COMP 395 -2 Deep Learning: Partial Derivatives and Gradients Practice

Name: ______________________________ Date: ________________

Part A: Single Partial Derivatives

For each function, compute only the partial derivative indicated.

1. Let $f(x, y) = 4x^3 - 7y^2 + 5xy$

Find $\dfrac{\partial f}{\partial x}$

2. Let $g(x, y) = x^2 \ln(y) + 3y^4$

Find $\dfrac{\partial g}{\partial y}$

3. Let $h(x, y) = \sin(x) + y\cos(x) + y^3$

Find $\dfrac{\partial h}{\partial x}$

Part B: Both Partial Derivatives and Geometric Interpretation

4. Let $f(x, y) = x^2 + 4y^2$

This function describes a surface in 3D space (an elliptic paraboloid).

(a) Find $\dfrac{\partial f}{\partial x}$

(b) Find $\dfrac{\partial f}{\partial y}$

(c) Evaluate both partial derivatives at the point $(1, 2)$.

(d) **Geometric interpretation:** In your own words, explain what $\left.\dfrac{\partial f}{\partial x}\right|_{(1,2)}$ tells you about the surface at the point $(1, 2, f(1, 2))$.

Hint: Imagine slicing the surface with a plane where $y = 2$ (constant). What does the partial derivative describe about that slice?

Part C: The Gradient Vector

5. Let $f(x_1, x_2, x_3, x_4) = (x_1 - 2)^2 + (x_2 + 1)^3 + e^{2x_3} + x_4^2$

(a) Find all four partial derivatives:

$$\frac{\partial f}{\partial x_1} = \qquad \textit{(chain rule)}$$

$$\frac{\partial f}{\partial x_2} = \qquad \textit{(chain rule)}$$

$$\frac{\partial f}{\partial x_3} = \qquad \textit{(chain rule)}$$

$$\frac{\partial f}{\partial x_4} =$$

(b) Write the gradient vector ∇f as a column vector (in terms of x_1, x_2, x_3, x_4):

Hint: The gradient collects all partial derivatives into one vector.

$$\nabla f = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

(c) Evaluate the gradient at the point $(3, 0, 0, 2)$:

$$\nabla f\big|_{(3,0,0,2)} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

(d) **Interpretation:** Answer the following about the gradient you computed:

i. The gradient vector points in the direction of ________________________________.
Hint: If you were standing on the surface and wanted f to increase as fast as possible, which way would you walk?

ii. If you wanted f to *decrease* as fast as possible, you would move in the direction of ____________________.

iii. The *magnitude* of the gradient, $\|\nabla f\|$, tells you ________________________________.
Hint: A larger magnitude means the surface is changing more rapidly. What does that correspond to physically?