Week 9 Homework

This is a preview of the published version of the quiz

Started: Jul 2 at 7:50am

Quiz Instructions

Question 1

1 pts

(Lesson 7.1: Introduction to Random Variate Generation.) Unif(0,1) PRNs can be used to generate which of the following random entities?

- a. Exp(λ) random variates Via Inverse Transform
- b. Nor(0,1) random variates A-R Method
- c. Triangular random variates Via Inverse Transform
- d. Bern(p) random variates Inverse Transform
- e. Nonhomogeneous Poisson processes AR Method
- f. All of the above --- and just about anything else!

Question 2

1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) If X is an Exp(λ) random variable with c.d.f. $F(x) = 1 - e^{-\lambda x}$, what's the distribution of the random variable $\underbrace{1 - e^{-\lambda X}}_{F(X)}$?

Inverse Transform Theorem

Let X be a cts random variable with c.d.f. $F(x)$.

Then $F(X) \sim U(0,1)$

- a. Unif(0,1)
- b. Nor(0,1)
- c. Triangular
- d. Exp(λ)

e. None of the above

Question 3

1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) If U is a Unif(0,1) random variable, what's the distribution of $-\frac{1}{\lambda}\ln(U)$?

a. Unif(0,1)

$F(X) = U \Rightarrow X = F^{-1}(U)$

b. Nor(0,1)

$X = -\frac{1}{\lambda}\ln U \Rightarrow -\lambda X = \ln U$

c. Triangular

$\Rightarrow U = e^{-\lambda X}$

d. Exp(λ)

Where it is also true that $1 - U = e^{-\lambda X}$

e. None of the above

$$\underbrace{1 - e^{-\lambda X}}_{\text{cdf for Exp}(\lambda)} = U$$

Question 4

1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) Suppose that $U_1, U_2, \ldots, U_{5000}$ are i.i.d. Unif(0,1) random variables. Using Excel (or your favorite programming language), simulate $X_i = -\ln(U_i)$ for $i = 1, 2, \ldots, 5000.$ Draw a histogram of the 5000 numbers. What p.d.f. does the histogram look like?

a. Uniform

b. Normal

c. Triangular

d. Exponential

e. Bernoulli

Question 5

1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) Suppose the c.d.f. of X is $F(x) = x^3/8$, $0 \le x \le 2$. Develop a generator for X and demonstrate with $U = 0.54$.

$F(X) = U \Rightarrow X^3/8 = U$

$X^3 = 8U \Rightarrow X = 2\sqrt[3]{U} = 2U^{1/3}$

$U = 0.54 \Rightarrow X = 2U^{1/3} = 1.629$

- a. $X = U^3/8 = 0.0197$
- b. $X = 8U^3 = 1.260$
- c. $X = 2U^{1/3} = 1.629$ ✓
- d. $X = 4U^{1/3} = 3.257$
- e. $X = 8U^{1/3} = 6.515$

Question 6

1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) If X is a Nor(0,1) random variate, and $\Phi(x)$ is the Nor(0,1) c.d.f., what is the distribution of $\Phi(X)$?

$X \sim \text{Nor}(0,1)$ and $\Phi(x)$ is Nor(0,1) c.d.f.

$\Phi(x)$ maps values of x to interval $[0,1]$

$\Phi : x \longrightarrow [0,1]$

$\Phi(X)$ effectively gives the probability that a standard normal variable is less than or equal to X.

- a. Uniform ✓
- b. Normal
- c. Triangular
- d. Exponential
- e. Bernoulli

Question 7

1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) If U is a Unif(0,1) random variate, and $\Phi(x)$ is the Nor(0,1) c.d.f., what is the distribution of $2\Phi^{-1}(U) + 3$?

If $Z \sim \text{Nor}(0,1)$ and you want $X \sim \text{Nor}(\mu, \sigma^2)$

take $X \rightarrow \mu + \sigma Z$

$2\Phi^{-1}(u) + 3$ means

$\sigma = 2$, $\mu = 3$

- a. Unif(0,1)
- b. Unif(3,2)

c. Nor(0,1)

$X \sim$ Nor(3,4)

d. Nor(3,2)

e. Nor(3,4)

Question 8

1 pts

(Lesson 7.4: Inverse Transform --- Discrete Examples.) How would you simulate the sum of two 6-sided dice tosses? (Note that $\lceil \cdot \rceil$ is the round-up function; and all of the U's denote PRNs.)

a. $12U$

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b. $\lceil 12U \rceil$

c. $6U_1 + 6U_2$

d. $\lceil 6U_1 \rceil + \lceil 6U_2 \rceil$

e. None of the above

Question 9

1 pts

(Lesson 7.4: Inverse Transform --- Discrete Examples.) If U is Unif(0,1), how can we simulate a Geom(0.6) random variate?

p

a. $\lceil \ln(U)/\ln(0.4) \rceil$

$\left\lceil \frac{\ln(1-u)}{\ln(1-p)} \right\rceil \sim \left\lceil \frac{\ln u}{\ln(1-p)} \right\rceil$

b. $\lceil \ln(1-U)/\ln(0.4) \rceil$

c. $\lceil \ln(U)/\ln(0.6) \rceil$

d. $\lceil \ln(1-U)/\ln(0.6) \rceil$

e. Both (a) and (b)

f. Both (c) and (d)

Question 10

1 pts

(Lesson 7.6: Convolution.) Suppose that U and V are PRNs. Let $X = U + V$. Simulate this 5000 times, and draw a histogram of the 5000 numbers. What p.d.f. does the histogram look like?

