Report Plan

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1 Background

- Definition of IVPs (standard 1st order form).
- Where they come from (modelling natural phenomena).
- Reducing nth order IVPs to standard (vector) form.
- How evolutionary problems modelled by PDEs can be reduced to discretised IVPs.

2 Numerical Methods

- One-step methods, explicit vs implicit.
- Local truncation error and the order of a method.
- Stability: region of stability, A-stability and L-stability.
- Runge-Kutta methods.
- Linear multi-step methods: Adams, BDF, Nystrom.
- Predictor corrector methods and pairings.
- Variable time-mesh.

3 Software Design

- Choice of python and numpy.
- Why choose to use object-oriented framework (future considerations).
- Briefly discuss design using UML.
- Difficulties that this presented in development.
- Graphing results (very brief).

4 Analysis of Numerical Methods

- Use scalar examples to demonstrate stability, stiffness and natural characteristics of each method.
  - ⋆ Non-stiff example, showing Forward Euler method vs Adams-Bashforth method with same step-size graph Local truncation errors too.
  - ⋆ Stiff example, showing Forward Euler vs Backward Euler with multiple step-sizes (emphasising region of stability).
  - ⋆ Stiff example, similarly for Adams-Bashforth vs Adams-Moulton.

- ⋆ Stiff example, show Adams-Moulton vs BDF2 graph local truncation errors.

- Showing comparison of predictor-corrector pairings with fixed mesh graph local truncation errors.
  - ⋆ Plot graph of step-sizes against local-truncation errors.
- Showing comparison of predictor-corrector pairings with variable mesh.
  - ⋆ Demonstrate how milnes device is used with set tol and its impact on accuracy and stability of methods.

5 Case Studies

- Single pendulum (no damping-simple harmonic motion).
  - ⋆ Demonstrate natural damping feature of fixed step Forward-Backward Euler predictor-corrector, vs Adams-Bashforth-Moulton-2 predictor corrector (no damping).
- Damped single pendulum.
- Damped forced pendulum.
- Double pendulum (chaotic motion).

6 Future Development

- Choice of step-size update function and local truncation error estimate in predictor-corrector (limitations to Milnes device).
- Refactoring code to use Generalised LMMs and k-step BDF methods with variable coefficients so that the order of methods can be updated on-the-fly dependent on local truncation error estimates.
- Refactoring code to not store the entirety of results in memory automated garbage collection or saving to disc.