# ISYE6501x Homework 8

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# Question 11.1

Using the crime data set uscrime.txt from Questions 8.2, 9.1, and 10.1, build a regression mode

1. Stepwise regression
2. Lasso
3. Elastic net For Parts 2 and 3, remember to scale the data first – otherwise, the regression c scales and the constraint won't have the desired effect.

For Parts 2 and 3, use the glmnet function in R.

Notes on R: • For the elastic net model, what we called λ in the videos, glmnet calls "alpha"; you by varying alpha from 1 (lasso) to 0 (ridge regression) [and, of course, other values of alpha in b like glmnet(x,y,family="mgaussian",alpha=1) the predictors x need to be in R's matrix format, rat You can convert a data frame to a matrix using as.matrix – for example, x <- as.matrix(data[,1:n a value of T, glmnet returns models for a variety of values of T.

Source (StatQuest): [https://www.youtube.com/watch?v=ctmNq7FgbvI (https://www.youtube.con](https://www.youtube.com/watch?v=ctmNq7FgbvI)

In [1]:

```
library(caret)
library(glmnet)
```

```
Loading required package: ggplot2

Loading required package: lattice

Loading required package: Matrix

Loaded glmnet 4.1-6
```

In [2]:

```
crime <- read.table("uscrime.txt", header=TRUE)
```

In [3]:

```
head(crime)
```

A data.frame: 6 × 16

| | M | So | Ed | Po1 | Po2 | LF | M.F | Pop | NW | U1 | U2 | Wealth | Ineq | Prob |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | <dbl> | <int> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <int> | <dbl> | <dbl> | <dbl> | <int> | <dbl> | <dbl> |
| **1** | 15.1 | 1 | 9.1 | 5.8 | 5.6 | 0.510 | 95.0 | 33 | 30.1 | 0.108 | 4.1 | 3940 | 26.1 | 0.084602 |
| **2** | 14.3 | 0 | 11.3 | 10.3 | 9.5 | 0.583 | 101.2 | 13 | 10.2 | 0.096 | 3.6 | 5570 | 19.4 | 0.029599 |
| **3** | 14.2 | 1 | 8.9 | 4.5 | 4.4 | 0.533 | 96.9 | 18 | 21.9 | 0.094 | 3.3 | 3180 | 25.0 | 0.083401 |
| **4** | 13.6 | 0 | 12.1 | 14.9 | 14.1 | 0.577 | 99.4 | 157 | 8.0 | 0.102 | 3.9 | 6730 | 16.7 | 0.015801 |
| **5** | 14.1 | 0 | 12.1 | 10.9 | 10.1 | 0.591 | 98.5 | 18 | 3.0 | 0.091 | 2.0 | 5780 | 17.4 | 0.041399 |
| **6** | 12.1 | 0 | 11.0 | 11.8 | 11.5 | 0.547 | 96.4 | 25 | 4.4 | 0.084 | 2.9 | 6890 | 12.6 | 0.034201 |

# Base Model and Step Variable Selection

Source: https://www.youtube.com/watch?v=bgJfXMBEfZc

In [33]:

```
linear_model = lm(Crime~., data=crime)
summary(linear_model)
```

```
Call:
lm(formula = Crime ~ ., data = crime)

Residuals:
    Min      1Q  Median      3Q     Max 
-395.74  -98.09   -6.69  112.99  512.67 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -5.984e+03  1.628e+03  -3.675 0.000893 ***
M            8.783e+01  4.171e+01   2.106 0.043443 *  
So          -3.803e+00  1.488e+02  -0.026 0.979765    
Ed           1.883e+02  6.209e+01   3.033 0.004861 ** 
Po1          1.928e+02  1.061e+02   1.817 0.078892 .  
Po2         -1.094e+02  1.175e+02  -0.931 0.358830    
LF          -6.638e+02  1.470e+03  -0.452 0.654654    
M.F          1.741e+01  2.035e+01   0.855 0.398995    
Pop         -7.330e-01  1.290e+00  -0.568 0.573845    
NW           4.204e+00  6.481e+00   0.649 0.521279    
U1          -5.827e+03  4.210e+03  -1.384 0.176238    
U2           1.678e+02  8.234e+01   2.038 0.050161 .  
Wealth       9.617e-02  1.037e-01   0.928 0.360754    
Ineq         7.067e+01  2.272e+01   3.111 0.003983 ** 
Prob        -4.855e+03  2.272e+03  -2.137 0.040627 *  
Time        -3.479e+00  7.165e+00  -0.486 0.630708    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 209.1 on 31 degrees of freedom
Multiple R-squared:  0.8031,	Adjusted R-squared:  0.7078 
F-statistic: 8.429 on 15 and 31 DF,  p-value: 3.539e-07
```

