Week 10 Homework

ⓘ This is a preview of the published version of the quiz

Started: Jul 2 at 7:50am

Quiz Instructions

Question 1 — 1 pts

(Lesson 7.12: The Box-Muller Method.) Suppose U_1 and U_2 are i.i.d. Unif(0,1) with $U_1 = 0.1$ and $U_2 = 0.8$. Use the "cosine" version of Box-Muller to generate a single Nor(-1,4) random variate. Don't forget to use radians instead of degrees!

μ σ^2 $Z_1 = \sqrt{-2\ln(U_1)}\cos(2\pi U_2) = \sqrt{-2\ln(0.1)}\cos(2\pi(0.8))$

- a. -0.326 $= 0.663139\ldots$
- b. 0 $X \sim Nor(-1,4) \Rightarrow X = \mu + \sigma Z_1$
- c. 0.326 ✓ $= -1 + 2(0.663139)$
- d. 0.663 $= -1 + 1.326278$
- e. 1.96 $\simeq 0.326$ //

Question 2 — 1 pts

The Distribution

(Lesson 7.13: Generating Order Statistics.) Consider i.i.d. Exp(λ) random variables $X_1, X_2, \ldots, X_n$, and let $Y = \max_i(X_i)$. How can we generate Y using just *one* PRN? $X_1, X_2, \ldots, X_n \sim Exp(\lambda)$ | $F(x) = 1 - e^{-\lambda x}$ ← Exp(λ) cdf

$G(y)$ is cdf of $Y = \max_i(X_i)$

- a. $-\frac{1}{\lambda}\ln(1-U)$ $G(y) = P(Y<y) = P(\max_i(X_i) < y)$
- b. $-\frac{1}{\lambda}\ln(U)$ $= P(\text{all } X_i\text{'s} < y) = [P(X_1 < y)]^n$
- c. $-\frac{1}{\lambda}\ln(1-U^{1/n})$ ✓ $= [F(y)]^n$.
- d. $-\frac{1}{\lambda}\ln(U^{1/n})$ Now set $G(y) = U \Rightarrow [F(y)]^n = U$

$[1 - e^{-\lambda y}]^n = U \Rightarrow 1 - e^{-\lambda y} = U^{1/n}$

$e^{-\lambda y} = 1 - u^{1/n}$

- e. $-\left[\frac{1}{\lambda}\ln(1-U)\right]^{1/n}$

$-\lambda y \ln e = 1 - u^{1/n}$

$y = -\frac{1}{\lambda}\left[1 - u^{1/n}\right]$

Question 3

1 pts

(Lesson 7.14: Multivariate Normal Distribution.) Suppose I have a matrix $A = \begin{array}{c} \begin{matrix} \sigma_{11} & \sigma_{12} \end{matrix} \\ \begin{pmatrix} 2 & -1 \\ -1 & 4 \end{pmatrix} \\ \begin{matrix} \sigma_{21} & \sigma_{22} \end{matrix} \end{array}$. Find the lower triangular matrix C such that $A = CC'$ and tell me what the entry c_{21} is (use the results from class).

↑ Cholesky Matrix

$$C = \begin{pmatrix} \sqrt{\sigma_{11}} & 0 \\ \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} & \sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}} \end{pmatrix}$$

- a. -1
- ✓ b. -0.7071
- c. 0
- d. 0.3536
- e. 1

$$c_{21} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} = \frac{-1}{\sqrt{2}} = -0.7071$$

Question 4

1 pts

NHPP

(Lesson 7.16: Nonhomogeneous Poisson Processes.) Suppose that the arrival pattern to a parking lot over a certain time period is an NHPP with $\lambda(t) = 2t$. Use simulation to find the probability that there will be exactly 3 arrivals between times $t = 0$ and 2.

$\lambda(t) = 2t$

$N(2) - N(0) \sim \text{Pois}\left(\int_0^2 2t\,dt\right)$

$\sim \text{Pois}(4)$

$\int_0^2 2t\,dt = \left[\frac{2t^2}{2}\right]_0^2$

$P(N(2) - N(0) = 3) = \frac{e^{-4}(4)^3}{3!}$

$\boxed{P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}}$ $= 0.19536$

≈ 0.195

- a. 0
- ✓ b. 0.195
- c. 0.5
- d. 0.805
- e. 1

Question 5

1 pts

(Lesson 7.19: Brownian Motion.) Let $\mathcal{W}(t)$ denote a Brownian motion process at time t. Calculate $\text{Cov}(\mathcal{W}(3),\ \mathcal{W}(5))$.

