quad \nabla\times E=-j\omega\mu H \longrightarrow (3)$

$\nabla\cdot(\mu H)=0$ $\quad \nabla\times H=\sigma E+j\omega\varepsilon E \longrightarrow (4)$ $\quad \frac{\partial}{\partial t}=j\omega$.

$\nabla\times E=-\frac{\partial B}{\partial t}$ Applying curl on eqn (3)

$\frac{\partial}{\partial t}=j\omega$ $\quad \nabla\times(\nabla\times E)=\nabla\times(-j\omega\mu H)$

$$\nabla(\nabla\cdot E)-\nabla^2 E=-j\omega\mu(\nabla\times H)=-j\omega\mu(\sigma E+j\omega\varepsilon E)$$

$\nabla\times H=\sigma E+\frac{\partial D}{\partial t}$ $\quad -\nabla^2 E=-j\omega\mu(\sigma+j\omega\varepsilon)E$

$$\boxed{\nabla^2 E=\gamma^2 E}$$ Helmholtz equation. $\quad \gamma^2=j\omega\mu(\sigma+j\omega\varepsilon)\rightarrow(5)$

$\gamma=\sqrt{j\omega\mu(\sigma+j\omega\varepsilon)}$

propagation constant

$\gamma=\alpha+j\beta$

$(\alpha+j\beta)^2=\gamma^2=\alpha^2+2j\alpha\beta-\beta^2\longrightarrow(6)$

from (5) & (6)

$\alpha^2+2j\alpha\beta-\beta^2=j\omega\mu(\sigma+j\omega\varepsilon)$

$\alpha^2+2\alpha j\beta-\beta^2=j\omega\mu\sigma-\omega^2\mu\varepsilon$

Equating real & imaginary parts

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon \longrightarrow (7)$$

$$2\alpha\beta = \omega\mu\sigma$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha} \longrightarrow (8)$$

Substitute eqn (8) in (7)

$$\alpha^2 - \frac{\omega^2\mu^2\sigma^2}{4\alpha^2} = -\omega^2\mu\varepsilon$$

$$\alpha^2 + \omega^2\mu\varepsilon - \frac{\omega^2\mu^2\sigma^2}{4\alpha^2} = 0$$

$$\alpha^4 + \omega^2\mu\varepsilon\alpha^2 - \frac{\omega^2\mu^2\sigma^2}{4} = 0$$

This is a quadratic equation in α^2

$\alpha^2 =$

$$\alpha^2 = \frac{-\omega^2\mu\varepsilon \pm \sqrt{(\omega^2\mu\varepsilon)^2 - 4\frac{-\omega^2\mu^2\sigma^2}{4}}}{2}$$

$$= \frac{-\omega^2\mu\varepsilon}{2} \pm \frac{\sqrt{(\omega^2\mu\varepsilon)^2 + (\omega^2\mu\varepsilon)^2\left(\frac{\sigma^2}{\omega^2\varepsilon^2}\right)}}{2}$$

$$= \frac{-\omega^2\mu\varepsilon}{2} \pm \frac{\omega^2\mu\varepsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\left[-1 \pm \sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}\right] \longrightarrow (9)$$

$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}$

$1+\frac{\sigma^2}{\omega^2\varepsilon^2}$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]}} \longrightarrow (10)$$

$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}$

$\omega\sqrt{\frac{\mu\varepsilon}{2}}$

$$(7) \Rightarrow \beta^2 = \alpha^2 + \omega^2\mu\varepsilon$$

$$= \frac{\omega^2\mu\varepsilon}{2}\left[-1 \pm \sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}\right] + \omega^2\mu\varepsilon$$

$$= \frac{\omega^2\mu\varepsilon}{2}\left[-1 + \sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 2\right]$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)}} \longrightarrow (12)$$

Special case — For good conductors

$$\frac{\sigma}{\omega\varepsilon} \gg 1$$

from equation (10) $\quad \alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$

$$\alpha = \omega\sqrt{\cancel{\frac{\mu\varepsilon}{2}\left(\sqrt{\frac{\omega^2\varepsilon^2+\sigma^2}{\omega^2\varepsilon^2}}-1\right)}} = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]} \qquad \sigma \gg \omega\varepsilon,\ \sigma^2 \gg \omega^2\varepsilon^2$$

