A robust approach to food aid supply chain optimisation for minimising nutrient deficit

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1 Introduction

After 13 years of conflict, Syria faces a dire food crisis, being the sixth most food-insecure country in the world (Action Against Hunger, 2024). Around 14 million Syrians are food insecure, driven by war, economic collapse, and rising inflation (Action Against Hunger, 2024). Basic necessities have become unaffordable for many, leaving millions at risk of starvation. The ongoing crisis demands global attention as humanitarian efforts work to address the growing need. However, with limited resources, organisations need to find the most cost-effective solutions to maximise their ability to provide relief.

de Moor et al. (2024) sought to solve this issue by presenting multiple linear programming models to determine the most cost-effective method of managing food aid supply chains in unstable market environments. An optimised supply chain management plan is critical in crises such as Syria, where resources are scarce and the effective distribution of aid can hugely benefit those that require it.

This report aims to implement three of the models outlined by de Moor et al. (2024), while improving on them by correcting an issue whereby each beneficiary would always receive a specified nutrient deficit, regardless of food availability.

2 Models

2.1 Nominal Optimisation (NO)

The Nominal model is a linear optimisation model without uncertainty, originally described in Peters et al. (2022), and forms the basis for the robust approaches. The purpose of this model is to fulfill demand for food aid at a minimum cost and thus increase the efficiency of the operation. It is formulated as a network flow model where a node can either be a procurement (supplier), transshipment or delivery node, or some combination of the three. Transshipment nodes are responsible for controlling the flow of resources from suppliers to delivery node. Supplier nodes are delineated into three subsets, international suppliers are located outside of Syria, regional suppliers are located within Syria and can supply commodities to any connecting node within the network, and local suppliers are also in Syria but can only supply to the node in which they are located; with each node having a predefined set of arcs connecting it to its neighbouring transshipment and delivery nodes. Furthermore, we also have a set of commodities, (i.e., the different foods available for purchase), nutrients (i.e., the different vitamins and dietary components necessary for sustaining health) and the time periods over which the model will optimise. The sets used in the each model are described in Table 1.

Set	Description	Definition	Cardinality
$\mathcal{N}$	All nodes		N
$\mathcal{N_S}$	All suppliers	$\subset \mathcal{N}$	N_S
$\mathcal{N_{SI}}$	International suppliers	$\subset \mathcal{N_S}$	N_{SI}
$\mathcal{N_{SR}}$	Regional suppliers	$\subset \mathcal{N_S}$	N_{SR}
$\mathcal{N_{SL}}$	Local suppliers	$\subset \mathcal{N_S}$	N_{SL}
$\mathcal{N_T}$	Transshipment nodes	$\subset \mathcal{N}$	N_T
$\mathcal{N_D}$	Delivery nodes	$\subset \mathcal{N}$	N_D
$\mathcal{N_{ST}}$	$\mathcal{N_S} \cup \mathcal{N_T}$	$\subset \mathcal{N}$	N_{ST}
$\mathcal{N_{TD}}$	$\mathcal{N_T} \cup \mathcal{N_D}$	$\subset \mathcal{N}$	N_{TD}
$\mathcal{K}$	Commodities		K
$\mathcal{L}$	Nutrients		L
$\mathcal{T}$	Time periods		T

Table 1: Set notation

The required capacities, costs and other data are defined in Table 2 below, and the decision variables are defined as shown in Table 3.

Data	Description
c^H_{it}	Handling capacity (mt) at node $i \in \mathcal{N_{TD}}$ in time period $t \in \mathcal{T}$
c^T_{ijt}	Transportation capacity (mt) from node $i \in \mathcal{N_{ST}}$ to node $j \in \mathcal{N_{TD}}$ in time period $t \in \mathcal{T}$
c^P_{ikt}	Procurement capacity (mt) for commodity $k \in \mathcal{K}$ at node $i \in \mathcal{N_S}$ in time period $t \in \mathcal{T}$
p^P_{ikt}	Cost (USD/mt) of procuring 1 mt of commodity $k \in \mathcal{K}$ at supply node $i \in \mathcal{N_S}$ in time period $t \in \mathcal{T}$
p^T_{ijkt}	Cost (USD/mt) of moving 1 mt of commodity $k \in \mathcal{K}$ from node $i \in \mathcal{N_{ST}}$ to node $j \in \mathcal{N_{ST}}$ in time period $t \in \mathcal{T}$
p^H_i	Cost (USD/mt) of handling 1 mt of commodity $k \in \mathcal{K}$ at node $i \in \mathcal{N_{TD}}$
p^S_{it}	Cost (USD/mt) of storing 1 mt of resources at node $i \in \mathcal{N_T}$ in time period $t \in \mathcal{T}$
d_{it}	Number of beneficiaries at delivery node $i \in \mathcal{N_D}$ in time period $t \in \mathcal{T}$
β_{kl}	Nutritional value per 100g of commodity $k \in \mathcal{K}$ for each nutrient $l \in \mathcal{L}$
η_l	Nutritional requirement (g/beneficiary/day) of each nutrient $l \in \mathcal{L}$
γ_{ij}	Duration (days) for shipping from node $i \in \mathcal{N_{ST}}$ to $j \in \mathcal{N_{TD}}$
δ_t	Number of days in time period $t \in \mathcal{T}$
τ_{ij}	$\lfloor \frac{\gamma_{ij} + \frac{\delta_t}{2}}{\delta_t} \rfloor$. Rescaling factor for shipping duration from days to time periods
α	$10,000$. Conversion from metric tonnes (mt) to 100 grams
sf_l	Maximum shortfall in nutrient $l \in \mathcal{L}$ as a fraction of total nutrition required

Table 2: Data

Variable	Description
F_{ijkt}	Amount (mt) of commodity $k \in \mathcal{K}$ transported between node $i \in \mathcal{N_{ST}}$ and node $j \in \mathcal{N_{TD}}$ in time period $t \in \mathcal{T}$
R_{kt}	Daily ration (100g) of commodity commodity $k \in \mathcal{K}$ in time period $t \in \mathcal{T}$ to a single beneficiary
S_{lt}	Realised shortfall of nutrient $l \in \mathcal{L}$ in time period $t \in \mathcal{T}$ to all beneficiaries as a fraction of total nutrition required

Table 3: Variables

The objective of the Nominal model is to minimise total costs, i.e., to minimise the sum of procurement (PC), transportation (TC), handling (HC) and storage costs (SC), as shown in Equations 1-5.

$$\mathrm{PC} = \sum_{i\in\mathcal{N}_\mathcal{S}} \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} p^P_{ikt} F_{ijkt} \tag{1}$$

$$\mathrm{TC} = \sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} p^T_{ijkt} F_{ijkt} \tag{2}$$

$$\mathrm{HC} = \sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} p^H_{j} F_{ijkt} \tag{3}$$

$$\mathrm{SC} = \sum_{i\in\mathcal{N}_\mathcal{T}} \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} p^S_{i} F_{ijkt} \tag{4}$$

$$\min\ \mathrm{PC} + \mathrm{TC} + \mathrm{HC} + \mathrm{SC} \tag{5}$$

Subject to the following constraints:

$$\sum_{j\in\mathcal{N}_{\mathcal{TD}}} F_{ijkt} = \sum_{j\in\mathcal{N}_{\mathcal{ST}}} F_{ijk,t-\tau_{ji}} \qquad \forall i \in \mathcal{N}_\mathcal{T}, k \in \mathcal{K}, t \in \mathcal{T} \tag{6}$$

$$\sum_{i\in\mathcal{N}_{\mathcal{ST}}} \alpha F_{ijkt} = d_{jt}\delta_t R_{kt} \qquad \forall j \in \mathcal{N}_\mathcal{D}, k \in \mathcal{K}, t \in \mathcal{T} \tag{7}$$

$$\sum_{j\in\mathcal{N}_{\mathcal{TD}}} F_{ijkt} \leq c^P_{ikt} \qquad \forall i \in \mathcal{N}_\mathcal{S}, k \in \mathcal{K}, t \in \mathcal{T} \tag{8}$$

$$\sum_{k\in\mathcal{K}} F_{ijkt} \leq c^T_{ijt} \qquad \forall i \in \mathcal{N}_{\mathcal{ST}}, j \in \mathcal{N}_{\mathcal{TD}}, t \in \mathcal{T} \tag{9}$$

$$\sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{k\in\mathcal{K}} F_{ijkt} \leq c^H_{jt} \qquad \forall j \in \mathcal{N}_{\mathcal{TD}}, t \in \mathcal{T} \tag{10}$$

$$\sum_{k\in\mathcal{K}} \beta_{kl} R_{kt} \geq \eta_l(1 - S_{lt}) \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{11}$$

$$S_{lt} \leq sf_l \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{12}$$

$$F_{ijkt} \geq 0 \qquad \forall i \in \mathcal{N}_{\mathcal{ST}}, j \in \mathcal{N}_{\mathcal{TD}}, k \in \mathcal{K}, t \in \mathcal{T} \tag{13}$$

$$R_{kt} \geq 0 \qquad \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{14}$$

Where Constraint 6 balances flow across transshipment nodes. Constraint 7 ensures the flow of commodities at delivery nodes meets the demand per commodity. Constraints 8-10 are capacity constraints for the amounts procured, transported and handled respectively. Constraint 11 ensures the total nutritional value of each nutrient given in each time period meets or exceeds that nutrients required intake, unless a shortfall S_{lt} is present. Constraint 12 prevents the shortfall of each nutrient in each time period from exceeding the maximum allowed shortfall of that nutrient. Constraints 13 and 14 are non-negativity constraints for the flow F and ration R variables respectively.

