Theory and Phenomena of Superfluid Helium

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Abstract

This report details the theory and applications of superfluid helium-4. First, the quantum mechanical nature of superfluid behavior is described. Relevant concepts such as elementary excitations, Bose-Einstein condensation, and quantized vorticity are defined and used to explain superfluid phenomena. The two-fluid model is described extensively as a tool for conceptualizing the apparently contradictory behavior of superfluids in laboratory experiments. Ramifications of the two-fluid mode, such as second sound, are also included. Finally, these concepts are applied to explain some modern applications of superfluids and superfluid concepts.

Keywords: superfluid; quantum; excitation; condensate; vortex

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1 Introduction

The study of superfluid behavior began in the early 20th century when helium was first condensed into liquid form by the Dutch physicist Kammerlingh Onnes in 1908 [11]. Continued research in the following decades soon found that liquefied helium held abnormal properties. Independently in 1938, physicists Pyotr Kapitza, John Allen, and Don Misener discovered that helium-4 had become a different fluid at temperatures less than 2.17 Kelvin. This new state exhibited both inviscid flow and the existence of quantized vortices, leading to what is now known as a superfluid [2]. It was postulated shortly after by German physicist Fritz London that superfluidity could have a connection to superconductivity, suggesting that the electron gas in these metals demonstrate similar properties [11]. This then lead to the suggestion that superfluidity was a macroscopic representation of quantum mechanics, where the true significance of this idea did not emerge until the 1950's.

Much of this paper will draw upon research into liquid helium to describe superfluid behavior. This is because superfluid behavior requires extremely low temperatures to be observed. Specifically, the superfluid properties of helium-4 will be discussed. As the only element to remain a liquid down to absolute zero, helium-4 is the most common example of a superfluid. No other element will remain a liquid at the temperatures required for superfluid behavior to arise [3].The goal of this review is to describe the salient features of superfluidity and explain how these features arise. Exotic states of matter, such as a Fermi gas, can also exhibit superfluid properties [10]. Such states are highly complex and were not deemed relevant to this basic analysis of superfluid theory and behavior.

2 The Origin of Superfluidity

To explain why helium is special, recall the microscopic behavior of matter. The internal energy of matter at room temperature is almost entirely contained in the translational and rotational energy of atoms and molecules. At absolute zero, the state of minimum internal energy, atomic motion cannot completely stop. The absence of motion would imply a perfectly defined momentum, and such a condition is a violation of the Heisenberg uncertainty principle. Because zero internal energy is not a realistic concept, the term "zero-point" energy was coined to describe the effective "zero" from which all higher energy states are referenced. This concept is essential for understanding the unique properties of helium, since it is this zero point motion that enables helium to remain a liquid at arbitrarily low temperatures.

Elementary chemistry defines an ideal gas as one in which inter-molecular (or inter-atomic) forces can be ignored. Only at low temperatures ($<$30 K), or high pressures ($>$1000 atm), does this assumption need to be relaxed [1]. When the temperature is sufficiently low, attractive inter-molecular forces begin to have a significant influence on atomic motion. In most elements, inter-atomic forces overcome the kinetic energy of the atoms when the temperature drops below a certain value. At this point, a phase change from gas to liquid occurs. Lower the temperature further and the inter-atomic forces are strong enough to form a more rigid structure, which is the phase transition from liquid to solid. This is not the case in helium [3]. From the phase diagram below, it is apparent that two states of the liquid phase exist: helium I, which has the conventional properties of a liquid, and helium II, which has superfluid properties. These two phases are separated by what is known as the λ point.