## Observation

- $R^2$ is 0.8031 which is very good
- F-statistic has p-value which is very small
- This model is a significant model

Now we want to build a smaller model using Step function.

In [34]:

```
model.final=step(linear_model)
```

```
Start:  AIC=514.65
Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + 
    U2 + Wealth + Ineq + Prob + Time

         Df Sum of Sq     RSS    AIC
- So      1        29 1354974 512.65
- LF      1      8917 1363862 512.96
- Time    1     10304 1365250 513.00
- Pop     1     14122 1369068 513.14
- NW      1     18395 1373341 513.28
- M.F     1     31967 1386913 513.74
- Wealth  1     37613 1392558 513.94
- Po2     1     37919 1392865 513.95
<none>                1354946 514.65
- U1      1     83722 1438668 515.47
- Po1     1    144306 1499252 517.41
- U2      1    181536 1536482 518.56
- M       1    193770 1548716 518.93
- Prob    1    199538 1554484 519.11
- Ed      1    402117 1757063 524.86
- Ineq    1    423031 1777977 525.42

Step:  AIC=512.65
Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 + 
    Wealth + Ineq + Prob + Time

         Df Sum of Sq     RSS    AIC
- Time    1     10341 1365315 511.01
- LF      1     10878 1365852 511.03
- Pop     1     14127 1369101 511.14
- NW      1     21626 1376600 511.39
- M.F     1     32449 1387423 511.76
- Po2     1     37954 1392929 511.95
- Wealth  1     39223 1394197 511.99
<none>                1354974 512.65
- U1      1     96420 1451395 513.88
- Po1     1    144302 1499277 515.41
- U2      1    189859 1544834 516.81
- M       1    195084 1550059 516.97
- Prob    1    204463 1559437 517.26
- Ed      1    403140 1758114 522.89
- Ineq    1    488834 1843808 525.13

Step:  AIC=511.01
Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 + 
    Wealth + Ineq + Prob

         Df Sum of Sq     RSS    AIC
- LF      1     10533 1375848 509.37
- NW      1     15482 1380797 509.54
- Pop     1     21846 1387161 509.75
- Po2     1     28932 1394247 509.99
- Wealth  1     36070 1401385 510.23
- M.F     1     41784 1407099 510.42
<none>                1365315 511.01
- U1      1     91420 1456735 512.05
- Po1     1    134137 1499452 513.41
- U2      1    184143 1549458 514.95
- M       1    186110 1551425 515.01
- Prob    1    237493 1602808 516.54
- Ed      1    409448 1774763 521.33
- Ineq    1    502909 1868224 523.75
```

```
Step:  AIC=509.37
Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + NW + U1 + U2 + Wealth + 
    Ineq + Prob

         Df Sum of Sq     RSS    AIC
- NW      1     11675 1387523 507.77
- Po2     1     21418 1397266 508.09
- Pop     1     27803 1403651 508.31
- M.F     1     31252 1407100 508.42
- Wealth  1     35035 1410883 508.55
<none>                1375848 509.37
- U1      1     80954 1456802 510.06
- Po1     1    123896 1499744 511.42
- U2      1    190746 1566594 513.47
- M       1    217716 1593564 514.27
- Prob    1    226971 1602819 514.54
- Ed      1    413254 1789103 519.71
- Ineq    1    500944 1876792 521.96

Step:  AIC=507.77
Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + U1 + U2 + Wealth + Ineq + 
    Prob

         Df Sum of Sq     RSS    AIC
- Po2     1     16706 1404229 506.33
- Pop     1     25793 1413315 506.63
- M.F     1     26785 1414308 506.66
- Wealth  1     31551 1419073 506.82
<none>                1387523 507.77
- U1      1     83881 1471404 508.52
- Po1     1    118348 1505871 509.61
- U2      1    201453 1588976 512.14
- Prob    1    216760 1604282 512.59
- M       1    309214 1696737 515.22
- Ed      1    402754 1790276 517.74
- Ineq    1    589736 1977259 522.41

Step:  AIC=506.33
Crime ~ M + Ed + Po1 + M.F + Pop + U1 + U2 + Wealth + Ineq + 
    Prob

         Df Sum of Sq     RSS    AIC
- Pop     1     22345 1426575 505.07
- Wealth  1     32142 1436371 505.39
- M.F     1     36808 1441037 505.54
<none>                1404229 506.33
- U1      1     86373 1490602 507.13
- U2      1    205814 1610043 510.76
- Prob    1    218607 1622836 511.13
- M       1    307001 1711230 513.62
- Ed      1    389502 1793731 515.83
- Ineq    1    608627 2012856 521.25
- Po1     1   1050202 2454432 530.57

Step:  AIC=505.07
Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Wealth + Ineq + Prob

         Df Sum of Sq     RSS    AIC
- Wealth  1     26493 1453068 503.93
<none>                1426575 505.07
- M.F     1     84491 1511065 505.77
- U1      1     99463 1526037 506.24
```

```
- Prob    1    198571 1625145 509.20
- U2      1    208880 1635455 509.49
- M       1    320926 1747501 512.61
- Ed      1    386773 1813348 514.35
- Ineq    1    594779 2021354 519.45
- Po1     1   1127277 2553852 530.44

Step:  AIC=503.93
Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob

       Df Sum of Sq     RSS    AIC
<none>              1453068 503.93
- M.F   1    103159 1556227 505.16
- U1    1    127044 1580112 505.87
- Prob  1    247978 1701046 509.34
- U2    1    255443 1708511 509.55
- M     1    296790 1749858 510.67
- Ed    1    445788 1898855 514.51
- Ineq  1    738244 2191312 521.24
- Po1   1   1672038 3125105 537.93
```