2.2 Robust Optimisation(RO)

The Robust model attempts to capture uncertainty in the procurement prices of the regional and local suppliers as those are historically the most unstable (Action Against Hunger, 2024). By defining a safety parameter Ω, we were able to limit the amount of uncertainty the Robust model would account for. Then an uncertainty vector ζ is generated from the ellipsoidal uncertainty set $\mathcal{V}$ (Equation 15) which denotes a fluctuation in procurement price of a given commodity at a given time. The matrix Σ denotes the covariance matrix of procurement prices per market and is positive semi-definite.

$$\mathcal{V} = \{\zeta : \zeta^\top \Sigma^{-1} \zeta \leq \Omega^2\} \tag{15}$$

This vector is applied to all suppliers within the region that carry that commodity at that time period, as it is assumed price uncertainty is constant through the region. The new procurement prices are defined as shown in Equation 16 below, where θ denotes the Nominal value of procurement prices at international suppliers and μ denotes the Nominal value of procurement prices at local and regional suppliers. This means that pricing for all suppliers in the first time period is known, but every time period following is uncertain.

$$p^P_{ikt} = \begin{cases} \theta_{ikt} & , i \in \mathcal{N}_{\mathcal{SI}} \\ \mu_{ikt} & , i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}, t = 1 \\ \mu_{ikt} + \zeta_{m_i kt} & , i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}, t \geq 2 \end{cases} \tag{16}$$

To apply this uncertainty a new variable q is defined which encapsulates all procurement costs in regional and local suppliers with uncertainty (at $t \geq 2$) via the following constraint:

$$(\mu + A\zeta)^t F^P \leq q \qquad \forall \zeta \in \mathcal{V} \tag{17}$$

Where $\mu, F^P \in \mathbb{R}^{(N_{SR}+N_{SL})K(T-1)}$ are vectors of dimension determined by the product of the size of the sets of local and regional markets, commodities and uncertain time periods. The elements of μ are procurement prices in local and regional markets without uncertainty and elements of $\mathcal{F}^p$ are the summation of all the flow occurring from a supplier node to its neighbouring transshipment and delivery nodes for a given time and commodity:

$$F^P_{ikt} = \sum_{j \in \mathcal{N}_{\mathcal{TD}}} F_{ijkt} \qquad \forall i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}, t \in \mathcal{T} \setminus \{1\} \tag{18}$$

The transformation matrix $A \in \mathbb{R}^{(N_{SR}+N_{SL})K(T-1)\times MK(T-1)}$ is a linear transformation matrix such that $(A\zeta)_{ikt} = \zeta_{m_i kt}$, which takes the uncertainty per market and applies it to each suppliers procurement price if that supplier exists in that market. Thus Constraint 17 can be rewritten as a second order cone constraint without uncertainty by considering its LHS as a random variable with expected value $\mu^T F^P$ and standard deviation $\sqrt{(A^\top F^P)^\top \Sigma A^\top F^P}$:

$$\mu^\top F^P + \Omega\sqrt{(A^\top F^P)^\top \Sigma A^\top F^P} \leq q \tag{19}$$

Utilising the definition of q gives a new objective function of the Robust model:

$$\begin{aligned} \min\ q + &\sum_{i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \mu_{ik1} F_{ijk1} + \sum_{i \in \mathcal{N}_{\mathcal{SI}}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \theta_{ikt} F_{ijkt} \\ &+ \text{PC} + \text{TC} + \text{HC} + \text{SC} \end{aligned} \tag{20}$$

Note that while the Robust solution has multiple optimal solutions based on scenario (defined by the values of the ζ), this model determines the optimal solution to the worst case scenario.

2.3 Pareto Robust Optimisation (PRO)

The Pareto Robust formulation (Iancu & Trichakis, 2014) calculates the optimal solution to the expected scenario of market prices. First the RO model is solved for optimal objective bound q^* which is used for the following constraint:

$$q = q^* \tag{21}$$

This provides an upper bound for the flow during the worse case scenario represented in constraint 19. Then the RO is solved for objective function 5 as the expected value of market prices is equivalent to the market prices without uncertainty.

3 Implementation

All three models outlined above were implemented in Python 3.11 using the Gurobi 11 library, Gurobipy. Futhermore, for each model, we set the maximum shortfall of each nutrient to be 0.1, to allow for a maximum 10% nutrient deficit.

3.1 Data Procurement

In order to effectively use the models described in the original paper, we first needed to retrieve a sufficient data set. The primary author of de Moor et al. (2024) was able to provide us with a data set that contained market pricing information from January 2017 to October 2021. Furthermore, we were able to find the initial source of this data was the United Nations World Food Programme (WFP) website (World Food Programme, 2017-2021). The WFP releases monthly Market Price Watch Bulletins which, until June 2022, contained a summary of local and regional commodity pricing for most major population centres in Syria. However, since this data is stored in PDF files such that it cannot be directly copied, we elected not to use the extra 8 months of data available to us. This left us with a data set of 58 months worth of data to use. However, for reasons that will be explained in Section 3.3, we chose to only use the first 20 months of data. Since the transportation, handling and storage capacities as well as handling and storage costs vary significantly per Non-Government Organisation (NGO), no sufficiently reasonable data could be sourced. As such large artificial values for this data were chosen to ensure the model wasn't artificially constrained.

3.2 Nominal Optimisation (NO)

3.2.1 NO Implementation

The implementation of the Nominal model was relatively straightforward as it is a standard linear optimisation model, with the primary challenges being data procurement and formatting. It should be noted that due to the formatting of most of the constraint data, it results in the capacity constraints having a large right-hand side. However, this cannot be prevented without lowering the artificial capacities and costs, which would in turn impact the solution.

The NO is computationally efficient on the primary data set, however this decreases the more it has to rely on regional and local markets (which can occur when international costs are increased or international capacity is constrained). This is due to the increased complexity of optimal flow paths through the network, instead of having to only flow from international supplier to delivery nodes. It is however quite scalable with time, with only a small notable time difference between running on 20 and 52 time periods.

3.2.2 NO Results

The Nominal model resulted in an optimised objective function of approximately 13.71 million USD. The plan consisted of purchasing commodities mostly from the three international suppliers Amman, Beirut and Gaziantep (Figure 1). However, two of the local suppliers, Deir ex-Zur and Al-Hasakah, were used to supply their local delivery nodes.

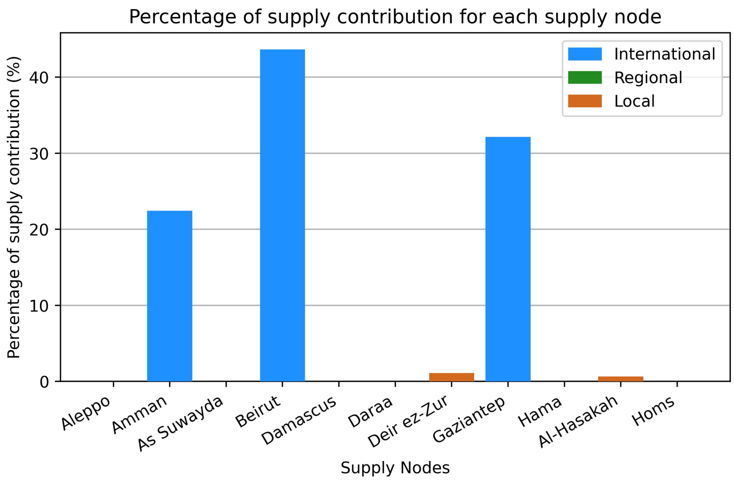

Figure 1: Bar chart showing the contribution of each supplier to the network as a percentage of the total supplier contribution for Nominal optimisation (NO) model.

The distribution of commodities purchased is shown in Figure 2, where we can see that the most commonly purchased commodity is oil. This is likely due to the high energy content of oil (885 kcal/100g), which is more than double the next highest energy content, that of sugar (400 kcal/100g). This choice by the model is likely intended to fulfill the large energy requirement (2100 kcal/100g) of each beneficiary.

