BONUS Week 12 Homework

ⓘ This is a preview of the published version of the quiz

Started: Jul 2 at 7:55am

Quiz Instructions

Question 1

1 pts

(Lesson 9.2: A Mathematical Interlude.) BONUS: Suppose that $X_1, X_2, \ldots$ is a stationary (steady-state) stochastic process with covariance function $R_k \equiv Cov(X_1, X_{1+k})$, for $k = 0, 1, \ldots$. We know from class that the variance of the sample mean can be represented as

$Var(\bar{X}_n) = \frac{1}{n}\left[R_0 + 2\sum_{k=1}^{n-1}\left(1 - \frac{k}{n}\right)R_k\right]$. → Auto Regressive

We also know from class that for a simple AR(1) process, we have $R_k = \phi^k$, $k = 0, 1, 2, \ldots$ Compute $Var(\bar{X}_n)$ for an AR(1) process with $n = 3$ and $\phi = 0.8$.

- a. -0.831
- b. -0.5
- c. 0
- d. 0.5
- e. 0.831 ✓

$$Var(\bar{X}_3) = \frac{1}{3}\left[R_0 + 2\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)R_k\right]$$

$$= \frac{1}{3}\left[0.8^0 + 2\left(\sum_{k=1}^{2}\left(1-\frac{k}{n}\right)\phi^k\right)\right]$$

$$= \frac{1}{3}\left[1 + 2\left(\left(1-\frac{1}{3}\right)(0.8)^1 + \left(1-\frac{2}{3}\right)(0.8)^2\right)\right]$$

$$= \frac{1}{3}\left[1 + 2\left(\frac{8}{15} + \frac{16}{75}\right)\right]$$

$$= \frac{1}{3}\left(\frac{187}{75}\right) = \frac{187}{225} = 0.831$$

Question 2

1 pts

(Lesson 9.2: A Mathematical Interlude.) BONUS: TRUE or FALSE? Using the notation of the previous question,

$$\lim_{n\to\infty} nVar(\bar{X}_n) = R_0 + 2\sum_{k=1}^{\infty} R_k = \sum_{k=-\infty}^{\infty} R_k.$$

- True ✓

$$\sigma_n^2 \equiv n\,Var(\bar{Y}_n) = R_0 + 2\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)R_k$$

$$\sigma^2 = \lim_{n\to\infty}\sigma_n^2 = \lim_{n\to\infty} n\,Var(\bar{Y}_n) \overset{*}{=} R_0 + 2\sum_{k=1}^{\infty} R_k$$

$= \sum_{K=-\infty}^{\infty} R_k$

False True by definition.

Question 3

1 pts

(Lesson 9.7: Properties of Batch Means.) (You can do this problem without watching Lesson 9.7. You can do it!) Then let's do it.

BONUS: Consider the output analysis method of non overlapping batch means. Assuming that you have a sufficiently large batch size, it can be shown that when the number of batches b is even, the expected width of the 95% two-sided confidence interval for μ is proportional to

$$\frac{t_{0.025,b-1}}{\sqrt{b(b-1)}} \frac{\left(\frac{b-2}{2}\right)!}{\left(\frac{b-3}{2}\right)\left(\frac{b-5}{2}\right)\cdots\frac{1}{2}}$$

Using the above equation, determine which of the following values of b gives the smallest expected width.

a. b=4 $\frac{t_{0.025,3}}{\sqrt{4\times3}}\left(\frac{1!}{\frac{1}{2}}\right) = \frac{3.182}{\sqrt{12}}\left(\frac{1}{1/2}\right) = 1.837$

b. b=6 $\frac{t_{0.025,5}}{\sqrt{6\times5}}\left(\frac{2!}{\frac{3}{2}\times\frac{1}{2}}\right) = \frac{2.571}{\sqrt{30}}\left(\frac{2}{3/4}\right) = 1.2517$

c. b=8 $\frac{t_{0.025,7}}{\sqrt{8\times7}}\left(\frac{3!}{\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}}\right) = \frac{2.365}{\sqrt{56}}\left(\frac{6}{15/8}\right) = 1.0113$

d. 2b or not 2b, that is a question.

e. Do b do b do. See https://www.youtube.com/watch?v=Fd_3EkGr0-4 at time 2:22.

Quiz saved at 7:55am

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