MODULE-3

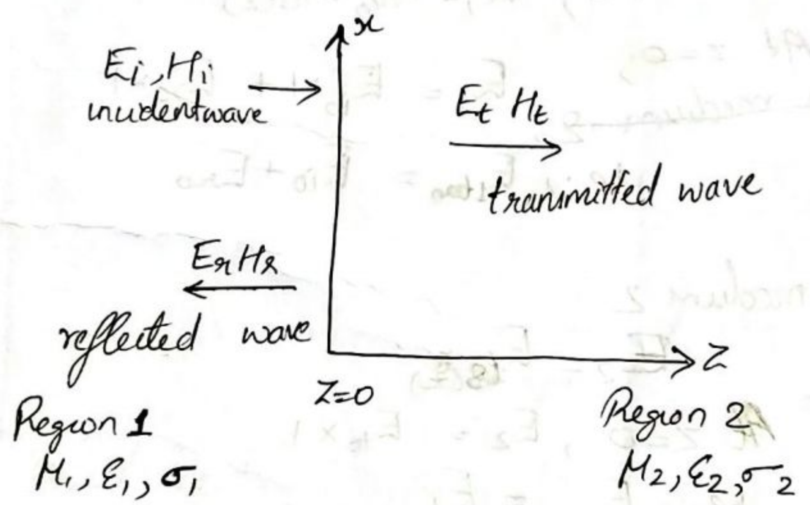
REFLECTION & REFRACTION OF PLANE WAVE

Reflection of Planewave at normal incidence.

When a plane wave from one medium meets a different medium, it is partly reflected & partly transmitted. The proportion of the incident wave that is reflected or transmitted depends on the ~~corrective~~ consicutive parameters (ε, μ, σ) of the two media involved.

Incident plane wave is normal to the boundary b/w the media.

Suppose that a plane wave propagating along the +z direction is incident normally on the boundary z=0 b/w medium 1 ($z<0$) [$\sigma_1, \varepsilon_1, \mu_1$] & medium 2 ($z>0$) characterised by $\sigma_2, \varepsilon_2, \mu_2$ as shown.



$E = \eta H$

Incident wave

(E_i, H_i) is traveling along $+a_z$ in medium 1

$$E_{is}(z) = E_{io}\, e^{-\gamma_1 z} a_x$$

$$H_{is}(z) = H_{io}\, e^{-\gamma_1 z} a_y$$

$$H_{is}(z) = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z} . a_y$$

Transmitted wave E_t & H_t is travelling along $+a_z$ direction in medium 2.

$$E_{ts} = E_{to} e^{-\gamma_2 z} a_x$$

$\gamma = \alpha + j\beta$

$$H_{ts} = H_{to} e^{-\gamma_2 z} a_y$$

$$= \frac{E_{to}}{\eta_2} e^{-\gamma_2 z} a_y$$

Reflected wave

E_r, H_r is travelling along $-a_z$ in medium 1

2

$$E_{rs(z)} = E_{ro} e^{\gamma_1 z} . a_x$$

$$H_{rs(z)} = H_{ro} e^{\gamma_1 z} . (-1 a_y)$$

$$= -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} a_y$$

At the interface $z=0$, $E_{1tan} = E_{2tan}$ $H_{1tan} = H_{2tan}$

$$\cancel{E_i = E_{io}}$$

$$\cancel{E_t = E_{to}}$$

$$\cancel{E_r = E_{ro}}$$

Total field in medium 1, $E_1 = E_{is(z)} + E_{rs(z)}$

~~Total field in medium 2~~, At $z=0$, $E_1 = E_{io} \times 1 + E_{ro} \times 1$

ie., $E_{1tan} = E_{io} + E_{ro}$

Total field in medium 2

$$E_2 = E_{ts(z)}$$

At $z=0$, $E_2 = E_{to} \times 1$

ie., $E_{2tan} = E_{to}$

Boundary condition requires that $E_{1tan} = E_{2tan}$

$$E_{io} + E_{ro} = E_{to} \longrightarrow ①$$

$H_1 = H_{is(z)} + H_{rs(z)}$

At $z=0$, $H_1 = H_{io} + H_{ro}$

ie., $H_1 = \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}$

$H_{1tan} = \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}$

$H_2 = H_{ts(z)}$

$H_2 = \frac{E_{to}}{\eta_2}$

$H_{2tan} = \frac{H_{to}}{\eta_2}$

Boundary condition requires that $H_{1tan} = H_{2tan}$

ie., $\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$

$$E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to} \longrightarrow (2)$$

