A dynamic splitting approach to fibre-to-the-home passive optical distribution network design

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Abstract

In this paper, I investigate the optimisation of fibre-to-the-home (FTTH) passive optical distribution networks through mixed-integer programming (MIP) formulations. Building upon existing single-splitting (SSP), double-splitting (DSP), and mixed-splitting (MSP) models, I propose a novel extension termed the Type-R formulation, which allows dynamic selection of splitting ratios for optical splitters. This modified formulation introduces greater flexibility into the network design and provides additional cost saving by more precisely matching splitter capacity to demand. Computational experiments on randomly generated large-scale network instances demonstrate that the Type-R formulations consistently yield lower objective costs compared to fixed-ratio models, albeit at the expense of increased computation time. Notably, the single-splitting Type-R model (SSP-R) achieves the most favourable trade-off between cost efficiency and solution time, outperforming both standard (MSP) and Type-R mixed-splitting models (MSP-R). These findings suggest that Type-R formulations offer a promising direction for more cost-effective FTTH network deployment, particularly in large-scale settings.

1. Introduction

The proliferation of data-driven applications and the shift toward digital economies have intensified the global demand for high-capacity broadband infrastructure. In Australia, the roll-out of the NBN required an initial funding of 49 billion AUD, demonstrating the need for cost saving optimisation (Sinclair, 2023). In this paper, I address the issue of optimising fibre-to-the-home (FTTH) passive optical distribution networks, which deliver optical fibre connections directly to end users. FTTH systems offer high bandwidth, low latency, and future-proof scalability, positioning them as a strategic asset in both urban and rural telecommunications planning.

A standard FTTH architecture is composed of a feeder network and a distribution network. The feeder network connects the Optical Line Terminal (OLT)—typically housed in a central office—to one or more Optical Distribution Points (ODPs). From each ODP, the distribution network branches out to serve multiple Optical Network Units (ONUs), often structured as a rooted tree with the ODP at the origin. This design is particularly well-suited to passive optical network (PON) technologies, which are characterized by the absence of active components in the field (Grötschel et al., 2014).

A major cost determinant in FTTH deployment lies in the expenditure associated with laying fibre-optic cables. A naïve point-to-point (P2P) configuration—assigning a dedicated fibre path from the ODP to each ONU—yields prohibitively high costs due to the extensive cabling required. As a cost-effective alternative, point-to-multipoint (P2MP) networks incorporate passive optical splitters, which allow a single fibre to be shared among multiple endpoints. Splitters introduce a trade-off: while they reduce fibre usage, they entail additional installation costs and can degrade signal strength, thus necessitating a carefully balanced network design (Lyu et al., 2018).

Existing literature on FTTH P2MP network design generally consists of two main problem areas: single-splitting problem (SSP) and double-splitting problem (DSP). SSP defines a FTTH problem in which all input signals undergo a single level of splitting, whereas DSP denotes a problem in which all input signals undergo two levels of splitting before reaching an ONU. The advantage of the double-splitting model over single-splitting is the added flexibility, by allowing fibres to be split a second time. This however, can potentially bloat costs due to the required use of two splitters for each signal. Instead, Agarwal and Jayaswal (2025) defined the mixed-splitting problem (MSP) as a FTTH problem in which all signals may undergo either single-splitting or double-splitting. This gives two options for how a signal may reach a demand node, clearly increasing the level of flexibility available within the solution.

2. Network Model

Define a tree graph consisting of a set of nodes N and set of directed arcs A between nodes. These arcs are directed outwards from the root node and represent the trenches in which fibre-optic cables may be run and the nodes represent the containers housing the optical splitters (cabinets, manholes, etc.). The ODP is represented by the root node in the network tree (node 0), with all traffic entering the network flowing out from the root node. The network tree can be viewed as an arborescence rooted at the ODP (Ito et al., 2023), allowing each node (except the root node) to be identified by its unique incoming arc, and each arc by its head and tail nodes.

The MSP allows for use of both single splitters and double splitters, hence I define the following three types of splitters: Type-A splitters perform a single level of splitting with the maximum allowable splitting ratio, and Type-B1 and B2 are the primary and secondary double splitters respectively. B1 splitters must be placed earlier in the network tree (closer to the root node) than B2 splitters. Together, a B1 and B2 splitter pair is referred to as a single Type-B splitter.

The traffic on an arc is defined as the total number of optical fibres needed on that arc. Fibres are needed to carry the required signal from the root node to the demand nodes that are connected to the subtree rooted at the head node of each arc. There are three types of traffic in the network, Type-1, Type-2 and Type-3. Type-1 traffic is traffic that has not passed through any splitters, and Type-3 traffic has passed through a Type-A or B (passed through both primary B1 then secondary B2) splitters. Type-2 traffic has passed through only a Type-B1 splitter. Type-A and B1 splitters may only take Type-1 traffic, producing Type-3 and 2 traffic respectively, and Type-B2 splitters may only take Type-2 traffic, producing Type-3 traffic. Furthermore, no Type-3 traffic is permitted to enter splitters of any type, as it has already undergone the maximum permissible splitting. Demand nodes may only take traffic Type-3 to satisfy their demand. This means that the traffic type at each node indicates the level of splitting the signal has undergone since leaving the root node. It is important to note that any number of splitters of each type may be assigned to each node.

For the purposes of the below formulation, I will index the three splitter types A, B1 and B2 as 1, 2 and 3 respectively. Furthermore, I define the following sets and data in Table 1.

N	Set of nodes (0 is the root node)
N_i	Set of nodes rooted at node $i \in N$
L	Set of leaf nodes
P_i	Set of predecessor nodes of node $i \in N$
A	Set of arcs
C	Set of cable types
$S = \{1, 2, 3\}$	Set of splitter types
$K = \{1, 2, 3\}$	Set of traffic types
d_i	Demand (number of fibres) at node $i \in N$
h_a	Head node of arc $a \in A$
t_a	Tail node of arc $a \in A$
$O_i = \{a \in A : t_a = i\}$	Subset of arcs directed out of node $i \in N$
$e_i = \{a \in A : h_a = i\}$	The unique arc directed into node $i \in N$
l_a	Length of arc $a \in A$
α_c	Cost per unit length of cable type $c \in C$
β_c	Capacity (no. fibres) of cable type $c \in C$
γ_s	Cost of splitter types $s \in S$
r_s	Ratio of splitter type $s \in S$