- a. 0
- b. 2
- c. 3
- d. 5
- e. 8

$$\text{Cov}(\mathcal{W}(3), \mathcal{W}(5)) = \min(3,5)$$
$$= 3$$

It's that simple due to the Wiener Process

Question 6

1 pts

(Lesson 7.19: Brownian Motion.) Let $\mathcal{W}(t)$ denote a Brownian motion process at time t and define a Brownian bridge by $\mathcal{B}(t) = \mathcal{W}(t) - t\mathcal{W}(1)$ for $0 < t < 1$. Find the variance of the area under a bridge, i.e., $Var\Big(\int_0^1 \mathcal{B}(t)\,dt\Big)$. I'm a nice guy, so I'll get you started...

simi bridge?

$$Var\Big(\int_0^1 \mathcal{B}(t)\,dt\Big) = Cov\Big(\int_0^1 \mathcal{B}(s)\,ds,\ \int_0^1 \mathcal{B}(t)\,dt\Big)$$
$$= \int_0^1\int_0^1 Cov\big(\mathcal{B}(s),\ \mathcal{B}(t)\big)\,ds\,dt$$

- a. -1/2
- b. 0
- c. 1/12
- d. 1/2
- e. 1

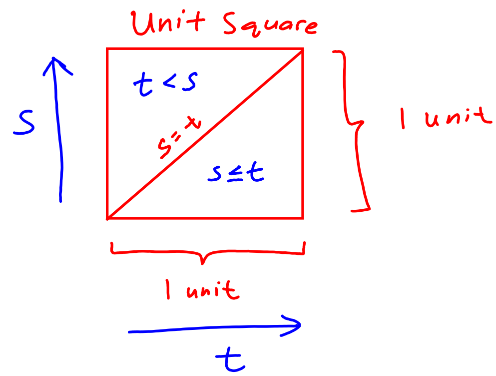
$$\int_0^1\int_0^1 \text{cov}(B(s),B(t))\,ds\,dt = \int_0^1\int_0^1 [\min(s,t) - st]\,ds\,dt$$
$$= \int_0^1\int_0^1 \min(s,t)\,ds\,dt - \int_0^1\int_0^1 st\,ds\,dt$$
$$= \underbrace{\int_0^1\int_0^t s\,ds\,dt}_{\min(s,t)=s} + \underbrace{\int_0^1\int_t^1 s\,ds\,dt}_{\min(s,t)=t} - \int_0^1 \Big[t\Big(\frac{s^2}{2}\Big)\Big]_0^1 dt$$
$$= \int_0^1 \Big[\frac{s^2}{2}\Big]_0^t dt + \int_0^1 \Big[\frac{s^2}{2}\Big]_t^1 dt - \int_0^1 \frac{t}{2}\,dt$$
$$= \int_0^1 \frac{t^2}{2}\,dt + \int_0^1 \frac{1}{2} - \frac{t^2}{2}\,dt - \Big[\frac{t^2}{4}\Big]_0^1$$
$$= \Big[\frac{t^3}{6}\Big]_0^1 + \Big[\frac{1}{2}t - \frac{t^3}{6}\Big]_0^1 - \frac{1}{4}$$
$$= \frac{1}{6} + \frac{1}{2} - \frac{1}{6} - \frac{1}{4}$$
$$= \frac{1}{4} \quad \times$$

Corrected Working Below

$$\int_0^1\int_0^1 \operatorname{cov}\big(B(s),B(t)\big)\,ds\,dt = \int_0^1\int_0^1 \big[\min(s,t)-st\big]\,ds\,dt$$

$$= \int_0^1\int_0^1 \min(s,t)\,ds\,dt - \underbrace{\int_0^1\int_0^1 st\,ds\,dt}_{\text{this is } \frac{1}{4} \text{ as calculated above}}$$

$$= \underbrace{\int_0^1\int_0^t s\,ds\,dt}_{s\le t} + \underbrace{\int_0^1\int_t^1 t\,ds\,dt}_{t<s} - \frac{1}{4}$$



$$= \int_0^1 \left[\frac{s^2}{2}\right]_0^t dt + \int_0^1 \big[ts\big]_t^1\,dt - \frac{1}{4}$$

$$= \int_0^1 \frac{t^2}{2}\,dt + \int_0^1 t - t^2\,dt - \frac{1}{4}$$

$$= \left[\frac{t^3}{6}\right]_0^1 + \left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^1 - \frac{1}{4}$$

$$= \frac{1}{6} + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12} \;//$$

The unit square region is divided into two triangles with the line $s=t$ as the boundary.

For $s \le t$, which is below the line, the minimum value is s so we integrate s wrt s and then wrt t.

For $t < s$, which is above the line, the minimum value is t so we integrate t wrt s first and then wrt t.

Question 7

1 pts

(Lesson 7.19: Brownian Motion.) As we discussed in class, you can use Brownian motion to estimate option prices for stocks. I'm not going to have you simulate that, but I'm going to give you a quick look-up assignment. As I write this on May 10, 2020, IBM is currently selling for about $122.99 per share. Suppose I'm interested in guaranteeing that I can buy a share of IBM for at most $145 on Sept. 20, 2020. Look up (maybe using something like FaceTube on the internets) the corresponding stock option price. [You don't have to write down an answer for this problem, but I'd like you to do the look-up anyway.]

As I was writing this solution sheet, the option price was $2.72 --- but this is obviously subject to change depending on how the market does.

- a. I swear I looked it up :)

Not saved

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