Adding (1) & (2)

$$2 E_{io} = E_{to} + \frac{\eta_1}{\eta_2} E_{to}$$

$$2 E_{io} = \left(1 + \frac{\eta_1}{\eta_2}\right) E_{to}$$

$$E_{to} = \frac{2\eta_2 E_{io}}{\eta_1 + \eta_2} \longrightarrow (3)$$

Transmission coff is defined as the ratio of magnitude transmitted wave to incident wave.

Transmission coefficient, $\boxed{\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_1 + \eta_2}} \longrightarrow (4)$

Substracting (2) from (1)

$$2 E_{ro} = E_{to} - \frac{\eta_1}{\eta_2} E_{to}$$

$$2 E_{ro} = \left(1 - \frac{\eta_1}{\eta_2}\right) E_{to}$$

$$E_{ro} = \frac{(\eta_2 - \eta_1)}{2\eta_2} E_{to}$$

~~$E_{to} = \frac{2\eta_2}{\eta} E_{to}$~~

$$E_{ro} = \frac{2(\eta_2-\eta_1)}{2\eta_2} \times \frac{2\eta_2}{\eta_1+\eta_2} E_{io}$$

Γ & τ are dimensionless.

$$E_{ro} = \frac{\eta_2-\eta_1}{\eta_1+\eta_2} E_{io} \longrightarrow (5)$$

Reflection coefficient is defined as the ratio of magnitudes of reflected wave to incident wave.

(gamma → Γ)

$$\boxed{\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_1+\eta_2}} \longrightarrow 6$$

$$\boxed{0 \le \Gamma \le 1}$$

$$\boxed{1+|\Gamma| = \tau}$$

- Case (i) Medium 1 is perfect dielectric (lossless, $\sigma_1 = 0$)

Medium 2 is a perfect conductor ($\sigma_2 = \infty$)

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$$

$$\Gamma = \frac{0-\eta_1}{\eta_1+0}$$

$$\eta_2 = 0 \quad \because \sigma_2 = 0$$

$$\Gamma = \left(\frac{-\eta_1}{\eta_1}\right) = -1$$

$$\frac{E_{ro}}{E_{io}} = -1$$

$$\Rightarrow E_{ro} = -E_{io}$$

Phasor form of electric field intensity in medium 1

$$E_{1s} = E_{10} e^{-\gamma_1 z} a_x + E_{ro} e^{\gamma_1 z} a_x$$

$\gamma_1 = (\alpha + j\beta_1)$

$\alpha = 0$

$$= E_{10} e^{-j\beta_1 z} a_x + E_{ro} e^{j\beta_1 z} a_x$$

$E_{ro} = -E_{io}$

$$E_{1s} = \left(E_{10} e^{-j\beta_1 z} - E_{io} e^{j\beta_1 z}\right) a_x$$

$$E_{1s} = E_{10}^{-}(e^{j\beta_1 z} - e^{-j\beta_1 z})\, a_x$$

$$E_{1s} = -E_{10}\, 2j \sin\beta_1 z \cdot a_x$$

$$E_{1s} = -2E_{10} j \sin\beta_1 z \cdot a_x$$

Real instantaneous form of electric field is obtained by multiplying above equation by $e^{j\omega t}$ and taking the real part.

$$E_1 = E_{(z,t)} = Re\{E_{1s}\, e^{j\omega t}\}$$

Field in a perfect conductor must vanish. There is no transmitted wave.

The totally reflected wave combines with the incident wave to form a standing wave.

(3) Standing wave consist of two travelling waves (E_i & E_r) of equal amplitude but in opposite direction.

Standing wave in medium 1 is

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$$E_{1s} = -2E_{10} j \sin\beta_1 z \cdot a_x$$

Reflection of plane wave at Oblique Incidence.

Consider a linearly polarized plane wave obliquely incident on a boundary b/w two medium as shown in figure.