Table 1: Set and data notation

The MSP formulation requires the tracking of cable types, splitter numbers and traffic types. Hence, define the following decision variables:

$$x_a^k \in \mathbb{Z}, \quad \text{Flow of traffic type } k \text{ on arc } a \qquad \forall k \in K,\ a \in A$$

$$z_i^s \in \mathbb{Z}, \quad \text{Number of splitters of type } s \text{ at node } i \qquad \forall s \in S,\ i \in N$$

$$y_a^c = \begin{cases} 1, & \text{if cable type } c \text{ in installed on arc } a \\ 0, & \text{otherwise} \end{cases} \qquad \forall c \in C,\ a \in A$$

The above notations and variables can be used to define the following MIP formulations for MSP as described in Agarwal and Jayaswal (2025):

$$[\text{MSP}] \min \quad \sum_{i \in N} \sum_{s \in S} \gamma_s z_i^s + \sum_{a \in A} \left(l_a \sum_{c \in C} \alpha_c y_a^c \right) \tag{1}$$

$$\text{s.t.} \quad x_{e_i}^1 \geq \sum_{a \in O_i} x_a^1 + z_i^1 + z_i^2 \qquad \forall i \in N \setminus \{0\} \tag{2}$$

$$x_{e_i}^2 + r_2 z_i^2 \geq \sum_{a \in O_i} x_a^2 + z_i^3 \qquad \forall i \in N \tag{3}$$

$$x_{e_i}^3 + r_1 z_i^1 + r_3 z_i^3 \geq \sum_{a \in O_i} x_a^3 + d_i \qquad \forall i \in N \tag{4}$$

$$\sum_{k \in K} x_a^k \leq \sum_{c \in C} \beta_c y_a^c \qquad \forall a \in A \tag{5}$$

$$\sum_{c \in C} y_a^c = 1 \qquad \forall a \in A \tag{6}$$

$$x_a^k \geq 0 \qquad \forall k \in K,\ a \in A \tag{7}$$

$$z_i^s \geq 0 \qquad \forall s \in S,\ i \in N \tag{8}$$

The objective function (1) minimises the total cost of splitters and total cost of installed cables. Constraints (2) - (4) are flow balance constraints for the three traffic types within the network. Constraint (2) ensures that the amount Type-1 traffic (i.e. traffic yet to be split) entering each node is at least equal to the amount of Type-1 traffic leaving the node plus the number of Type-A and B1 splitters on the node. This constraint is enforced on every node except the root node (ODP) since it has no entering arc and enforcing it would disallow any traffic to exit the root node. Constraint (3) ensures the the amount of Type-2 traffic entering each node plus the additional Type-2 traffic from Type-B1 splitters is at least equal to the total Type-2 traffic exiting the node plus the number of Type-3 splitters on the node. Constraint (4) ensures that the amount of Type-3 traffic entering the each node plus the amount of Type-3 traffic from Type-A and B2 splitters is at least equal to the amount of Type-3 traffic exiting the node plus the demand at the node. Constraint (5) ensures that the capacity of the cable placed at each arc can satisfy the amount of traffic flowing through that arc. Constraint (6) ensures that only a single cable type is chosen for each arc and Constraints (7) and (8) are non-negativity constraints.

As MSP is a generalisation of SSP and DSP, I define SSP and DSP with the following formulations.

$$
\begin{aligned}
\text{[SSP] min} \quad & \sum_{i \in N} \sum_{s \in S} \gamma_s z_i^s + \sum_{a \in A} \left(l_a \sum_{c \in C} \alpha_c y_a^c \right) \\
\text{s.t.} \quad & \text{Constraints (2) - (8)} \\
& z_i^2 = 0 \qquad \forall i \in N \qquad (9) \\
& z_i^3 = 0 \qquad \forall i \in N \qquad (10) \\
& x_a^2 = 0 \qquad \forall a \in A \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\text{[DSP] min} \quad & \sum_{i \in N} \sum_{s \in S} \gamma_s z_i^s + \sum_{a \in A} \left(l_a \sum_{c \in C} \alpha_c y_a^c \right) \\
\text{s.t.} \quad & \text{Constraints (2) - (8)} \\
& z_i^1 = 0 \qquad \forall i \in N \qquad (12)
\end{aligned}
$$

Constraints (9) to (11) effectively disable all double-splitting variables, limiting the solution to use exclusively single splitters. Similarly, Constraint (12) disables all single-splitting variables, limiting the solution to use exclusively double splitters.

3. Type-R Formulation

An area the literature fails to address is the idea of dynamic choice of splitting ratio. As they currently stand, SSP, DSP and MSP all rely on a predefined splitting ratio for each type of splitter. Hence, I define a Type-R formulation as one that allows for the dynamic choice of splitting ratio for each splitter in the network. In this section, I describe the logic behind a Type-R variation of the above formulation, and outline the full MIP formulation of this problem type.

3.1. Logic and Illustrative Example

The aim of a Type-R formulation is to further reduce network setup cost, so to explain the logic behind this formulation, I present the following illustrative example. Take Constraint (4) for some arbitrary leaf node $i \in L$:

$$x^3_{e_i} + r_1 z^1_i + r_3 z^3_i \geq \sum_{a \in O_i} x^3_a + d_i \tag{13}$$

Since i is a leaf node, there are no arcs directed out of this node (since $O_i = \emptyset \; \forall i \in L$). For this example, I will only consider single-splitting. Furthermore, let there be 3 fibres of traffic Type-3 entering node i, and let i have a demand of 8. Equation (13) then becomes the following:

$$3 + r_1 z^1_i \geq 8 \tag{14}$$

In my computational experiments, I use $r_1 = 32$ for all instances. Therefore, minimisation would result in the following inequality.

$$3 + 32 \cdot 1 \geq 8$$

While this inequality does hold true, this solution overshoots the required demand by 27 fibres, wasting fibres and potentially bloating costs by using high capacity splitters. If we were to instead allow for dynamic choice of splitting ratio (limited to powers of 2), Equation (14) would instead take the following form.

$$3 + \sum_{r \in R} r \cdot z^1_{ir} \geq 8$$

where $R = \{2, 4, 8\}$ and $z^1_{ir} \in \mathbb{Z}$ decides the number of single splitters with ratio r on node i. This would minimise to one of the following three solutions (depending on associated splitter costs):

$$3 + 2 \cdot 3 \geq 8 \tag{15}$$

$$3 + 4 \cdot 2 \geq 8 \tag{16}$$

$$3 + 8 \cdot 1 \geq 8 \tag{17}$$

Depending on the cost structure of smaller capacity splitters, i.e. if a smaller capacity splitter is cheaper than a larger capacity splitter, this can have a substantial impact on the overall cost of the network. However there is the risk of increasing symmetry within the formulation with this modification. While all three of these solutions satisfy the constraint, it is obvious that (16) and (17) are identical solutions if splitter cost is linearly proportional to splitting ratio (i.e. a splitter of ratio 8 costs twice as much as the same splitter type of ratio 4). This is because the cost of a single splitter of ratio 8 would be equal to the cost of two splitters of ratio 4, creating symmetrical solutions (Margot, 2010). Therefore, to avoid symmetry in the solutions, cost of splitters must not be linear in splitting ratio (Ghoniem and Sherali, 2011).

