

# BONUS Week 13 Homework

⚠ This is a preview of the published version of the quiz

Started: Jul 2 at 7:56am

## Quiz Instructions

### Question 1

1 pts

Opposite of CRN - Suppose  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are iid unbiased estimators for some parameter  $\theta$ .

If we induce negative correlation between  $\hat{\theta}_1$  and  $\hat{\theta}_2$ , then the average of the two is also unbiased and may have very low variance.

(Lesson 10.7: Antithetic Random Numbers.) BONUS: Suppose  $A$  and  $B$  are two identically distributed, unbiased, antithetic estimators for the mean  $\mu$  of some random variable, and let  $C = (A + B)/2$ . Which of the following is true?

☐ a.  $E[C] < \mu$ .

$$\text{Var}\left(\frac{\hat{\theta}_1 + \hat{\theta}_2}{2}\right) < \frac{\text{Var}(\hat{\theta}_1)}{2}$$

☐ b.  $E[C] = \mu$  and  $\text{Var}(C) = \text{Var}(A)$ .

☐ c.  $E[C] = \mu$  and  $\text{Var}(C) = \text{Var}(A)/2$ .

☒ d.  $E[C] = \mu$  and  $\text{Var}(C) < \text{Var}(A)/2$ .

C is also unbiased by definition, so  $E[C] = \mu$ .

☐ e.  $E[C] = \mu$  and  $\text{Var}(C) > \text{Var}(A)/2$ .

### Question 2

1 pts

$M_{\text{BEM}}$  refers to single-stage procedure by Bedhofer.

(Lesson 10.16: Multinomial Procedure.) BONUS: Suppose that we want to know which of <sup>k=3</sup> Coke, Pepsi, and Dr. Pepper is the most popular. We would like to make the correct selection with probability of at least  $P^* = 0.90$  in the event that the ratio of the highest-to-second-highest preference probabilities happens to be at least  $\theta^* = 1.4$ . If we use procedure  $M_{\text{BEM}}$ , then the corresponding table in the notes (with  $k = 3$ ) tells us to take 126 samples (taste tests). Suppose we take those samples sequentially and after 100 have been taken it turns out that 65 people prefer Coke, 25 love Pepsi, and 10 like Dr. Pepper. What to do?

$$P_{[1]} = 0.10 \\ \text{Dr. Pepper}$$

$$P_{[2]} = 0.25 \\ \text{Pepsi}$$

$$P_{[3]} = 0.65 \\ \text{Coke}$$

$$P_{[3]} = 0.65 - 2.6$$

$$P_{[2]} = 0.25$$

$$\frac{P_{[2]}}{P_{[1]}} = \frac{0.25}{0.10} = 2.5 > \theta^*$$

KIV {

- ☒ a. Keep the experiment going until you get 126 people. (only 100 taken)
- ☐ b. Stop the test now and declare with confidence of at least 90% that Coke is the most-preferred. *I can stop the test because the extra 26 people can't help Pepsi to win. But  $\frac{65}{25+26} = 1.274 < 1.4$*
- ☒ c. Stop the test now, but without any probability statement.
- ☒ d. Put a little rum in the Coca Cola samples and start the test again.

[www.youtube.com/watch?v=zGxL2uNr7bk](http://www.youtube.com/watch?v=zGxL2uNr7bk)

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