Damping of low pressure acoustic pressure w
isotropic newtonian fluids

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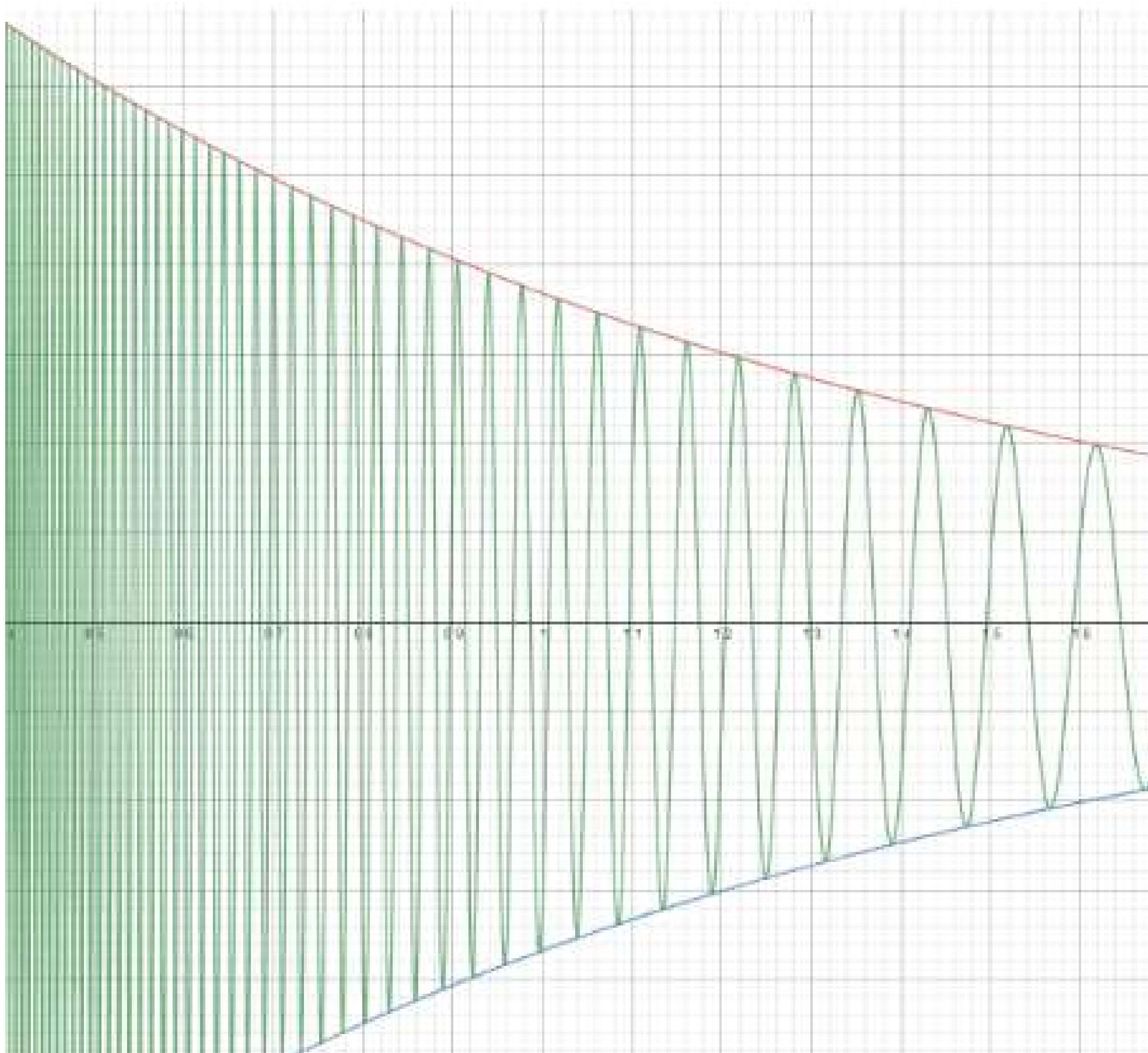


Chart of variables

ρ	Density
P	Pressure
v	Velocity as a scalar(one dimension)
$\underline{V}$	Velocity as a vector, underlined or bold letters are vectors.
λ	Volume viscosity
μ	Viscosity
∇	Eulerian gradient
∇_0	Lagrangian gradient(gradient with respect to material coordinates)
$\frac{D}{Dt}$	Material time derivative(derivative with respect to time holding X fixe
$\frac{\partial}{\partial t}$	Eulerian time derivative(derivative with respect to time holding x fixe
p,P	Lower case letters denote eulerian frame of reference, while upperc lagrangian frames.
t	Time