3.2. Type-R MIP Formulation

I define the following set and data for the Type-R formulation, with the maximum splitting ratio capped at 32 as this was the largest splitting ratio in the original formulation.

R = {2,4,8,16,32}	Set of splitting ratios
γ_r^s	Cost of splitter type $s \in S$ with ratio $r \in R$

Table 2: Set and parameter notation for Type-R formulations

I also re-define the splitter choice variable, z.

$z_{ir}^s \in \mathbb{Z}$ Number of splitters of type $s \in S$ at node $i \in N$ with ratio $r \in R$

Let SSP-R, DSP-R and MSP-R denote single-splitting, double-splitting and mixed-splitting Type-R formulations respectively. MSP-R is formulated as follows:

$$
\begin{aligned}
\text{[MSP-R] min} \quad & \sum_{i\in N}\sum_{s\in S}\sum_{rinR}\gamma_r^s z_{ir}^s + \sum_{a\in A}\left(l_a\sum_{c\in C}\alpha_c y_a^c\right) && && (18)\\
\text{s.t.} \quad & x_{e_i}^1 \geq \sum_{a\in O_i} x_a^1 + \sum_{r\in R}\left(z_{ir}^1 + z_{ir}^2\right) && \forall i\in N\setminus\{0\} && (19)\\
& x_{e_i}^2 + \sum_{r\in R} r z_{ir}^2 \geq \sum_{a\in O_i} x_a^2 + \sum_{r\in R} z_{ir}^3 && \forall i\in N && (20)\\
& x_{e_i}^3 + \sum_{r\in R} r\left(z_{ir}^1 + z_{ir}^3\right) \geq \sum_{a\in O_i} x_a^3 + d_i && \forall i\in N && (21)\\
& \sum_{k\in K} x_a^k \leq \sum_{c\in C}\beta_c y_a^c && \forall a\in A && (22)\\
& \sum_{c\in C} y_a^c = 1 && \forall a\in A && (23)\\
& x_a^k \geq 0 && \forall k\in K,\ a\in A && (24)\\
& z_{ir}^s \geq 0 && \forall s\in S,\ i\in N,\ r\in R && (25)
\end{aligned}
$$

The objective function in (18) serves the same purpose as the objective function in (1), in which it minimises the total splitter cost and the total cable cost. Constraints (19) - (21) are flow balance constraints for the three traffic types in the network. Constraint (22) - (24) are unchanged from MSP and Constraint (25) is the updated non-negativity constraint for the splitter decision variable.

Similar to MSP, MSP-R is the generalisation of SSP-R and DSP-R. Hence, SSP-R and DSP-R can be formulated as the following:

$$
\begin{aligned}
\text{[SSP-R] min} \quad & \sum_{i\in N}\sum_{s\in S}\sum_{rinR}\gamma_r^s z_{ir}^s + \sum_{a\in A}\left(l_a\sum_{c\in C}\alpha_c y_a^c\right) && && \\
\text{s.t.} \quad & \text{Constraints (19) - (25)} && && \\
& z_{ir}^2 = 0 && \forall i\in N,\ r\in R && (26)\\
& z_{ir}^3 = 0 && \forall i\in N,\ r\in R && (27)\\
& x_a^2 = 0 && \forall a\in A && (28)
\end{aligned}
$$

$$\text{[SSP-R]} \min \quad \sum_{i\in N}\sum_{s\in S}\sum_{rinR}\gamma_r^s z_{ir}^s + \sum_{a\in A}\left(l_a \sum_{c\in C}\alpha_c y_a^c\right)$$

$$\text{s.t.} \quad \text{Constraints (19) - (25)}$$

$$z_{ir}^1 = 0 \qquad \forall i \in N,\ r \in R \qquad (29)$$

Constraints (26) - (28) disable all double-splitting variables, limiting the solution to use exclusively single splitters. Similarly, Constraint (29) disables all single-splitting variables, limiting the solution to use exclusively double splitters.

4. Computational Experiments

All computational experiments are performed on an Intel(R) Core(TM) i7-13700H processor at 2.40 GHz (14 Cores, 20 Threads), 16GB LPDDR5 4800 MT/s Memory, and Windows 11 Home Operating System. All models are implemented in Python 3.12.9 using the Gurobi 12.0.1 Python package (gurobipy) as the LP solver. I set the target optimality gap to zero and gave the solver a time limit of one hour (3600 seconds).

4.1. Random Instance Generation

Kim et al. (2011) provide five benchmark instances based on existing networks in Seoul, South Korea. However, these instances only contain networks of 19 to 24 nodes. As this does not present a challenge to Gurobi 12 and the models presented above, larger test instances were required to be generated (results for benchmark instances are provided in Appendix A). To this end, I generated 20 instances each of networks containing 66, 76 and 101 nodes using the following method provided by Agarwal and Jayaswal (2025).

The positions of the network nodes are determined by randomly generating their x and y coordinates using a uniform distribution between 0 and 100. The network root node (node 0) is fixed at location $(50, 50)$. To construct a tree network, the parent of each node is selected as follows: starting with the node farthest from node 0 whose

Splitter type (s)	**1**	**2**	**3**
Splitter ratio (r_s)	32	8	4
Splitter cost (γ_s)	80	30	20

Table 3: Splitters ratios and costs

Cable type (c)	**1**	**2**	**3**	**4**	**5**
Cable capacity (β_c)	4	10	20	40	80
Cable cost (α_c)	3	5	7	10	15

Table 4: Cable capacities and cost per unit length for each cable type

parent is not yet assigned, we choose its parent to be the closest available node whose child connections are still unassigned.

As in Agarwal and Jayaswal (2025), we relax the assumption that only leaf nodes contain demand, allowing demand to originate at any node (excluding node 0). For standard formulations (SSP, DSP and MSP), all instances use fixed splitting ratios as described in Table 3. Both standard and Type-R instances include five cable types, with their capacities and costs detailed in Table 4.

