Week 4 Homework

ⓘ This is a preview of the published version of the quiz

Started: Jul 2 at 7:48am

Quiz Instructions

Question 1 — 1 pts

(Lesson 3.1: Solving a Differential Equation.) Suppose that $f(x) = e^{2x}$. We know that if h is small, then

$f'(x) \approx \frac{f(x+h)-f(x)}{h}$.

Using this expression with $h = 0.01$, find an approximate value for $f'(1)$.

$$\frac{e^{2(1+0.01)} - e^{2(1)}}{0.01} = \frac{e^{2(1.01)} - e^{2}}{0.01} = 14.926$$

- a. 1
- b. 2.72
- c. 7.38
- ✓ d. 14.93

Question 2 — 1 pts

(Lesson 3.1: Solving a Differential Equation.) Suppose that $f(x) = e^{2x}$. What is the actual value of $f'(1)$?

$f'(x) = 2e^{2x}$

- a. 1
- b. $e \approx 2.72$
- c. $e^2 \approx 7.39$
- ✓ d. $2e^2 \approx 14.78$
- e. 14.93

Question 3

1 pts

(Lesson 3.1: Solving a Differential Equation.) Consider the differential equation $f'(x) = (x+1)f(x)$ with $f(0) = 1$. What is the exact formula for $f(x)$?

$\frac{df}{dx} = (x+1)f \Rightarrow \frac{df}{f} = (x+1)dx$

$\int \frac{1}{f} df = \int (x+1) dx$

$\Rightarrow \ln f = \frac{1}{2}x^2 + x + c$

$f = c_1 e^{\frac{x^2}{2}+x}$ } $c_1 = e^c$

$x=0, \; f(x) = 1$

$1 = c_1$

$f = e^{\frac{x^2}{2}+x}$

- a. $f(x) = e^x$
- b. $f(x) = e^{2x}$
- c. $f(x) = \exp\{\frac{x^2}{2} + x\}$
- d. $f(x) = \exp\{x^2 + 2x\}$

Question 4

CHECK

1 pts

$f = e^{\frac{x^2}{2}+x}$

(Lesson 3.1: Solving Differential Equations.) Consider the differential equation $f'(x) = (x+1)f(x)$ with $f(0) = 1$. Solve for $f(0.20)$ using Euler's approximation method with increment $h = 0.01$ for $x \in [0, 0.20]$.

$\hookrightarrow i = 20$

$f'(x) = (x+1)f(x)$

$f(x+h) \approx f(x) + hf'(x)$

$= f(x) + h(x+1)f(x)$

$= (1+hx+h)f(x)$

$f(x+2h) = f((x+h)+h) = (1+hx+h^2+h)f(x)$

$f(x+3h) = f((x+h)+2h) = (1+hx+h^2+h^2+h)f(x)$

$= (1+hx+2h^2+h)f(x)$

$f(0+20(0.01)) = (1+0.01(0)+(20-1)(0.01)^2 + 0.01)(1)$

$= 1.0119$

- a. $f(0.20) \approx 0.0$
- b. $f(0.20) \approx 1.0$
- c. $f(0.20) \approx 1.24$
- d. $f(0.20) \approx 2.49$

Question 5

1 pts

$b=3, \; a=1$

(Lesson 3.2: Monte Carlo Integration.) Suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\, dx$. If $U_1, U_2, \ldots, U_n$ are i.i.d. Unif(0,1)'s, what's a good approximation $\bar{I}_n$ for I?

$I_i = (3-1)\, g(1+(3-1)U_i)$

Qn 4 $\quad f'(x) = (x+1)f(x) \quad$ from Qn 3 $\quad f(x) = e^{\frac{x^2}{2}+x}$

$f(0)=1$

straight up I should subst $x = 0.2$ into $f(x)$

$$f(0.2) = e^{\frac{0.2^2}{2}+0.2} = 1.246 \simeq 1.24$$

Using Euler's method $\quad$ note: $f'(x) = (x+1)f(x)$

$$f(x+h) \simeq f(x) + hf'(x)$$

$$= f(x) + h(x+1)f(x) \quad \left[\text{because } f'(x) = (x+1)f(x)\right]$$

$$= f(x)[1 + h(x+1)]$$

Given $f(0)=1$ ie $x=0$ and $f(x)=1$

x	$f(x)$ approx
0	$f(0) = 1$
0.01	$f(0+h) = f(0)[1+0.01(0+1)]$ $= 1[1.01] = 1.01$
0.02	$f(0+2h) = f(0)[1+0.02(0+1)]$ $= 1[1+0.02]$ $= 1.02$
⋮ 0.20	$f(0.20) = f(0+20h)$ $= f(0)[1+0.2(0+1)]$ $= 1.2$

KIV

$= 2\, g(1+2U_i)$

a. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$ $\quad = 2\,\frac{1}{2+2U_i} = \frac{2}{2+2U_i} = \frac{1}{1+U_i}$

b. $\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$

c. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+2U_i}$ $\quad \bar{I}_n = \frac{1}{n}\sum_{i=1}^{n} I_i = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$

Alternative

d. $\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+2U_i}$ $\quad \bar{I}_n = \frac{b-a}{n}\sum_{i=1}^{n} g(a+(b-a)U_i) = \frac{3-1}{n}\sum_{i=1}^{n} g(1+(3-1)U_i)$

e. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+3U_i}$ $\quad = \frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+1+2U_i} = \frac{2}{n}\sum_{i=1}^{n}\frac{1}{2+2U_i} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$

Question 6

1 pts

(Lesson 3.2: Monte Carlo Integration.) Again suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\,dx$. You may have recently discovered that the MC estimator is of the form $\bar{I}_n = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$. $\bar{I}_n = \frac{3-1}{n}\sum_{i=1}^{n}\frac{1}{1+1+2U_i}$