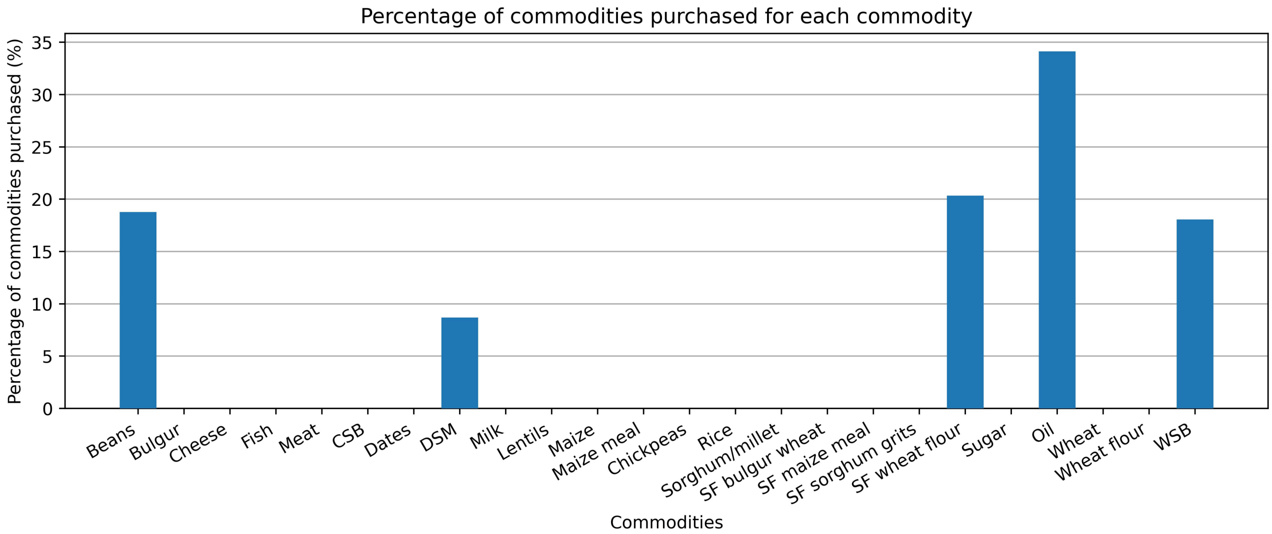

Figure 2: Bar chart showing quantity of each commodity purchased in optimised plan as a percentage of the total amount purchased for Nominal optimisation (NO) model.

It is evident from Figure 1 that there is a strong preference for international supply. With the exception of sugar and lentils, every commodity that can be purchased from local and regional suppliers is substantially more expensive than the same commodity from international suppliers. On average, local and regional markets are approximately 91% more expensive than international. Furthermore, international suppliers can also provide additional commodities that local and regional suppliers cannot, such as soya-fortified wheat flour. These may act as substitutes for local and regional commodities at potentially lower procurement prices.

By adjusting the international procurement pricing to be five times it's original amount, the supplier distribution as shown in Figure 3 was produced. As we can see, the model no longer primarily favoured the international suppliers, with a much higher contribution from local and regional suppliers. Furthermore, the optimised cost increased to approximately 50.10 million USD, highlighting the essential nature of low-cost international markets. A similar effect can be observed when increasing the international prices for both the Robust (RO) and Pareto Robust (PRO) models. As Daraa makes up for approximately 26% of the total number of beneficiaries within the aid network, it is reasonable that the local market in Daraa makes up approximately 18% of the total supplier contribution, as all of this would directly serve the beneficiaries within that node.

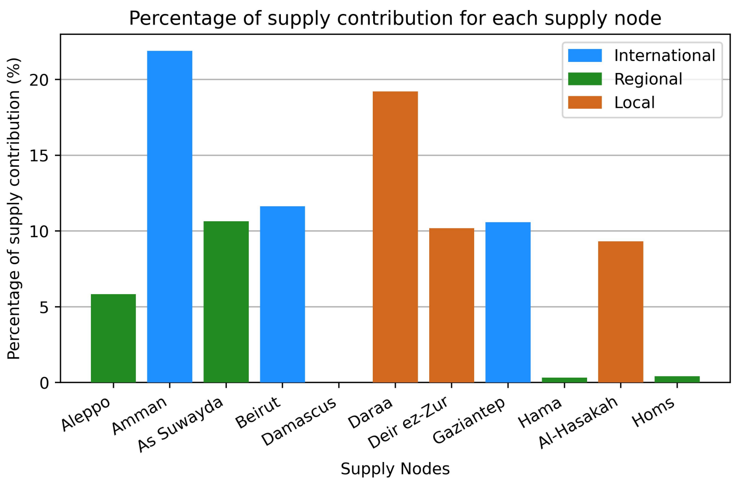

Figure 3: Bar chart showing the contribution of each supplier to the network as a percentage of the total supplier contribution for Nominal optimisation (NO) model. International procurement costs have been set to five times their original amount.

As the model is designed to minimise operating costs, the realised shortfall for each nutrient was set to its maximum allowed value. This is because a higher shortfall allows the model to purchase less commodities in order to save money. This means that every beneficiary experienced a 10% nutrient deficit in each time period. This issue is addressed in our expansion of these models in Section 4.

3.3 Robust Optimisation (RO)

3.3.1 RO Implementation

The implementation of the RO model faced three primary issues, reformatting Constraint 19 to a function in Gurobi 11, calculating covariance matrix Σ and transformation matrix A, as well as implementing the matrices and vectors from Constraint 19 in a form that could be indexed in the same manner as the Nominals data and variables.

For the implementation of Equation 19, while Gurobi 11 is capable of handling quadratic and bi linear constraints, it cannot process constraints with square roots of summations or products of Gurobi variables, as is found in the standard deviation of 2nd order cone. Thus it had to be rearranged into the following form:

$$\Omega^2(A^\top F^P)^\top \Sigma A^\top F^P \leq (q - \mu^\top F^P)^2 \tag{22}$$

In order to make it numerically easier to compute and also test, this constraint was broken up into one primary constraint with three secondary constraints:

$$\Omega^2 Z \leq \beta^2 \text{ s.t.} \tag{23}$$
$$A^\top F^P = \gamma \tag{24}$$
$$\gamma \top \Sigma \gamma = Z \tag{25}$$
$$Q - \mu^\top F^P = \beta \tag{26}$$

With new variables being introduced to store the calculated values to make them easily accessible from the model. Where $\sqrt{Z}$ represents the total standard deviation of purchased commodity prices across all time periods and $\beta - Q$ being equivalent to the expected value of total local and regional procurement costs.

The second primary challenge we faced implementing the RO was the generation and implementation of the transformation matrix A and covariance matrix Σ, due to the original paper being vague on this matter. On consultation with the original papers author, we surmised it was a sparse matrix with its index's being a linear combination of the sets of its dimension and its element being determined by:

$$A_{ikt,mkt} = 1 \text{ if } i \in m, \forall m \in M, k \in \mathcal{K}, t \in \mathcal{T} \tag{27}$$

First we attempted to implement A as a dictionary using a pair of tuples as its keys. While this allowed it to be easily formulated with our Gurobi variables, it resulted in a data structure that was impractically large, exceeding 1 gigabyte when we attempted to use over 50 time periods. While this approach might have been more viable in a more efficient programming language, we instead decided implement it as a matrix, however the issue became a problem of how to index a 2-dimensional matrix using a linear combination of indexes. Taking the product of the indices did not retain order as indexes were not unique, so we decided on the following formulation which allowed us to index over the entire matrix:

$$A[m \cdot K \cdot T + k \cdot T \cdot t,\ i \cdot K \cdot T + k' \cdot T + t']\ \forall m \in M,\ i \in NSL \cup NSR,\ t, t' \in \mathcal{T},\ k, k' \in \mathcal{K} \tag{28}$$

From our current understanding, the covariance matrix should be created from samples created from our data and a given ζ, where then the covariance matrix and zeta vectors are checked using Equation 15 to ensure they fall within the ellipsoidal uncertainty. Unfortunately all efforts to generate this matrix using the given data were unsuccessful. We were able to generate a matrix of sufficient size, however, this matrix was never positive semi-definite. As this is a required property of a covariance matrix, this can't be correct. Upon consultation we were able to obtain a small-sized covariance matrix utilised in the original paper. By expanding this small scale matrix into a block diagonal matrix of the desired dimensions, we achieved a positive semi-definite covariance matrix as required. However, we were unable to ascertain the ordering of the values required to index over them. Eventually after trialling several linear combinations, we settled on a method similar to Equation 28, which produced the most reasonable results.

