Exam 2

Due Mar 31 at 11:59pm **Points** 100 **Questions** 40
Available Mar 15 at 8am - Mar 31 at 11:59pm **Time Limit** 120 Minutes

Submission Details:

Time:	98 minutes
Current Score:	92.5 out of 100
Kept Score:	92.5 out of 100

Instructions

This test is 120 minutes. You're allowed two cheat sheets (4 sides total). You will find tables attached on the last page of this exam (scroll to the end). All questions are worth 2.5 points.

You're allowed the following items:
- Pencil / pen and scratch paper.
- A reasonable calculator.
- Two cheat sheets (4 sides total).
- Tables. (Available at the end of test questions too)

Note that you are ***not*** allowed to use Arena (or any other software), even though I'm asking questions about it.

Good luck! I want you to make this test sorry that it ever tried to mess around with you!!!

Question 1

2.5 / 2.5 pts

Suppose X and Y are continuous random variables with joint p.d.f. $f(x,y) = 6xy^2$, $0 \leq x \leq 1, 0 \leq y \leq 1$. Find $E[-4X + 2]$.

- (a) -11/3
- (b) -7/3
- (c) -2
- Correct! (d) -2/3
- (e) 7/3

Question 2

2.5 / 2.5 pts

Suppose X and Y are discrete random variables with $E[X] = 0, E[X^2] = 100, E[Y] = 2,$ $E[Y^2] = 5,$ and $E[XY] = -2$. Find the correlation between X and Y.

- (a) -0.5
- (b) -0.4
- Correct! (c) -0.2
- (d) 0
- (e) 0.8

Question 3

2.5 / 2.5 pts

Suppose that X_1, X_2, X_3, X_4 are i.i.d. Nor(0,1). Find $Pr(X_1 + X_2 - X_3 - X_4 > -2)$.

- (a) 0.023
- (b) 0.16
- (c) 0.50
- Correct! (d) 0.84
- (e) 1.00

Question 4

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, you can use a single **CHANCE** module to route customers to the resource having the smallest queue.

- ○ True
- Correct! ◉ False

Question 5

2.5 / 2.5 pts

TRUE or FALSE? In Arena, the **Schedule** spreadsheet can be used to establish either resource capacity schedules or arrival rate schedules.

- Correct! ◉ True
- ○ False

Question 6

2.5 / 2.5 pts

In an Arena program, how would you be able to do a **DELAY** ?

- (a) From within a **PROCESS** module in the Basic Process template.
- (b) Using a **DELAY** module in the Advanced Process template.
- (c) Using a **DELAY** block in the Blocks template.
- (d) (a) and (b) only
- (e) **(a) and (b) and (c)**

Correct!

Question 7

2.5 / 2.5 pts

What random variable do you get from the Arena expression **2*NORM(3, 3)** ?

- (a) Normal with mean 6 and variance 6.
- (b) Normal with mean 6 and variance 12.
- (c) Normal with mean 6 and variance 18.
- (d) Normal with mean 6 and variance 24.

Correct!

- (e) Normal with mean 6 and variance 36.

Question 8

0 / 2.5 pts

How do you generate a discrete random variable X in Arena such that

Pr(X = 1) = 0.3, Pr(X = 2.5) = 0.5, and Pr(X = 8) = 0.2 ?

You Answered

- (a) DISC(0.3, 1, 0.5, 2.5, 0.2, 8).

Correct Answer

- (b) DISC(0.3, 1, 0.8, 2.5, 1.0, 8).
- (c) DISC(0.3, 0.5, 0.2; 1, 2.5, 8).
- (d) CONT(0.3, 0.8, 1.0; 1, 2.5, 8).
- (e) CONT(0.3, 0.8, 1.0; 1, 2.5, 8).

Question 9

2.5 / 2.5 pts

What is an equivalent alternative ARENA expression for **-2*LN(1-UNIF(0,1)) -2*LN(1-UNIF(0,1))**?

- TRIA(0, 1, 2).
- EXPO(2)
- EXPO(4)
- Correct! ERLA(2,2)
- NORM(2, 4)

Question 10

0 / 2.5 pts

TRUE or FALSE ?

In Arena, it is possible to set the maximum capacity of a queue by using a **QUEUE** block found in the **Blocks** template.

- Correct Answer — True
- You Answered — False

Question 11

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, it is possible to use more than one resource at a time in a **PROCESS** module.

Correct!

- (•) True
- ( ) False

Question 12

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, it is easy to generate a *negative* interarrival time using the **CREATE** module.

- ( ) True

Correct!

- (•) False

Question 13

2.5 / 2.5 pts

In Arena, what typically happens when you do not **RELEASE** a **SEIZE'd** resource?

Correct!

- (a) A long line forms in front of the resource.
- (b) The resource automatically adds capacity (more servers).
- (c) The resource turns off.
- (d) The simulation immediately stops.

Question 14

2.5 / 2.5 pts

TRUE or FALSE ?