$\gamma = \alpha + j\beta$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}-1\right)} \qquad \sigma \gg \omega\varepsilon$$

$$= \omega\sqrt{\frac{\sigma\mu}{2\omega}}$$

$$\alpha = \sqrt{\frac{\sigma\mu\omega^2}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

|||ly $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ $\qquad \boxed{\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}}$

Propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\varepsilon)} = \sqrt{j\omega\mu\sigma}$$

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Wave velocity (v_p) phase velocity

It is defined as the velocity in which the phase of the wave propagates.

$$\boxed{v_p = \frac{dz}{dt} = \frac{\omega}{\beta}}$$

for good conductors, $v_p = \frac{\omega}{\beta}$

$$v_p = \frac{\omega}{\sqrt{\omega\mu\sigma/2}}$$

$$\boxed{v_p = \sqrt{\frac{2\omega}{\mu\sigma}}}$$

Group velocity $\Rightarrow \frac{d\omega}{d\beta}$.

Wave length

It is the distance that must be traveled by the wave to change the phase by 2π radiance

$$\lambda = \frac{2\pi}{\beta} = \frac{v}{f}$$

$\beta = \frac{2\pi}{\lambda}$ $\quad \frac{\sigma}{\omega\varepsilon} = \tan 2\theta_n$

Intrinsic Impedance

$$\boxed{\eta = \frac{j\omega\mu}{\gamma}} \Rightarrow \text{perfect cond.}$$

$\gamma = j\omega\mu\sigma$ $\quad \gamma = j\omega\left(\frac{\mu}{\varepsilon}\right)$ $\quad \sigma = \frac{1}{\varepsilon}$

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \quad \left(\frac{\sigma}{\omega\varepsilon} >> 1\right)$$

$\sqrt{j} = -1$ $\quad j^2 = -1$ $\quad j = \sqrt{-1}$

For good conductors, $\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

For perfect dielectric, $\sigma = 0$, $\alpha = 0$
(lossless dielectr) $\quad \eta = \sqrt{\mu/\varepsilon} \angle 0^\circ$ (lossless)

$\left|\frac{\sigma}{\omega\varepsilon}\right| << 1$

Imperfect (lossy) dielectric; $\sigma \neq 0, \sigma \neq \infty, \alpha \neq 0$

* Skindepth or depth of penetration (δ)

It is the measure of the depth to which an electromagnetic wave can penetrate the medium. It is defined as the distance or depth from the surface of the conductor at which the magnitude of the field is $\frac{1}{e}$ times its value at the surface.

$$\boxed{e^{-\alpha\delta} = e^{-1}}$$

$$-\alpha\delta = -1$$

$$\alpha\delta = 1.$$

$$\delta = \frac{1}{\alpha}$$

for good conductors

$$\delta = \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$= \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi f\mu\sigma}}$$

$$= \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}}$$

~~Dielectric~~ Depth of penetration increases with λ as lossy dielectric.

Skin depth is inversely proportional to frequency

~~Electrostatic bounda~~

Skin effect

module 6.

At high frequency skin deapth is very small for good conductors. Therefore at high frequency all the fields and currents are confined to the skin of the conductor (ie, a very thin layer near the surface of the conductor). This is called skin effect.

- To transmit very high frequency signals it is enough to have a skin of conducting coating on a wire.

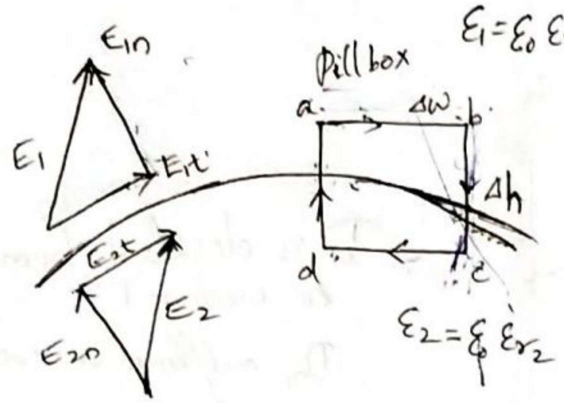
Application:-

Electrical machines and communication systems.

Electrostatic boundary Conditions

If the field exist in a region consisting of two different medium, the conditions that the field must satisfy at the interphase separating the media are called boundary condition.