For Type-R formulations, all instances use dynamic splitting ratios as defined in the set R. Additionally the costs are defined as follows:

$$\gamma_r^s = \begin{cases} 2.5r + 0.1|r - 32|, & \text{if } s = 1 \\ 3.75r + 0.1|r - 8|, & \text{if } s = 2 \\ 5r + 0.1|r - 4|, & \text{if } s = 3 \end{cases} \qquad \forall s \in S,\ r \in R \tag{30}$$

The definition of these splitter costs is mostly arbitrary, as they are a placeholder for real-world installation costs. However, it is important to note that splitters of each type with the same ratio as in the standard formulations have had their costs unchanged, e.g. $\gamma_{32}^1 = \gamma_1 = 80$.

While each model (standard and Type-R) was tested on all large instances, for the purposes of comparison to Agarwal and Jayaswal (2025), I only present full SSP results for 101 node instances, full DSP results for 76 node instances, and full MSP results for 66 node instances. For comprehensive results, refer to Appendix B.

5. Computational Results

In the following sections, I outline the cost saving benefits of the Type-R formulations, as well as describe the benefits of Gurobi. For each instance, I report the objective value (Obj.) and computation (CPU) time for the corresponding standard and Type-R formulations. Furthermore, for each instance I also report the percentage saving of the objective value (% Sav.) for the corresponding Type-R formulation. In each table, the "Average" row reports the mean computation time and percentage saving.

5.1. Single-Splitting

The results presented in Table 5 demonstrate the cost saving ability of the Type-R formulation, with objective values on average being 6.67% lower than in the standard formulation. This does come with the trade-off of instances taking on average 35.74 times as long to solve with SSP-R. Both SSP and SSP-R were able to be solved to optimality for all instances of 66, 76 and 101 nodes.

In Agarwal and Jayaswal (2025), their fastest single-splitting models, SSP1 and SSP2 solved to optimality in an average of 9.9 and 10.2 seconds respectively. Their slowest single-splitting formulation, SSP0 (which I denote simply as SSP), when implemented in Gurobi solved to optimality in an average of 1.36 seconds. Furthermore, my Type-R formulation, SSP-R, had an average solving time of 6.67 seconds. These results are likely as an effect of Gurobi's cutting plane algorithm, which is explored in greater detail in Section 5.5.

Inst.	SSP		SSP-R		
	Obj.	**CPU**	**Obj.**	**% Sav.**	**CPU**
1	8640.95	1.39	8068.11	6.63	34.09
2	8542.24	1.36	7837.05	8.26	16.92
3	8607.07	2.39	7831.55	9.01	15.62
4	8697.32	0.66	7926.02	8.87	19.36
5	8642.12	0.99	8084.24	6.46	21.89
6	9027.88	0.85	8508.72	5.75	117.37
7	9760.69	2.58	9019.65	7.59	26.98
8	8347.69	0.66	7865.1	5.78	34.12
9	9480.68	0.78	8864.13	6.5	20.44
10	8711.19	3.09	8085.76	7.18	57.77
11	9048.73	1.74	8493.55	6.14	21.51
12	9170.23	2.71	8686.39	5.28	304.16
13	8812.55	0.86	8315.42	5.64	19.99
14	8774.55	0.59	8346.52	4.88	15.79
15	8806.85	0.9	8192.99	6.97	20.34
16	8901.11	2.27	8373.99	5.92	140.51
17	8647.7	0.9	8065.99	6.73	39.78
18	8612.51	0.7	8234.17	4.39	15.49
19	8567.54	0.93	7998.05	6.65	15.43
20	9108.9	0.92	8315.68	8.71	14.58
Average		1.36		6.67	48.61

Table 5: 101 Node Solutions for single-splitting formulations. For full results for instances with 101 nodes, refer to Table B.15 for standard formulations and Table B.16 for Type-R formulations.

5.2. Double-Splitting

The results in Table 6 further demonstrate the cost saving ability of the Type-R formulation, with objective values on average being 15.32% lower than that of the standard formulation. This is a substantial increase in saving from the 101 node SSP formulations. This is likely due to the required use of double splitters in DSP. Because each signal is required to be split twice, and the cost per splitting ratio of Type-B1 and B2 splitters is higher than Type-A splitters, there is likely more opportunity to save costs by using splitters of smaller splitting ratio. Although DSP-R demonstrated substantial cost savings over DSP, SSP-R still solved to lower objective values for every instance. DSP was able to solve all instances of 66, 76 and 101 nodes to optimality, while DSP-R could solve all but one instance of 101 nodes to optimality.

5.3. Mixed-Splitting

Table 7 again demonstrates the cost saving ability of the Type-R formulation, with MSP-R having an average cost saving of 6.51% over MSP. As with the SSP-R and DSP-R, this cost saving does come with the disadvantage of longer solve times. For small instances like 66 nodes, this isn't a major issue as the average solve time is only 43.32 seconds. However this does increase by a substantial amount for larger instances. MSP was able to solve all instances of 66, 76 and 101 nodes to optimality, while MSP-R could solve all instances of 66 and 76 nodes, and 15 of the 20 instances of 101 nodes.

Inst.	DSP		DSP-R		
	Obj.	**CPU**	**Obj.**	**% Sav.**	**CPU**
1	9806.46	2.6	8367.81	14.67	30.8
2	9386.27	5.4	7858.99	16.27	40.92
3	8681.62	4.36	7365.37	15.16	21.28
4	8938.82	1.95	7571.08	15.3	24.35
5	9454.24	1.8	8059.69	14.75	69.39
6	9714.63	8.12	8135.26	16.26	66.81
7	9734.15	2.85	8175.18	16.02	16.53
8	9148.81	3.24	7780.64	14.95	72.07
9	9341.91	2.16	7945.4	14.95	53.33
10	9706.92	3.78	8025.22	17.32	46.36
11	9789.62	5.43	8274.65	15.48	109.4
12	9820.4	4.47	8351.25	14.96	42.14
13	9530.41	2.08	8026.85	15.78	23.13
14	8960.57	3.58	7644.37	14.69	47.81
15	8978.04	6.52	7627.92	15.04	53.09
16	9607.75	2.46	8095.39	15.74	39.44
17	8545.29	1.33	7410.64	13.28	22.14
18	9967.51	5.21	8424.07	15.48	153.31
19	9435.61	3.98	8057.98	14.6	59.83
20	9506.23	3.22	8013.31	15.7	62.21
Average		3.73		15.32	52.72

Table 6: 76 Node Solutions for double-splitting formulations. For full results for instances with 76 nodes, refer to Table B.13 for standard formulations and Table B.14 for Type-R formulations.

