MODULE - 4

UNIFORM TRANSMISSION LINE

Transmission line

Transmission lines are commonly used in power distribution (at low frequencies) & in communications (at high frequencies).

A transmission line basically consists of two or more parallel conductors used to connect a source to ~~load~~ load.

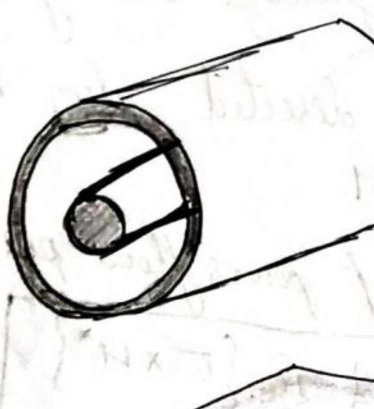
Typical transmission line include

a) coaxial cable
b) Two-wire line
c) a parallel-plate or planar line
d) a wire above the conducting plane
e) a micro strip line.

These lines are portrayed in figure.

(a)

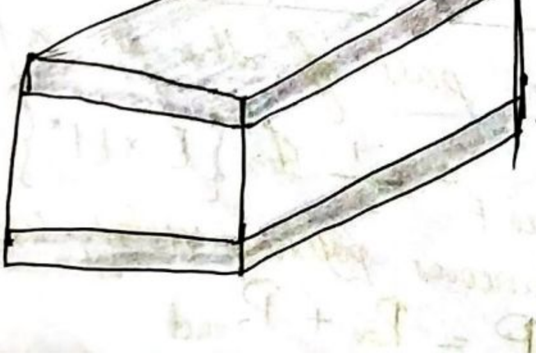
coaxial



(b)

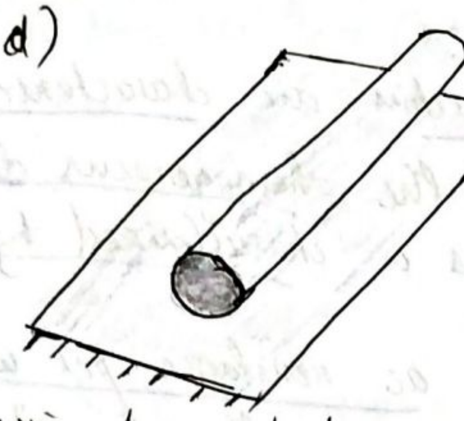
two wire

(c)



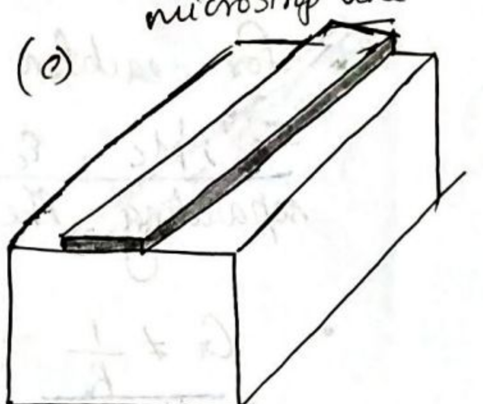
planar.

d)



wire above conducting plane

(e)



- Each of these lines consists of 2 conductors in parallel.

- coaxial cables are routinely used in electrical laboratories & in connecting TV sets to antenna.
- Microstrip lines are particularly important in integrated circuits, where metallic strips connecting electronic elements are deposited on dielectric substrates

* For frequencies up to 300 MHz, a two-wire transmission lines are insufficient. ~~Above~~ Up to frequencies about 3000 MHz coaxial cable is used to reduce radiation loss. For still higher frequencies, we use waveguides

Uniform transmission line features

- Area of crossection of the two conductors are equal throughout the lines ~~of~~ & conductors have constant separation.
- em properties of the conductors are ~~the~~ same throughout the line.
- The intervening medium has the same property throughout the line

- For each line, the conductors are characterized by $\sigma_c, \mu_c, \varepsilon_c = \varepsilon_0$, & the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

- $G \neq \frac{1}{R}$, R is the ac resistance per unit length of the conductors comprising the line & G is the conductance per unit length due to the dielectric medium separating the conductors.

- The effects of internal inductance $L_{in} = R/\omega$ are negligible at the high frequency at which common systems operate.

- For each line $LC = \mu\varepsilon$ & $\frac{G}{C} = \frac{\sigma}{\varepsilon}$

R, L, G, C are called line parameters or primary constants of a line.

Transmission line Parameters

The line parameters R, L, G & C are not discrete or lumped. Rather they are distributed as shown in figure. By this, we mean that the parameters are uniformly distributed along the entire length of the line.