Introduction

This paper seeks to answer the question "how far can sound waves travel" when dispersi and what happens to the waves as they are damped? The acoustic wave equation is one o equations for modeling propagation of pressure waves. However, the derivation of the ac equation requires many strict assumptions in order to ensure its accuracy. One of the big the acoustic wave equation is its ability to describe dispersion via kirchhoff's formula[5] weakness is that the wave equation doesn't account for damping, it shows that a wave ca forever, which in a physical sense isn't possible since the waves are bound to lose energy due to physical processes.

This paper will answer the question above by figuring out how we can modify the wave include damping effects and restricting it to one dimension to not allow dispersion. This first developing a model based on physical intuition of pressure waves which will lead to equations. Secondly, the wave equation will be derived from the Lagrangian form while attention to the required assumptions and analyzing how they affect the applicability of t equation, which leads to the derivation of the damped wave equation. Thirdly, a solution wave equation will be derived and analyzed, showing what pressure waves look like as t damped. Finally, the main part of the question above will be answered by deriving a sim attenuation from our former solution, which shows how the maximum pressure of a puls time, showing how far at least a wave can travel.

Motivation/What is known about modeling pressure waves

There are two main types of pressure waves, shocks and sound waves. Sound waves are

One of the simplest models for sound wave propagation is just the acoustic wave equati

$-p_{tt} + c^2 \Delta p = 0$

$x \in R^n \qquad p(x,0) = \alpha(x)$

$p = (x, t) \qquad p_t(x,0) = \beta(x)$

The solution to this wave equation can be found for any initial conditions α and β with I
formula[5], which shows in higher dimensions waves will naturally disperse and decrea
However, the right handside to the equation is always equal to zero, meaning the wave e
include any form of damping, it models a wave free of any physical damping.

Commonly the field of acoustics models pressure waves by deriving PDEs from conser
Generally these PDEs come from low order taylor series approximations coming from c
relations and conservation laws. When pressure increases more terms are needed which
nonlinear terms. A nonlinear PDE derived by Rasmussen that shows how waves dampe
presented below.

[3] $\psi_{tt} - 2\psi_{xt}\psi_x + \psi_x\psi_x\psi_{xx} - b\psi_{xxt} = c^2\psi_{xx}$.

$\psi(x,0) = \mu(x)$, $\psi_t(x,0) = g(x)$ and $x \in (-\infty, \infty)$

$\psi_{tt} - c^2\psi_{xx} = 2\psi_{xt}\psi_x - b\psi_{xxt} - \psi_x\psi_x\psi_{xx}$. (canonical form)

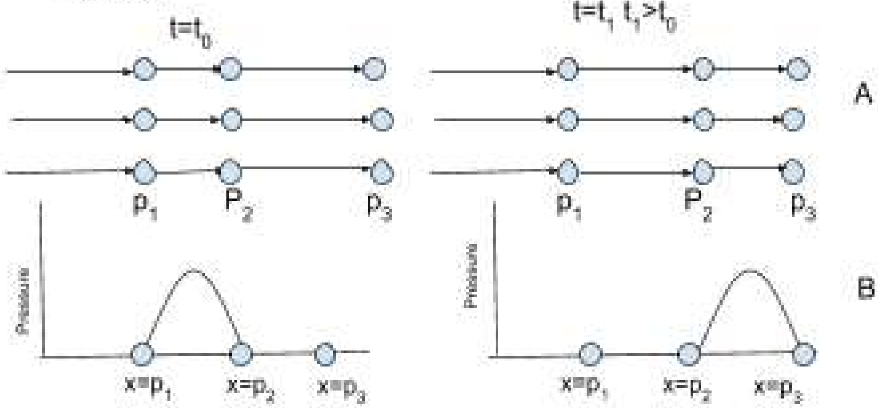
Where $\psi=\psi(x,t)$ is the velocity potential which is related to pressure and b is the diffusi

While there appears to be a great deal of research available on modeling sound wave at
and liquids, there is not a whole lot of information available on modeling sound attenua
model. Most of the research is concerned with local effects, such as how the waves red
smaller waves, rather than how far they can go before being weakened due to damping.
to the practicality of the problem in the introduction, most of the time dispersion greatl
to the point where their amplitude approaches zero very quickly. However, in one dime
cases this isn't possible, so our primary source of attenuation is damping. In the next se
begin forming a model for pressure wave propagation, based on physical intuition of lo