The incident wave makes an angle θ_i with the z axis and the reflected wave makes an angle of θ_r, and the transmitted wave (refracted wave) makes an angle of θ_t. Consider two cases i) the electric field $\perp^r$ to the plane of incidence (xz plane).

ii) the electric field parallel to the plane of incidence.

These waves are said to be perpendicularly polarized & parallel polarized respectively.

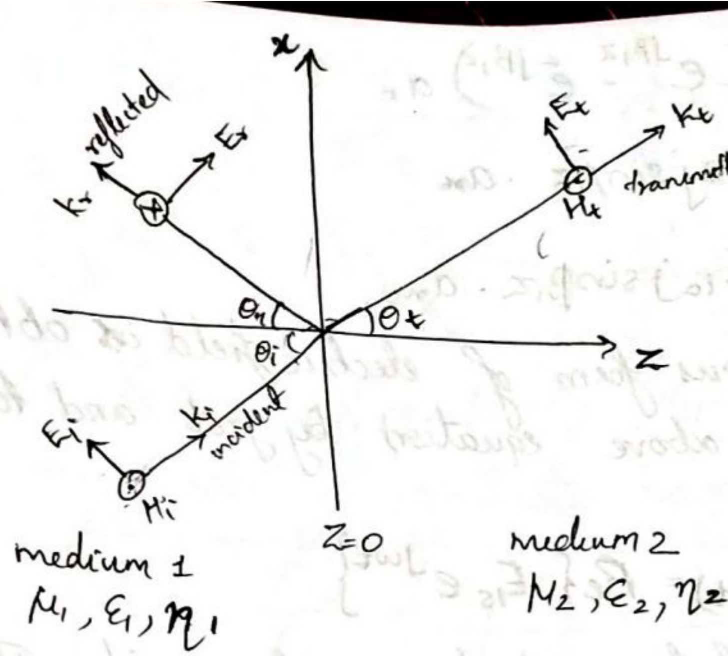

figure: Oblique incidence with E parallel to the plane of incidence.

Case (i) Parallel Polarization

E field lies on the xz-plane, plane of incidence

(6) boundary condition: $\theta_r = \theta_i$, E & H continuous at boundary Z=0

$E_{1t} = E_{2t}$ ①⇒ $E_{10}\cos\theta_i + E_{r0}\cos\theta_r = E_{t0}\cos\theta_t$

$$(E_{i0} + E_{r0})\cos\theta_i = E_{t0}\cos\theta_t \longrightarrow ①$$

$H_{1t} = H_{2t}$ ②⇒ $$E_{i0} - E_{r0} = \frac{\eta_1}{\eta_2}E_{t0} \longrightarrow ②$$

~~Adding ① & ②~~ ①⇒ $$E_{i0} + E_{r0} = E_{t0}\frac{\cos\theta_t}{\cos\theta_i} \longrightarrow ③$$

$$\cancel{(E_{10} + E_{r0}) + E_{i0} - E_{r0} = E_{t0}\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}E_{t0}}$$

$$\cancel{E_{i0}(\cos\theta_i + 1) + E_{r0}(\cos\theta_i - 1) = E_{t0}\left(\cos\theta_t + \frac{\eta_1}{\eta_2}\right)}$$

② + ③

$$2E_{i0} = E_{t0}\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}E_{t0}$$

$$2E_{i0} = E_{t0}\left[\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}\right] = E_{t0}\left[\frac{\eta_2\cos\theta_t + \eta_1\cos\theta_i}{\eta_2\cos\theta_i}\right]$$

$$E_{t0} = 2E_{i0}\frac{\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\boxed{\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}} \longrightarrow (4)$$ Transmission coefficient

(3) - (2) $\Rightarrow$

$$2E_{ro} = E_{to}\frac{\cos\theta_t}{\cos\theta_i} - \frac{\eta_1}{\eta_2}E_{to}$$

$$= E_{to}\left[\frac{\cos\theta_t}{\cos\theta_i} - \frac{\eta_1}{\eta_2}\right]$$

$$2E_{ro} = \left[2E_{io}\frac{\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\right]\left[\frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_i}\right]$$

$$\frac{E_{ro}}{E_{io}} = \cancel{E_{io}}\left[\frac{\eta_2^2\cos\theta_i\cdot\cos\theta_t - \eta_1\eta_2\cos^2\theta_i}{\eta_2^2\cos\theta_t\cos\theta_i + \eta_1\eta_2\cos^2\theta_i}\right]$$

$$\boxed{\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}} \longrightarrow (5)$$ Reflection coefficient