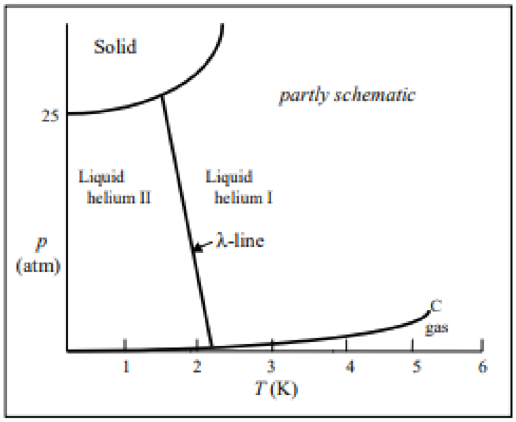

Figure 1: Phase diagram of helium-4 [13]

In Figure 1, it can be seen that for pressures below about 25 atm, even at the absolute zero of temperature, the liquid phrase remains. Because the inter-atomic forces are so weak, even the zero-point energy is enough to overcome them. The other inert gasses have larger atomic masses, which prevents the zero point motion from being sufficient to overcome inter-atomic forces. In addition, the Third Law of Thermodynamics states that any system in equilibrium must not contain any entropy at absolute zero. As a result, this system must be completely ordered. For bosonic matter at absolute zero, this state of complete order means that all particles are at the lowest possible energy level. It is this perfectly ordered state that gives rise to superfluidity [12].

3 Quantization and Energy Levels

A brief discussion of fundamental physics must be discussed before the origins of superfluid behavior can be treated. A key concept in quantum mechanics is the idea of quantization: that state variables can only take on a discrete set of values. A key result is that the energy of a molecule or atom can only exist at set values. Depending on the species or type of particle, there can only be a set number of particles that are allowed at a given energy level. Particles with non-integer quantum spin (a type of angular momentum), called fermions, follow this idea that an energy level can be ”filled up”. This rule is called the Pauli exclusion principle. Particles with integer spins are called bosons and do not follow this rule. This has the important consequence that any number of bosons can be at a given energy level. For very low temperatures, matter made of fermions cannot have many particles at the zero-point energy because of the Pauli exclusion principle. By contrast, matter made of bosons can potentially have all particles at the zero-point energy. It is for this reason that superfluids can only be made of bosons. No matter how cold you go, fermions simply cannot get enough particles into low energy states to manifest superfluid behavior [10]. Why is it so important for superfluid behavior that most particles occupy the lowest energy state? Because, as the energy gets lower, the gaps between energy levels grow larger. If all particles are at the lowest excitation, it will not be a trivial task to raise them to a higher energy state. If the particles cannot be raised to a higher energy level, then the fluid cannot dissipate energy.

3.1 Elementary Excitations

The state of an individual particle is described in quantum mechanics by the Schrdinger equation. In theory, the description of any substance can be calculated by solving this equation for all of its constituent particles. In the case of a macroscopic system, it is practically impossible to solve the Schrdinger equation for every particle simultaneously [3]. While the energy levels discussed earlier are illustrative of the principles of

quantization, that discussion was with regard to individual atoms. Practical calculations for macroscopic systems will require a more coarse grained descriptions. This is where the concept of elementary excitations becomes important. Just as the energy of an atom is quantized, only permitting certain energy levels, so too can energy states of *groups* of particles be quantized. In the case of an atom, the energy level is for a given particle. In what sense can a group of particles be treated like a single particle?
All quantum mechanical particles can be accurately described as waves via the DeBroglie wavelength. Electrons, protons, and even heavy nuclei can all be meaningfully described as waves in a field. Using this principle, excitations in groups of particles can be treated as particles even though they are waves. The energy of these waves is quantized in the same sense that atomic energy levels are quantized. In the case of mechanical compression waves, these ”elementary excitations” are called phonons. By definition, phonons only refer to very low energy compression waves. Higher energy mechanical excitations are referred to as rotons [6] [3] [10]. Using the wave-particle duality discussed above, the phonon and roton can be usefully treated as particles, referred to as quasiparticles. These quasiparticles behave in the same way as molecules in an ideal gas. This will be a key idea in the two-fluid model, where the ”superfluid component” refers to the zero-point energy helium-4, while the ”normal” component is treated as an ideal gas made up of elementary excitations (phonons and rotons) that is capable of diffusing energy. This idea will be developed further in Section 4.
These two modes of energetic excitation lead to the idea of two superimposed heat capacities: one for the phonons and one for the rotons. With two dominant modes of excitation, a given input of energy can be stored in either phonons or rotons. Temperature dictates this distribution of energy between modes. The amount of energy that is bound in phonons scales with T^4, while energy in rotons scales by $e^{-E_1/(kt)}$, where k is Boltzmann's constant and E_1 is the lowest possible roton energy. It can be shown that the roton part of the heat capacity plays a dominant role all the way down to about 1 Kelvin. Below this temperature, the roton contribution drops off quickly. This manifests as a steep drop in heat capacity below 1 K [6], as can be seen in Figure 2 where the heat capacity of liquid helium-4 is shown.

