Assignment 1
Discrete models

Finn Joel Bjervig[*], August Forsman[†] and Erik Turesson[‡]

1MA256 - Modeling Complex Systems,
Department of Mathematics,
Uppsala university, Sweden

April 12, 2021

[*]Electronic address: `jobj8920@student.uu.se`
[†]Electronic address: `aufo8456@student.uu.se`
[‡]Electronic address: `ertu2293@student.uu.se`

1 Introduction

The popularity is represented by two variables p_c and p_l which denotes the number of votes for each party. This is measured on a domain $[0, 1]$ where $0 \leq p_c \leq 1; 0 \leq p_l \leq 1; 0 \leq p_c + p_l \leq 1$. There is also a proportion of neutral voters that is defined by $1 - p_c - p_l = p_n$ i.e the people not voting for either the liberal or conservative party. We assume that between each polling, the rate of voter transition between two parties are proportional to the difference in popularity between the two parties in the previous polling.

2 Popular Opinion Dynamics A

At each new polling, calculate and updatethe popularity of each party by considering the six possible transitions between them. The transition rate K_p is a scaling constant that determines how fast the transitions between the parties are. For example, the transition from conservative to liberal ($c \to l$)between time-step $k-1$ and k is modeled as

$$p_{c\to l}^{k-1}(p_c^{k-1}, p_l^{k-1}) = \begin{cases} p_c^{k-1} K_p (p_l^{k-1} - p_c^{k-1}), & if\ p_l^{k-1} > p_c^{k-1} \\ 0 & otherwise \end{cases} \tag{1}$$

Subsequently, we update the proportions as

$$p_c^k = p_c^{k-1} + p_{l\to c}^{k-1} + p_{n\to c}^{k-1} - p_{c\to l}^{k-1} - p_{c\to n}^{k-1} \tag{2}$$

$$p_n^k = p_n^{k-1} + p_{c\to n}^{k-1} + p_{l\to n}^{k-1} - p_{n\to c}^{k-1} - p_{n\to l}^{k-1} \tag{3}$$

$$p_l^k = p_l^{k-1} + p_{c\to l}^{k-1} + p_{c\to l}^{k-1} - p_{l\to c}^{k-1} - p_{l\to n}^{k-1} \tag{4}$$

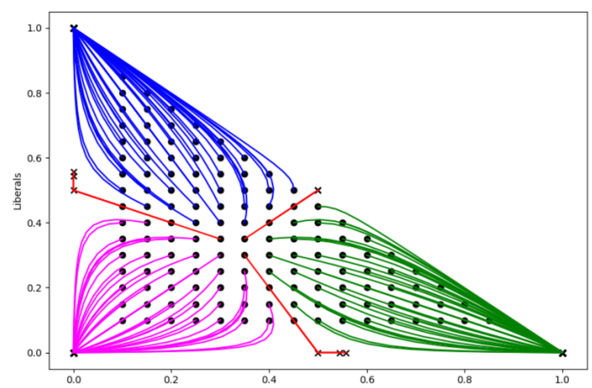

Figure 1: The phase space for varying initial conditions in *winner-takes-all* system. Each dot represents a different initial voter share for the conservative and liberal parties (with the remainder being neutrals), and each cross is the final solution. The basins of attraction for the neutral (cyan), liberal (blue), and conservative (green) majority attractors are colored in.

As seen in the figure 1 above there are attractors in $p_l,\ p_c,\ p_n = 0, 1$. Due to the setup of this system, it is inevitable that the party that is the most popular in the beginning, also ends up with all the votes. The basins of attraction are thus divided into the regions in the phase space corresponding to where each party is the largest. For example, $p_n > p_l, p_c$ defines the basin of attraction towards the attractor of $p_n = 1, p_c = 0, p_l = 0$. There are also attractors corresponding to unstable solutions at $(0.5, 0)$, $(0, 0.5)$, and $(0.5, 0.5)$, ie. when we have an even split between the largest parties, or at $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ where all parties are equally popular. Varying the value of K_p will not result in a different phase space in this scenario, as it will just give faster updates while still following the same paths.

2.1 Choice of strategy

The fate of the system is highly dependent on the initial popularity of a party. It is hard to choose a specific strategy since it is an example of a *winner-takes-all* system where the largest party wins the largest amount of votes. The model indicates that a party should always try to find new methods of gaining voters and try to be as large as possible before an impending polling/election.

3 Popular Opinion Dynamics B

In addition to the voter shifts from differing popularity in part A, we model the parties (i.e. the conservatives and the liberals but not the neutrals) running some advertising campaigns. For this, we assume that the effectiveness of the campaigns are inversely proportional to their current popularity. For example, the number of voters transitioning from liberal to conservative due to the conservative campaigns are modeled as

$$v_{l\to c}^{k-1} = p_l^{k-1} K_v \frac{1}{p_c^{k-1}} \tag{5}$$

where K_v is some constant related to the effectiveness of the campaigns. We then remodel the updates from equations 2, 3, 4 to include the campaigns as well

$$p_c^k = p_c^{k-1} + p_{l\to c}^{k-1} + p_{n\to c}^{k-1} - p_{c\to l}^{k-1} - p_{c\to n}^{k-1} + v_{l\to c}^{k-1} + v_{n\to c}^{k-1} - v_{c\to l}^{k-1} \tag{6}$$

$$p_n^k = p_n^{k-1} + p_{c\to n}^{k-1} + p_{l\to n}^{k-1} - p_{n\to c}^{k-1} - p_{n\to l}^{k-1} - v_{n\to c}^{k-1} - v_{n\to l}^{k-1} \tag{7}$$

$$p_l^k = p_l^{k-1} + p_{c\to l}^{k-1} + p_{c\to l}^{k-1} - p_{l\to c}^{k-1} - p_{l\to n}^{k-1} + v_{c\to l}^{k-1} + v_{n\to l}^{k-1} - v_{l\to c}^{k-1} \tag{8}$$