Parallel polarization $\rightarrow \parallel$

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Equation (4) & (5) are called Fresnel's equation.

or.

$$\boxed{\begin{aligned} E_{ro} &= \Gamma_{\parallel} E_{io} \\ E_{to} &= \tau E_{io} \end{aligned}}$$

$v_{01} = \frac{c}{v_{01}} \cdot c$

According to Snell's law, $\theta_r = \theta_i$

$n = c\sqrt{\mu\varepsilon}$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}$$

$$n_1 = c/v_{01} \qquad n_2 = c/v_{02}$$

$$\frac{v_{02}}{v_{01}} = \frac{\cancel{1}/\sqrt{\mu_2\varepsilon_2}}{\cancel{1}/\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

$$\frac{\frac{c}{v_{01}}}{\frac{c}{v_{02}}}$$

$$v_{01} = \frac{1}{\sqrt{\mu_1\varepsilon_1}}$$

$$\frac{c}{v_{01}} \times \frac{v_{02}}{c}$$

$$\frac{n_1}{n_2} = \frac{c}{v_{01}} \cdot \frac{c}{v_{02}} = \frac{v_{02}}{v_{01}} = \frac{1/\sqrt{\mu_1\varepsilon_1}}{1/\sqrt{\mu_2\varepsilon_2}} = \cancel{\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}}$$

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}} \longrightarrow (a)$$

(○) $\boxed{V_0 = \frac{c}{k}}$ α. — $k \to$ popagation vector or wave number vector.

numbery

Brewster angle or Polarizing angle ($\theta_{B\parallel}$)

Brewster angle is a particular angle at which ~~a~~ no reflection takes place and the wave is totally transmitted into medium 2. i.e., $E_{r_0}=0$ and $\Gamma_{\parallel}=0$

Setting $\theta_i = \theta_{B\parallel}$ & $\Gamma_{\parallel}=0$ in eqn (5)

$$\Rightarrow \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_{B\parallel}}{\eta_2\cos\theta_t + \eta_1\cos\theta_{B\parallel}} = 0$$

Numerator is zero

$$\eta_2\cos\theta_t - \eta_1\cos\theta_{B\parallel} = 0$$

$$\Rightarrow \eta_2\cos\theta_t = \eta_1\cos\theta_{B\parallel}$$

Squaring

$$\eta_2^2\cos^2\theta_t = \eta_1^2\cos^2\theta_{B\parallel}$$

$$\eta_2^2(1-\sin^2\theta_t) = \eta_1^2(1-\sin^2\theta_{B\parallel}) \qquad \eta = \sqrt{\mu/\epsilon}$$

$$\frac{\mu_2}{\epsilon_2}\left(1 - \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_{B\parallel}\right) = \frac{\mu_1}{\epsilon_1}\left(1-\sin^2\theta_{B\parallel}\right) \qquad \sin\theta_t = \sin\theta_{B\parallel}\sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}$$

$$\frac{\mu_2}{\epsilon_2} - \frac{\mu_2}{\epsilon_2}\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_{B\parallel} = \frac{\mu_1}{\epsilon_1} - \frac{\mu_1}{\epsilon_1}\sin^2\theta_{B\parallel}$$

$$-\frac{\mu_1\epsilon_1}{\epsilon_2^2}\sin^2\theta_{B\parallel} + \frac{\mu_1}{\epsilon_1}\sin^2\theta_{B\parallel} = \frac{\mu_1}{\epsilon_1} - \frac{\mu_2}{\epsilon_2}$$

$$\sin^2\theta_{B\parallel}\left[\frac{\mu_1}{\varepsilon_1}-\frac{\mu_1\varepsilon_1}{\varepsilon_2^2}\right]=\frac{\mu_1}{\varepsilon_1}-\frac{\mu_2}{\varepsilon_2}$$