We wish to select, from among the candidate models, the model that minimizes the information with lower AIC scores to minimize information loss.

In [37]:

```
summary(model.final)
```

```
Call:
lm(formula = Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob,
    data = crime)

Residuals:
    Min      1Q  Median      3Q     Max
-444.70 -111.07    3.03  122.15  483.30

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -6426.10    1194.61  -5.379 4.04e-06 ***
M              93.32      33.50   2.786  0.00828 **
Ed            180.12      52.75   3.414  0.00153 **
Po1           102.65      15.52   6.613 8.26e-08 ***
M.F            22.34      13.60   1.642  0.10874
U1          -6086.63    3339.27  -1.823  0.07622 .
U2            187.35      72.48   2.585  0.01371 *
Ineq           61.33      13.96   4.394 8.63e-05 ***
Prob        -3796.03    1490.65  -2.547  0.01505 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 195.5 on 38 degrees of freedom
Multiple R-squared:  0.7888,    Adjusted R-squared:  0.7444
F-statistic: 17.74 on 8 and 38 DF,  p-value: 1.159e-10
```

## Observation

- The final model only has 8 predictors as opposed to the original 15
- The final model with the lowest AIC score actually ended up having a reduced $R^2$ score

- But we observe that all predictors are significant due to the p-values (t test)
- The F-statistic is also significant