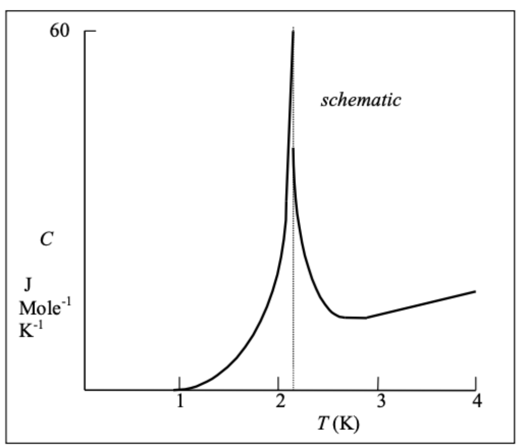

Figure 2: The heat capacity of liquid helium-4 [13]

At temperatures above 1 K, there is sufficient energy (E_1) to excite the higher energy roton states. The resultant increase in available energy states leads to an observed increase in specific heat above 1 K [5, 3]. Figure 2 demonstrates that the transition into the superfluid state induces a spike in the heat capacity where at the transition point it approaches infinity. Specific heat capacities that exhibit this type of phenomena are common in systems with an order-disorder transition, including superconductors. This further supports the theory that superfluid behavior is closely correlated to a quantum mechanical ordering.

4 The Two Fluid Model

In a landmark paper by Lev Landau in 1941, *Theory of the Superfluidity of Helium II* [6], helium II is described with a two-fluid model, which is regarded as a purely phenomenological description. It is emphasized that there are not *actually* two fluids present. In this model, the superfluid phase can be regarded as the superposition of two fluids, each with independent velocity fields. The normal fluid component carries all of the thermal energy and entropy in the system, via elementary excitations, while the superfluid component carries no thermal energy and is inviscid [10, 3]. Each of these fluids carry their own densities, ρ_n and ρ_s, respectfully, and each vary with temperature uniquely, as shown in Figure 3. Though a pressure gradient will generally drive both fluids along the same path, a temperature gradient will affect each fluid independently. As can be seen in Figure 3, an increase in temperature will increase the density of the normal fluid yet have the opposite effect on the superfluid component. As a result, the superfluid liquid is driven towards the higher temperature while the normal fluid flows in the opposite direction towards lower temperatures [12].
What is the physical meaning behind this two-fluid model? Because helium II will always have *some* excited phonon and/or roton states, the fluid can be divided into an excitation-free condensate and a "normal" fluid that contains all of the excitations [3]. Suppose that these excitations were set into motion at some arbitrary velocty v, leaving the rest of the fluid at rest. Given the energy-momentum properties of these excitations, the momentum density J_e associated with these excitations can be found and is presented in Equation 1.

$$J_e = \rho_e v < \rho v \tag{1}$$

This inequality is only valid for extremely low temperature domains, which happen to coincide with those under the lamda point. As a result, the drifting excitations carry an effective density that is lower than the total density, allowing them to travel without causing the entire fluid to drift alongside it. This independent motion of the excitations are known as the normal fluid component of the two-fluid model [12].