1) b/w Dielectric & Dielectric



$E_1 = E_{1n} + E_{1t}$

$E_2 = E_{2n} + E_{2t}$

$\oint_C E \cdot dl = 0$ (static electric field is conservative).

$$\int_a^b E\cdot dl + \int_b^c E\cdot dl + \int_c^d E\cdot dl + \int_d^a E\cdot dl = 0$$

$$E_{1t} \cdot \Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

As $\Delta h \to 0$

$E_{1t} \cdot \Delta W - E_{2t}\Delta W = 0$

$(E_{1t} - E_{2t})\Delta w = 0$

$\Delta W \neq 0$

$E_{1t} - E_{2t} = 0$

$$\boxed{E_{1t} = E_{2t}}$$

The tangential component of E are the same on the two sides of the boundary.

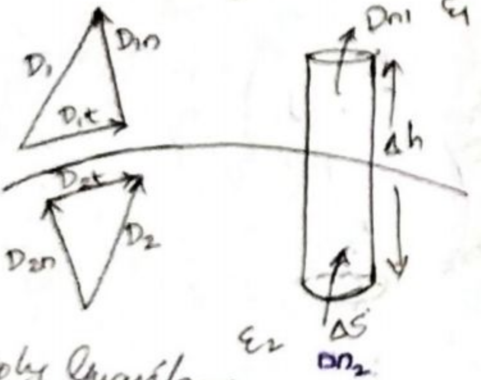
ie., E is said to be continuous across the boundary

$E_{1t} = E_{2t}$

$$E_{1t} = \frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2} = E_{2t}$$

ie $\boxed{\dfrac{D_{1t}}{\varepsilon_1} = \dfrac{D_{2t}}{\varepsilon_2}}$ D_t is discontinuous across the surface.

For determining $D_{n1} = D_{n2}$, consider a cylindrical Guassian surface.



D is directed from medium 2 to medium 1.

D_{n1} outflux is +ve

D_{n2} inward flux is -ve

Apply Guass law

$$\oint_S D \cdot ds = Q \qquad \Delta h \to 0 \qquad \Delta Q = \rho_s \Delta S$$

$$-D_{n2}\Delta S + D_{n1}\Delta S = \rho_s \Delta S$$

If we place deliberately charge density ρ_s on the surface

if $\rho_s = 0$

$$-D_{n2}\Delta S + D_{n1}\Delta S = 0$$

$$(-D_{n_2} + D_{n_1})\Delta S = 0$$

$$-D_{n_2} + D_{n_1} = 0$$

$$\boxed{D_{n_1} = D_{n_2}}$$

Normal component of D is continuous across the interface.

i.e., D_n undergoes no change at the boundary

$$D_{n_1} = \varepsilon_1 E_{n_1} = D_{n_2} = \varepsilon_2 E_{n_2}$$

ie., the normal component of E is discontinuous across the interface.

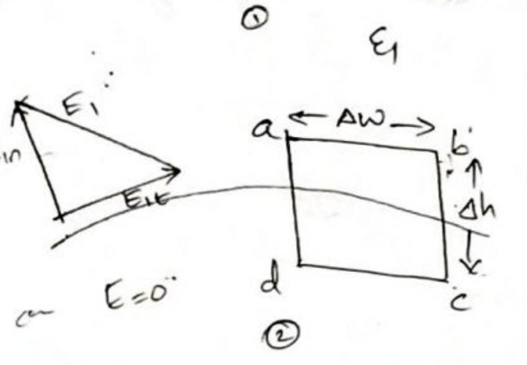
if $\rho_s \neq 0$

then $-D_{n2}\Delta S + D_{n_1}\Delta S = \rho_s \Delta S$

$$-D_{n_2} + D_{n_1} = \rho_s$$

$$D_{n_1} - D_{n_2} = \rho_s$$

2) b/w Conductor – Dielectric boundary condition.



medium 1 → dielectric

med 2 → conductor

Conductor → $E = 0$

$\oint E \cdot dl = 0$

to the closed path abcda

$$\int_a^b E\,dl + \int_b^c + \int_c^d + \int_d^a = 0$$

$E_2 = 0$

$$E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

$$E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - 0\times\frac{\Delta h}{2} - 0\times\Delta w + 0\times\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

$\Delta h \to 0$

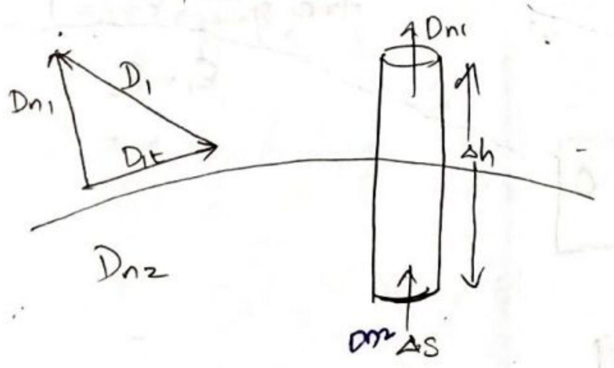
$$E_{1t}\Delta w = 0$$

$$\boxed{E_{1t} = 0}$$

$$\boxed{D_{t_1} = \varepsilon E_{t_1} = 0}$$

Tangential component of electric field intensity is continuous across the interface.