# Stepwise Regression

Source: https://www.youtube.com/watch?v=vFH--Xdt3Pk (https://www.youtube.com/watch?v=vl
Source: https://www.rdocumentation.org/packages/caret/versions/2.27/topics/train (https://www.rdocumentation.org/packages/caret/versions/2.27/topics/train)

In [5]:

```
set.seed(1)

train_rows <- sample(1:nrow(crime),as.integer(0.7*nrow(crime),replace=F))

train = crime[train_rows,]

test = crime[-train_rows,]

train_control <- trainControl(method ="cv", number =10)

stepwise <- train(Crime ~., data = crime , method ="lmStepAIC", trControl = train_control

stepwise$results
```

A data.frame: 1 × 7

| | parameter <chr> | RMSE <dbl> | Rsquared <dbl> | MAE <dbl> | RMSESD <dbl> | RsquaredSD <dbl> | MAESD <dbl> |
|---|---|---|---|---|---|---|---|
| 1 | none | 253.266 | 0.5832771 | 216.4972 | 90.06367 | 0.3322636 | 84.15968 |

In [6]:

```
stepwise$finalModel
```

```
Call:
lm(formula = .outcome ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq +
    Prob, data = dat)

Coefficients:
(Intercept)            M           Ed          Po1          M.F           U1
   -6426.10        93.32       180.12       102.65        22.34     -6086.63
         U2         Ineq         Prob
     187.35        61.33     -3796.03
```

In [7]:

```
summary(stepwise)
```

```
Call:
lm(formula = .outcome ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq +
    Prob, data = dat)

Residuals:
    Min      1Q  Median      3Q     Max 
-444.70 -111.07    3.03  122.15  483.30 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6426.10    1194.61  -5.379 4.04e-06 ***
M              93.32      33.50   2.786  0.00828 ** 
Ed            180.12      52.75   3.414  0.00153 ** 
Po1           102.65      15.52   6.613 8.26e-08 ***
M.F            22.34      13.60   1.642  0.10874    
U1          -6086.63    3339.27  -1.823  0.07622 .  
U2            187.35      72.48   2.585  0.01371 *  
Ineq           61.33      13.96   4.394 8.63e-05 ***
Prob        -3796.03    1490.65  -2.547  0.01505 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 195.5 on 38 degrees of freedom
Multiple R-squared:  0.7888,	Adjusted R-squared:  0.7444 
F-statistic: 17.74 on 8 and 38 DF,  p-value: 1.159e-10
```

The following line is from StatQuest ([https://www.youtube.com/watch?v=ctmNq7FgbvI](https://www.youtube.com/watch?v=ctmNq7FgbvI)) 8:56

alpha0 represents RIDGE Regression

In [43]:

```
alpha0.fit<-cv.glmnet(x_train,y_train, type.measure="mse", alpha=0, family="gaussian")
alpha0.fit
```

```
Call:  cv.glmnet(x = x_train, y = y_train, type.measure = "mse", alpha = 0,      family = "gaussian") 

Measure: Mean-Squared Error 

    Lambda Index Measure     SE Nonzero
min 0.1444    90  0.5436 0.1477      15
1se 1.2274    67  0.6812 0.2150      15
```

In [44]:

```
alpha0.predicted<-predict(alpha0.fit, s=alpha0.fit$lambda.1se, newx=x_test)
```

In [45]:

```
mean((y_test-alpha0.predicted)^2)
```

```
0.420748688472113
```

---

Observation:

- this method is the same as what I did in the previous section.
- $R^2$ is 0.7888 which is good
- 8 significant predictors are selected out of 15
- F- statistic is also significant
- MSE is 0.420748688472113

