Week 13 Homework

ⓘ This is a preview of the published version of the quiz

Started: Jul 2 at 7:51am

Quiz Instructions

Question 1

1 pts

(Lesson 10.1: Introduction to Comparison of Systems.) Suppose we are dealing with i.i.d. normal observations with unknown variance. Which of the following is true about a 95% confidence interval for the mean μ?

Not true. though n increases, the S_X may also change.

- a. If you take more observations, the CI will always shrink. ✗

not "certain" but "sure"

- b. We are 95% sure that our CI will actually contain the unknown value of μ. ✓

never "exactly" but "about"

- c. If we calculate 100 of these CI's, exactly 95 will actually contain μ. ✗
- d. A 99% CI based on the same data will be shorter than the corresponding 95% CI. ✗

Wrong. It will be longer/wider

Question 2

Two-sample case
$X_1, X_2, \ldots, X_n \overset{iid}{\sim} Nor(\mu_X, \sigma_X^2)$
$Y_1, Y_2, \ldots, Y_m \overset{iid}{\sim} Nor(\mu_Y, \sigma_Y^2)$

1 pts

Two systems implies σ_X^2 and σ_Y^2 are unequal and unknown. Use approximate CI method.

(Lesson 10.3: Confidence Intervals for the Difference in Two Means.) We are studying the waiting times arising from two queueing systems. Suppose we make 4 independent replications of both systems, where the systems are simulated independently of each other.

replication	system 1 (X)	system 2 (Y)
1	10	25
2	20	10
3	5	40
4	30	30

$\bar{X} = \frac{10+20+5+30}{4} = 16.25$

$\bar{Y} = \frac{25+10+40+30}{4} = 26.25$

$S_X^2 = \frac{6.25^2+3.75^2+11.25^2+13.75^2}{3} = \frac{368.75}{3} = 122.9167$

$S_Y^2 = \frac{1.25^2+16.25^2+13.75^2+3.75^2}{3} = \frac{468.75}{3} = 156.25$

The Waite-Satterthwaite Equation.

DOF $V = \dfrac{\left(\frac{S_X^2}{n} + \frac{S_Y^2}{m}\right)^2}{\frac{(S_X^2/n)^2}{n+1} + \frac{(S_Y^2/m)^2}{m+1}} - 2 = \dfrac{\left(\frac{122.9167}{4} + \frac{156.25}{4}\right)^2}{\frac{(122.9167/4)^2}{5} + \frac{(156.25/4)^2}{5}} - 2 = \dfrac{4870.8779}{494.0322} - 2 = \lfloor 7.859 \rfloor \approx 7$ (round down)

Assuming that the average waiting time results from each replication are approximately normal, find a two-sided 95% CI for the difference in the means of the two systems.

$\alpha = 0.05$

- a. [-30, -5] (-17.5)
- b. [-15, 5] (-5)
- c. [-29.76, 9.76] (-10) ✓
- d. [-35, 10] (-12.5)
- e. [5, 30] (17.5)

$\mu_X - \mu_Y \in \bar{X} - \bar{Y} \pm \underbrace{t_{\alpha/2, v}}_{t_{0.025,7} = 2.3646} \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}$

$= -10 \pm 2.3646\sqrt{69.7917}$

$= -10 \pm 19.754$

$[-29.754, 9.754]$

Question 3

CRN is a Variance Reduction technique to tighten the CI.

1 pts

Observations are paired. Use paired CIs.

(Lesson 10.6: Common Random Numbers.) This is sort of the same as Question 2, except we have now used common random numbers to induce positive correlation between the results of the two systems.

↳ not independent ?

$n = 4$, $n-1 = 3$

replication	system 1	system 2	Difference
1	10	25	−15
2	20	30	−10
3	5	10	−5
4	30	40	−10

$\bar{D} = \frac{-15-10-5-10}{4} = -10$

$S_D^2 = \frac{5^2+0^2+5^2+0^2}{4-1} = 16.667$

$\mu_D \in \bar{D} \pm t_{0.025,3}\sqrt{\frac{16.667}{4}}$

$= -10 \pm 3.182\sqrt{4.16675}$

$= -10 \pm 6.4953$

Again find a two-sided 95% CI for the difference in the means of the two systems.

- a. [-30, -5] (-17.5) ✗
- b. [-15, 5] (-5) ✗
- c. [-16.5, -3.5] (-10) ✓
- d. [-18.5, -1.5] (-10)
- e. [5, 15] (10) ✗

$[-16.49, -3.5047]$

Possible options (c, d)

Question 4

1 pts

Bechhofer

(Lesson 10.11: Single-Stage Normal Means Procedure.) Suppose that you want to pick that one of three normal populations having the largest mean. We'll assume that the variances of the three competitors are all known to be equal to $\sigma^2 = 4$. (Ya, I know that this is a crazy, unrealistic assumption, but let's go with it anyway, okey dokey?) I want to choose the best of the three populations with probability of correct selection of 95% whenever the best population's mean happens to be at least $\delta^\star = 1$ larger than the second-best population's. How many observations from each population does Bechhofer's procedure $\mathcal{N}_B$ tell me to take before I can make such a conclusion?

common known variance

$P(CS)$

↳ δ^* is the indifferent zone parameter.

$k=3$, $p^* = 95\% = 0.95$, $\sigma = 2$, $\delta^* = 1$

$\delta^*/\sigma = \frac{1}{2}$

- a. 22
- b. 30 ✓
- c. 35
- d. 58
- e. 361

Just use the table

Question 5

KIV

1 pts

(Lesson 10.11: Single-Stage Normal Means Procedure.) In the above problem, suppose that we take the necessary observations and we come up with the following sample means: $\bar{X}_1 = 7.6$, $\bar{X}_2 = 11.1$, and $\bar{X}_3 = 3.6$. What do we do?

Select the population that yielded the largest sample mean

$\bar{X}_{[k]} = \max\{\bar{x}_1, \bar{x}_2, \bar{x}_3\}$ as the one associated with $\mu_{[k]}$

- a. Pick population 1 and say that we are right with probability at least 95%
- b. Pick population 2 and say that we are right with probability at least 95% ✓
- c. Take more observations and re-evaluate.
- d. Estimate the variance and re-evaluate.
- e. Not enough information to make a conclusion.

Not saved

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