D is from medium ② to medium ① ∴ D_{n2} is −ve, D_{n1} is +ve



In order to find the normal component, consider the cylindrical Gaussian surface.

In conductor, $D_{n2} = 0$

$\oint D \cdot ds = Q$ to the cylindrical Gaussian surface.

and letting $\Delta h \to 0$

$$D_n\,\Delta S - 0 \times \Delta S = \rho_S\,\Delta S = \Delta Q$$

$-D_{n_2}\Delta S + D_{n_1}\Delta S = \rho_s \Delta S$ across the ~~surface of~~ interface

if ρ_s is deliberately placed across the

$$D_{n_1}\Delta S = \rho_s \Delta S$$

$$D_{n_1} = \rho_s$$

if $\rho_s = 0$

$$\boxed{D_{n_1} = 0}$$

$$\boxed{D_{n_1} = \varepsilon_1 E_{1n} = 0}$$

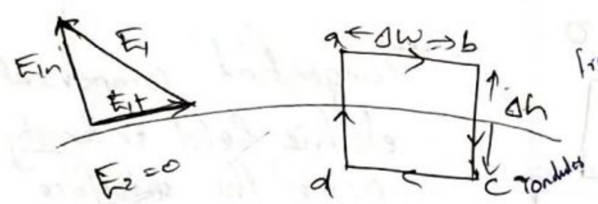
~~About condu~~ a perfect conductor, $\rho_v = 0$, $E = 0$ (within conductor)

$\therefore E = -\nabla V$, there can be no potential difference b/w any two points in the conductor.

Electric filed intensity E must be external to the conductor and must be normal to its surface.

iii) Conductor - Freespace boundary condition.

$$\varepsilon_1 = \varepsilon_0$$



$$\oint E \cdot dl = 0$$

$\vdots$

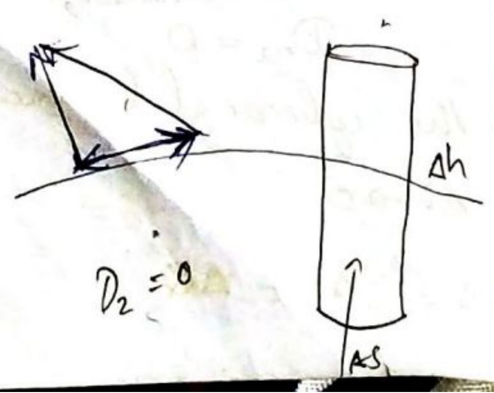
$$\boxed{E_{t_1} = 0}$$

$$\boxed{D_{1t} = \varepsilon_0 E_{t_1} = 0}$$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r}$

freespace $\rightarrow \varepsilon_r = 1$

$\varepsilon_1 = \varepsilon_0$



$\boxed{D_{n1} = 0}$

$\boxed{D_{n1} = \varepsilon_0 E_{1n} = 0}$

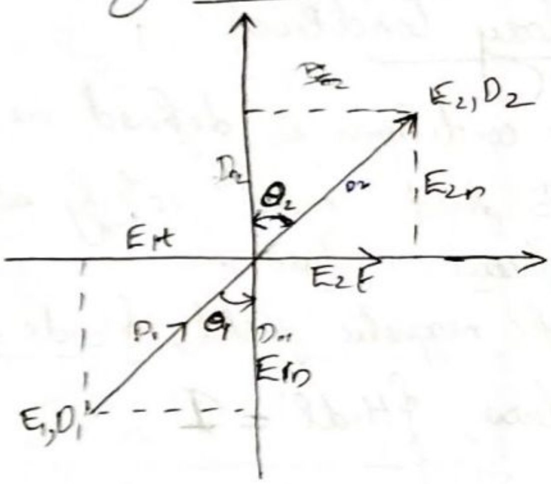
If $\rho_s = 0$

$D_n = \varepsilon_0 E_n = 0$

$\rho_s \neq 0$

$D_{n1} = \varepsilon_0 E_{1n} = \rho_s$

Derive law of Refraction in context to electrostatic boundary condition b/w Dielectric - Dielectric media.