$$\sin^2\theta_{B\parallel}\left[\frac{\mu_1\varepsilon_2^2-\mu_1\varepsilon_1^2}{\varepsilon_1\varepsilon_2^2}\right]=\frac{\mu_1\varepsilon_2-\mu_2\varepsilon_1}{\varepsilon_1\varepsilon_2}$$

$$\sin^2\theta_{B\parallel}=\frac{\mu_1\varepsilon_2-\mu_2\varepsilon_1}{\varepsilon_1\varepsilon_2}\times\frac{\varepsilon_1\varepsilon_2^2}{\mu_1\varepsilon_2^2-\mu_1\varepsilon_1^2}$$

$$=\frac{(\mu_1\varepsilon_2-\mu_2\varepsilon_1)\,\varepsilon_2}{\mu_1\varepsilon_2^2-\mu_1\varepsilon_1^2}$$

$$=\frac{\mu_1\varepsilon_2\left(1-\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right)\varepsilon_2}{\mu_1\varepsilon_2^2\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)}$$

$$\boxed{\sin^2\theta_{B\parallel}=\frac{1-\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}}$$

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Dielectric media are not only lossless but also non-magnetic

ie $\mu_1=\mu_2=\mu_0$ $(\mu_r=1)$

$$\sin^2\theta_{B\parallel}=\frac{1-\varepsilon_1/\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2}$$

$$=\frac{1-\varepsilon_1/\varepsilon_2}{(1-\varepsilon_1/\varepsilon_2)(1+\varepsilon_1/\varepsilon_2)}$$

$$=\frac{1}{1+\varepsilon_1/\varepsilon_2}$$

$$\sin^2\theta_{B\parallel}=\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}$$

$$\sin\theta_{B\parallel}=\sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

$$\cos^2\theta_{B\parallel}=1-\sin^2\theta_{B\parallel}=1-\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}=\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}$$

$$\cos\theta_{B\parallel} = \sqrt{\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}}$$

$$\tan\theta_{B\parallel} = \frac{\sin\theta_{B\parallel}}{\cos\theta_{B\parallel}} = \frac{\sqrt{\varepsilon_2/\varepsilon_1+\varepsilon_2}}{\sqrt{\varepsilon_1/\varepsilon_1+\varepsilon_2}} = \sqrt{\varepsilon_2/\varepsilon_1} = \frac{n_2}{n_1}$$

$$\boxed{\theta_{B\parallel} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

← It is used to control polarization of emitted light.