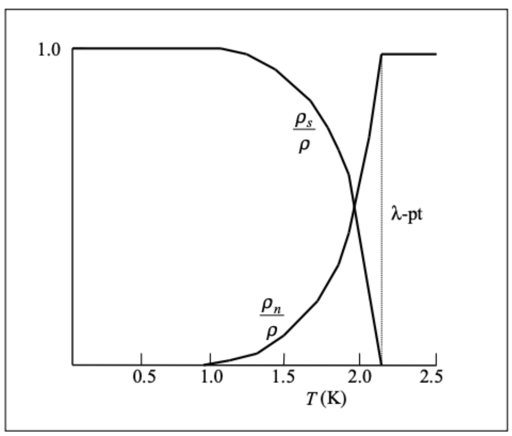

Figure 3: Temperature dependence of the normal and superfluid density components of the two-fluid model [13]

To understand the roots of the superfluid component, it is necessary to review a phase transition known in condensed matter physics as Bose-Einstein condensation (BEC). BEC describes the state of low-density dilute gasses made up of bosons, cooled to temperatures close to absolute zero [11]. At this state, the bosons become "condensed" into the lowest energy quantum state. As a result, microsopic quantum phenomena become apparent macroscopically. This ordering is what occurs below the lambda-point and is responsible for the state of the superfluidity component [3].
The existence of two independent fluids naturally leads to the conception of two sound modes of longitudinal wave propagation, which have been observed [5]. While it was never meant to describe two *actual*

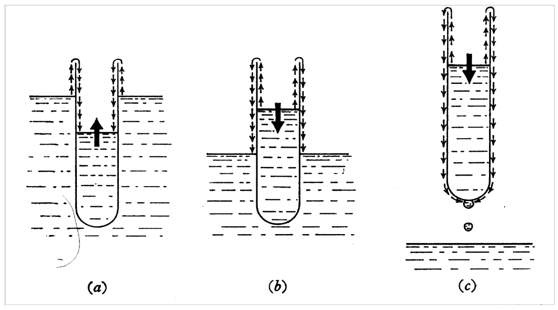

Figure 4: Due to the invicid property of the superfluid, it is able to flow through the film of liquid into the container [13]

fluids, such a description helps to explain apparently contradictory behavior in helium II. In a famous experiment conducted by Georgian physicist Elephter Andronikashvili, helium II was placed in a torsion pendulum where the period of oscillation was measured as a function of temperature. He found that normal fluid component was coupled to the disc system and contributed to the moment of inertia, while the superfluid component did not [10]. Another example of this dual fluid behavior can be seen when superfluid helium is passed through extremely narrow capillaries.

The most well-known feature of superfluid behavior is the ability to flow without resistance, or viscosity. This phenomenon can now be qualitatively understood with the concepts described so far. Viscosity arises because velocity gradients within the fluid are diffused as thermal energy. Inviscid behavior is most evident in capillary flow. At low velocities, the interaction between the superfluid part and the capillary wall is insufficient to result in energy dissipation. In general, the momentum of slowly flowing superfluid helium is insufficient to excite the higher energy roton energy states. Only at fluid velocities $v < (2\nabla/\mu)^{\frac{1}{2}}$, can rotons be excited. Velocities that satisfy $v < c$, where c is the acoustic speed, are required to excite phonon modes. Only with sufficient velocity to excite phonon and/or roton modes can viscous dissipation take place. This is the origin of the idea of a critical velocity. Above this critical velocity, rotons do not form either. Rather, the energy dissipation occurs through turbulence. Vortices are also quantized. The quantization of vortices was first proposed by Lars Osnager in 1949 [8]. Below the critical velocity, the energy to create the lowest momentum vortex is not available. Thus, with all available modes of energy dissipation unavailable, viscous dissipation does not occur.

Below the critical velocity, momentum can therefore not be dissipated as heat. At low velocities, this lack of bulk momentum transfer to internal excitations results in inviscid behavior. Hence, the superfluid part will be able to flow without friction due to the absence of viscous forces. This can be demonstrated even within flow in a narrow channel, where the normal fluid component cannot pass due to the presence of viscous forces [5]. This can be seen in what is known as ”film flow” where any solid surface in contact with the liquid is covered in a thin film about 30 nm thick, as a result of van der Walls forces. Normal fluid components cannot flow through this film very far, as it dissipates energy, losing momentum, while the superfluids properties allows it to flow through the film. A demonstration of this phenomena can be observed in Figure 4 [6], further showing how two-fluid behavior can be seen.