E_1 is incident on medium 1 at an angle θ_1 & E_2 is refracted in medium 2 with an angle θ_2

Consider charge free region, $\rho_s = 0$

$D_{n1} = D_{n2}$, $E_{1t} = E_{2t}$

$\cos\theta_1 = \frac{D_{n1}}{D_1}$, $D_{1n} = D_1 \cos\theta_1 \longrightarrow (1)$

$\cos\theta_2 = \frac{D_{n2}}{D_2}$, $D_{2n} = D_2 \cos\theta_2 \longrightarrow (2)$

$\sin\theta_1 = \frac{E_{1t}}{E_1}$, $E_{1t} = E_1 \sin\theta_1 \longrightarrow (3)$

$\sin\theta_2 = \frac{E_{2t}}{E_2}$, $E_{2t} = E_2 \sin\theta_2 \longrightarrow (4)$

Apply boundary condition

$(1) = (2) \Rightarrow D_1 \cos\theta_1 = D_2 \cos\theta_2 \longrightarrow (5)$

$(3) = (4) \Rightarrow E_1 \sin\theta_1 = E_2 \sin\theta_2 \longrightarrow (6)$

$$\frac{(6)}{(5)} \Rightarrow \frac{E_1 \sin\theta_1}{D_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{D_2 \cos\theta_2}$$

$$\frac{E_1}{D_1} \tan\theta_1 = \frac{E_2}{D_2} \tan\theta_2$$

$$\frac{E_1}{\varepsilon_1 E_1} \tan\theta_1 = \frac{E_2}{\varepsilon_2 E_2} \tan\theta_2 \quad \} \; D = \varepsilon E$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\cancel{E_2}}{\cancel{\varepsilon_1}} \frac{1}{\varepsilon_2} \times \varepsilon_1$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$

$\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$

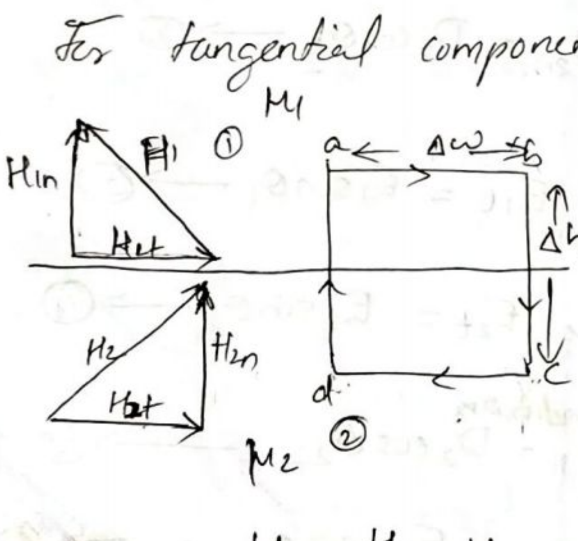
Magnetostatic boundary Condition

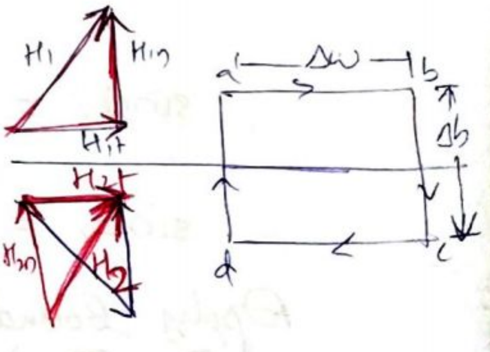
Magnetic boundary conditions is defined as the conditions that H or B field must satisfy at the boundary b/w two different medium.

Applying Gauss law for magnetic field, $\oint B \cdot ds = 0$ and Ampere's circuital law, $\oint_l H \cdot dl = I$

Consider the boundary b/w two magnetic media characterised by μ_1 and μ_2 as in figure.