---

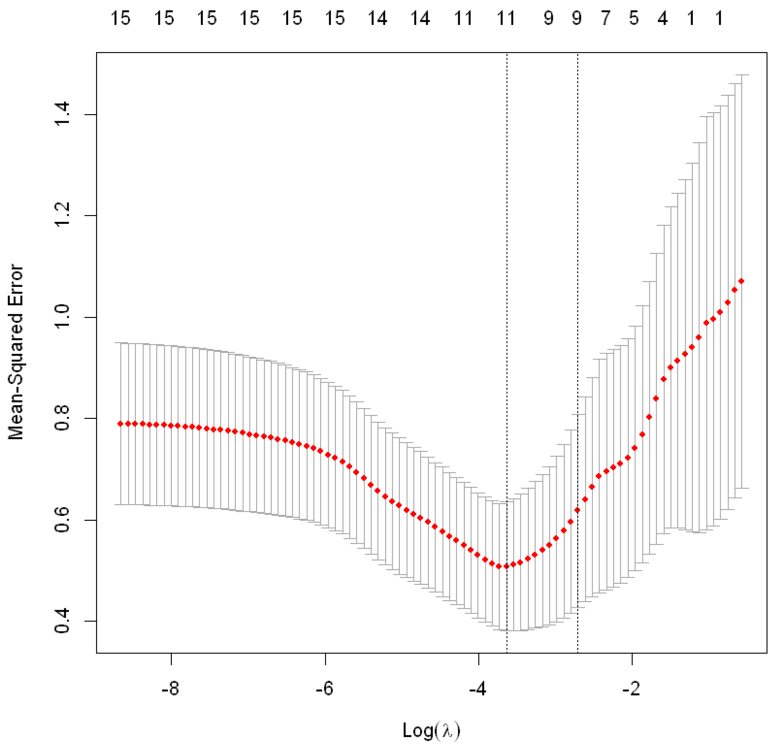
# LASSO

In [18]:

```
x_train<-scale(as.matrix(train)[,-16], center =TRUE, scale =TRUE)

y_train<-scale(as.matrix(train)[,16], center =TRUE, scale =TRUE)

lasso_model <- cv.glmnet(x_train, y_train, family="gaussian", alpha=1)
plot(lasso_model)
```



In [20]:

```
coef(lasso_model)
```

```
16 x 1 sparse Matrix of class "dgCMatrix"
                       s1
(Intercept)  3.403255e-16
M            2.245276e-01
So           .
Ed           2.054619e-01
Po1          6.927334e-01
Po2          .
LF           .
M.F          1.415977e-01
Pop          5.495775e-02
NW           3.017289e-02
U1           .
U2           2.965666e-02
Wealth       .
Ineq         3.855585e-01
Prob        -1.521280e-01
Time         .
```

In [21]:

```
lambda <- lasso_cv$lambda.min
cat(lambda)
```

```
0.02195409
```

In [22]:

```
best_lasso = glmnet(xtrain, ytrain, family ="gaussian", alpha =1, lambda = lambda)
coef(best_lasso)
```

```
16 x 1 sparse Matrix of class "dgCMatrix"
                       s0
(Intercept)  6.048164e-16
M            2.305511e-01
So           .
Ed           6.425983e-01
Po1          7.498446e-01
Po2          .
LF          -7.255044e-02
M.F          1.227830e-01
Pop          5.737205e-02
NW           1.713323e-01
U1           .
U2           1.077069e-01
Wealth       7.695443e-02
Ineq         8.017996e-01
Prob        -1.899131e-01
Time         .
```

In [23]:

```
# I need R Square
x_test<-scale(as.matrix(test)[,-16], center =TRUE, scale =TRUE)

y_test<-scale(as.matrix(test)[,16], center =TRUE, scale =TRUE)
```

The following line is from StatQuest (https://www.youtube.com/watch?v=ctmNq7FgbvI (https://w
v=ctmNq7FgbvI)) 11:59

alpha1 represents LASSO Regression

In [39]:

```
alpha1.fit <- cv.glmnet(x_train,y_train, type.measure="mse", alpha=1, family="gaussian")
alpha1.fit
```

```
Call:  cv.glmnet(x = x_train, y = y_train, type.measure = "mse", alpha = 1,      family = "gaussian")

Measure: Mean-Squared Error

     Lambda Index Measure     SE Nonzero
min 0.03837    30  0.6380 0.1294      10
1se 0.07358    23  0.7535 0.1800       9
```