Estimate the integral I by calculating $\bar{I}_n$ with the following 4 uniforms: $= \frac{2}{n}\sum_{i=1}^{n}\frac{1}{2+2U_i}$

$$U_1 = 0.3 \qquad U_2 = 0.9 \qquad U_3 = 0.2 \qquad U_4 = 0.7$$

a. 0 $\quad \frac{1}{4}\sum_{i=1}^{4}\frac{1}{1+U_i} = \frac{1}{4}\left(\frac{1}{1.3}+\frac{1}{1.9}+\frac{1}{1.2}+\frac{1}{1.7}\right)$

b. 0.2 $\quad = \frac{1}{4}(2.717115)$

c. 0.321 $\quad = 0.679$

d. 0.679

e. 0.8

Question 7

1 pts

(Lesson 3.2: Monte Carlo Integration.) Yet again suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\,dx$. What is the *exact* value of I?

a. 0.197 $\quad \int_1^3 \frac{1}{1+x}\,dx = \left[\ln(1+x)\right]_1^3$

b. 0.693

$= \ln 4 - \ln 2$

$= 0.693$

c. 1.386

d. 2.773

Question 8

1 pts

(Lesson 3.3: Making Some π.) Inscribe a circle in a unit square and toss $n = 1000$ random darts at the square. Suppose that 760 of those darts land in the circle. Using the technology developed in class, what is the resulting estimate for π?

$$\frac{\pi\left(\frac{1}{2}\right)^2}{1} = \frac{760}{1000}$$

$$\frac{\pi}{4} = 0.76 \Rightarrow \pi = 3.04$$

a. π

b. 4.0 (UGA answer)

c. 3.2

d. 3.04

e. 3.12

Arrive	Served At	Service Time	leave At
5	5	4	9
6	13	4	17
8	9	4	13

Question 9

1 pts

(Lesson 3.4: Single-Server Queue.) Consider a single-server Q with *LIFO (last-in-first-out)* services. Suppose that three customers show up at times 5, 6, and 8, and that they all have service times of 4. When does customer 2 leave the system?

	Arrive	ServeTime	leave Time
C_1	5	5	9
C_2	6	9	13
C_3	8	13	17

a. 3

b. 9

c. 13

d. 17

e. 19

Question 10 — 1 pts

$d=10,\ s=4,\ S=10,\ K=2,\ c=4,\ h=1,\ p=2$

(Lesson 3.5: (s, S) Inventory Model.) Consider our numerical example from the lesson. What would the third day's total profits have been if we had used a (4,10) policy instead of a (3,10)?

- a. -22
- b. -13
- c. 44
- d. 45
- e. 70

Day i	begin stock	D_i	End Inventory I_i	End Order Z_i	Sales Rev	Order Cost	Hold Cost	Penalty Cost	TOTAL REV
1	10	5	$10-5=5$	0	$d\times5=50$	0	$h\times5=5$	0	$50-5=45$
2	5	2	$5-2=3$	$10-3=7$	$d\times2=20$	$K+c(7)=30$	$h\times3=3$	0	$20-30-3=-13$
3	10	8	$10-8=2$	$10-2=8$	$d\times8=80$	$K+c(8)=34$	$h\times2=2$	0	$80-34-2=44$

Question 11 — 1 pts

CDF of $\exp(\lambda)$

$\boxed{1-e^{-\lambda x}} = U$

$1-U = e^{-\lambda x}$

$\ln(1-u) = -\lambda x \ln e$

$x = -\frac{1}{\lambda}\ln(1-u)$

(Lesson 3.6: Simulating Random Variables.) If U is a Unif(0,1) random number, what is the distribution of $-0.5\ell n(U)$?

u & $1-u$ both Uni(0,1)

$-\frac{1}{\lambda}\ln(1-u) \sim \text{Exp}(\lambda)$

$-\frac{1}{\lambda}\ln(u) \sim \text{Exp}(\lambda)$

- a. Who knows?
- b. Exp(2)
- c. Exp(1/2)
- d. Exp(-2)
- e. Exp$(-1/2)$

$-\frac{1}{\lambda} = -0.5$

$\lambda = \frac{1}{0.5} = 2$

Exp(2)

Question 12 — 1 pts

(Lesson 3.6: Simulating Random Variables.) If U_1 and U_2 are i.i.d. Unif(0,1) random variables, what is the distribution of $U_1 + U_2$? Hints: (i) I may have mentioned this in class at some point; (ii) You may be able to reason this out by looking at the distribution of the sum of two dice tosses; or (iii) You can use something like Excel to simulate $U_1 + U_2$ many times and make a histogram of the results.

- a. Unif(0,2)
- b. Normal
- c. Exponential
- d. Triangular

Question 13

1 pts

(Lesson 3.7: Spreadsheet Simulation.) I stole this problem from the Banks, Carson, Nelson and Nicol text (5th edition). Expenses for Joey's college attendance next year are as follows (in $):

Tuition = 8400
Dormitory = 5400
Meals $\sim$ Unif(900,1350)
Entertainment $\sim$ Unif(600,1200)
Transportation $\sim$ Unif(200,600)
Books $\sim$ Unif(400,800)

Here are the income streams the student has for next year:
Scholarship = 3000
Parents = 4000
Waiting Tables $\sim$ Unif(3000,5000)
Library Job $\sim$ Unif(2000,3000)

Use Monte Carlo simulation to estimate the expected value of the loan that will be needed to enable Joey to go to college next year.

- a. $2500
- b. $3250
- c. $3325
- d. $3450

refer to excel &
colab

e. $4000

Not saved

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