- a. Uniform
- b. Normal
- c. Triangular
- d. Exponential
- e. Bernoulli

Question 11

"Desert Island" $Y = \sum_{i=1}^{n} U_i$

1 pts

$Y \approx Nor\left(\frac{n}{2}, \frac{n}{12}\right)$

(Lesson 7.6: Convolution.) Suppose that $U_1, U_2, \ldots, U_{24}$ are i.i.d. PRNs. What is the approximate distribution of $X = 5 + 3\sum_{i=1}^{24} U_i$?

Using "desert island" generator

$Y = \sum_{i=1}^{24} U_i \Rightarrow Y \sim Nor\left(\frac{24}{2}, \frac{24}{12}\right)$

$Y \sim Nor(12, 2)$

$5 + 3Y$ where $Y \sim Nor(\underset{\mu}{12}, \underset{\sigma^2}{2})$

new mean: $5 + 3(12) = 41$

new var: $3^2(2) = 18$

- a. Uniform
- b. Nor(0,1)
- c. Nor(5,1)
- d. Nor(12,2)
- e. Nor(41,6)
- f. Nor(41,18)

Question 12

1 pts

(Lesson 7.6: Convolution.) If U_1, U_2, U_3 are PRNs, what's the distribution of $-2\ln(U_1^2 \underbrace{(1-U_2)^2}_{U_2^2} U_3^2)$? $Y \sim \text{Erlang}_n(\lambda)$

$Y = -\frac{1}{\lambda} \ln\left(\prod_{i=1}^{n} U_i\right)$

$-2\ln\left(U_1^2 U_2^2 U_3^2\right) = -2\ln(U_1 U_2 U_3)^2$

$= -4\ln\left(\prod_{i=1}^{3} U_i\right)$

$-\frac{1}{\lambda} = -4 \Rightarrow \lambda = \frac{1}{4}$, $n=3$

- a. Exp(1/2)
- b. Exp(4)
- c. Erlang$_2(1/2)$
- d. Erlang$_3(1/4)$ ✓
- e. Erlang$_3(2)$

Question 13

1 pts

$t(x) \geq f(x)$

(Lesson 7.7: Acceptance-Rejection --- Intro.) In general, the majorizing function $t(x)$ is itself a p.d.f. $f(x)$.

Wrong. Because the $\int_{\mathbb{R}} t(x)\,dx$ might not be equal to 1.

Only $\int_{\mathbb{R}} h(x)\,dx = \int_{\mathbb{R}} \frac{t(x)}{c}\,dx = 1$

- True
- False ✓

Question 14

1 pts

(Lesson 7.9: Acceptance-Rejection --- Continuous Examples.) Suppose that X is a continuous RV with p.d.f. $f(x) = 30x^4(1-x)$, for $0 < x < 1$. What's a good method that you can use to generate a realization of X?

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- a. Inversion
- b. Convolution
- c. Box-Muller
- d. Acceptance-Rejection ✓
- e. Composition

Question 15

1 pts

(Lesson 7.8: Acceptance-Rejection --- Proof.) Consider the constant $c = \int_{\mathbb{R}} t(x)\, dx = 5$. On average, how many iterations (trials) will the A-R algorithm require?

The number of trials until "success" $U \leq g(Y)$ is geom$(1/c)$ so mean number of trials is c

- a. 1/5
- b. 5 ✓
- c. 10
- d. 25
- e. None of the above

Question 16

1 pts

(Lesson 7.10: Acceptance-Rejection --- Poisson Distribution.) Suppose that $U_1 = 0.65, U_2 = 0.45, U_3 = 0.82, U_4 = 0.09$, and $U_5 = 0.26$. Use our acceptance-rejection technique from class to generate $N \sim \text{Pois}(\lambda = 3.7)$. (You may not need to use all of the uniforms.)

Sample until $e^{-\lambda} = e^{-3.7} = 0.02472 > \prod_{i=1}^{n} U_i$

- a. N=0
- b. N=1
- c. N=2
- d. N=3 ✓
- e. N=4

n	U_{n+1}	$\prod_{i=1}^{n} U_i$	$< e^{-3.7}$
0	0.65	0.65	no
1	0.45	0.2925	no
2	0.82	0.23985	no
3	0.09	0.02159	yes

Not saved

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