Basis for the problem that we're modeling

In this section a model will be developed that will lead into a Navier Stokes descriptio
pressure wave(sound wave). Since the question says a sound wave it is referring to a lo
or a compressive wave that is squeezing particles closer together. When thinking about
problem, these pressure waves will propagate as particles are pushed close together and
are stretched. As time passes the particles decompress and return to equilibrium causin
particles to become compressed and so forth until the pressure wave hits the end of the
two ways we can describe this process, we have model A and B below for a pressure p

Figure 0.1



shows all the particle's posi
B shows the pressure at a g
x=p. A is the lagrangian de
density as a function of pre
the eulerian description sho
function of density. Either
will work for modeling pre

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} + \frac{\partial p}{\partial x} = (\lambda + 2\mu)\, \frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$p = p(\rho)$$

We can map $p(\rho(t))$ to $\rho(p,t)$. Converting Eq 0.2 and Eq 0.3 to their lagrangian counterpar
and EqA.2 below. We can go in between models A and B, by definition of lagrangian and
coordinates. We also make stoke's assumption $\lambda = \frac{-2}{3}\mu$

$$\rho \frac{Dv}{Dt} + \frac{\partial p}{\partial x} = \frac{4\mu}{3} \frac{\partial^2 v}{\partial x^2}.$$

$$\frac{D\rho}{Dt} + \rho v_x = 0.$$

$$\rho(p) = \rho.$$

We also need to consider the assumptions necessary to apply Eq A.1 above.

The system behaves like a fluid, cannot support shear.
The system is near equilibrium.
Stoke's assumption.
The medium is an isotropic fluid.
Gravitational effects are negligible.

We now have a set of equations describing our model in figure 0.1. EqA.1 is the navier st
one dimension assuming negligible gravitational effects and Eq A.2 is the mass continuit

Deriving the wave equation from Navier Stokes

This section derives the wave equation from the lagrangian form of navier stokes, which Schreyer's[1] derivation based on the eulerian framework(which is method B above). Th is being derived in order to find a version that includes damping and to understand when will hold. The lagrangian framework is chosen over Eulerian in order to get a better und nonlinear terms that appear in the taylor series later on.

We start by rewriting the equation in terms of how far the density is displaced from a ref we assume $d\rho$ is small or that density fluctuations are small.

Assume $d\rho$ is small and v is low

Let $d\rho=\rho-\rho_0$

Substituting $\rho=d\rho+\rho_0$ into Eq A.1 we get

$$\frac{D(d\rho+\rho_0)}{Dt}+(d\rho+\rho_0)v_x=0$$

$$v_x=-\frac{1}{\rho_0}\frac{D(d\rho)}{Dt}.$$

where we neglect $d\rho\, v_x\cong 0$ by assumption 6.

Substituting $\rho=d\rho+\rho_0$ into Eqn A.2 we get

$$(d\rho+\rho_0)\frac{Dv}{Dt}+\frac{\partial p}{\partial x}=\frac{4\mu}{3}\frac{\partial^2 v}{\partial x^2}$$

$$(\rho_0)\frac{Dv}{Dt}+\frac{\partial p}{\partial x}=\frac{4\mu}{3}\frac{\partial^2 v}{\partial x^2}.$$

Taking the derivative with respect to x of Eq2 gives:

$$v_{xx} = \frac{1}{\rho_0}\frac{\partial D(d\rho)}{\partial x Dt}$$

Substituting v_{xx} into eqn 3 gives

$$\rho_0 \frac{Dv}{Dt} + \frac{\partial p}{\partial x} = \frac{-4\mu}{3}\frac{1}{\rho_0}\frac{\partial^2 D(d\rho)}{\partial x^2 Dt}.$$

Now Eqn4 still has a term of velocity in it that we need to eliminate. We will do this by t
derivative of Eqn4 with respect to x and substituting in the time derivative of Eqn2 in for

$$v_{xt} = -\frac{1}{\rho_0}\frac{D^2(d\rho)}{Dt^2}$$

$$\rho_0 v_{xt} + \frac{\partial^2 p}{\partial x^2} = \frac{-4\mu}{3}\frac{1}{\rho_0}\frac{\partial D^2(d\rho)}{\partial x Dt^2}$$