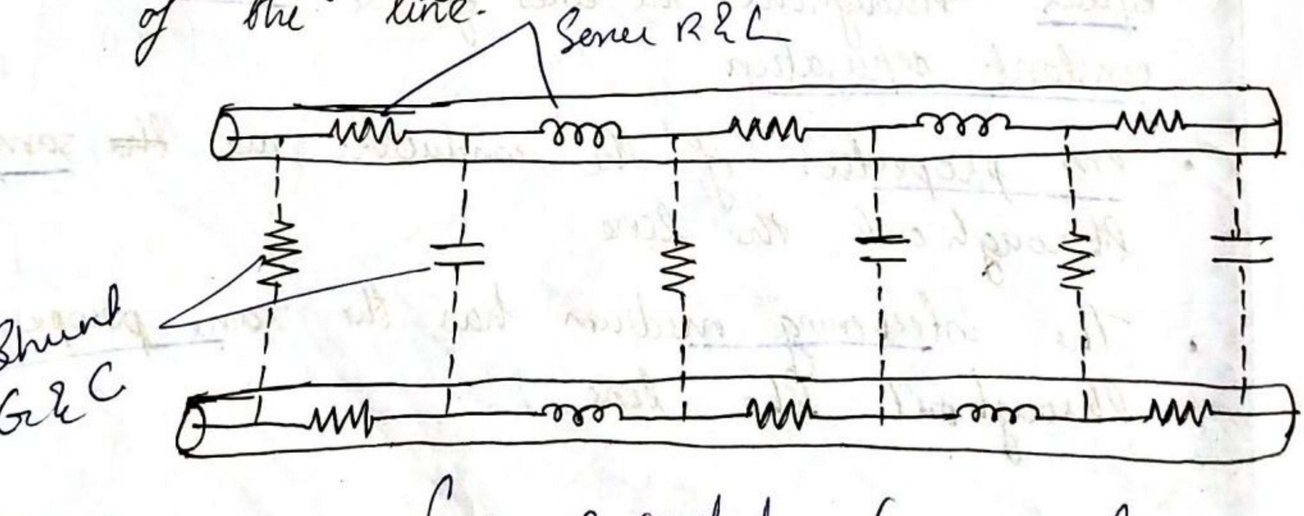


fig. 2 conductor transmission line

i) Series resistance (R) (Ω/m)

The finite conductivity and radiation loss can be modeled as a series resistance per unit length. It is the resistance of both the wires per unit length. It is a frequency dependent parameter.

ii) Shunt capacitance per unit length (C)

Two conducting wires separated by a distance situated in a dielectric medium give rise to a capacitance that acts in parallel with the wires. ~~The loop capacitance~~ (F/m)

iii) Series Inductance (L)

A current carrying conductor has an associated magnetic field. Both the growth & decay of current is opposed and hence it possesses inductance. It acts in series

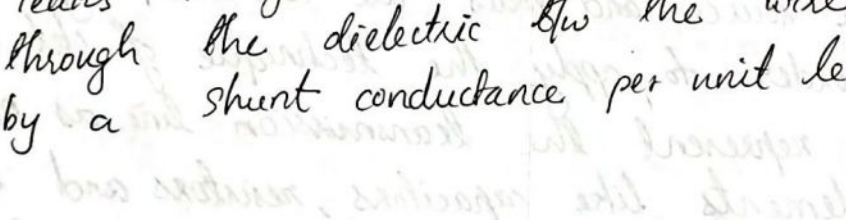
iv) Shunt leakage capacitance (G)

If the conductors are not perfectly insulated, current leaks through the medium. This leakage of current through the dielectric b/w the wires is represented by a shunt conductance per unit length.

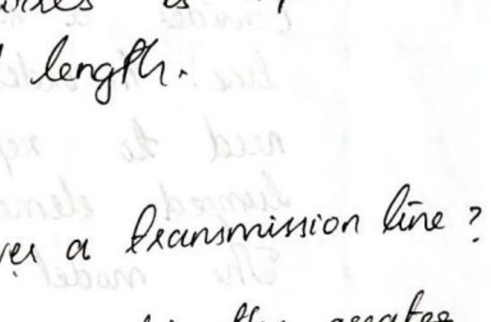
UQ Discuss the flow of em power over a transmission line?

Consider the coaxial line connecting the ~~greater~~ generator or source to the load as in figure.

(b) E & H fields on the coaxial line.