Inst.	MSP		MSP-R		
	Obj.	**CPU**	**Obj.**	**% Sav.**	**CPU**
1	5896.06	1.69	5444.02	7.67	7.89
2	6049.29	6.78	5764.25	4.71	56.09
3	6322.3	3.92	5886.99	6.89	140.31
4	6542.93	3.47	6112.66	6.58	23.8
5	6547.08	4.2	6238.15	4.72	19.32
6	6637.51	3.23	6139.91	7.5	25.06
7	6387.62	1.92	5920.2	7.32	13.76
8	6592.49	1.79	6132.49	6.98	30.88
9	5996.16	0.94	5714.47	4.7	9.11
10	6297.73	5.86	5877.31	6.68	38.87
11	6588.91	5.88	6185.76	6.12	26.85
12	6348.62	3.2	5841.59	7.99	10.31
13	6941.65	5.34	6470.34	6.79	30.55
14	6436.41	4.29	6030.39	6.31	22.44
15	6190.66	4.28	5842.36	5.63	27.78
16	6359.05	3.43	5989.17	5.82	29.73
17	6531.59	4.32	6134.54	6.08	35.19
18	6807.76	3.6	6350.63	6.71	24.37
19	6485.6	13.11	5983.0	7.75	112.07
20	6192.51	4.12	5738.94	7.32	181.99
Average		4.27		6.51	43.32

Table 7: 66 Node Solutions for mixed-splitting formulations. For full results for instances with 66 nodes, refer to Table B.11 for standard formulations and Table B.12 for Type-R formulations.

5.4. Cost Saving

In Agarwal and Jayaswal (2025), on benchmark instances their MSP implementation only provided an average cost saving of 0.5% over the SSP formulation. This is a substantially lower saving than the savings described above for Type-R formulations and

Model	66 Nodes		76 Nodes		101 Nodes	
	% Sav.	CPU	% Sav.	CPU	% Sav.	CPU
SSP	0	0.5	0	0.67	0	1.36
MSP	1.42	4.27	1.34	11.45	1.08	32.11
SSP-R	7.49	6.72	7.3	8.88	6.67	48.61
MSP-R	7.84	43.32	7.59	98.51	6.78	1181.41

Table 8: Mean percentage cost saving (over SSP) and mean computation time for SSP, MSP, SSP-R and MSP-R.

their respective standard formulations. However, in my testing of larger instances for all formulations, I observed the results displayed in Table 8. For instances of 66 nodes, MSP only provided a 1.42% cost saving over SSP. However, SSP-R achieved a cost saving of 7.49% while only having a marginally higher computation time. Although MSP-R does provide a higher average cost saving of 7.84%, the extreme jump in computation time may outweigh the benefit. MSP-R took approximately 6.45 times as long to create only a 0.35% increase in cost saving.

This result is also evident in larger instances of 76, with MSP providing a relatively small cost saving. Once again SSP-R was able to achieve a substantially higher cost saving (7.3%) compared to MSP, while having a lower average computation time. Similar to instances of 66 nodes, MSP-R provides a marginally higher cost saving over SSP-R while having a substantially higher average solve time. MSP-R on average took 11.09 time longer to result in only an additional 0.29% cost saving.

The benefits of SSP-R are only demonstrated further for instances of 101 nodes. SSP-R again provides a substantially higher cost saving of 6.67% when compared to 1.08% for MSP. In testing these instances, MSP-R was able to solve 15 of the 20 instances to optimality within the one hour time limit. This resulted in a large average solve time of 1181.41 seconds compared to 48.61 for SSP-R. Additionally, the cost saving for MSP-R was only higher than that of SSP-R by 0.11%.

I infer this to mean that as the instances get larger, the trade-off between cost saving and solve time for MSP-R decreases. As demonstrated for larger instances, the difference in cost savings between SSP-R and MSP-R grows smaller while the difference in computation times increases at a drastic rate. Hence, if mixed-splitting is not a requirement for a large network, then SSP-R provides the most time-efficient method of saving costs.

5.5. Gurobi Cuts

In the results described above, I demonstrated that my implementation in Gurobi was able to vastly outperform previous implementations of the same formulations. Besides my computer being marginally more powerful, the one major difference between our implementations is my use of Gurobi, while Agarwal and Jayaswal (2025) used CPLEX. It appears that Gurobi has a superior cutting plane algorithm than CPLEX. This is evident by the fact that disabling cut generation in Gurobi for these models resulted in much longer solve times for even small instances.

The two main cuts Gurobi added to the formulation were Mixed-Integer Rounding (MIR) and Gomory Cuts. For standard formulations, MIR and Gomory cuts made up 59.30% and 18.99% respectively of total cuts added. For Type-R formulations, MIR and Gomory cuts made up 69.87% and 12.24% respectively of total cuts added. Disabling these specific types of cuts significantly impacts performance, suggesting they offer the greatest speed-ups to the formulations.

6. Conclusion and Future Research

In this paper, I studied the problem of splitter allocation and cable selection in FTTH passive optical distribution networks. I implemented a mixed-integer programming formulation of a generalised mixed-splitting model (MSP), which can be modified for use as a single-splitting (SSP) or double-splitting formulation (DSP). To lower the objective values of the solutions I proposed a Type-R variation of these models, in which the splitting ratio of each allocated splitter can be chosen dynamically to fit the demand needs. The solution time of the standard formulations far outperformed those from the literature and the Type-R formulations were able to provide meaningful decreases in objective value for test instances. A trade-off between solution time and cost savings was established for MSP-R, leading to the conclusion that SSP-R is the most time-efficient method for saving costs.

The natural progression of this research is integrating the models presented above with larger-scale real-world instances, while accounting for potential physical constraints that come with constructing optical distribution networks. These could include cable trench size and depth, splitter cabinet size, weather conditions, etc. Furthermore, Gouveia et al. (2015) present a formulation for unconstrained splitting stages, in which fibres may be split any number of times necessary to meet demand. It would be interesting to examine the effect of a multi-objective formulation, minimising the the cost and number of times a signal is split, thereby minimising the degradation of the signal.