One of the primary issues with the current implementation is that the number of constraints and variables scales dramatically with regards to all sets. Even for 20 time periods, the generation of the γ and Z constraint takes a non trivial amount of time, rising to around 5 minutes for 50 time periods, which is after reformatting these constraints to reduce generation time (a naive implementation took upwards of 30 minutes). This appears to be due to the constraints 24 and 25 that are directly dependent on the sizes of Σ and A, both large matrices whose dimensions scale with increases in K and T. The size of the problem coupled with the issues with the conversion of quadratic constraints to a large number of bilinear constraints (as outlined below) make the extendability of the model somewhat poor as the computation time rapidly becomes impractical, although still producing a feasible result.

The other primary issue with the RO is that it has a large number of non-convex quadratic constraints which are generally difficult to solve compared to convex (Gurobi Optimization: Variables and Constraints and Objectives, 2024). Convex constraints are those of the form:

$$x^T Q x \leq b, \tag{29}$$

$$x^T Q x \leq y^2, \tag{30}$$

$$x^T Q x \leq yk, \tag{31}$$

Such that Q is Positive Semi-Definite, x is a vector of variables, y and k are non-negative variables and b is some constant.

While Constraint 23 is of a similar form to convex Constraint 30, Gurobi pre-solve is unable to convert it into this form, both quadratic constraints are converted into a large number of bilinear constraints (nearly 30,000 for only 20 time periods). Bilinear constraints have to be solved via spatial branching where the Mixed-Integer Programming (MIP) solver computes an approximation in the neighbourhood of each node in branch and bound. This results in less efficient tree exploration and may explain why the model tends to stagnate at lower MIP gaps, since this method is less efficient for finding a global solution. To this end, running the model with a small MIP gap can dramatically increase performance which might be advisable for real world applications. This would also potentially explain why Gurobi pre solve has difficulties with the model, with it failing to convert the formulation into a more easily solvable convex form. Bilinear constraint also benefit from tight variable bounds for good convergence which unfortunately is a numerical issue that the quadratic constraints suffer from. It might be possible reformulate this constraint to a convex form via expressing Z as the product of two other variables yk , but that was unfortunately outside the scope of available resources.

3.3.2 RO Results

The Robust model resulted in an optimal cost of approximately 13.79 million USD, making it slightly more expensive than the Nominal solution. This is an expected result, as the Robust model is designed to optimised operational costs under the worst-case stochastic market scenario. This means we would expect a higher optimal value, as the model accounts for potential increases in procurement prices.

As shown in Figure 4, the Robust model resulted in a similar procurement strategy as the Nominal model. The main source of commodities was international suppliers, with a very small contribution coming from the local market within Deir ez-Zur. This is to be expected, as local and regional market uncertainty would make the model favour international suppliers, whereby avoiding most, market fluctuation.

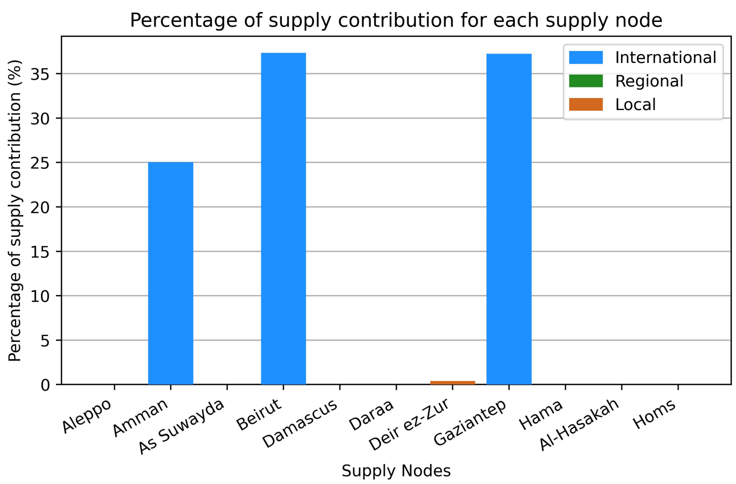

Figure 4: Bar chart showing the contribution of each supplier to the network as a percentage of the total supplier contribution for Robust optimisation (RO) model.

The distribution of commodities purchased are shown in Figure 5. This distribution is almost identical to that of the Nominal model, with the one exception being a slightly higher purchasing percentage for beans. As the solution showed a preference for international suppliers, it makes sense that commodities such as soya-fortified (SF) wheat flour and enriched dried-skim milk (DSM) make up a large proportion of the total purchasing, as opposed to their locally available counterparts (milk and wheat flour respectively). Additionally, as with the Nominal solution, the realised shortfall for each nutrient was set to its maximum allowed value.

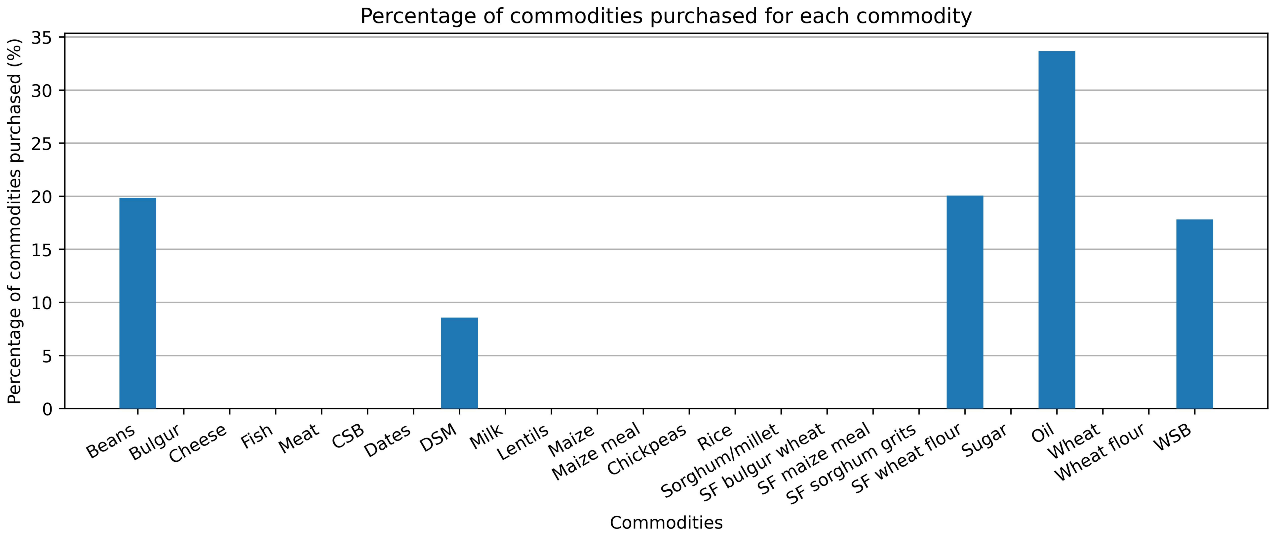

Figure 5: Bar chart showing quantity of each commodity purchased in optimised plan as a percentage of the total amount purchased for Robust optimisation (RO) model.

3.4 Pareto Robust Optimisation (PRO)

3.4.1 PRO Implementation

The implementation of the PRO was relatively simple as it expands the RO to a two-step optimisation with an additional constraint based on the RO solution. In order to ease the numerics of the upper bound constraint, a small tolerance was placed on 21:

$$q^*(1-\epsilon) <= q, \ 0 < \epsilon \ll q^* \tag{32}$$

The initial step of the PRO inherits all the numerical issues of the RO as to be expected. However because the quadratic constraints are bounded by a constant, instead of being minimised by an objective function, Gurobi is able to solve the second step as a non-convex Mixed Integer Quadratically Constrained Problem (MIQCP). This can be solved by a more efficient heuristic and converges faster as a result. This is evidenced by the second step of the PRO taking on average 34 seconds to solve to optimality for 20 time periods compared to the RO step taking on average 1160 seconds.

3.4.2 PRO Results

The Pareto Robust model resulted in an optimal cost of approximately 13.71 million USD, making it approximately the same solution as the Nominal model, and thus substantially lower than the Robust solution. This was expected as the aim of the PRO was to minimise the operational costs of the supply network in stochastic conditions for expected scenario scenario. This is contrasted with the worst-case scenario approach of the Robust model, explaining why it gave such a similar result to the Nominal.

As with the Nominal model, the Pareto Robust solution displays a strong preference for international suppliers, however the contribution from each international supplier is markedly different. The Nominal solution uses Beirut, Gaziantep and Amman in descending order of contribution. However, the Pareto Robust solution uses Amman, Beirut and Gaziantep in descending order of contribution, indicating Pareto Robust model was able to achieve a similar objective value using an alternative plan.

