# Week 2 Homework

ⓘ This is a preview of the published version of the quiz

Started: Jul 2 at 7:47am

# Quiz Instructions

Please answer all the questions below.

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## Question 1

**1 pts**

(Lesson 2.5: Probability Basics.) If $P(A) = P(B) = P(C) = 0.6$ and $A, B,$ and $C$ are independent, find the probability that exactly one of $A, B,$ and $C$ occurs.

- a. 0.144
- b. 0.288
- c. 0.576
- d. 0.6
- e. I'm from The University Of Georgia. Is the answer -3?

$(0.6)(0.4)(0.4) \times {}^3C_1$

$= 0.288$

## Question 2

**1 pts**

(Lesson 2.5: Probability Basics.) Toss 3 dice. What's the probability that a "4" will come up exactly twice?

- a. 5/72
- b. 1/2
- c. 13/16
- d. 1/8

$\left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) \times {}^3C_2$

$= \frac{5}{72}$

## Question 3

1 pts

(Lesson 2.7: Great Expectations.) Suppose that $X$ is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathbf{E}[X]$.

- a. -1
- b. 3
- c. 1
- d. 2.2

$0.2(-1) + 0.8(3)$

$= 2.2$

## Question 4

1 pts

(Lesson 2.7: Great Expectations.) Suppose that $X$ is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{Var}[X]$.

- a. -1
- b. 1
- c. 2.56
- d. 5.12

$\mathrm{Var}[X] = E[X^2] - (E[X])^2$

$E[X^2] = \sum_x x^2 f(x)$

$= (-1)^2(0.2) + 3^2(0.8)$

$= 7.4$

$E[X] = 2.2$

## Question 5

1 pts

(Lesson 2.7: Great Expectations.) Suppose that $X$ is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathbf{E}[3 - \frac{1}{X}]$
.

- a. 3

$E\left[3 - \frac{1}{X}\right] = \sum_x \left(3 - \frac{1}{x}\right) f(x)$

b. $\infty$

$= \left(3 - \frac{1}{-1}\right)0.2 + \left(3 - \frac{1}{3}\right)(0.8)$

c. -2

d. 44/15

$= \frac{44}{15}$

## Question 6

1 pts

(Lesson 2.7: Great Expectations.) Suppose X is a continuous random variable with p.d.f. $f(x) = 4x^3$ for $0 \leq x \leq 1$. Find $\mathrm{E}[1/X^2]$.

a. 2/3

b. 1

c. 3/2

d. 2

$E\left[\frac{1}{x^2}\right] = \int_{\mathbb{R}} \frac{1}{x^2}(4x^3)\,dx$

$= \int_0^1 4x\,dx$

$= \frac{4x^2}{2}\Big|_0^1$

## Question 7

1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose $X$ is the result of a 5-sided die toss having sides numbered $-2, -1, 0, 1, 2$. Find the probability mass function of $Y = X^2$.

| | | | | | |
|---|---|---|---|---|---|
| $X$ | -2 | -1 | 0 | 1 | 2 |
| $X^2$ | 4 | 1 | 0 | 1 | 4 |

a. $\mathrm{P}(Y = 1) = \mathrm{P}(Y = 4) = 1/2$

$X^2 : P(4) = P(1) = \frac{2}{5}$

b. $\mathrm{P}(Y = 1) = \mathrm{P}(Y = 2) = 1/2$

c. $P(Y = 0) = \frac{1}{5},\ and\ P(Y = 1) = P(Y = 4) = \frac{2}{5}$

d. $\mathrm{P}(Y = -2) = \mathrm{P}(Y = -1) = \mathrm{P}(Y = 0) = \mathrm{P}(Y = 1) = \mathrm{P}(Y = 2) = 1/5$

## Question 8

1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose $X$ is a continuous random variable with p.d.f. $f(x) = 2x$ for $0 < x < 1$. Find the p.d.f. $g(y)$ of $Y = X^2$. (This may be easier than you think.)