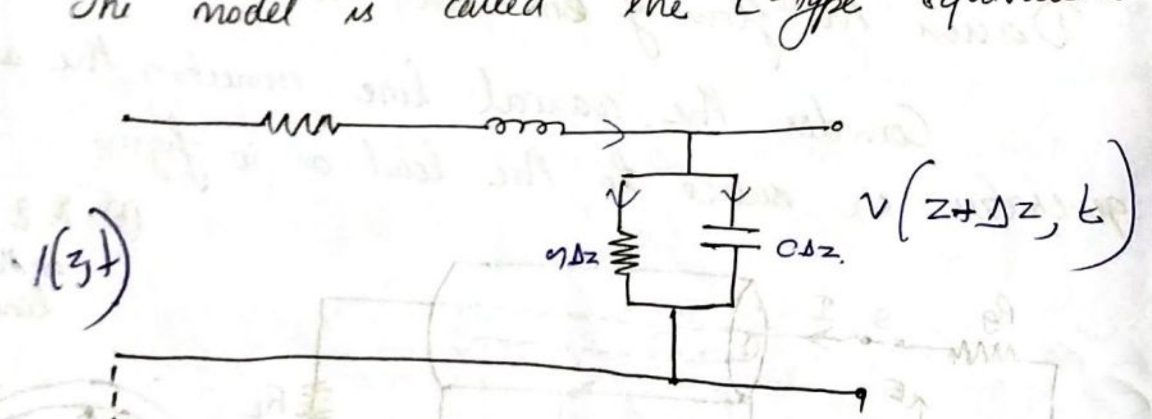
(a) Coaxial line connecting the generator to the load



When the switch S is closed, the inner conductor is made +ve w.r.t the outer one so that the E field is radially outward. According to ampere's law, the H field encircles the current carrying conductor as in fig. The poynting vector E×H points along the transmission line. Thus closing the switch simply establishes a disturbance, which appears as a transverse em wave propagating along the line. This wave is a non-uniform plane wave & by means of its power is transmitted through the line.

Transmission line Equation or Telegraphy equation

A two conductor transmission line supports a TEM wave. E & H on the line are perpendicular to each other and are transverse to the direction of propagation. Consider a source and load are connected by a transmission line. In order to apply the technique of ckt theory, we need to represent the transmission line as n/w of lumped elements like capacitors, resistors and inductors. The model is called the L-type equivalent ckt.



Consider an incremental portion of length Δz of a two conductor transmission line. Assume that the wave propagates along the positive z direction from generator to the load.

KVL,

$$V(z,t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial}{\partial t} I(z,t) + V(z+\Delta z, t)$$

$$V(z,t) - V(z+\Delta z,t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t}$$

$$-\left[V(z+\Delta z,t) - V(z,t)\right] = \Delta z\left[R\, I(z,t) + L\frac{\partial I(z,t)}{\partial t}\right]$$

$$-\frac{\left[V(z+\Delta z,t) - V(z,t)\right]}{\Delta z} = R I(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$\lim \Delta z \to 0$

$$-\frac{\partial V(z,t)}{\partial z} = R I(z,t) + L\frac{\partial I(z,t)}{\partial t} \longrightarrow (1)$$

KCL,

$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$I(z,t) = I(z+\Delta z,t) + G\Delta z\, V(z+\Delta z,t) + C\Delta z \frac{\partial}{\partial t} V(z+\Delta z,t)$$

$$I(z,t) - I(z+\Delta z,t) = \Delta z\left[G V(z+\Delta z,t) + C\frac{\partial}{\partial t} V(z+\Delta z,t)\right]$$

$$-\frac{\left[I(z+\Delta z,t) - I(z,t)\right]}{\Delta z} = G V(z+\Delta z,t) + C\frac{\partial}{\partial t} V(z+\Delta z,t)$$

As $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = G V(z,t) + C\frac{\partial V(z,t)}{\partial t} \longrightarrow (2)$$

$\boxed{\frac{\partial}{\partial t} = j\omega}$ $\quad \frac{d}{dt} = j\omega$

Assume

$$V(z,t) = Re\left\{V_{s(z)} e^{j\omega t}\right\} \;\&\; I(z,t) = Re\left\{I_{s(z)} e^{j\omega t}\right\}$$

Now eqn (1) $\Rightarrow$

$$-\frac{\partial}{\partial z} Re\left\{V_{s(z)} e^{j\omega t}\right\} = R\, Re\left\{I_{s(z)} e^{j\omega t}\right\} + L\frac{\partial}{\partial t} Re\left\{I_{s(z)} e^{j\omega t}\right\}$$

$$-\frac{\partial}{\partial z} Re\left\{V_{s(z)} e^{j\omega t}\right\} = Re\left\{I_{s(z)} e^{j\omega t}\right\} R + L\, Re\left\{I_{s(z)} e^{j\omega t}\right\} j\omega$$

$$-\frac{\partial}{\partial z} V_{s(z)}\, Re\left\{e^{j\omega t}\right\} = R I_{s(z)}\, Re\left\{e^{j\omega t}\right\} + j\omega L I_{s(z)}\, Re\left\{e^{j\omega t}\right\}$$

$$-\frac{\partial}{\partial z} V_{s(z)} = R I_{s(z)} + j\omega L\, I_{s(z)}$$

$$-\frac{\partial}{\partial z} V_{s(z)} = (R+j\omega L)\, I_{s(z)} \longrightarrow ③$$