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Appendix A. Benchmark Instances Results

# Nodes	Ins.	SSP		DSP		MSP	
		Obj.	CPU	Obj.	CPU	Obj.	CPU
19	1	12485.0	0.19	16800.0	0.24	12485.0	0.41
19	2	12215.0	0.09	18342.0	0.44	12044.0	0.23
22	3	13545.0	0.06	20173.0	0.26	13545.0	0.19
20	4	12644.0	0.06	18204.0	0.29	12644.0	0.11
24	5	12967.0	0.12	21013.0	0.26	12966.0	0.28
	Average		0.1		0.3		0.24

Table A.9: Benchmark instances solutions for standard formulations

# Nodes	Ins.	SSP-R		DSP-R		MSP-R	
		Obj.	CPU	Obj.	CPU	Obj.	CPU
19	1	7946.2	0.14	7263.4	0.63	7263.4	0.99
19	2	6112.8	0.13	6237.9	0.64	6066.5	0.8
22	3	9225.0	0.16	7891.4	0.52	7891.4	1.54
20	4	8042.4	0.22	7357.6	0.63	7357.6	0.74
24	5	6251.8	0.22	6353.6	0.85	6221.6	1.2
	Average		0.17		0.65		1.05

Table A.10: Benchmark instances solutions for Type-R formulations

Appendix B. Large Instances Results

# Nodes	Ins.	SSP				DSP				MSP			
		Obj.	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**
66	1	6000.48	0.39	62.67	37.33	7503.98	1.0	42.7	57.3	5896.06	1.69	61.84	38.16
66	2	6150.76	0.61	63.58	36.42	7946.93	2.98	39.85	60.15	6049.29	6.78	62.47	37.53
66	3	6458.33	1.1	61.6	38.4	8286.28	4.13	41.71	58.29	6322.3	3.92	59.98	40.02
66	4	6725.36	0.86	60.75	39.25	8467.39	2.04	39.53	60.47	6542.93	3.47	59.5	40.5
66	5	6632.26	0.61	61.4	38.6	8692.89	1.77	39.49	60.51	6547.08	4.2	59.07	40.93
66	6	6748.42	0.5	64.44	35.56	8532.34	1.75	40.23	59.77	6637.51	3.23	61.58	38.42
66	7	6396.23	0.49	63.73	36.27	7999.51	3.46	43.0	57.0	6387.62	1.92	61.96	38.04
66	8	6665.63	0.31	59.19	40.81	8975.84	4.03	38.17	61.83	6592.49	1.79	58.13	41.87
66	9	6177.52	0.39	62.44	37.56	7904.18	1.21	41.17	58.83	5996.16	0.94	60.64	39.36
66	10	6465.36	0.57	61.64	38.36	8219.77	0.96	38.93	61.07	6297.73	5.86	57.76	42.24
66	11	6635.71	0.42	61.42	38.58	8728.69	1.39	39.97	60.03	6588.91	5.88	59.48	40.52
66	12	6428.4	0.3	62.67	37.33	8230.84	1.08	42.53	57.47	6348.62	3.2	60.62	39.38
66	13	6999.04	0.35	59.99	40.01	9254.75	2.33	38.84	61.16	6941.65	5.34	58.8	41.2
66	14	6549.97	0.31	57.25	42.75	8550.86	1.55	39.66	60.34	6436.41	4.29	59.92	40.08
66	15	6240.78	0.26	61.54	38.46	8237.7	3.48	40.4	59.6	6190.66	4.28	60.59	39.41
66	16	6394.14	0.44	56.21	43.79	8674.45	3.75	37.52	62.48	6359.05	3.43	56.28	43.72
66	17	6584.33	0.35	58.69	41.31	8629.3	3.93	40.67	59.33	6531.59	4.32	58.51	41.49
66	18	6881.21	0.33	58.15	41.85	9161.69	2.43	39.64	60.36	6807.76	3.6	57.99	42.01
66	19	6635.85	0.88	60.22	39.78	8520.06	1.6	37.79	62.21	6485.6	13.11	57.91	42.09
66	20	6229.37	0.45	61.47	38.53	7863.21	1.24	44.17	55.83	6192.51	4.12	60.92	39.08
	Average		0.5	60.95	39.05		2.31	40.3	59.7		4.27	59.7	40.3

Table B.11: 66 Node Solutions for standard formulations. Columns "Obj." denotes objective values, "CPU" denotes computation times, "Cable %" and "Splitter %" denote the percentage contributions of respectively cable cost and splitter cost to the total cost (objective value).

# Nodes	Ins.	SSP-R				DSP-R				MSP-R			
		Obj.	CPU	Cable %	Splitter %	Obj.	CPU	Cable %	Splitter %	Obj.	CPU	Cable %	Splitter %
66	1	5465.33	0.92	62.91	37.09	6410.4	7.77	39.79	60.21	5444.02	7.89	60.05	39.95
66	2	5764.25	4.77	63.75	36.25	6731.28	17.43	36.96	63.04	5764.25	56.09	63.75	36.25
66	3	5913.45	6.34	62.8	37.2	6932.56	20.36	36.6	63.4	5886.99	140.31	61.71	38.29
66	4	6136.22	7.93	63.16	36.84	7206.18	18.64	36.26	63.74	6112.66	23.8	61.93	38.07
66	5	6292.27	8.79	62.85	37.15	7328.55	13.03	36.18	63.82	6238.15	19.32	60.91	39.09
66	6	6151.11	8.49	62.1	37.9	7240.7	13.76	35.5	64.5	6139.91	25.06	59.92	40.08
66	7	5946.24	4.77	63.38	36.62	6866.88	17.14	40.94	59.06	5920.2	13.76	61.1	38.9
66	8	6132.49	5.3	59.35	40.65	7587.52	38.67	34.37	65.63	6132.49	30.88	59.35	40.65
66	9	5714.47	2.04	62.29	37.71	6704.6	10.3	37.83	62.17	5714.47	9.11	62.29	37.71
66	10	5911.06	8.03	60.83	39.17	7007.39	59.2	35.33	64.67	5877.31	38.87	59.66	40.34
66	11	6200.98	5.54	62.07	37.93	7312.9	20.57	36.49	63.51	6185.76	26.85	61.17	38.83
66	12	5920.88	4.12	62.49	37.51	7008.06	11.59	38.86	61.14	5841.59	10.31	60.59	39.41
66	13	6503.43	7.09	61.1	38.9	7747.66	62.88	34.58	65.42	6470.34	30.55	59.5	40.5
66	14	6043.12	3.1	61.0	39.0	7299.73	43.98	36.78	63.22	6030.39	22.44	59.52	40.48
66	15	5867.2	5.15	62.89	37.11	6940.64	25.36	36.85	63.15	5842.36	27.78	60.47	39.53
66	16	6005.89	8.52	58.6	41.4	7345.07	45.4	34.23	65.77	5989.17	29.73	56.12	43.88
66	17	6167.74	10.08	61.06	38.94	7233.92	63.31	35.76	64.24	6134.54	35.19	60.36	39.64
66	18	6371.41	3.71	59.19	40.81	7642.29	13.63	33.99	66.01	6350.63	24.37	58.83	41.17
66	19	6004.78	13.53	59.98	40.02	7236.89	55.25	34.21	65.79	5983.0	112.07	59.26	40.74
66	20	5745.35	16.1	63.82	36.18	6733.16	20.3	41.2	58.8	5738.94	181.99	63.34	36.66
	Average		6.72	61.78	38.22		28.93	36.63	63.37		43.32	60.49	39.51

Table B.12: 66 Node Solutions for Type-R formulations.