Substituting v_{xt} gives

$$-\frac{D^2(d\rho)}{Dt^2} + \frac{\partial^2 P}{\partial x^2} = \frac{-4\mu}{3}\frac{1}{\rho_0}\frac{\partial D^2(d\rho)}{\partial x Dt^2}.$$

Now we have successfully eliminated velocity as a variable. We are now left with two un
Recall that $\rho=\rho(p)$ we can use a taylor series to relate p to ρ to eliminate ρ as a variable.

$$\rho(p)=\rho_0+\rho_p(P_0)(p-p_0)+\tfrac{1}{2}\rho_{pp}(p_0)(p-p_0)^2+\tfrac{1}{6}\rho_{ppp}(p_0)(p-p_0)^3+\ldots$$

$$d\rho=\rho-\rho_0 \cong A(p-p_0)+\tfrac{1}{2}B(p-p_0)^2 = Adp+\tfrac{1}{2}Bdp^2 \text{ where } A=\rho_p(p_0) \text{ and } B=\rho_{pp}(p_0)$$

$d\rho \cong Adp$.

Substituting eq 6 into eq 5 gives

$$-A\frac{D^2(dp)}{Dt^2} + \frac{\partial^2 P}{\partial x^2} = \frac{-4u}{3}\frac{1}{\rho_0}\frac{\partial D^2(Adp)}{\partial x Dt^2}$$

Rearranging and calling dp p gives Eqn8 the damped wave equation. Where υ is the kin
viscosity $\upsilon = \frac{4u}{3}\frac{1}{\rho_0}$

$$-Ap_{tt} + \frac{\partial^2 P}{\partial x^2} = -\frac{4}{3}\upsilon p_{xtt} .$$

Recall A= $\frac{d\rho}{dp}$ (p_0) this says how density changes in response to pressure changes. This is
of the material over the density at its reference density and pressure.

$\frac{d\rho}{dp}$ (p_0)=A= $\frac{\rho}{B} = \frac{1}{c^2}$ where c is the speed of sound

Relabeling A as $\frac{1}{c^2}$ and neglecting υ we get the wave equation

$$-\frac{1}{c^2}p_{tt} + \frac{\partial^2 p}{\partial x^2} = 0.$$

Where p is the displacement of pressure from its equilibrium pressure p_0. Results are con
schreyer[1]'s derivation based on eulerian coordinates.

So we have derived the acoustic wave equation for a fluid. We are particularly interested

[7]$u_{tt}=c^2u_{xx}+A^2u_{txx}$ where $c^2=\frac{\lambda_s+2\mu_s}{\rho}$ and $A^2=\frac{\lambda_f+2\mu_f}{\rho}$

Now u is based on pressure or u(p)=u

Using taylor series, we can write displacement in terms of pressure

$u=u_0+u_p(p_0)dp=kdp$

We get

$kdp_{tt}=kc^2dp_{xx}+A^2dp_{txx}$

$p_{tt}=c^2p_{xx}+A^2p_{txx}$. Eq10

So in order to use this equation, we must assume that the material is mostly linear in its deformation since it's based on hooke's law, if the pressure causes strain outside of the linear elastic range, this will no longer be an accurate model. Also note that the p_{txx} has substantially more influence in a solid rather than a liquid, showing initial waves can dampen very quickly in viscoelastic solids, however this is offset by the larger speed of sound in solids. Eqn 8 will serve as a rough model for showing how a wave pulse.

Analyzing the assumptions

This section will analyze the assumptions we made in order to derive the wave equation with damping, in order to determine which systems it will be applicable towards and what the results will tell us. In order to derive the wave equation we made the following assumptions A1 small pressure, weak nonlinearity B/A<1, isotropic medium,stokes assumption(for liquids) and linear elastic deformation for solids(hooke's law).

The strongest assumption above is A6, since it's responsible for eliminating the velocity term in Eq3 and Eq2. Now if the pressure changes are no longer small, say in the instance of a "shock" the terms in Eq3 and Eq2 will no longer be zero and will not be accurate. This will require the model to be near equilibrium in the event of a shock to allow the shockwave to disperse into multiple smaller waves.

The isotropic assumption is what was required in order to use the navier stokes equation decreases the number of unknowns needed to find the stress tensor. If the medium is not there could be a change in behavior of the model which could cause pressure waves to re shocks or even dampen.