III^{ly}

$$② \Rightarrow -\frac{\partial I_{s(z)}}{\partial z} = (G+j\omega C)\, V_{s(z)} \longrightarrow ④$$

Now differentiating eqn ③ w.r.t z.

$$-\frac{\partial^2 V_{s(z)}}{\partial z^2} = (R+j\omega L)\frac{\partial I_{s(z)}}{\partial z}$$

$$-\frac{\partial^2}{\partial z^2} V_s(z) = -(R+j\omega L)(G+j\omega C)\, V_s(z)$$

$$-\frac{\partial^2}{\partial z^2} V_s(z) = -\gamma^2 V_s(z) \qquad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$-\frac{\partial^2 V_{s(z)}}{\partial z^2} + \gamma^2 V_{s(z)} = 0$$

$$-\left[\frac{\partial^2}{\partial z^2} V_s(z) - \gamma^2 V_s(z)\right] = 0$$

$$\boxed{\frac{\partial^2}{\partial z^2} V_s(z) - \gamma^2 V_s(z) = 0} \longrightarrow ⑤$$

Similarly differentiating eqn (4) w.r.t z, we get

$$\boxed{\frac{\partial^2}{\partial z^2} I_{s(z)} - \gamma^2 I_{s(z)} = 0} \longrightarrow ⑥$$

Equation ⑤ and eqn ⑥ are called telegraphy equations or transmission line equations

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$\alpha \rightarrow$ in Neper/meter

$\beta \rightarrow$ radians/meter.

The general solution of transmission line equation is

$$\left.\begin{aligned} V_{s(z)} &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I_{s(z)} &= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{aligned}\right\} \longrightarrow ⑦$$

$V_0^+ e^{-\gamma z} \rightarrow$ forward travelling wave / incident wave.

$V_0^- e^{\gamma z} \rightarrow$ backward travelling wave / reflected wave.

Derivation of characteristic Impedance.

(3) $-\frac{\partial V_s(z)}{\partial z} = (R+j\omega L) I_{s(z)}$

Substituting eqn (7)

$$-\frac{\partial}{\partial z}\left[V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right] = (R+j\omega L)\left[I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right]$$

$$-\left[V_0^+ e^{-\gamma z}(-\gamma) + V_0^- e^{\gamma z}(\gamma)\right] = (R+j\omega L)\left[I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}\right]$$

$$\gamma\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right] = (R+j\omega L) I_{s(z)}$$

$$\frac{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}{I_{s(z)}} = \frac{R+j\omega L}{\gamma} = Z_0$$

$$\frac{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}{Z_0} = I_{s(z)} \longrightarrow (8)$$

where $Z_0 = \frac{R+j\omega L}{\gamma}$

or $Z_0 = \frac{R+j\omega L}{\gamma} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$Z_0 = R_0 \pm j X_0$$

$X_0 \rightarrow$ reactive part of impedance reactance

$Y_0 \rightarrow$ characteristic admittance $Y_0 = \frac{1}{Z_0}$

Derivation of Input Impedance.

Consider the receiving end of transmission line. In practice, it is convenient to find $z=0$ at the receiving end. The terminating impedance Z_R [R stands for receiving end) is located at $z=0$

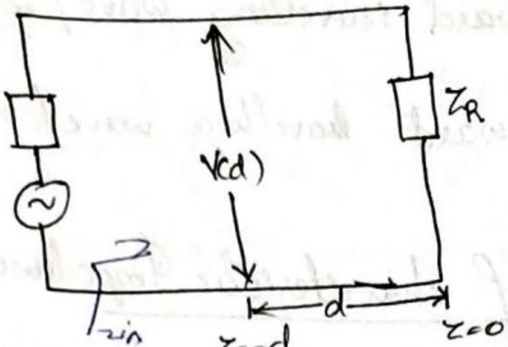


Fig: Load is located at $z=0$. A general point is located at a distance 'd' from the load end. The voltage and current at this position are $V_{(d)}$ & $I_{(d)}$

At the receiving end, $z=0$, voltage and current can be represented as V_R & I_R.