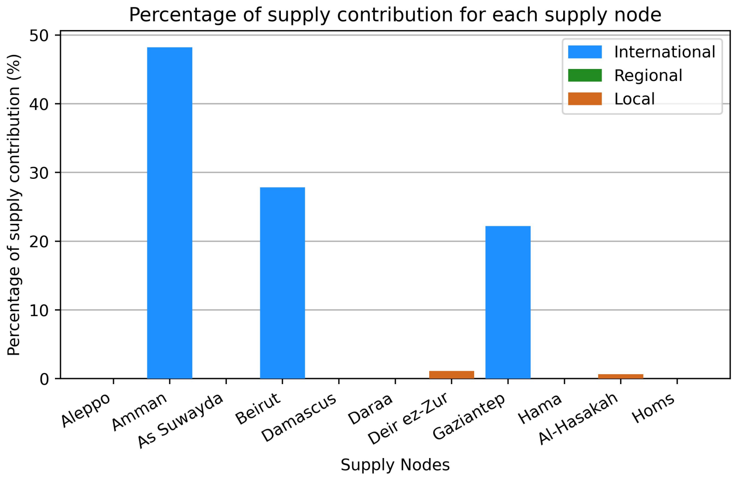

Figure 6: Bar chart showing the contribution of each supplier to the network as a percentage of the total supplier contribution for Robust optimisation (PRO) model.

Figure 7 shows the commodity purchasing distribution of the PRO solution. Furthermore, it also shows that the distribution of commodities purchased is exactly the same as that of the Nominal model. This implies that whilst the same commodities where purchased, they were purchased in different quantities from each supplier (as per the changed supplier contribution). In addition, since the procurement costs are consistent across international suppliers, the total procurement cost for both models is identical, and thus the total of transportation costs must also be equal (as we have disregarded storage and handling costs). As international suppliers are extremely limited in which nodes they can transport to (Gaziantep can only directly ship to Aleppo for example), and as their transportation costs are all different, these costs must still balance out. This is an extremely important fact, because it implies that there is more than one optimal plan to be found from this data.

As with the Nominal and Robust models, the realised shortfall for each nutrient was set to it's maximum allowed value in the optimal solution.

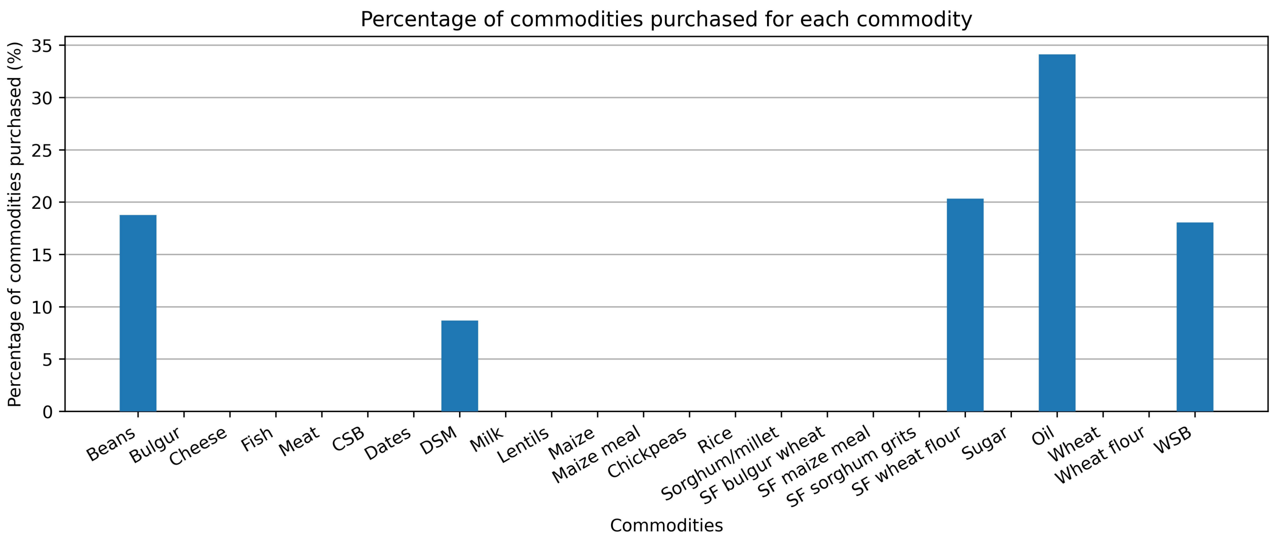

Figure 7: Bar chart showing quantity of each commodity purchased in optimised plan as a percentage of the total amount purchased for Robust optimisation (PRO) model.

4 Expansion

4.1 Issues with original models

Each model presented in de Moor et al. (2024) has an issue with the implementation of the shortfall variable S_{lt}. This variable is intended to represent the allowed daily nutrient deficit for each nutrient to each beneficiary, i.e., the fraction of each required nutrient the model can choose to not ration in order to save money.

The issue with the implementation is the lack of any constraint or cost on increasing shortfall, resulting in realised shortfall always being set to the maximum allowed value, effectively causing a constant nutrient deficit for each beneficiary. As each model is intended to minimise operational/procurement costs and thus maximise the amount of food that can be purchased over time and thus the amount of nutrients given beneficiary, this is an issue that needs to be addressed.

4.2 Our expansion

This issue could be solved by adding a penalty to the realised shortfall in the objective function, potentially representing publicity damage for the organisation implementing the solution. However incorporating a penalty for publicity damage would require additional assumptions and data that fall outside the scope of this study. Instead, we chose to implement a secondary objective function using Gurobi's multiple objective support.

Gurobi has two ways to implement multiple objectives, hierarchical and blended. The hierarchical approach assigns a priority to each objective function and optimises for the objectives in decreasing priority order, but each subsequent objective optimisation cannot degrade the previous objective value (Gurobi Optimization: Working With Multiple Objectives, 2024). Hence, it can be conceptualised as each objective being being solved for its optimal solution and then acting as a constraint for the remaining objectives. While it is possible to specific a tolerance the previous objective can be degraded by, it proved unreliable during testing.

Our initial attempt at hierarchical solution was to minimise the realised shortfall with a higher priority than the cost minimisation. This presented an issue whereby it would always minimise the realised shortfall S_{lt} to 0 for all time periods and nutrients. Effectively altering the constraint in Equation 11 to the following:

$$\sum_{k\in\mathcal{K}} \beta_{kl} R_{kt} \geq \eta_l \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{33}$$

We then reversed the priority order, which gave the same result as the original Nominal optimisation by setting the realised shortfall to the maximum allowed value. Hence, we needed another solution.

The blended approach works by creating a single linear objective from a linear combination of all objective functions, each multiplied by some weight. In the case of the Nominal solution, we have a blended objective of the form:

$$w_1(\min\ \text{NO}) + w_2(\min \sum_{l\in\mathcal{L}} \sum_{t\in\mathcal{T}} S_{lt}) \tag{34}$$

Where NO is the Nominal objective function as defined in Equations 1-5, and w_1 and w_2 are the weightings for each objective. From this point, w_1 will be referred to as primary weighting, w_2 as shortfall weighting. This model is referred to as the Blended Nominal (BNO) model. When the shortfall weighting is set to 0, it is obvious to see that this will completely disregard that minimisation and instead just result in the standard Nominal solution. This process was also used for the Robust and Pareto Robust models, creating the Blended Robust (BRO) and Blended Pareto Robust (BPRO) models.

For the each model, we ran multiple tests with varying primary weighting and shortfall weighting. After many tests, we settled on setting the primary weighting to $w_1 = 10^{-5}$, and running each model with shortfall weightings ranging from 0 to 3.5 with increments of 0.1, where a weighting of 3.5 results in $S_{lt} = 0$. This allowed us to examine the impacts of weighting shortfall penalties over a range, which in turn can help in deciding a reasonable trade-off for the organisation implementing the solution.

4.3 Blended Nominal Optimisation (BNO)

Due to the simplicity of the secondary objective, the blended objective results in very little increase in numerical complexity and for a given weightings w_1, w_2 the BRO has the same computational efficiency and scalability as the NO.

As shown in Figure 8, the distribution of each supplier contribution remains mostly consistent to the standard Nominal solution. As the shortfall weighting is increased, the main contributing suppliers are once again the international suppliers. Furthermore, we can see in Figure 9 that there is also little variation in the distribution of commodities purchased to satisfy the demands.