Stoke's assumption is related to our assumption of small pressure, it is shown to fail duri ultrasonic pulses [9], moreover our model to not be able to model shocks in fluids, such sonic booms, which was already excluded due to small pressure. Finally, In order to deri equation for a solid, hooke's law was needed which implies linear elastic deformation. In damped wave equation for a solid should hold as long as the pressure is not capable of p nonlinear elastic deformation. However, this equation will not capture the formation of s the primary wave redisperses.

A summary of the assumptions and their consequences is consolidated below

Assumption chart(Figure 0.3) **Results/physical restrictions**

A1	The medium is a fluid	The medium cannot support shear stress and model is needed for a rigid object.
A2	The system is near equilibrium	The system needs to be relaxed, no shocks, fr cavitation effects should appear. Excludes exp earthquake shocks near epicenters or systems of entropy and high potential energy.
A3	Stokes assumption	May not work for pressure waves in the ultraso fluids.[9]
A4	Isotropic medium	Uniform material, same strain in all orientation stress. Will not work for liquid crystals or some

A7	Low nonlinearity parameter, weakly nonlinear	The coefficient of "B" should be low enough to effects low. For example animal fat has a valu while a monatomic gas is B/A=0.6[10], making nonlinear and a monatomic gas more linear.
A8	Viscous effects are negligible	This is the last step that gives the wave equati applied to the damped wave equation

Solving the equations

The last section showed that it is appropriate to use the damped wave equation to model as long as the initial wave pulse is low. We begin solving the damped wave equation by variables by treating it as a boundary value problem with reflective conditions. We are o how the addition of the damping term attenuates the wave.

$$-p_{tt} + c^2 p_{xx} = -\tfrac{4\nu}{3} p_{txx}$$

$$p(x,0) = Z(x) \qquad p(a,t) = 0$$

$$p_t(x,0) = 0 \qquad p(b,t) = 0$$

$$x \in (a,b)$$

Since we have a homogenous linear bounded PDE we can use separation of variables or function "p" is a product of 2 functions. Z(x) is the initial wave pulse.

$$p = \Phi(x)G(t)$$

Substituting Eq6 into Eq5 and rearranging we get

Both of these ODEs have solutions that can be found with a characteristic equation.

$\mathbf{\Phi}$=a*sin($\sqrt{\lambda}$ x)+b*cos($\sqrt{\lambda}$ x)

Now considering boundary conditions P(a,t)=P(b,t)=$\mathbf{\Phi}$(a)=$\mathbf{\Phi}$(b)=0 we can find the eiger

$\lambda = \frac{n^2\pi^2}{(b-a)^2}$ and $\mathbf{\Phi}$=a*sin($\sqrt{\lambda}$)

Someone could also use cosine as a solution as well, however we're sticking with sine. I
a solution for $\mathbf{\Phi}$, next we look at finding a solution to G. Writing G as a characteristic ec
solving for r with the quadratic formula we get Eq11 below.

$$G'' + \lambda[c^2 G + \tfrac{4}{3} v G'] = 0$$

$$r^2 + \lambda[c^2 + \tfrac{4}{3} v r] = 0 \quad where\ G = Aexp(r_1 t) + Bexp(r_2 t)$$

$$r = \tfrac{1}{2} v\, \lambda \pm \sqrt{(\tfrac{4}{3} v)^2 \lambda^2 - 4c^2\lambda}$$

$$\alpha = (\tfrac{4}{3} v)^2 \lambda^2 - 4c^2\lambda$$

$$G = e^{-3/2v\lambda t}[Asin(\sqrt{-\alpha}/2t) + Bcos(\sqrt{-\alpha}/2t)] \quad \alpha<0 \text{ or } n<Q$$

$$G = e^{-3/2v\lambda t}[Ae^{\sqrt{-\alpha}/2t} + Be^{\sqrt{-\alpha}/2t}(\sqrt{-\alpha}/2t)] \quad \alpha>0 \text{ or } n>Q.$$

Where Q is a natural number that makes the eigenvalues large enough to make the soluti
giving us two solutions. Going back to Eq6 and multiplying $\mathbf{\Phi}$ and G we get the eigenfu
equation is linear we can represent the general solution as a combination of these if neec

$P_n = \mathbf{\Phi}_n G_n = e^{-3/2v\lambda t}[Asin(\sqrt{-\alpha}/2t) + Bcos(\sqrt{-\alpha}/2t)]$ (a*sin($\sqrt{\lambda}$ x))

$$P=\sum_{n=1}^{Q} e^{-3/2v\lambda} A_n[cos(\sqrt{-\alpha}/2t)+\frac{3v\lambda A}{\sqrt{-\alpha}} B_n sin\sqrt{-\alpha}/2t)]sin(\sqrt{\lambda}x).$$