V_R can be found out from (7) and I_R can be found out from eqn (8)

$$V_{S(z=0)} = V_R \,, \quad I_{S(z=0)} = I_R$$

$$V_R = V_0^+ + V_0^- \longrightarrow (9)$$

$I_R \approx I_0$

$$I_R = \left(V_0^+ - V_0^-\right)\frac{1}{Z_0} \longrightarrow (10)$$

$$(10) \Rightarrow Z_0 I_R = V_0^+ - V_0^-$$

Adding (9) & (10)

$\boxed{I_R = \frac{V_g}{Z_{in}+Z_g}}$

$$V_R + Z_0 I_R = V_0^+ + V_0^- + V_0^+ - V_0^-$$

$$2V_0^+ = V_R + Z_0 I_R$$

$$\boxed{V_0^+ = \frac{1}{2}\left(V_R + Z_0 I_R\right)}$$

Substracting (9) & (10)

$$V_R - Z_0 I_R = 2V_0^-$$

$$\boxed{V_0^- = \frac{1}{2}\left[V_R - Z_0 I_R\right]}$$

Substituting V_0^+ & V_0^- in eqn (7) & (8)

$⑦ \Rightarrow V_{s(z)} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$ $\qquad z = -d$

$$V_{s(z=-d)} = V(d) = \frac{1}{2}(V_R + Z_0 I_R) e^{+\gamma d} + \frac{1}{2}(V_R - Z_0 I_R) e^{-\gamma d}$$

$$V(d) = \frac{1}{2} V_R e^{+\gamma d} + \frac{1}{2} Z_0 I_R e^{+\gamma d} + \frac{1}{2} V_R e^{-\gamma d} - \frac{1}{2} Z_0 I_R e^{-\gamma d}$$

$$= \frac{V_R(e^{\gamma d} + e^{-\gamma d})}{2} + \frac{Z_0 I_R (e^{\gamma d} - e^{-\gamma d})}{2}$$

$$\boxed{= V_R \cosh \gamma d + Z_0 I_R \sinh \gamma d} \longrightarrow ⑪$$

$*\rightarrow)\ ⑧ \Rightarrow \boxed{I_{s(z)} = \frac{1}{Z_0}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})}$

$$I_{s(z=-d)} = I(d) = \frac{1}{Z_0} \times \frac{1}{2}(V_R + Z_0 I_R) e^{\gamma d} - \frac{1}{Z_0} \times \frac{1}{2}[V_R - Z_0 I_R] e^{-\gamma d}$$

$$= \frac{1}{2Z_0} V_R e^{\gamma d} + \frac{1}{2Z_0} Z_0 I_R e^{\gamma d} - \frac{1}{2Z_0} V_R e^{-\gamma d} + \frac{1}{2Z_0} Z_0 I_R e^{-\gamma d}$$

$$= \frac{V_R(e^{\gamma d} - e^{-\gamma d})}{2 Z_0} + \frac{I_R(e^{\gamma d} + e^{-\gamma d})}{2}$$

$$I_d = \frac{1}{Z_0} V_R \sinh \gamma d + I_R \cosh \gamma d \longrightarrow ⑫$$

The impedance of the line of length 'd'. The line impedance viewed from DQ towards load end;

$$Z_{in} = Z(d) = \frac{V(d)}{I(d)}$$

$$Z_{in} = \frac{V_R \cosh \gamma d + Z_0 I_R \sinh \gamma d}{\frac{V_R}{Z_0} \sinh \gamma d + I_R \cosh \gamma d}$$

$$= \frac{\cosh \gamma d \left(V_R + Z_0 I_R \frac{\sinh \gamma d}{\cosh \gamma d}\right)}{\cosh \gamma d \left(\frac{V_R}{Z_0} \frac{\sinh \gamma d}{\cosh \gamma d} + I_R\right)}$$

$$= \frac{V_R + Z_0 I_R \tanh \gamma d}{\frac{V_R}{Z_0} \tanh \gamma d + I_R}$$

$$= \frac{I_R\left(\frac{V_R}{I_R} + Z_0 \tanh \gamma d\right)}{I_R\left(\frac{V_R}{Z_0 I_R} \tanh \gamma d + 1\right)}$$

$$\frac{V_R}{I_R} = Z_R$$

$$Z_{in} = \frac{Z_R + Z_0 \tanh \gamma d}{\frac{Z_R \tanh \gamma d}{Z_0} + Z_0}$$

$$\boxed{Z_{in} = \frac{Z_0 \left[Z_R + Z_0 \tanh \gamma d\right]}{Z_0 + Z_R \tanh \gamma d}}$$

A transmission line can be considered as a device to realize an impedance $Z(d)$ from Z_R with suitable selection of the length 'd' of the line. Thus we consider a transmission line as an impedance transformer.