# Nodes	Ins.	SSP				DSP				MSP			
		Obj.	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**
76	1	7233.04	0.48	59.08	40.92	9806.46	2.6	39.12	60.88	7194.0	5.24	58.58	41.42
76	2	7293.45	0.7	60.51	39.49	9386.27	5.4	42.36	57.64	7156.67	9.6	62.41	37.59
76	3	6616.46	1.26	58.89	41.11	8681.62	4.36	38.15	61.85	6457.9	4.98	57.88	42.12
76	4	6979.13	0.66	64.47	35.53	8938.82	1.95	41.94	58.06	6894.26	10.57	60.69	39.31
76	5	7129.48	0.54	58.48	41.52	9454.24	1.8	38.44	61.56	7097.21	22.22	58.58	41.42
76	6	7218.49	1.27	58.99	41.01	9714.63	8.12	37.62	62.38	7159.82	49.2	59.5	40.5
76	7	7408.38	0.73	61.13	38.87	9734.15	2.85	39.8	60.2	7325.21	5.25	60.41	39.59
76	8	7079.71	0.72	59.32	40.68	9148.81	3.24	39.01	60.99	7024.7	7.14	58.29	41.71
76	9	7104.83	0.4	61.72	38.28	9341.91	2.16	38.45	61.55	7055.42	4.46	59.75	40.25
76	10	7499.21	0.61	60.53	39.47	9706.92	3.78	40.15	59.85	7426.18	6.86	60.41	39.59
76	11	7583.2	1.04	60.97	39.03	9789.62	5.43	40.04	59.96	7500.09	29.85	60.8	39.2
76	12	7608.02	0.66	60.04	39.96	9820.4	4.47	38.39	61.61	7462.69	9.57	59.0	41.0
76	13	7338.92	0.51	61.85	38.15	9530.41	2.08	38.51	61.49	7140.5	6.76	59.95	40.05
76	14	7019.71	0.6	63.53	36.47	8960.57	3.58	39.85	60.15	6918.5	11.85	60.25	39.75
76	15	7064.87	0.52	63.76	36.24	8978.04	6.52	41.19	58.81	6962.98	7.71	61.65	38.35
76	16	7354.43	0.59	61.93	38.07	9607.75	2.46	38.28	61.72	7215.12	5.68	59.67	40.33
76	17	6912.24	0.46	61.81	38.19	8545.29	1.33	41.61	58.39	6698.37	4.11	60.59	39.41
76	18	7509.84	0.46	58.45	41.55	9967.51	5.21	38.6	61.4	7446.33	6.71	58.23	41.77
76	19	7337.69	0.51	64.02	35.98	9435.61	3.98	40.97	59.03	7293.38	13.26	60.92	39.08
76	20	7177.44	0.58	60.99	39.01	9506.23	3.22	38.67	61.33	7109.05	7.89	60.89	39.11
	Average		0.67	61.02	38.98		3.73	39.56	60.44		11.45	59.92	40.08

Table B.13: 76 Node Solutions for standard formulations.

# Nodes	Ins.	SSP-R				DSP-R				MSP-R			
		Obj.	CPU	Cable %	Splitter %	Obj.	CPU	Cable %	Splitter %	Obj.	CPU	Cable %	Splitter %
76	1	6790.9	6.13	57.59	42.41	8367.81	30.8	35.89	64.11	6790.9	22.37	57.59	42.41
76	2	6788.32	8.83	63.0	37.0	7858.99	40.92	37.96	62.04	6735.78	158.95	59.31	40.69
76	3	6109.31	7.23	60.09	39.91	7365.37	21.28	34.88	65.12	6099.28	72.73	58.94	41.06
76	4	6491.29	7.55	63.96	36.04	7571.08	24.35	38.16	61.84	6447.31	72.34	62.29	37.71
76	5	6664.88	6.02	61.52	38.48	8059.69	69.39	35.0	65.0	6643.72	19.07	60.1	39.9
76	6	6642.64	13.39	58.59	41.41	8135.26	66.81	32.8	67.2	6642.64	219.83	58.59	41.41
76	7	6851.81	7.9	61.54	38.46	8175.18	16.53	35.49	64.51	6851.81	60.3	61.54	38.46
76	8	6497.7	8.86	61.0	39.0	7780.64	72.07	36.66	63.34	6497.7	82.09	61.0	39.0
76	9	6677.27	7.07	61.52	38.48	7945.4	53.33	35.09	64.91	6667.49	45.15	61.13	38.87
76	10	6871.79	6.74	61.63	38.37	8025.22	46.36	35.62	64.38	6871.79	103.5	61.63	38.37
76	11	7036.95	17.49	61.76	38.24	8274.65	109.4	35.32	64.68	6980.32	178.42	59.82	40.18
76	12	6955.74	6.8	60.78	39.22	8351.25	42.14	34.67	65.33	6946.82	45.6	59.18	40.82
76	13	6736.4	6.87	60.33	39.67	8026.85	23.13	34.0	66.0	6729.35	43.07	57.9	42.1
76	14	6447.1	10.32	62.28	37.72	7644.37	47.81	35.73	64.27	6423.93	81.92	59.51	40.49
76	15	6597.09	11.93	63.69	36.31	7627.92	53.09	38.52	61.48	6587.61	451.89	62.78	37.22
76	16	6835.34	7.25	62.17	37.83	8095.39	39.44	34.7	65.3	6808.08	36.36	61.66	38.34
76	17	6407.19	6.15	62.12	37.88	7410.64	22.14	39.36	60.64	6345.89	45.09	59.99	40.01
76	18	6987.01	16.53	60.29	39.71	8424.07	153.31	34.28	65.72	6958.11	127.52	58.07	41.93
76	19	6760.87	5.28	61.28	38.72	8057.98	59.83	38.55	61.45	6760.87	42.93	61.28	38.72
76	20	6758.21	9.18	61.58	38.42	8013.31	62.21	35.31	64.69	6708.08	61.09	58.1	41.9
	Average		8.88	61.34	38.66		52.72	35.9	64.1		98.51	60.02	39.98

Table B.14: 76 Node Solutions for Type-R formulations.