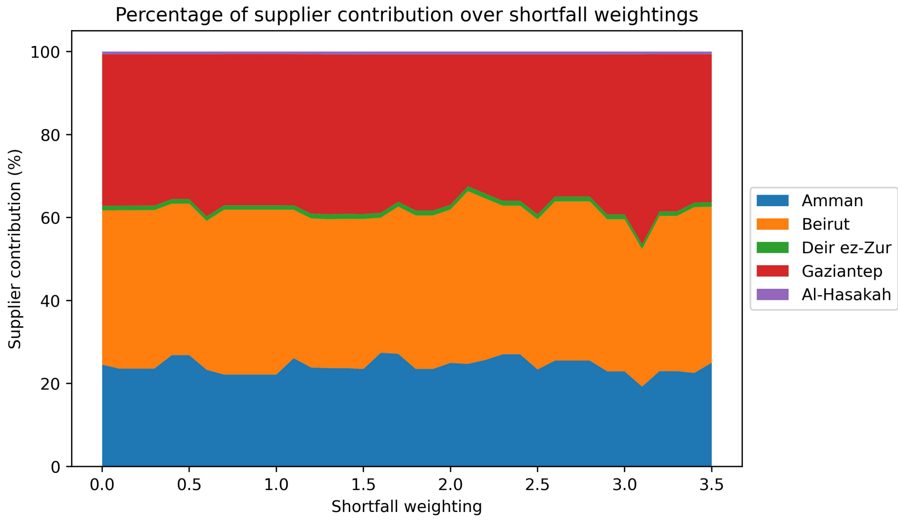

Figure 8: Stacked area plot showing the contribution of each supplier as a percentage of the total supplier contribution, for shortfall weighting in the range 0 to 3.5. For the Blended Nominal (BNO) model. Any suppliers that did not contribute to the network have been disregarded.

Figure 9: Stacked area plot showing the quantity of each commodity purchased as a percentage of the total amount purchased, for shortfall weighting in the range 0 to 3.5. For the Blended Nominal (BNO) model. Any commodities that were not purchased have been disregarded.

When the shortfall weighting is set to $w_2 = 0$, we observe the same outcome as the standard Nominal model (as predicted), with an optimised cost of 13.71 million USD. And, as shown in Figure 10, the optimised cost increases as the shortfall weighting increases. This is to be expected, as when the shortfall weighting is increased, the optimised solution puts a stronger priority on minimising the shortfall, increasing the purchasing amount required to meet the increased demand. Furthermore, once the weighting reaches $w_2 = 3.2$, the cost stops increasing at it's maximum of 15.23 million USD. The cost does not increase in any uniform way, but instead has multiple points at which it jumps up, and multiple regions during which it plateaus.

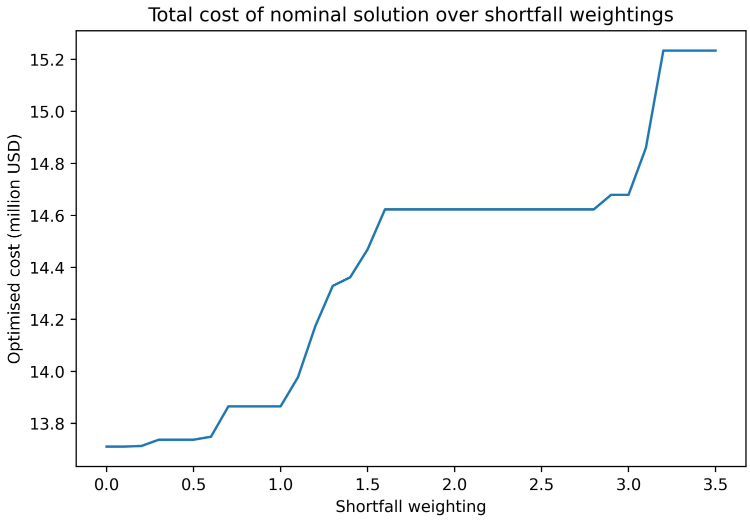

Figure 10: Plot of optimised cost (million USD) of Blended Nominal solution for shortfall weighting in the range 0 to 3.5. For the Blended Nominal optimisation (BNO) model.

By observing the behaviour of the realised shortfall for each nutrient, as shown in Figure 11, we observe that the increases in the optimised cost directly align with the decreases in realised shortfall for each commodity. Whenever we observe a decline in realised nutrient deficit, we observe a proportional increase in optimised cost. Furthermore, energy (kcal) resists the impacts of increasing the shortfall weighting more than any other nutrient. This is a large issue as having a sufficient daily intake of energy is one of the most important things someone can do to prevent starvation (Keys et al., 1950).

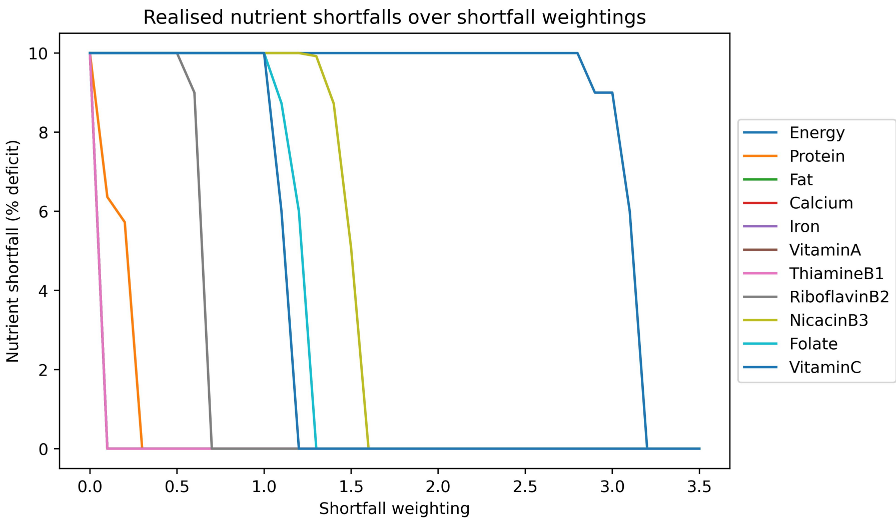

Figure 11: Plot of realised shortfall as a percentage nutrient deficit, for each nutrient for a shortfall weighting in the range 0 to 3.5. For the Blended Nominal (BNO) optimisation model.

To solve this, we can set the maximum allowed shortfall for energy to a lower value, thereby allowing a lower energy deficit. However, through testing, we found that energy still resists the shortfall weighting increase more than any other nutrient. Meaning the only effective way of minimising the energy shortfall is by setting it to a sufficiently low value. This set value would likely be decided by the organisation implementing the solutions of these models.

4.4 Blended Robust Optimisation (BRO)

Much like the BNO, the BRO inherent the same scalability and numerical issues as the RO, however the blended objective didn't seem to have any notable further impact. This results in a model with poor scalability and long compute times when run over the full weighting scale although usage of a small MIP gap can still produce relatively accurate results with significantly decreased runtime.

As shown in Figure 12, the distribution of each supplier contribution remains mostly consistent to the standard Robust solution, however local supplier usage drops under 0.5% and thus isn't displayable soon as any shortfall weighting is applied. . Furthermore, Figure 13 shows similar lack of variation as the BRO solution.

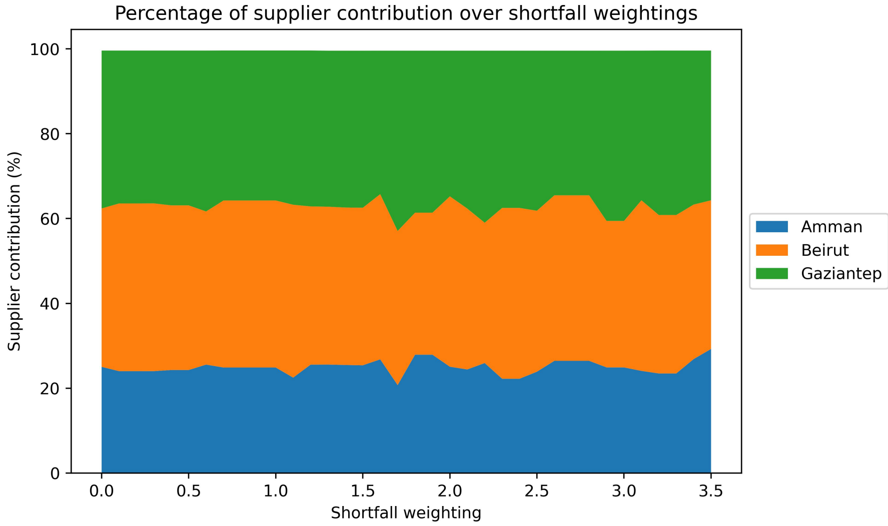

Figure 12: Stacked area plot showing the contribution of each supplier as a percentage of the total supplier contribution, for shortfall weighting in the range 0 to 3.5. For the Blended Robust (BRO) model. Any suppliers that did not contribute to the network have been disregarded.