Special cases :-

i) If the line is short circuited at the receiving end

$$\text{ie.,} \quad Z_R = 0, \quad Z_{SC} = Z_0 \tanh \gamma d$$

ii) Open circuited at the receiving end,

$$Z_R = \infty, \quad Z_{OC} = Z_0 \coth \gamma d$$

$$Z_{SC} Z_{OC} = Z_0 \tanh \gamma d \times Z_0 \coth \gamma d$$

$$= Z_0 \tanh \gamma d \, \frac{1}{\tanh \gamma d} \times Z_0$$

$$= Z_0^2$$

$$Z_0 = \sqrt{Z_{SC} \cdot Z_{OC}}$$

ie., the characteristic impedance Z_0 of the line is the geometric mean of the opencircuited and short circuited impedances.

iii) $Z_R = Z_0$

$$Z_{in} = \frac{Z_0(Z_0 + Z_0 \tanh \gamma d)}{Z_0 + Z_0 \tanh \gamma d}$$

$$Z_{in} = Z_0$$

The line is terminated in its characteristic impedance. Z_0 of a line may be defined as that load impedance for which the input impedance of the line equals the load impedance itself for all lengths of the line.

$\alpha = \frac{\omega \mu_0 \sigma}{2}$

$= 2\pi f \times$

$E =$

$J = \sigma E$

$\sigma = 20\,V$

$f = 9375$

Transmission line parameters.

i) Primary constants $\downarrow$ R, L, G, C

ii) Secondary constants γ, $Z_0 \rightarrow$ characteristic impedance

$\uparrow$ propagation constant

Case i)

Lossless line: A transmission line is said to be lossless, if the conductors of the line are perfect ($\sigma_c \simeq \infty$) & the dielectric medium separating the conductors is lossless ($\sigma \simeq 0$).

$$R = 0 = G$$

This is necessary condition for a line to be lossless. For lossless lines, we neglect R & G.

1) Characteristic Impedance

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{L/C} = R_0 + jX_0$$

$R_0 = \sqrt{L/C}$ ie Z_0 is purely resistive ie, a resistive load of value $R_L = R_0$ can match the line.

uQ 2) Propagation constant

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = j\omega\sqrt{LC} \quad \{\alpha + j\beta\}$$

$$\beta = \omega\sqrt{LC} \qquad \alpha = 0$$

3) Phase velocity

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

It is designed that powerlossless, no attenuation for a wave propagates along a lossless line.

Distortionless line

Condition $\boxed{\frac{R}{L} = \frac{G}{C}}$ heaviside condition

It is one in which, α is frequency independent & the phase constant β is linearly dependent on frequency.

i) Z_0

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R\left(1+\frac{j\omega L}{R}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} = \sqrt{R/G} = \sqrt{L/C}$$

$$= R_0 + jX_0$$

$$Z_0 = R_0 = \sqrt{L/C} = \sqrt{R/G} \;\&\; X_0 = 0$$

ii) γ, propagation constant

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{R\left(1+\frac{j\omega L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1+j\omega\frac{C}{G}\right)^2} = \sqrt{RG}\left(1+j\frac{\omega C}{G}\right) \quad \} \; R/G = L/C$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \sqrt{RG}$$

$$\beta = \sqrt{RG}\,\frac{\omega C}{G} = \sqrt{RG}\,\frac{\sqrt{C}\sqrt{C}}{\sqrt{G}\sqrt{G}}\,\omega$$

$$\beta = \sqrt{R/G}\,\sqrt{C}\sqrt{C}\,\omega$$

$$\beta = \sqrt{L/C}\,\sqrt{C}\sqrt{C}\,\omega = \underline{\sqrt{LC}\,\omega}$$

β is linearly dependent on ω.

For a distortionless line, V_p is independent of frequency.

A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

- Heavyside condition is very difficult to realize in practice $\boxed{\frac{R}{G} = \frac{L}{C}}$. The left hand side $\frac{R}{G}$ is ~~abut~~ about thousand times greater than the right hand side $\frac{L}{C}$ for telephone lines. One of the methods to reduce distortion is to increase the value of inductance L.

Reflection and Transmission at the end of a line.

A transmission line is expected to propagate the signal from the generator to the load without any loss. A txn line terminated its characteristic impedance absorbs all the power at the load. For a transmission line which is terminated in an impedance other than characteristic impedance ($Z_R \neq Z_0$) a part of supplied power is reflected back from the load. Hence the voltage at any point is the superposition of incident & reflected waves. This leads to standing waves depending upon the transmission load. If there is no impedance mismatch ($Z_R = Z_0$), then no reflection at all. We need to consider the standing waves at 3 cases.

i) $Z_R = 0$ short circuited line

ii) $Z_R = \infty$ open circuited line

iii) Z_R is arbitrary (other than 0 or ∞).