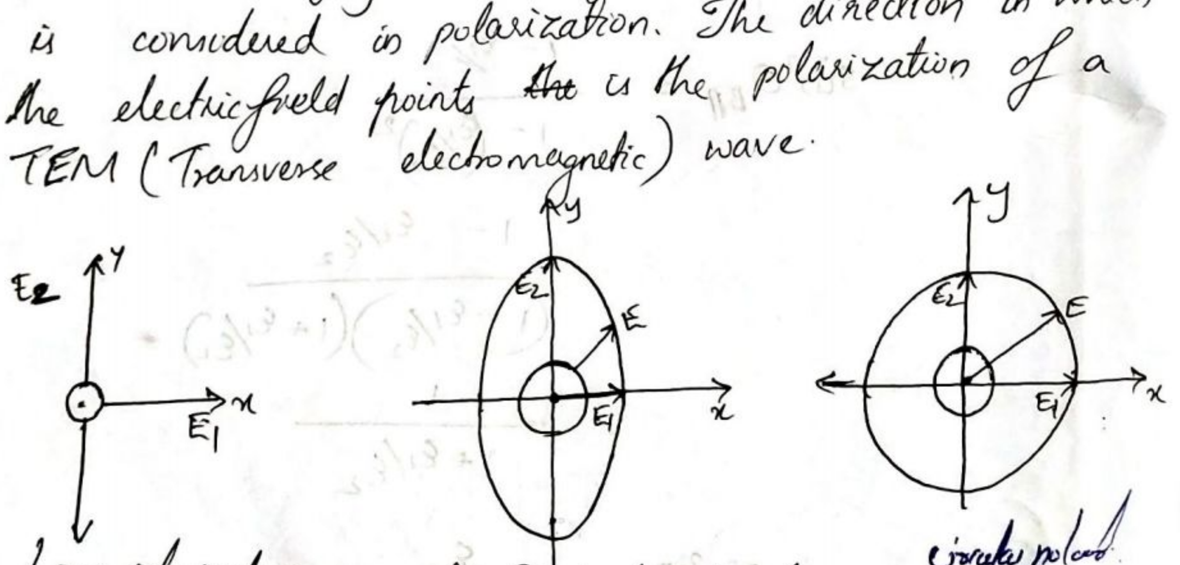
Polarization

The brewster angle is sometimes called the polarizing angle since a wave composed of both perpendicular & parallel components. ~~and~~ A wave incident at the brewster angle produces a reflected wave with only a perpendicular component. Thus circularly polarized wave incident at the brewster angle becomes linearly polarized on reflection.

* Polarization

(Derive polarization ellipse ?)

The time varying behavior of electromagnetic radiation is considered in polarization. The direction in which the electric field points ~~the~~ is the polarization of a TEM (Transverse electromagnetic) wave.



(a) linear polarization (b) Elliptical polarization circular polar

i) $E_1 = 0$, $AR = \infty$ } the wave is linearly polarized in y direction — linear polarization

ii) $E_2 = 0$, the wave is linearly polarised in x direction

iii) If $\delta = 0$ & $E_1 = E_2$, the wave is also linearly polarized but in a plane at an angle of 45° w.r.t x axis. This angle is tilt angle.

IV) $E_1 = E_2$ & $\delta = +90$, left circularly polarized

$\delta = -90$, right circularly

$AR = 1$

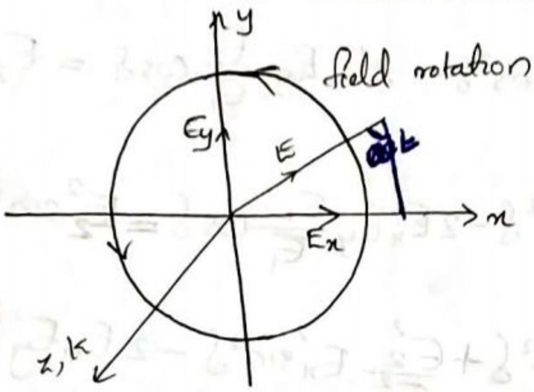
Axial ratio of polarization ellipse

$1 \leq AR \leq \infty$

Axial ratio (AR): The ratio of major to minor axises of the polarization ellipse is called the axial ratio.

$$AR = \frac{E_2}{E_1}$$

Note: As the wave propagates in the forward z direction the (E) field vector rotates counter clockwise in the xy plane.



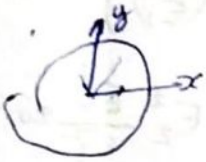


fig: electric field in the xy plane of a right circularly polarized plane wave. $\delta = -\pi/2$

~~Con~~: If the wave is traveling in the +ve z direction the electric field components in the x and y directions are

$$E_x = E_1 \sin(\omega t - \beta z) \longrightarrow ①$$

$$E_y = E_2 \sin(\omega t - \beta z + \delta) \longrightarrow ②$$

$E_1 \rightarrow$ amplitude of wave linearly polarized in x direction

$E_2 \rightarrow$ amplitude of wave linearly polarized in y direction

δ - time phase angle by which E_y leads E_x

Combining ① & ② gives instantaneous total vector field E.

$$E = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \sin(\omega t - \beta z + \delta)\, a_y$$

At $z = 0$

$$E_y = E_2 \sin(\omega t + \delta) = E_2(\sin\omega t \cos\delta + \cos\omega t \sin\delta)$$

$$\sin\omega t = \frac{E_x}{E_1}.$$

$$\cos\omega t = \sqrt{1-\frac{E_x^2}{E_1^2}}$$

$$E_y = E_2\left(\frac{E_x}{E_1}\cos\delta + \sqrt{1-\frac{E_x^2}{E_1^2}}\sin\delta\right)$$

$$E_y - \frac{E_2}{E_1}E_x\cos\delta = E_2\sqrt{1-\frac{E_x^2}{E_1^2}}\sin\delta$$

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Squaring and expanding the terms.