Where

$A_n=2\int_a^b Z(x)sin(\sqrt{\lambda}x)dx$

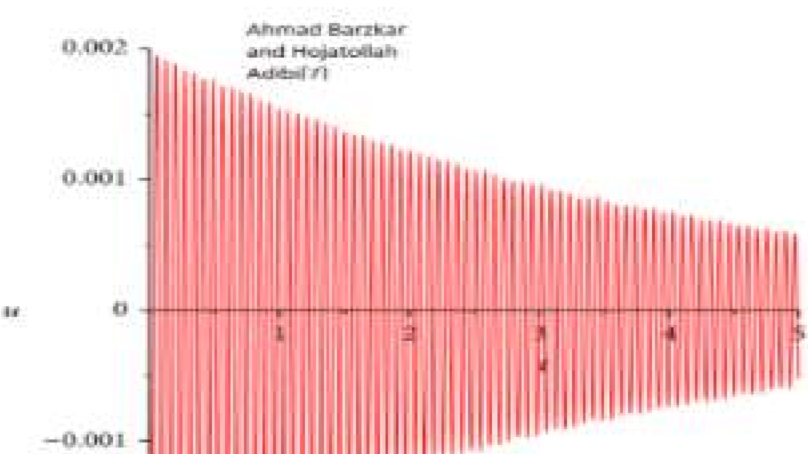
$\lambda=\frac{n^2\pi^2}{(b-a)^2}=\frac{n^2\pi^2}{L^2}$

$\alpha=\frac{16}{9}v^2\lambda^2-4c^2\lambda$

$Q=Step[\sqrt{\frac{36Lc^2}{16v^2\pi^2}}]$

Next we can see the solution by using matlab(code in appendix) giving videos. Note the distorted, the waves are traveling much farther than they appear. The videos serve to sho damping term affect the wave equation.

1. (Room temperature)Water [4] $v=10^{-6}\ m^2/s$
2. (Cooler temperature)Oil [4] $v=0.09m^2/s$
3. (Room temperature)Kerosene [4] $v=2.4*10^{-4}m^2/s$



There appears to be strong dampin kerosene models of pressure wave appears to offer reasonable values. kinematic viscosity term has a stro more viscous fluids. Another obse wave pulse appears to be "stretchi our initial condition was a soundw

which when considering the fast speed of sound in a solid, is similar to attenuation in a f
considering distances traveled.

After solving the damped wave equation with separation of variables by treating it as a b
problem we saw that the pulse decrease in amplitude or strength in time and increased in
wavelength. It can be inferred from Eq13 that this damping is related to a decaying expo
implies that a sound wave traveling through a medium will increase in wavelength and d
amplitude over time. These results will only hold if the conditions on Figure 0.3 are sati

Wave attenuation by dispersion

This section will derive a relationship for describing wave attenuation due to dispersion
looking at the maximum value of a wave pulse as time progresses. In a simplified case t
consider how the maximum value of the wave changes in time, we can do this by setting
the wave pulse or max($Z(x)$) and using Eqn11 and Eqn6

p=max($Z(x)$)* $e^{-3/2v\lambda t}[Asin(\sqrt{-\alpha}/2t)+Bcos(\sqrt{-\alpha}/2t)]$ ≤max($Z(x)$)(A+B) $e^{-3/2v\lambda t}$.

p(0)=max(Z(x))

$p_t(0)=0$

1= $Asin(\sqrt{-\alpha}/2*0)+Bcos(\sqrt{-\alpha}/2*0)$ B=1

0=Acos(0)+sin(0) A=0

p=max($Z(x)$)* $e^{-3/2v\lambda t}cos(\sqrt{-\alpha}/2t)]$ ≤max($Z(x)$) $e^{-3/2v\lambda t}$.