# Nodes	Ins.	SSP				DSP				MSP			
		Obj.	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**
101	1	8640.95	1.39	59.26	40.74	11631.69	8.85	35.35	64.65	8499.02	16.87	55.05	44.95
101	2	8542.24	1.36	58.79	41.21	11294.52	5.33	36.43	63.57	8352.6	14.62	56.9	43.1
101	3	8607.07	2.39	57.24	42.76	11261.46	8.32	36.42	63.58	8503.92	33.17	56.73	43.27
101	4	8697.32	0.66	60.45	39.55	11372.35	5.33	38.8	61.2	8611.4	14.24	59.36	40.64
101	5	8642.12	0.99	59.27	40.73	11609.42	9.09	36.0	64.0	8560.03	12.19	57.24	42.76
101	6	9027.88	0.85	60.12	39.88	12130.54	7.41	36.94	63.06	8960.53	57.39	60.05	39.95
101	7	9760.69	2.58	57.38	42.62	13251.85	9.13	36.46	63.54	9621.9	20.71	56.45	43.55
101	8	8347.69	0.66	62.62	37.38	10874.65	8.17	40.69	59.31	8213.11	16.64	61.52	38.48
101	9	9480.68	0.78	58.65	41.35	13022.26	8.79	37.18	62.82	9432.98	9.74	57.38	42.62
101	10	8711.19	3.09	60.51	39.49	11626.13	34.21	36.87	63.13	8626.39	126.59	58.85	41.15
101	11	9048.73	1.74	60.22	39.78	12265.04	6.93	36.57	63.43	8943.7	12.83	57.96	42.04
101	12	9170.23	2.71	57.25	42.75	12333.75	80.93	36.19	63.81	9078.25	137.32	55.83	44.17
101	13	8812.55	0.86	58.24	41.76	11983.61	7.55	36.83	63.17	8756.45	21.92	57.52	42.48
101	14	8774.55	0.59	57.15	42.85	11804.71	5.47	35.87	64.13	8636.49	6.17	56.93	43.07
101	15	8806.85	0.9	59.12	40.88	11883.0	7.78	37.47	62.53	8721.23	13.0	58.95	41.05
101	16	8901.11	2.27	58.66	41.34	11930.13	18.7	37.55	62.45	8815.2	60.83	58.14	41.86
101	17	8647.7	0.9	57.45	42.55	11722.6	10.62	36.53	63.47	8549.35	13.84	57.54	42.46
101	18	8612.51	0.7	56.34	43.66	12083.48	10.33	34.54	65.46	8598.02	13.37	56.5	43.5
101	19	8567.54	0.93	57.98	42.02	11535.2	10.42	37.76	62.24	8519.52	26.97	57.16	42.84
101	20	9108.9	0.92	59.6	40.4	12040.84	6.97	37.96	62.04	9002.07	13.75	57.01	42.99
	Average		1.36	58.82	41.18		13.52	36.92	63.08		32.11	57.65	42.35

Table B.15: 101 Node Solutions for standard formulations.

# Nodes	Ins.	SSP-R				DSP-R				MSP-R			
		Obj.	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**	**Obj.**	**CPU**	**Cable %**	**Splitter %**
101	1	8068.11	34.09	58.93	41.07	9986.01	545.8	32.06	67.94	8071.69	(0.49%)	58.51	41.49
101	2	7837.05	16.92	59.58	40.42	9571.05	44.31	32.26	67.74	7837.05	49.48	59.58	40.42
101	3	7831.55	15.62	58.38	41.62	9680.77	264.2	33.52	66.48	7831.55	108.69	58.38	41.62
101	4	7926.02	19.36	59.89	40.11	9702.95	100.09	34.84	65.16	7921.8	299.87	59.42	40.58
101	5	8084.24	21.89	59.35	40.65	9866.81	246.07	32.75	67.25	8084.24	560.77	59.35	40.65
101	6	8508.72	117.37	59.12	40.88	10372.63	3500.11	33.3	66.7	8499.33	(0.31%)	57.77	42.23
101	7	9019.65	26.98	58.03	41.97	11133.33	133.11	30.67	69.33	9019.65	254.45	58.03	41.97
101	8	7865.1	34.12	62.22	37.78	9197.45	80.15	35.57	64.43	7826.73	1785.24	59.35	40.65
101	9	8864.13	20.44	58.12	41.88	10964.51	169.97	31.53	68.47	8861.64	273.66	56.96	43.04
101	10	8085.76	57.77	59.85	40.15	9888.58	2011.78	33.31	66.69	8071.86	(0.36%)	58.69	41.31
101	11	8493.55	21.51	59.42	40.58	10233.92	134.96	31.67	68.33	8491.78	308.54	58.25	41.75
101	12	8686.39	304.16	59.11	40.89	10468.03	(0.07%)	31.32	68.68	8621.61	(0.39%)	57.01	42.99
101	13	8315.42	19.99	58.4	41.6	10102.91	900.34	31.87	68.13	8266.12	344.72	57.29	42.71
101	14	8346.52	15.79	59.0	41.0	10111.96	651.68	33.1	66.9	8330.35	320.53	57.76	42.24
101	15	8192.99	20.34	58.99	41.01	9824.46	74.92	31.78	68.22	8192.99	140.05	58.99	41.01
101	16	8373.99	140.51	60.53	39.47	10061.68	3395.15	32.78	67.22	8384.93	(0.94%)	60.56	39.44
101	17	8065.99	39.78	58.22	41.78	9925.74	976.2	32.22	67.78	8065.99	854.69	58.22	41.78
101	18	8234.17	15.49	57.85	42.15	10234.22	111.33	31.26	68.74	8212.27	174.83	56.89	43.11
101	19	7998.05	15.43	59.61	40.39	9715.33	55.22	33.07	66.93	7998.05	107.13	59.61	40.39
101	20	8315.68	14.58	59.54	40.46	10200.06	74.94	33.12	66.88	8315.68	45.51	59.54	40.46
	Average		48.61	59.21	40.79		853.52	32.6	67.4		1181.41	58.51	41.49

Table B.16: 101 Node Solutions for Type-R formulations. Percentages under the “CPU” columns represent the percentage optimality gap where the instance could not be solved within the time limit of one hour.