$$\left(E_y - \frac{E_2}{E_1}E_x\cos\delta\right)^2 = \left[E_2\sqrt{1-\frac{E_x^2}{E_1^2}}\sin\delta\right]^2$$

$$E_y^2 + \left(\frac{E_2}{E_1}\right)^2 E_x^2\cos^2\delta - 2E_xE_y\frac{E_2}{E_1}\cos\delta = E_2^2\left[1-\left(\frac{E_x}{E_1}\right)^2\right]\sin^2\delta$$

$$E_y^2 + \left(\frac{E_2}{E_1}\right)^2 E_x^2\cos^2\delta - 2E_xE_y\frac{E_2}{E_1}\cos\delta = E_2^2\sin^2\delta - E_2^2\frac{E_x^2}{E_1^2}\sin^2\delta$$

$$E_y^2 + \left(\frac{E_2}{E_1}\right)^2 E_x^2\cos^2\delta + \frac{E_2^2}{E_1^2}E_x^2\sin^2\delta - 2E_xE_y\frac{E_2}{E_1}\cos\delta = E_2^2\sin^2\delta$$

$$E_y^2 + \left(\frac{E_2}{E_1}\right)^2 E_x^2\left[\cos^2\delta + \sin^2\delta\right] - 2E_xE_y\frac{E_2}{E_1}\cos\delta = E_2^2\sin^2\delta$$

$$E_y^2 + \left(\frac{E_2}{E_1}\right)^2 E_x^2 - 2E_xE_y\frac{E_2}{E_1}\cos\delta = E_2^2\sin^2\delta$$

Taking E_2^2 common,

$$\frac{E_y^2}{E_2^2} + \frac{E_x^2}{E_1^2} - 2\frac{E_xE_y}{E_1E_2}\cos\delta = \sin^2\delta$$

$$\frac{E_y^2}{E_2^2\sin^2\delta} + \frac{E_x^2}{E_1^2\sin^2\delta} - 2\frac{E_xE_y}{E_1E_2}\frac{\cos\delta}{\sin^2\delta} = 1$$

$$\frac{E_x^2}{E_1^2\sin^2\delta} - 2\frac{E_xE_y}{E_1E_2}\frac{\cos\delta}{\sin^2\delta} + \frac{E_y^2}{E_2^2\sin^2\delta} = 1$$

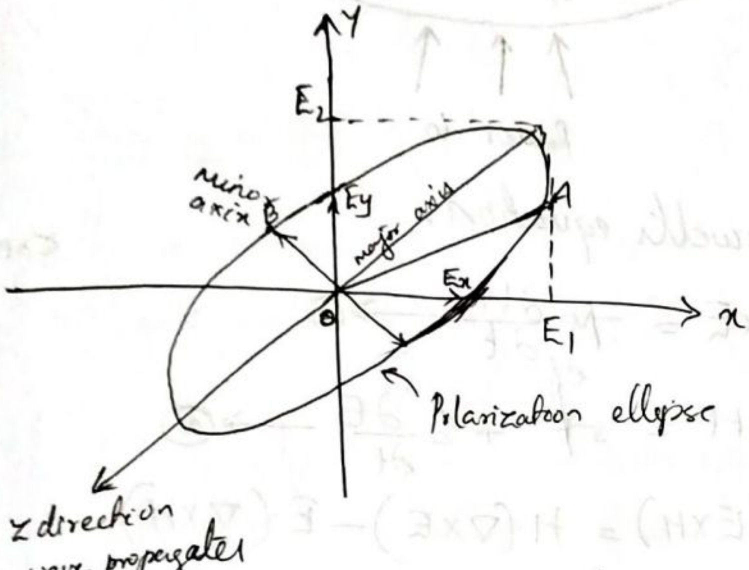
$$aE_x^2 - bE_xE_y + cE_y^2 = 1 \longrightarrow (1)$$

It describes a polarization ellipse where

$$a = \frac{1}{E_1^2 \sin^2\delta}$$

$$b = \frac{2\cos\delta}{E_1 E_2 \sin^2\delta}$$

$$C = \frac{1}{E_2^2 \sin^2\delta}$$



The line segment OB is called semi minor axis and OA is semi major axis.