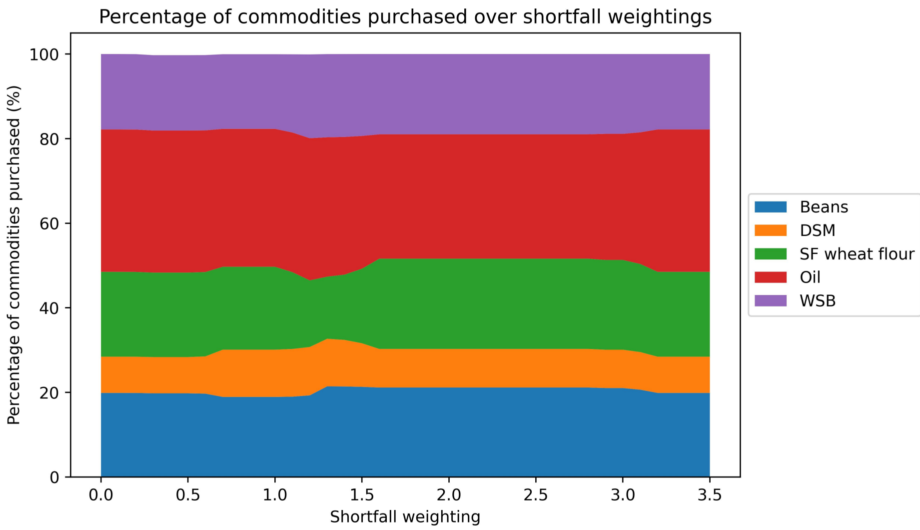

Figure 13: Stacked area plot showing the quantity of each commodity purchased as a percentage of the total amount purchased, for shortfall weighting in the range 0 to 3.5. For the Blended Robust (BRO) model. Any commodities that were not purchased have been disregarded.

When the shortfall weighting is set to $w_2 = 0$, results remain consistent with the RO at 13.79 million USD. And, as shown in Figure 14, the optimised cost increases as the shortfall weighting increases. The BRO optimised cost remains higher than the BNO for all w_2 which is expected as the BRO is optimising the worse case scenario of procurement prices, however the relative difference seems to vary over w_2 weightings. Since relative difference seems consistent as the objective values plateau and narrow when optimal values are rapidly increasing, this may be due to the properties of the RO having multiple optimal solutions which can be used to decrease the rate of objective increase as weightings increase.

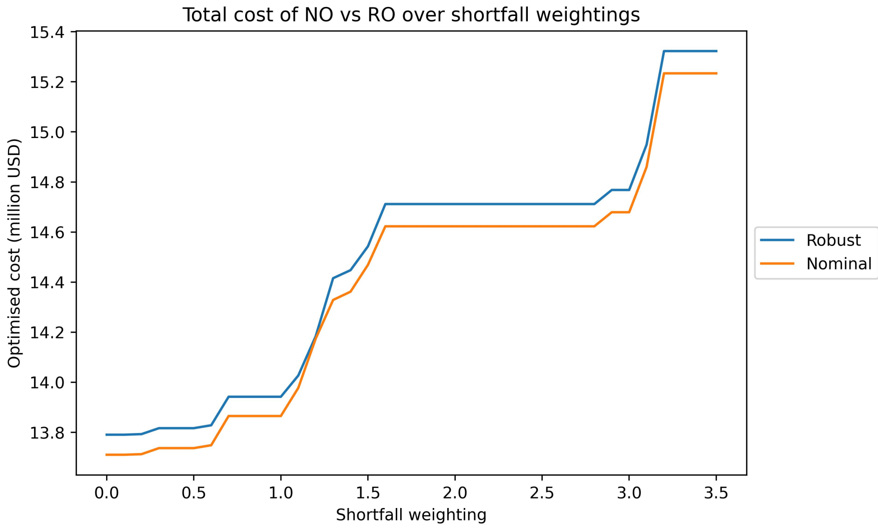

Figure 14: Plot of optimised cost (million USD) of Blended Robust solution vs Blended Nominal Solution for shortfall weighting in the range 0 to 3.5. For the Blended Robust optimisation (BRO) model.

By observing the behaviour of the realised shortfall for each nutrient, as shown in Figure 15, it appears quite similar to the BNO results, which seems consistent since both the BRO and BNO are predominately picking from international suppliers and those procurement costs remain consistent between models.

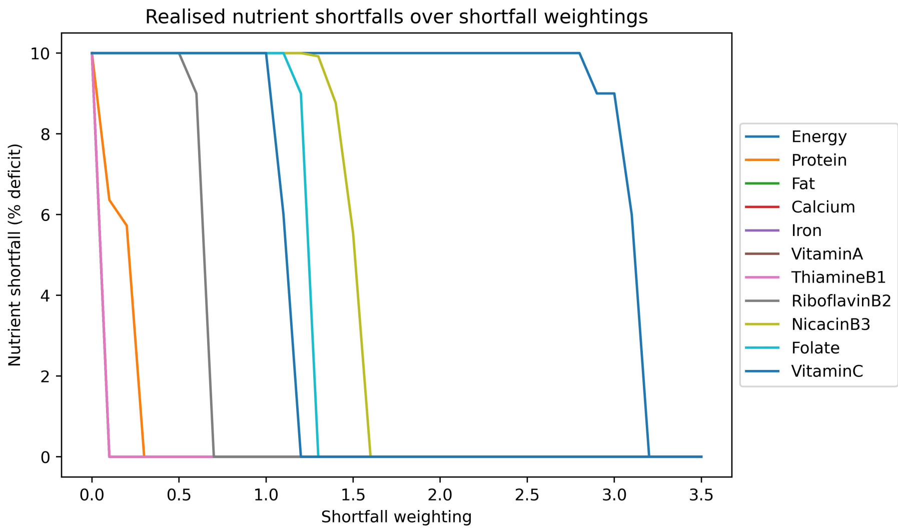

Figure 15: Plot of realised shortfall as a percentage nutrient deficit, for each nutrient for a shortfall weighting in the range 0 to 3.5. For the Blended Robust (BRO) optimisation model.

4.5 Blended Pareto Robust Optimisation (BPRO)

As the BPRO involves solving the RO first, a hierarchical constraint was placed on the Robust objective minimizing shortfalls first, to ensure that step minimizes the worse case scenario possible. Which objective value can then be used as an upper bound across all weightings of the PRO second step. Because the RO component only has to be solved once and with the relative efficiency of the PRO second step, the BPRO has significantly better scalability and compute times than the BRO.

As shown in Figure 16, the distribution of each suppliers contribution remains mostly consistent throughout the weighting range, with only slight variations from the standard Pareto Robust solution. Furthermore, the suppliers that had the most meaningful contribution doesn't change throughout the weighting range. Let it be known that some other suppliers do make some contribution to the aid network, however it is such an insignificant amount that visualising it is not necessary. Similar to the suppliers, there is little variation within the distribution of commodities purchasing, as shown in Figure 17.

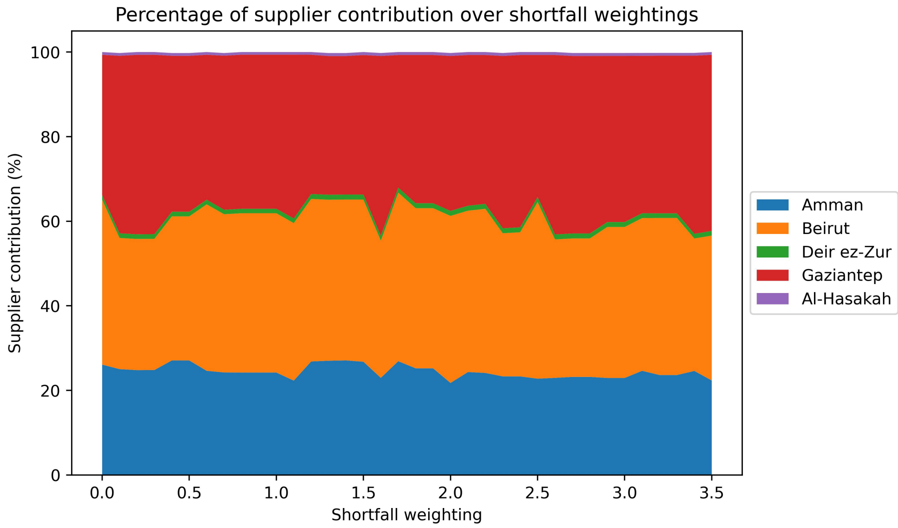

Figure 16: Stacked area plot showing the contribution of each supplier as a percentage of the total supplier contribution, for shortfall weighting in the range 0 to 3.5. For the Blended Pareto Robust (BPRO) model. Any suppliers that contributed less than 0.1% to the network have been disregarded.