Voltage Reflection Coefficient (Γ)

(at the load)

The voltage reflection coefficient Γ_L is the ratio of the voltage reflection wave to the incident wave at the load ie:) $\Gamma_L = \dfrac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}}$ (at load, $z=0$)

$$\boxed{\Gamma_L = \frac{V_0^-}{V_0^+}}$$

The voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave $\{z=-d\}$

ie. $\Gamma_{(d)} = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^- e^{-\gamma d}}{V_0^+ e^{+\gamma d}} = \frac{V_0^-}{V_0^+} e^{-\gamma d - \gamma d}$

$= \frac{V_0^-}{V_0^+} e^{-2\gamma d}$

$\Gamma_{(d)} = \Gamma_L e^{-2j\beta d}$ $\alpha = 0,\ \gamma = j\beta$

Γ_L is the reflection coefficient at the load end, $z=0$

$|\Gamma_{(d)}| = |\Gamma_L| =$ constant

ie, magnitude of voltage reflection coefficient remains constant throughout the lossless line. But the phase angle varies linearly with d.

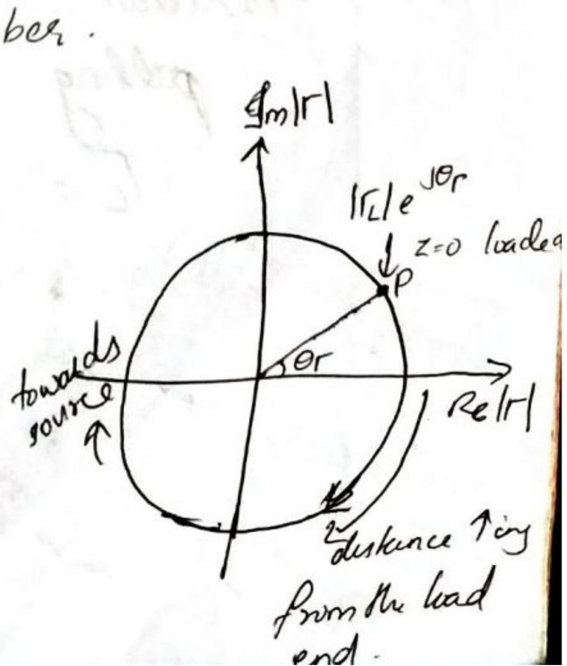
$$\boxed{\Gamma_L = |\Gamma_L| e^{j\theta_r}}$$

The reflection coefficient can be represented by a point on a circle in a complex plane.

Γ_L itself is a complex number.

$\Gamma_{(d)} = |\Gamma_L| e^{j\theta_r} \cdot e^{-2j\beta d}$

The reflection coefficient starts from P (z=0) at the load end and moves clockwise and the distance increases from the load end.



The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point i.e.,)

current reflection coefficient $= -\Gamma_L$

Impedance in terms of Γ_L

$$V_0^+ = \frac{1}{2}(V_R + Z_0 I_R)$$

$$V_0^- = \frac{1}{2}(V_R - Z_0 I_R)$$

At the load end $V_R = V_L$ & $I_R = I_L$

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L)$$

$$V_0^- = \frac{1}{2}(V_L - Z_0 I_L)$$

$$\Gamma_L = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}}$$

Reflection coefficient at the load end is given by putting $z = 0$

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{\frac{1}{2}(V_L - Z_0 I_L)}{\frac{1}{2}(V_L + Z_0 I_L)}$$

$$= \frac{I_L\left(\frac{V_L}{I_L} - Z_0\right)}{I_L\left(\frac{V_L}{I_L} + Z_0\right)} \qquad \left\{\frac{V_L}{I_L} = Z_L\right\}$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}} \longrightarrow (A)$$

$$\Gamma_L (Z_L + Z_0) = Z_L - Z_0$$

$$\Gamma_L Z_L + \Gamma_L Z_0 = Z_L - Z_0$$

$$\Gamma_L Z_0 + Z_0 = Z_L - \Gamma_L Z_L$$

$$Z_0(\Gamma_L + 1) = Z_L(1 - \Gamma_L)$$

$$\boxed{Z_L = \frac{Z_0(1+\Gamma_L)}{1-\Gamma_L}}$$

If $Z_L \neq Z_0$, there is a reflected wave

$Z_L = Z_0$, no reflected wave.