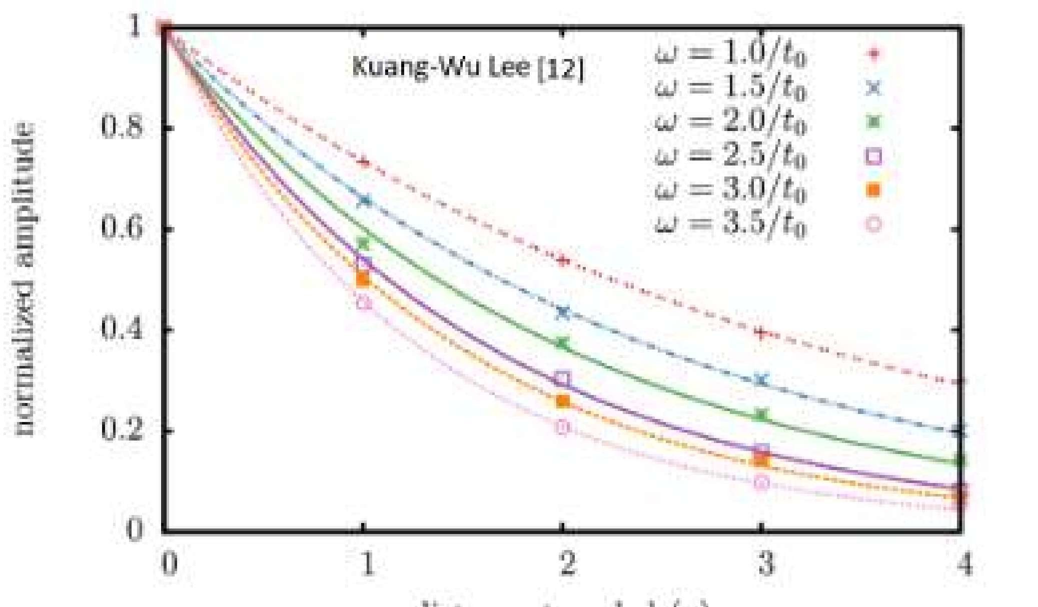
$$max(Z(x))e^{-3/2v\lambda t}=\frac{H}{100}max(Z(x)).$$

This is just a more specific form of the commonly used one in medical imaging below

[11]$A=A_0 e^{-\alpha z}$

where z is the distance, A_0 is the unattenuated value and a is the attenuation constant.

T_{damp} is the amount of time it takes for the wave to lose a % of its amplitude, while D_{damp} wave travels before it loses a % of its strength. Note λ represents a constant related to 1/wavelength(Eq13.d), which can be found for a given wave frequency and a material. I frequency will increase lambda since it will decrease the wavelength. Showing that high waves experience more damping while lower frequency waves experience less based on cannot correspond to values that would place the frequency of the wave in the ultrasonic made stoke's assumption. The equation can be rewritten by following our wave equation account for this. Eq19 is continuous with Lee's[12] results shown on the figure below, w attenuation based on frequency.



Choosing λ=170(square of the speed of sound in water over a wave with acoustic frequen
below the results in figure 0.4 below. Eq9 can be used to calculate the wave speed notice t
in solids, less in liquids and the least in gases.

Attenuation for different mediums(Figure 0.4)

Substance	t_{damp}(seconds)	Wave Speed(m/s)	D_{damp}
Water(90% attenuation)	413	1500	61km
Oil(cold)(50% attenuation)	0.03	1500	45m
Kerosene(50% attenuation)	27	1324	35km
Aluminum[7](50% attenuation)	2.8	6420	18km
Air(50% attenuation)	271	343	93km

It appears damping is stronger in solids and in more viscous fluids. Water appears to expe
attenuation, this is due to its extremely low viscosity, while oil and kerosene are much mo
substances. However Eq was created under many assumptions and has not been verified e
and has not had much analysis done on it, there is a high probability that it could be an ina
Results show sound waves travel farther in fluids rather than solids, dampen quicker in so
and waves travel the fastest in solids, slower in liquids and slowest in gases, and that high
waves dampen quicker than lower frequency ones.

Conclusion

In this paper we presented a model for longitudinal wave propagation in a fluid and deri wave equation from it. Our primary goal was to answer the question "how far can sound and what happens as they dampen. First we derived the damped wave equation from our equations while paying careful attention to our assumptions. Secondly, we analyzed the give us an idea when a solution will hold summarized in figure 0.3. Thirdly, we solved t equation as a boundary value problem with separation of variables, and when looking at observed as the waves dampen in time they decrease in amplitude and increase in wavel investigated the maximum value either an impulse or soundwave can have, and noticed t related to a decaying exponential where the decay rate is increased by the frequency of t Our final results show that waves can travel very far in ideal substances under the assum 0.3, around 20-90km before the waves lose about half of their strength(Figure 0.4) and t related to the initial frequency of the wave(Eq19) and the properties of the material, whi fluid is its kinematic viscosity.