An Arena **ASSIGN** module can be used to assign values to attributes *and* variables.

Correct!

- True
- False

Question 15

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, it is possible to use the *same* resource (worker) at two different workstations.

Correct!

- (•) True
- ( ) False

Question 16

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, it is possible to create *nonhomogeneous* Poisson arrivals (in which the arrival rate changes over time).

Correct!

- (•) True
- ( ) False

Question 17

2.5 / 2.5 pts

Suppose there are 5 people in the line called *joe.queue* and 3 people in the line called *tom.queue*. What is the value of the following Arena expression ?

(NQ(joe.queue) + (NQ(tom.queue) <= 10)) * (TNOW > -1)

- 3
- 4
- 5

Correct!

- (selected) 6
- 8

Question 18

2.5 / 2.5 pts

TRUE or FALSE ?

The Arena Call Center model discussed in class featured certain servers who would occasionally call back their customers.

Correct!

- (selected) True
- False

Question 19

2.5 / 2.5 pts

TRUE or FALSE ?

The Arena Inventory model discussed in class used “fake customers” to periodically make inventory ordering decisions.

Correct!

- (•) True
- ( ) False

Question 20

2.5 / 2.5 pts

TRUE or FALSE ?

In Arena, it is possible to pre-assign customers a sequence of stations to visit, and it is even possible for the sequence to have repeat visits at certain stations, and it is even possible to have different service-time distributions at those repeat stations!

Correct!

- (•) True
- ( ) False

Question 21

2.5 / 2.5 pts

Consider the linear congruential generator $X_{i+1} = (5X_i + 3) mod(12)$. Using $X_0 = 0,$ calculate the 48th integer X_{48}.

Correct!

- (selected) 0
- 2
- 3
- 6
- 9

Question 22

2.5 / 2.5 pts

Regarding Linear Congruential Generators (LCG's), select the statement that is **correct**.

- (a) The midsquare generator is an example of a good LCG.

Correct!

- (selected) (b) Some popular LCG's have cycle lengths that are over 2^{100}.
- (c) It is not possible to test LCGs for independence and uniformity.
- (d) Most LCG's in use produce numbers that are truly iid Unif(0,1).
- (e) The RANDU generator is an example of a good LCG.

Question 23

0 / 2.5 pts

Consider our desert island generator $X_{i+1} = 16807X_i \ mod(2^{31} - 1)$. If $X_0 = 123456789$, find the value of the *second PRN*, U_2.

- 0.184
- You Answered — 0.218
- 0.528
- 0.622
- Correct Answer — 0.956

Question 24

2.5 / 2.5 pts

Suppose that a Tausworthe generator gave you the series of six bits 110111. If you use all of these bits, what Unif(0,1) random number would that translate to ?

- 0.370
- 0.609
- 0.728
- Correct! — 0.859
- 0.984

Question 25

2.5 / 2.5 pts

Consider the following sequence of PRN's.

0.98 0.95 0.23 0.17 0.12 0.03 0.47 0.83 0.63
0.72 0.53 0.52

How many runs ***above and below the mean*** **(0.5)** does this sequence have?

(a) 2

Correct!

(b) 3

(c) 4

(d) 5

(e) 6

Question 26

2.5 / 2.5 pts

Suppose U_1, U_2, U_3, U_4 are i.i.d. Unif(0,1).
What is the probability that we will see exactly 1 run up and down ?

- (a) 1/24
- Correct! (b) 1/12
- (c) 1/4
- (d) 1/3
- (e) 1/2
- (f) None of the above

Question 27

2.5 / 2.5 pts

Suppose that we have a sequence of n = 100 PRN's, and we observe 70 runs *up and down*.
Using $\alpha = 0.01$, do we ACCEPT (i.e., fail to reject)
or REJECT the null hypothesis of independence ?

- Correct! ACCEPT null hypothesis
- REJECT null hypothesis

Question 28 2.5 / 2.5 pts

We are testing a certain COVID vaccine. Our null hypothesis is the assumption that the vaccine is safe. If we reject this null hypothesis but the medication is actually safe, what type of error have we committed ?

Correct!

- (•) Type I Error
- ( ) Type II Error
- Type III Error
- ( )

Question 29 2.5 / 2.5 pts

Suppose we sample 1000 PRN's and we wish to conduct a χ^2 goodness-of-fit test at level $\alpha = 0.01$ of the hypothesis that the numbers are Unif(0,1). Here are the results, divided into equal-probability intervals under H_0, as follows:

interval	O_i
$[0.00, 0.20]$	210
$(0.20, 0.40]$	225
$(0.40, 0.60]$	155
$(0.60, 0.80]$	???
$(0.80, 1.00]$	180

Use the tabled results to find the value of the g-o-f statistic, χ_0^2 .

- ( ) 12.25

Correct!