Case 1:)

If the line is shortcircuited at the load end is $Z_L = 0$,

$$\Gamma_L = |\Gamma_L| e^{j\theta_r} = -1$$

(A) $\rightarrow$ $$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = \frac{-Z_0}{Z_0} = -1 = e^{j180}$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{2}{0} = \infty$$

This means that the incident wave is reflected with same magnitude but of opposite phase. This gives $V_L = 0$ or $V_R = 0$.

Case 2:

If the line is open circuited at its load end, ie $Z_L = \infty$ $VSWR = \infty$

$$\Gamma_L = |\Gamma_L| e^{j\theta_r} = 1 \cdot e^{j0}$$

$\cos\theta + j\sin\theta = 1$
$\theta = 0$

$$\theta_r = 0$$

The incident wave is reflected without a change in magnitude and phase.

18/10/18

VSWR : Voltage Standing Wave Ratio

* If the line is terminated in a mismatch load, ($Z_R \neq Z_0$) then the line voltage shows a periodic variation with a non zero minimum. Thus the ratio of minimum voltage to maximum is a measure of impedance mismatch & hence the power transferred.

VSWR is defined as the ratio of maximum voltage to minimum voltage in a standing wave pattern on the line. It is denoted by S

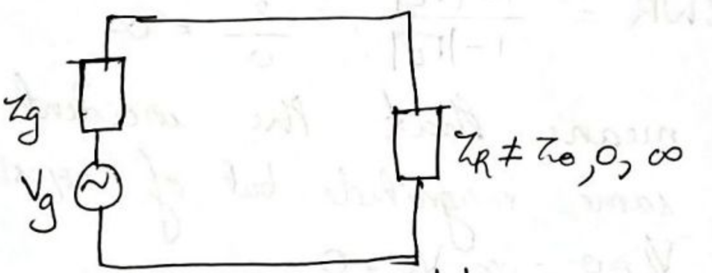
$$VSWR\,(S) = \frac{V_{max}}{V_{min}}$$

$$S = \frac{|V^+|\left[1 + |\Gamma_R|\right]}{|V^+|\left[1 - |\Gamma_R|\right]} = \frac{1 + |\Gamma_R|}{1 - |\Gamma_R|}$$

For perfect match , $\Gamma = 0$, $S = 1$

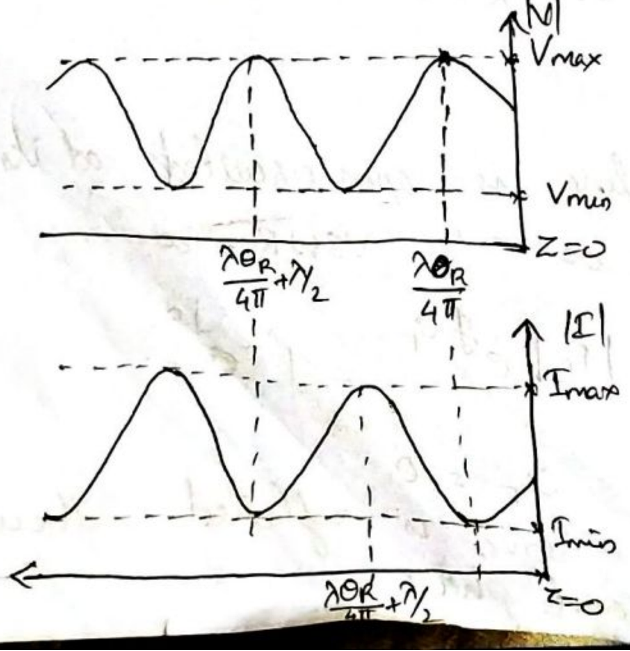
mismatch , $\Gamma = 1$, $S = \infty$

Variation of the line voltage and current when the load is mismatched.



$$\beta = \frac{2\pi}{\lambda}$$

$$d_{min} = \frac{\theta_R}{2\beta}$$



$$I_{max} = \frac{|V^+|}{Z_0}\left(1 + |\Gamma_R|\right)$$

$$I_{min} = \frac{|V^+|}{Z_0}\left(1 - |\Gamma_R|\right)$$

The line voltage maximum first occures at $d_{min} = \frac{\theta_R}{2\beta}$. The expression for maximum voltage

$$V_{max} = |V^+| \left[1 + |\Gamma_R|\right]$$

The line voltage minimum first occurs at $d_{min} = \frac{\theta_R + \pi}{2\beta}$

$$V_{min} = |V^+| \left[1 - |\Gamma_R|\right]$$

Normalized Impedance (z)

It is computationally convenient to work with impedances normalized w.r.t characteristic impedance denoted by lowercase letter 'z'.

$$z(d) = \frac{Z(d)}{Z_0}$$

$$Z_0 = R_0 + jX_0$$

$$r + jx = \frac{R + jX}{Z_0}$$