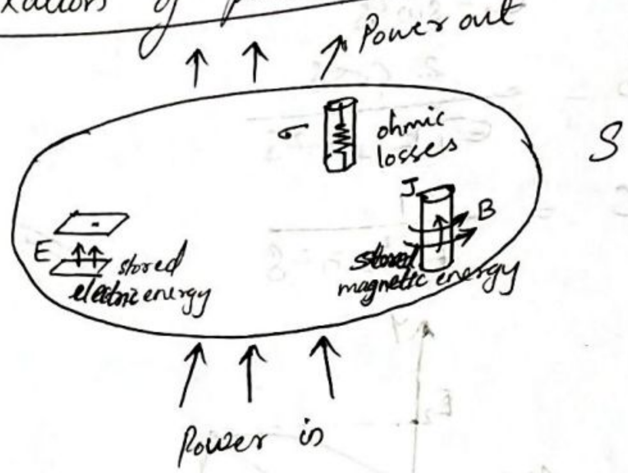
Poynting Theorem & Poynting Vector

When the electromagnetic wave ~~trav~~ travel in the freespace, the energy travels from source to destination takes place. Electromagnetic wave carry energy & momentum from the source to the receiver. In this case of EM waves, the power & energy relationships can be explained in terms of amplitudes of electric and magnetic fields.

* **Poynting Theorem**

It states that the net power flowing out of a given volume (V) is equal to the time rate of decrease in the energy stored within V minus the conduction losses.

The illustration of power balance for EM fields



From Maxwell's equation.

E×H → pointing vector

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \longrightarrow ①$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \longrightarrow ②$$

Taking $\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E \cdot (\nabla \times H)$

$$= H \cdot \left(-\mu \frac{\partial H}{\partial t}\right) - E \cdot \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\frac{\partial EE}{\partial t} = \frac{\partial E^2}{\partial t} = E \cdot \frac{\partial E}{\partial t} + E \frac{\partial E}{\partial t}$$

$$\frac{\partial E^2}{\partial t} = 2E \frac{\partial E}{\partial t}$$

$$= H \cdot \left(-\mu \frac{\partial H}{\partial t}\right) - \sigma E \cdot E \quad \varepsilon E \cdot \frac{\partial E}{\partial t}$$

$$= -\mu H \cdot \frac{\partial H}{\partial t} - \sigma E^2 - \varepsilon E \cdot \frac{\partial E}{\partial t}$$

$$\nabla \cdot (E \times H) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \sigma E^2 - \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t}$$

Taking volume integral on both side

$$\int_V \nabla \cdot (E \times H)\, dV = \int_V -\frac{\mu}{2} \frac{\partial H^2}{\partial t} dV - \int_V \sigma E^2 dV - \int_V \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} dV$$

$$\int_V \nabla \cdot E \times H\, dV \quad -\frac{\partial}{\partial t} \int_V \left(\frac{\mu}{2} H^2 + \frac{\varepsilon}{2} E^2\right) dV - \int_V \sigma E^2 dV$$

Applying Gauss's divergence theorem of on LHS

$$\oint_S E \times H\, dS = \int_V \nabla \cdot (E \times H)\, dV$$

$$\therefore \boxed{\int_S (E \times H)\,ds = \frac{-\partial}{\partial t}\int_V \left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right)dV - \int_V \sigma E^2 dV}$$

total power leaving the volume — energy stored in electric & magnetic field — conduction losses (ohmic power dissipated)

Poynting vector

The quantity $E \times H$ is known as pointing vector in ~~what~~ watts per square meter

i.e. $\boxed{P = E \times H}$

It represents the instantaneous power flow per unit area and it is called instantaneous poynting vector.

The complex poynting vector is $\boxed{P = \frac{1}{2}(E \times H^*)}$

H^* - complex conjugate of H.

It represents the instantaneous power density vector associated with the EM field at a given point. The integration of the poynting vector over any closed surface gives the net power flowing out of that surface.

The power flow is directed along the normal to the plane containing $E \times H$

The average part of power flow per unit area is

$$\boxed{P_{av} = \frac{1}{2} Re\{E \times H^*\}}$$

The reactive part of the power flow per unit area is

$$P_{react} = \frac{1}{2} Im\{E \times H^*\}$$

The total instantaneous poynting vector can be written as

$$P_{inst} = P = P_{av} + P_{react}$$