For tangential component of magnetic field.





$$H_1 = H_{1n} + H_{1t}$$

$$H_2 = H_{2n} + H_{2t}$$

$$\oint H \cdot dl = I$$

$I = K\Delta w$ — $K \rightarrow$ surface current density

$$\int_a^b H\cdot dl + \int_b^c H\cdot dl + \int_c^d + \int_d^a = K\Delta w$$

$$H_{1t}\Delta w - H_{1n}\frac{\Delta h}{2} - H_{2n}\frac{\Delta h}{2} - H_{2t}\Delta w + H_{2n}\frac{\Delta h}{2} + H_{1n}\frac{\Delta h}{2} = K\Delta w$$

limiting case As $\Delta h \to 0$

$$H_{1t}\Delta w - H_{2t}\Delta w = K\Delta w$$

$$H_{1t} - H_{2t} = K \quad , \; H_1 t = H_2 t + K.$$

$B = \mu H$

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = k$$

If $K = 0$

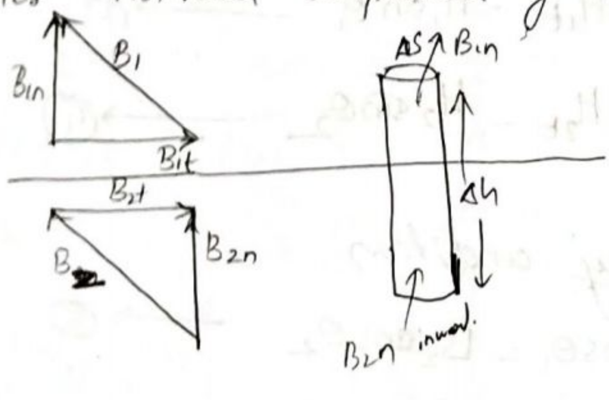
$$H_{1t} - H_{2t} = 0$$

$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

Tangential component H is continuous across the interface

For normal component of magnetic field.



$K \neq 0$

$$\oint B\cdot ds = 0$$

If there is a surface current density

$$-B_{2n}\Delta S + B_{1n}\Delta S = 0$$

$H_{t_2} = H_{t_1} - (a_n \times K)$

$$(-B_{2n} + B_{1n})\Delta S = 0$$

$$-B_{2n} + B_{1n} = 0$$

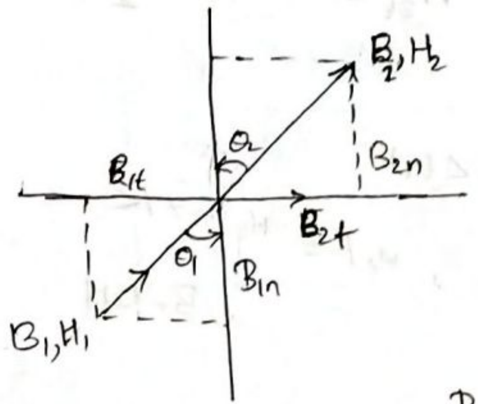
$$\boxed{B_{1n} = B_{2n}}$$

or

$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

i.e. Normal component of B is continuous across the interface.

Law of Refraction in contrast to Manetostatic interface



$$\underline{\underline{B_{1n} = B_{2n} \;\&\; H_{1t} = H_{2t}}}$$

$$\cos\theta_1 = \frac{B_{1n}}{B_1}, \quad B_{1n} = B_1\cos\theta_1 \longrightarrow (1)$$

$$\cos\theta_2 = \frac{B_{2n}}{B_2}, \quad B_{2n} = B_2\cos\theta_2 \longrightarrow (2)$$

$$\sin\theta_1 = \frac{H_{1t}}{H_1}, \quad H_{1t} = H_1\sin\theta_1 \longrightarrow (3)$$

$$\sin\theta_2 = \frac{H_{2t}}{H_2}, \quad H_{2t} = H_2\sin\theta_2 \longrightarrow (4)$$

module X

Apply boundary condition

$$(1) = (2) \Rightarrow B_1\cos\theta_1 = B_2\cos\theta_2 \longrightarrow (5)$$

$$(3) = (4) \Rightarrow H_1\sin\theta_1 = H_2\sin\theta_2 \longrightarrow (6)$$

$$\frac{(6)}{(5)} \Rightarrow \frac{H_1}{B_1}\tan\theta_1 = \frac{H_2}{B_2}\tan\theta_2$$

$$\frac{H_1}{\mu_1 H_1}\tan\theta_1 = \frac{H_2}{\mu_2 H_2}\tan\theta_2$$

$$\frac{1}{\mu_1}\tan\theta_1 = \frac{1}{\mu_2}\tan\theta_2$$

$\mu_1 = \mu_0\mu_{r1}$
$\mu_2 = \mu_0\mu_{r2}$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$