$f_X(x) = 2x \quad , \quad 0<x<1$

- a. $g(y) = 1$, for $0 < y < 1$
- b. $g(y) = y$, for $0 < x < 1$
- c. $g(y) = y^2$, for $-1 < y < 1$
- d. $g(y) = x^2$, for $0 < y < 1$

$Y = X^2 \Rightarrow X = \sqrt{Y} \Rightarrow \frac{dX}{dY} = \frac{1}{2\sqrt{Y}}$

$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right| = f_X(\sqrt{Y}) \cdot \left|\frac{1}{2\sqrt{Y}}\right|$

$= 2(\sqrt{Y}) \times \frac{1}{2\sqrt{Y}} = 1$

## Question 9

1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x, y) = 6x$ for $0 \le x \le y \le 1$. Find $\mathrm{P}(X < 1/2 \text{ and } Y < 1/2)$.

because $0 \le x \le y$

- a. 1
- b. 1/2
- c. 1/4
- d. 1/8

$P(X<\frac{1}{2} \text{ and } Y<\frac{1}{2}) = \int_0^{\frac{1}{2}}\int_0^{y} 6x \, dx \, dy$

$= \int_0^{\frac{1}{2}} \left[\frac{6x^2}{2}\right]_0^y dy = \int_0^{\frac{1}{2}} 3y^2 \, dy$

$= \frac{3y^3}{3}\Big|_0^{1/2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

## Question 10

CHECK

1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x, y) = 6x$ for $0 \le x \le y \le 1$. Find the marginal p.d.f. $f_X(x)$ of $X$.

- a. $6x(1 - x)$, for $0 \le x \le 1$
- b. $6x$, for $0 \le x \le 1$
- c. $6y$, for $0 \le x \le 1$
- d. $6x(1 - y)$, for $0 \le x \le 1$

$f_X(x) = \int_{\mathbb{R}} f(x,y)\, dy$

$= \int_x^1 6x \, dy = 6xy\Big|_x^1$

$= 6x - 6x^2$

$= 6x(1-x)$ //

Qn 8 alternative

pdf of $Y = X^2$ is $g(y)$

cdf of $Y$ is $P(Y \le y)$

$$= P(X^2 \le y)$$

$$= P(X \le \sqrt{y})$$

$$= \int_0^{\sqrt{y}} f(x)\,dx$$ ← pdf of X

$$= \int_0^{\sqrt{y}} 2x\,dx = \left[\frac{2x^2}{2}\right]_0^{\sqrt{y}}$$

$$= (\sqrt{y})^2 = y$$ ← cdf of Y $G(y)$

pdf of $Y$ is $\frac{d}{dy} y = 1$

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Qn 9 Alternative

$$P(X < \tfrac{1}{2} \text{ and } Y < \tfrac{1}{2}) = \int_0^{1/2} \int_x^{1/2} 6x \, dy \, dx$$

$$= \int_0^{1/2} \left[6xy\right]_x^{1/2} dx = \int_0^{1/2} \left(6x(\tfrac{1}{2}) - 6x^2\right) dx$$

$$= \int_0^{1/2} 3x - 6x^2 \, dx = \left[\frac{3x^2}{2} - \frac{6x^3}{3}\right]_0^{1/2}$$

$$= \frac{3}{2}\left(\frac{1}{2}\right)^2 - \frac{6}{3}\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

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Qn 10

$$\int_x^1 6x\,dy = \left[6xy\right]_x^1$$

$$= 6x - 6x^2$$

$$= 6(x - x^2) \text{ or } 6x(1-x)$$

## Question 11

**1 pts**

(Lesson 2.9: Jointly Distributed RVs.) YES or NO? Suppose $X$ and $Y$ have joint p.d.f. $f(x,y) = cxy/(1+x^2+y^2)$ for $0 < x < 1, 0 < y < 1$, and whatever constant $c$ makes the nasty mess integrate to 1. Are $X$ and $Y$ independent?

- a. Yes
- b. No

Not saved

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