- (•) 20.25
- ( ) 40.50
- ( ) 81
- ( ) 230

Question 30

2.5 / 2.5 pts

Suppose you are doing a χ^2 goodness-of-fit test with level of significance α = 0.05 and k =10 cells. Specifically, you are testing to see if your data is Unif(0,1); thus, there are no parameters to estimate, and so your g-o-f statistic χ_0^2 has k-1 degrees of freedom. If it happens that χ_0^2 = 120.0, do we ACCEPT (i.e., fail to reject) or REJECT the null hypothesis of uniformity?

- ACCEPT
- Correct! REJECT

Question 31

2.5 / 2.5 pts

Suppose X is a continuous random variable with p.d.f. $f(x) = 3x^2$ for $0 \leq x \leq 1$. Let U be a Unif(0,1) random variable. Which of the following functions of U will give you a realization of X?

- Correct! (a) $X = U^{1/3}$
- (b) $X = U^{1/2}$
- (c) $X = (1/3)U^{1/3}$
- (d) $X = (1/3)U^{1/2}$
- (e) $X = 3U^2$

Question 32

2.5 / 2.5 pts

Let's toss a pair of dice and look at the sum. The number of trials X it takes until I see a sum of 7 is obviously a *Geom(p)* random variable. Use the PRN $U = 0.5$ to generate X via inverse transform.

- (a) 3
- Correct! (b) 4
- (c) 6
- (d) 12
- (e) 24

Question 33

2.5 / 2.5 pts

Consider 24 PRNs, $U_1, U_2, \ldots, U_{24}$, and suppose that the average of those observations is $\bar{U} = \sum_{i=1}^{24} U_i/24 = 0.7$.
Use these PRNs to generate a single approximately Nor(0,1) random variate via our "desert island" technique.

- (a) -3.394
- (b) -1.517
- (c) 0.231
- (d) 1.96
- Correct! (e) 3.394

Question 34

2.5 / 2.5 pts

Name That Distribution!

If U_1 and U_2 are i.i.d. Unif(0,1), what's the distribution of $2(U_1 - U_2)$?

- (a) Unif(-2, 2)
- (b) Unif(0, 4)
- (c) Tria(-1, 0, 1)

Correct!

- (d) Tria(-2, 0, 2)
- (e) Tria(0, 2, 4)

Question 35

2.5 / 2.5 pts

Name That Distribution !

If X and Y are i.i.d. Nor(0, 1), with c.d.f. $\Phi(z)$, what's the distribution of $\Phi(X) + \Phi(Y)$?

- Unif(0, 1)
- Unif(0, 2)
- Nor(0, 2)
- Nor(0, 4)

Correct!

- Tria(0, 1, 2)

Question 36

2.5 / 2.5 pts

From the list of convolutions below, select the statement that is *incorrect*.

- The sum of *k* i.i.d. Bern(*p*) RVs has a Bin(*k*, *p*) distribution.
- The sum of 5 i.i.d. Exp(3) RVs has an Erlang distribution with mean 5/3.
- The sum of 30 i.i.d. Uniform RVs is approximately normal.
- The sum of 100 i.i.d. Exp(3) RVs is approximately normal.

Correct!

- The sum of 100 i.i.d. Cauchy RVs is approximately normal.

Question 37

2.5 / 2.5 pts

If $U_1, U_2,$ and U_3 are PRN's and $\Phi(x)$ is the standard normal c.d.f., find $Pr\left(2\Phi^{-1}(U_1) - \Phi^{-1}(U_2) + \Phi^{-1}(U_3) < 0\right)$.

- 0.16
- 0.24

Correct!

- 0.5
- 0.76
- 0.84

Question 38 2.5 / 2.5 pts

Regarding the Acceptance-Rejection RV generation method, select the incorrect statement:

(a) A-R attempts to gain efficiency by generating an easy "surrogate" random variable that is somewhat similar to the actual more-complicated random variable of interest.

(b) A-R can often be used even when a continuous RV doesn't have a closed-form inverse.

Correct!

(c) The Box-Muller method is an A-R method.

(d) The number of sampling cycles required by A-R is a geometric RV.

(e) The closer an A-R method's majorizing function is to the p.d.f. of interest, the better.

Question 39

2.5 / 2.5 pts

Suppose $U_1 = 0.12, U_2 = 0.93, U_3 = 0.50, U_4 = 0.11,$ and $U_5 = 0.49$.
Use these values and apply the Acceptance-Rejection technique to generate a realization of a $\text{Pois}(\lambda = 1.5)$ random variate. (You may not need to use all of the PRN's.)

Correct!

- (a) N = 0
- (b) N = 1
- (c) N = 2
- (d) N = 3
- (e) N = 4

Question 40

2.5 / 2.5 pts

If U_1 and U_2 are i.i.d. Unif(0,1), find the distribution of

$$X = 2 - \sqrt{-2ln(U_1)}\,[cos(2\pi U_2) + sin(2\pi U_2)]$$

- (a) Nor(0, 2)
- (b) Nor(0, 4)
- (c) Nor(2, 1)

Correct!

- (d) Nor(2, 2)
- (e) Nor(2, 4)