In [40]:

```
alpha1.predicted<-predict(alpha1.fit, s=alpha1.fit$lambda.1se, newx=x_test)
```

In [41]:

```
mean((y_test-alpha1.predicted)^2)
```

```
0.282616991992711
```

Observation:

- The best Lambda Value is 0.02195
- The MSE for LASSO is 0.282616991992711 (an improvement)

# Elastic Net

In [24]:

```
train_control <- trainControl(method ="repeatedcv", number =10, repeats =5, search ="rand

elastic_model <- train(Crime ~ .,data = as.matrix(scale(train)),method ="glmnet",preProce
```

```
Warning message in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo, :
"There were missing values in resampled performance measures."
```

In [25]:

```
elastic_model$bestTune
```

A data.frame: 1 × 2

| | alpha <dbl> | lambda <dbl> |
|---|---|---|
| 5 | 0.4507055 | 0.04054113 |

The following line is from StatQuest (https://www.youtube.com/watch?v=ctmNq7FgbvI) 12:59

alpha0.5 represents Elastic Net Regression

In [46]:

```
alpha0.5.fit<-cv.glmnet(x_train,y_train,type.measure="mse", alpha=0.5, family="gaussian")
```

In [47]:

```
alpha0.5.predicted<-predict(alpha0.5.fit, s=alpha0.5.fit$lambda.1se, newx=x_test)
```

In [48]:

```
mean((y_test-alpha0.5.predicted)^2)
```

0.292458616503691

Observation:

- The best alpha value is 0.4507055
- The best Lambda Value is 0.04054
- MSE for Elastic Net is 0.292458616503691 (slightly higher than LASSO)

# Preliminary Conclusion and Hyperparameter Tuning

At this point, LASSO is the best model

We still need to try different values of Alpha for Elastic Net

Source (StatQuest): https://www.youtube.com/watch?v=ctmNq7FgbvI (https://www.youtube.con

In [50]:

```
list.of.fits<-list()
for (i in 0:10){
    fit.name<-paste0("alpha",i/10)
    list.of.fits[[fit.name]]<-
    cv.glmnet(x_train,y_train,type.measure="mse", alpha=i/10,
             family="gaussian")
}
```

- When i=0, then alpha will be 0 and result in Ridge Regression
- When i=1, then alpha will be 0.1
- etc
- When i=10, then alpha=1, which results in Lasso Regression

In [53]:

```
results<-data.frame()
for (i in 0:10){
    fit.name<-paste0("alpha", i/10)
    predicted<-
    predict(list.of.fits[[fit.name]],
           s=list.of.fits[[fit.name]]$lambda.1se,newx=x_test)
    mse<-mean((y_test-predicted)^2)
    temp<-data.frame(alpha=i/10,mse=mse,fit.name=fit.name)
    results<-rbind(results,temp)
}
```

In [54]:

```
results
```

A data.frame: 11 × 3

| alpha | mse | fit.name |
|---|---|---|
| <dbl> | <dbl> | <chr> |
| 0.0 | 0.5048315 | alpha0 |
| 0.1 | 0.5991401 | alpha0.1 |
| 0.2 | 0.4128985 | alpha0.2 |
| 0.3 | 0.2759187 | alpha0.3 |
| 0.4 | 0.3342465 | alpha0.4 |
| 0.5 | 0.3073987 | alpha0.5 |
| 0.6 | 0.4251754 | alpha0.6 |
| 0.7 | 0.3090506 | alpha0.7 |
| 0.8 | 0.3025588 | alpha0.8 |
| 0.9 | 0.2676959 | alpha0.9 |
| 1.0 | 0.2826170 | alpha1 |

The best fit is when $\alpha = 0.9$ where the MSE is 0.2676959