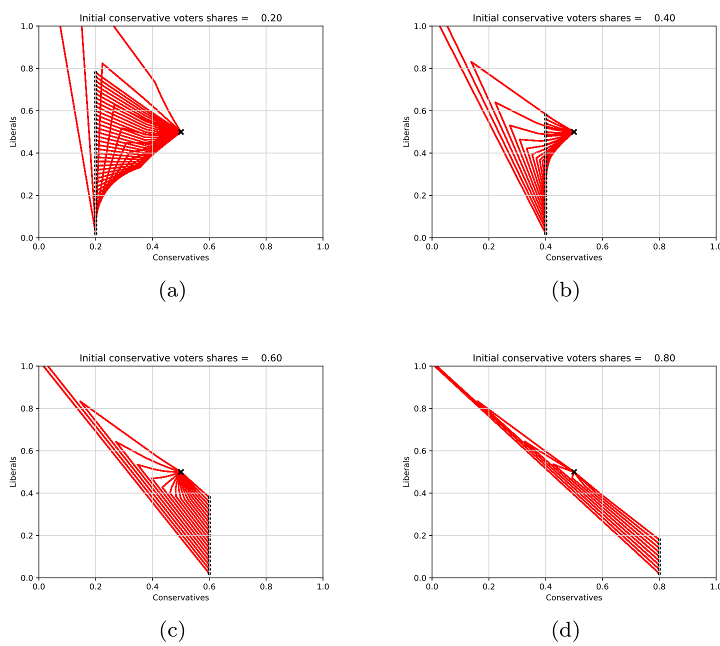

Figure 2: The campaign setup: dots represent the initial system, and the cross marks the attractor, where the system converges. The voter shares of the neutral party is not shown but can be calculated as $p_n = 1 - p_c - p_l$

It might be interesting to study how the campaign rate parameter K_v affects the outcomes. In figure 3 we present such scenarios. To begin with: when the rate is close to zero, the phase space tends to that of a system without campaigning. The dots represents the initial state, and the crosses marks the attractor, to where the system converges.

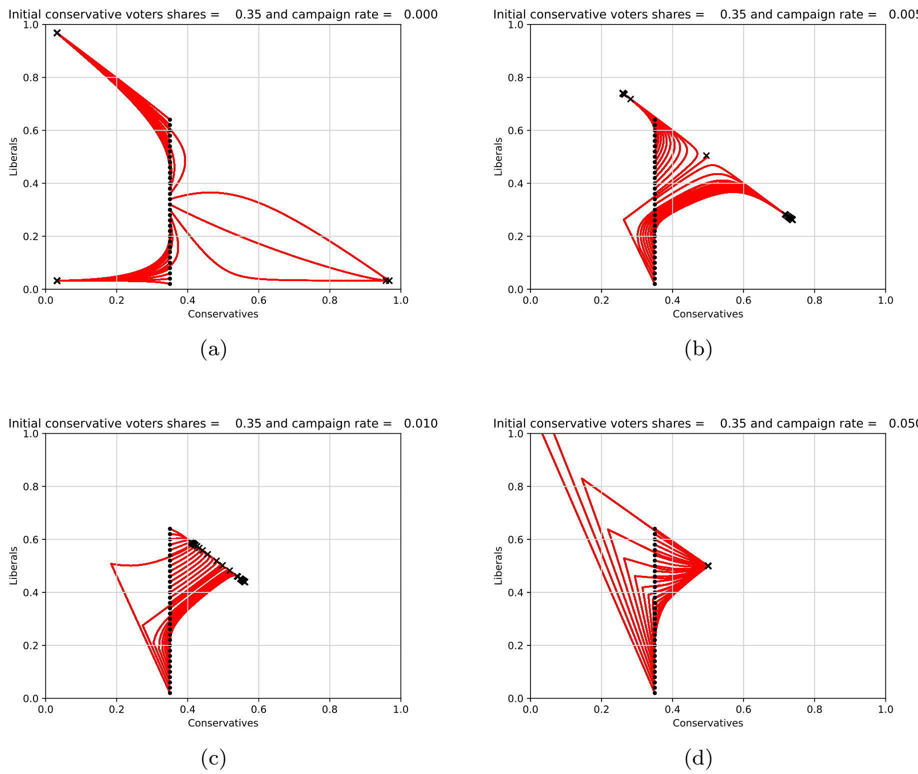

Figure 3: The phase space for the campaign setup with varying campaign rates. The voter shares of the neutral party is not shown but can be calculated as $p_n = 1 - p_c - p_l$

There are cases in figure 3 and figure 2 where the voter share is higher than 1 and therefore the curve is not shown in the plot. These are byproducts of numerical instabilities in the inverse campaign rates but do converge to the attractor in $(0.5, 0.5)$.