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Appendix:

Code for one dimensional attenuation:

```
clear
q=65 %sum constant Keep low!!
nx=200 %x steps
nt=500 %time steps
a=0 %left endpoint
b=10 %right endpoint
t0=0 %time start
tf=25 %time end
Ax=linspace(a,b,nx);
At=linspace(t0,tf,nt);
c=0.45;
v=0.0009/3;
f=0;
g=@(k) 15*sech(sqrt(20)/2*((k-3.6))).*sech(sqrt(20)/2*((k-1.6))); %initial conditio
%g=@(x) e^(-x^2);  %initial condition

for i=1:q
   fun= @(k) sqrt(c)*sech(sqrt(c)/2*((k-8.6))).*sech(sqrt(c)/2*((k-8.6))).*sin(i*pi*k/
   qq = integral(fun,a,b);
   an(i)=2*qq;
   bn(i)=8/((2*i-1)^3*pi^3);
end

R=zeros(nx,nt);

for xx=1:nx
   for tt=1:nt
      x=Ax(xx);
      t=At(tt);
      f=linspace(0,0,q);
      for n=1:q
         lambda=(n)^2*pi^2/(b-a)^2;
```

```
end

for xx=1:nx
    for tt=1:nt
        v=0;
        x=Ax(xx);
        t=At(tt);
        f=linspace(0,0,q);
        for n=1:q
            lambda=(n)^2*pi^2/(b-a)^2;
            alpha=lambda.^2*16*v^2-4*lambda*c^2;

f(n)=an(n)*((8*n^2*pi^2*v)/((b-a)^2*sqrt(-alpha))*sin(sqrt(-alpha)/2*t)+cos(sqrt(-al
qrt(lambda)*x)*exp(-lambda*4*v*t);
        end
        RR(xx,tt)=sum(f);
    end
end

RRR=abs(R-RR); %energy loss term

for i=1:nt
    AR(i)=sum(RRR(:,i));
end

%for i=2:nx-1
    %for ii=2:nt-1
        %RRR(i,ii)=(RRR(i,ii+1)-RRR(i,ii-1))/(2*(tf-t0));
    %end
%end

v = VideoWriter('oil');
open(v);
```

```
    ylim([-45 45])
    subplot(2,1,2);
    plot(Ax,transpose(abs(RR(:,kk)-R(:,kk))))
    ylabel 'Pressure loss'
    xlabel 'position(meters)'
    xlim([a b])
    ylim([0 max(max(abs(RR-R)))])
    frame = getframe(gcf);
    writeVideo(v,frame);
end
close(v);
```

Code for two dimensional attenuation:

```
clear
q=65 %sum constant VERY EXPENSIVE CAREFUL!
nx=20 %x steps
nt=50 %time steps
a=-10 %left endpoint
b=10 %right endpoint
t0=0.01 %time start
tf=100 %time end
Ax=linspace(a,b,nx);
At=linspace(t0,tf,nt);
c=2;
v=0.0009/3;
f=0;
g=@(k) 15*sech(sqrt(20)/2*((k-3.6))).*sech(sqrt(20)/2*((k-1.6))); %initial conditio
%g=@(x) e^(-x^2);  %initial condition
syms kk
syms k
syms t
r=exp(-(k-kk)^2/(4*pi*t))*exp(-kk^2);
rr=int(r,kk,-50,50);
rrr=1/2*(exp(-(k-c*t)^2)+exp(-(k+c*t)^2))+(4*v)/(6*sqrt(4*pi*t))*rr;
```

```
U=double(P);

%
%for i=1:nt
    %for i=1:nx
        %Ploss(i,j)=4*v/(6*sqrt(4*pi*At(i)).*integral(r,At(i)),-500,500));
    %end
%end
```

Code for checking solution

```
clear
syms x
syms t
syms v
syms lambda
syms l
syms c
alpha=lambda^2*16*v^2-4*lambda*c;
syms l;
syms pi;w
lambda=pi^2/l^2;
f=((2*lambda/(l^2*sqrt(-alpha))*sin(sqrt(-alpha)/2*t)+cos(sqrt(-alpha)/2*t))*exp(-la
n(sqrt(lambda)*x);

r=-diff(f,t,2)+c*diff(f,x,2)+4/3*v*diff(diff(f,x,2),t);

%
x=55
t=1
v=25
lambda=1
l=123
c=1
b=double(subs(r));
```