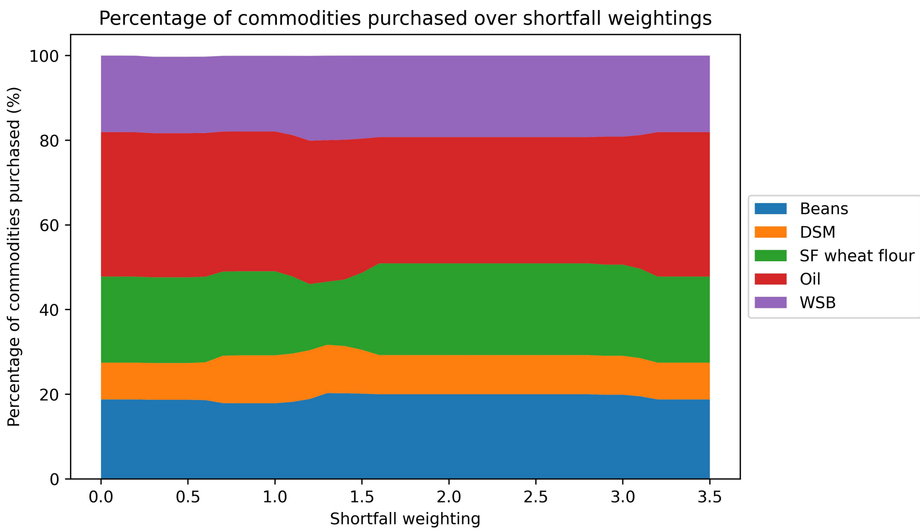

Figure 17: Stacked area plot showing the quantity of each commodity purchased as a percentage of the total amount purchased, for shortfall weighting in the range 0 to 3.5. For the Blended Pareto Robust (BPRO) model. Any commodities that mad up less than 0.1% of the total purchased quantity have been disregarded.

With a shortfall weighting of $w_1 = 0$, we are given an optimised cost of 13.7105 million USD, making it identical to the standard Pareto Robust solution. As shown in Figure 18 and Figure 10, Blended Nominal and Blended Pareto Robust models both result in the same optimal cost over the weighting range (within a margin of 2.9×10^{-14} million USD). Furthermore, we observe in Figure 19 that the realised shortfall for each nutrient are the same as those in the Blended Nominal solution. These facts gives further evidence for the existence of multiple optimal solutions.

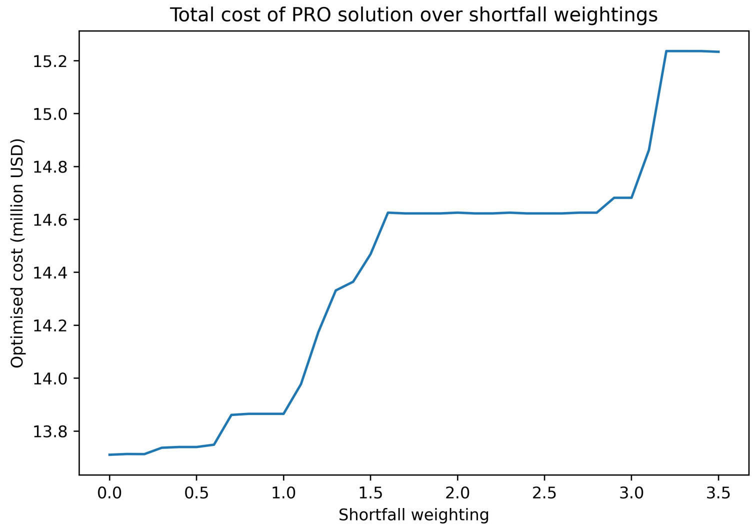

Figure 18: Plot of optimised cost (million USD) of Blended Nominal solution for shortfall weighting in the range 0 to 3.5. For the Blended Pareto Robust optimisation (BPRO) model.

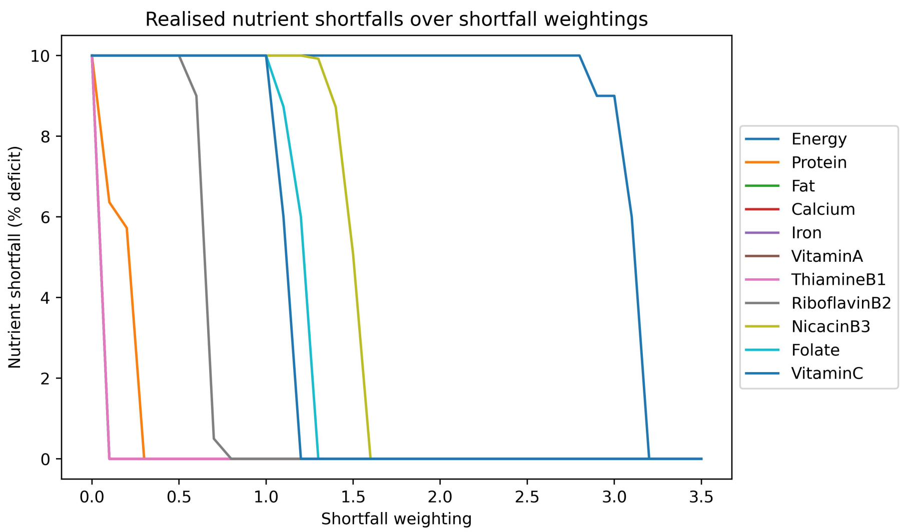

Figure 19: Plot of realised shortfall as a percentage nutrient deficit, for each nutrient for a shortfall weighting in the range 0 to 3.5. For the Blended Pareto Robust optimisation (BPRO) optimisation model.

5 Proposed Extensions

5.1 Adaptive International Prices

A primary assumption made by the paper is that international prices are without uncertainty, easily pourable and are generally cheaper than local and regional markets, however recent geo-political changes make this assumption less applicable. This could be accomplished by modifying the RO formulation as follows:

$$p^P_{ikt} = \begin{cases} \mu_{ikt} & , i \in \mathcal{N}_\mathcal{S}, t = 1 \\ \mu_{ikt} + \zeta_{m_i kt} & , i \in N_\mathcal{S}, t \geq 2 \end{cases} \tag{35}$$

With new objective function:

$$\min\ q + \sum_{i \in \mathcal{N}_\mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \mu_{ik1} F_{ijk1} \tag{36}$$

$$+ \mathrm{PC} + \mathrm{TC} + \mathrm{HC} + \mathrm{SC} \tag{37}$$

While changing the dimensions of the covariance, transformation and flow vectors to not exclude International markets.

5.2 Adaptive Robust Optimisation (ARO)

One of the primary drawbacks of the implemented solutions is that they do not allow for here and now decision making and cannot update the solution as market prices fluctuate. To address this the original paper implemented the Adaptive Robust Optimisation. In which the variable F_{ijkt} is redefined as a linear decision rule that includes the uncertainty vector ζ_{mkt} (Ben-tal et al, 2004) , which allows flow from suppliers at a given point in time to not rely on uncertainty in prices as the price is known at moment of procurement. Unfortunately the complexity of the this model was beyond what was reasonable to implement with available resources, however reviewing the formulation, it could be improved with the implementation of non-anticipatory constraints as currently the ARO has no restriction on altering commitments made previous time periods based on new information in the future.

5.3 Folding Horizon Approach

Another method of allowing for wait and see decisions proposed by the original paper is the folding horizons approach, where the NO and RO models are initially solved in the first time period to obtain an optimal plan for $\sqcup \in \mathcal{T}, t = 1$ and flexible commitments for $t \geq 2$. Then each subsequent time period actual procurement prices are revealed and the model is resolved, where commitments made for that time period can only be altered within a given percentage α determined by the following constraint:

$$\left| \sum_{j \in N_{TD}} F_{ijkt} - \sum_{j \in N_{TD} F^*_{ijkt}} \right| \leq \alpha \sum_{j \in N_{TD}} F^*_{ijkt} \tag{38}$$

The issue with this model is that apart from Constraint 38, there are mathematically no constraints on F_{ijkt} for previous time periods to prevent them from altering once new information is revealed in current time period $t' \in T$. To this end a modified version of a non anticipatory constraint could be added such as:

$$F_{ijkt} = F^*_{ijkt} \; \forall t < t' \in T \tag{39}$$

6 Conclusion

The purpose of this report was to implement three of the linear optimization models outlined in de Moor et al. (2024), and improve upon the formulation by addressing the issues with the shortfall variable. To this end, all three models have been implemented in Gurobi and a blended weighted objective was implemented for the NO, RO and PRO models that would allow NGOs to model the impact of prioritising reduced cost or preventing nutrient deficits. This would give them the ability to decide which weighting and optimal strategy best fits their resources. While the Nominal model remains quite efficient and scalable across multiple time periods and nodes, the Robust and Pareto models suffer from poor scalability due to their large matrices and non-convex quadratic constraints. So a linear optimiser more adept at dealing with such constraints, such as MOSEK (MOSEK ApS, 2024) might be preferable for solving them. Alternatively it might be possible to re-formate the quadratic constraints into a convex form as it would potentially decrease model complexity.

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