One more example of this two-fluid behavior can be seen with heat transport in superfluid helium. As mentioned before in Figure 3, the two components of the fluid will move in opposing directions when introduced to a temperature gradient. As only the normal component carries thermal energy, this can lead to an efficient means of thermal transport and has allowed researchers to isolate extremely cold temperatures [5].

5 Quantized Phenomena in Superfluid Helium

Since discovery, it has been proposed that the property of superlfuidity is quantum in nature. This nature was found to be strongly related to an area of condensed matter physics known as Bose-Einstein condensation. To understand this we must consider an ideal gas consisting of Bose particles , i.e. are quantum-mechanically indistinguishable but are not bound to how many particles can be in one physical state. If one were to determine the quantum state distribution of the particles an interesting property can be found: under a critical temperature a percentage of the particles are "condensed" into their lowest quantum state [13]. It has empirically been shown that the same phenomenon occurs within liquid superfluid Helium.
This can be mathematically demonstrated. Condensed atoms locked within a single quantum state are known as a condensate and can be represented using what is known as a condensate wave function (CWF) represented as:

$$\Psi = \Psi_0 \exp \frac{imvx}{h} \tag{2}$$

where mv is the respective momentum, h is the Planck constant. The local velocity of these condensed atoms is represented as:

$$\vec{v}_s = \frac{\hbar}{m} \nabla \phi \tag{3}$$

It is this velocity that is used to determine the velocity of the superfluid component of Helium within the two-fluid model.
It can now be shown that the microscopic occupation of a single quantum state of Bose-condensed helium can give rise to the macroscopic effects presented in superfluids. If one were to determine the local vorticity by taking the curl of the velocity component we obtain:

$$\nabla \times \vec{v}_s = \frac{\hbar}{m} \nabla \times \nabla \phi \tag{4}$$

This equation possesses the curl of a gradient and is therefore zero, demonstrating that the superfluid component is irrotational. Though this is zero, hydrodynamic circulation can present a finite value and is measured with:

$$\kappa = \oint_C = v_s \cdot dr \tag{5}$$

Substituting the superfluid component velocity equation into the hydrodynamic circulation yields:

$$\kappa = \frac{h}{m} \oint_C \nabla S \cdot dr = n \frac{2\pi h}{m} \tag{6}$$

Where n is an integer that must be used to satisfy the CWF condition of being single valued. This equation effectively demonstrated that superfluid circulation must be quantizied in the units of $\frac{2\pi h}{m}$. This circulation must be of macroscopic magnitude and therefore this provides the greatest evidence supporting that superfluids macroscopically demonstrates quantum mechanisms. Fundamentally this arises from the quantum conservation of angular momentum.
The relation with quantum phenomena can further be supported. Considering the two-fluid model, the normal component of a superfluid does experiences a friction, losing angular momentum and will ultimately stop. Assuming this is in a stationary regime, the superfluid component wont dissipate any energy, and will therefore keep on rotating. Consider the continuity equation for the superfluid part

$$\frac{\partial \rho_s}{\partial t} + \nabla \vec{j} = 0 \tag{7}$$

where ρ_s is the superfluid component density, and $\vec{j} = \rho_s \vec{v}_s$ is the momentum per unit volume for the super fluid component. Since the density ρ_s remains constant over time $\frac{\partial \rho_s}{\partial t} = 0$, the other terms is also zero: $\nabla \vec{j} = 0$. Expressing $\vec{j}$ in terms of equation 3 we can state

$$\nabla^2 \phi = 0 \tag{8}$$

Due to the cylindrical symmetry, the Laplace equation above will reduce to

$$\frac{\partial^2 \phi}{\partial \varphi^2} = 0 \tag{9}$$

Which has the solution

$$\phi = n\varphi \tag{10}$$

where φ is the azimuthal angle and n a constant factor from integration. This constant factor is not arbitrary by the fact that the wave function is proportional to $e^{i\phi}$ and must be singled valued, meaning that $\psi(\phi)$ must have a unique value for any given $\phi = n\varphi$. This condition confines the constant factor n to be a non zero integer. Considering cylindrical symmetry, the velocity of the superfluid in this coordinate system can be calculated as

$$\vec{v}_s = \frac{n\hbar}{m} \nabla \varphi = \frac{n\hbar\hat{\varphi}}{mr} \tag{11}$$

Integrating the vortex kinetic energy over the radius of the vortex will yield the energy of this type of vortex per unit length. This will allow for further investigation of its properties

$$E = \int_a^L 2\pi r \frac{1}{2} \rho_s v_s^2 dr = \frac{\pi \rho_s \hbar^2 n^2}{m^2} \ln \frac{L}{a} \tag{12}$$

where the integration bound L is the transverse length of the vortex vessel, and a a microscopic length of the distances between particles. This result shows that when a circulation is fixed, a vertex with $n > 1$ is unstable and will decay into n number of vortices with $n = 1$. During this process the energy decreaces since it goes from a factor of n^2 to n. Thereby vortices of factor $n = \pm 1$ are only taken into consideration, and are appropriately refered to as vortices and anti-vortices.

It has been proposed that the quanta of vorticity was given by $\hbar/m$, where $\hbar$ is Planck's constant and m is the mass of a single molecule. The energy of such a vortex line, per unit length, is proportional by a constant factor n^2. The quantizitation of these vorticies again supports the theory that this is a macroscopic demonstration of quantum effects [10].

6 Applications

Because superfluid helium-II has such remarkable properties, the areas in which it is applicable are wide and complex, and many applications are still to be discovered, for it is indeed, an abstract field. The major property of superfluid helium is its ability to remain liquid at temperatures close to $0K$. As shown previously in Figure 2, it also exhibits greatly elevated thermal conductivity at about $1.9 - 2.4$ K. These two properties result in a very effective cryogenic coolant, to the extent that the technique has been industrialized and operated with high reliability. Further the density is very low, about $145 kg/m^3$ (at λ-point), and its two component fluid behaviour of one normal fluid and one superfluid part, which has been demonstrated above will inhibit some rather counter intuitive heat transport; the superfluid part flows to the warmer region and the normal fluid towards the colder region. This low temperature operation is used to cool high field magnets made of technical superconductor alloy metals. Superconductors are materials where electrical current can flow with zero resistance. Superconductors are unable to resist electrical currents for the same reason that superfluids cannot dissipate momentum as heat. The leading material in the field of large scale superconductors is the metal Niobium Titanium (NbTi) because of relative low production cost, its mechanical strength and the fact that it can be bent and shaped without losing its properties. The lower operating temperature of these alloys by the cooling of superfluid helium improves the performance in means of the current density $[A/mm^2]$ versus the magnetic field plane $[T]$. NbTi magnets have been built operating at peak flux densities from under $1T$ to about $8-9T$ at a temperature of $4.2K$. Thus this technique is now utilized in fields reliant on high field magnetic apparatus, both in the commercial sector for Magnetic resonance imaging (MRI), and in laboratories and test facilities for condensed-matter physics, magnetic confinement fusion nuclear magnetic resonance and of course particle accelerators and colliders [9] [7] [4].

7 Conclusion

Ever since superfluidity found its origins in the mid 20th century, vast amounts of research have been conducted to understand the fundamental principles that govern this peculiar state of matter. To describe the inviscid behavior of helium-4 at these extremely low temperatures, a two-fluid model was developed. According to this model, a superfluid can be described as the superposition of a normal fluid component and a superfluid component, each with their own respective velocity field and density. This model is useful for describing the sometimes contradictory behavior of superfluids. The elementary excitations that are fundamental to understanding this model are described and used to explain the well known inviscid behavior of superfluids. Finally, some applications of superfluid properties are described with respect to industrial applications. This paper provides a well rounded discussion of fundamental superfluid phenomena, connected to modern application.

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