Adaptation to a Changing Environment by Means of the Feedback Thermodynamical Genetic Algorithm

Naoki MORI[1], Hajime KITA[2] and Yoshikazu NISHIKAWA[3]

[1] College of Engineering, Osaka Prefecture University, Sakai 599-8531, JAPAN
[2] Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology, Yokohama 226-8502, JAPAN
[3] Faculty of Information Science, Osaka Institute of Technology, Hirakata 573-0171, JAPAN

Abstract. In applications of the genetic algorithms (GA) to problems of adaptation to changing environments, maintenance of the diversity of the population is an essential requirement. Taking this point into consideration, the authors have proposed to utilize the thermodynamical genetic algorithm (TDGA) for the problems of adaptation to changing environments. The TDGA is a genetic algorithm that uses a selection rule inspired by the principle of the minimal free energy in thermodynamical systems. In the present paper, the authors propose a control method of the temperature, an adjustable parameter in the TDGA. The temperature is controlled by a feedback technique so as to regulate the level of the diversity of the population measured by entropy. The adaptation ability of the proposed method is confirmed by computer simulation taking time-varying knapsack problems as examples.

1 Introduction

Genetic Algorithms (GAs) are search and optimization techniques[1, 2] based on the mechanism of evolution by natural selection. Adaptation to changing environments is one of the most important applications of the GAs.

The GA approaches for adaptation to changing environments proposed so far are categorized into two types: If a environmental change is recurrent, i.e., if the environment appeared in the past reappears repetitively, to memorize the results of past adaptations and to utilize them as candidates of the solutions will be an effective strategy. We call such an approach *the memory-based approach.* In the context of GA research, several studies on this approach have been proposed. For example, methods using diploidy, structured gene have been proposed[2, 3]. The authors have also proposed a method taking this approach[4].

On the other hand, if an environmental change is unpredictable, GAs have to adapt to a novel environment using only their search ability. We call such an approach *the search-based approach.* If the environmental change is completely random, no method will be better than the method of restarting the GA for each environment. However, if the environmental change is moderate, to search the neighborhood of the recent solution will be an effective way. To achieve this strategy, maintenance of the diversity of the population plays an important role. That is, a GA must keep the results of adaptation to recent environments on one hand, and on the other hand, it must keep the diversity of the population to ensure their search ability.

There have been proposed several methods of the search-based approach. Grefenstette has proposed *the random immigrants*[5] where the population is partially replaced by randomly generated individuals in every generation. Since the randomly generated individuals are usually poor solutions, this method faces a difficulty of low online performance. Cobb has proposed the method of controlling the mutation rate called *the triggered hypermutation*[6], where the mutation rate is temporarily increased to a high value (called the hypermutation rate) whenever the time-averaged best performance of the GA declines. Since this method detects the environmental change using an assumption that the optimal value is always kept constant, its applicability is limited.

Both the random immigrants and the triggered hypermutation utilize random perturbation to increase the diversity of the population. We can generalize these two methods as *the random perturbation method* as follows:

1. The population is perturbed when the GA system receives *a perturbation signal* from environment.
2. Perturbation to the population is given by applying hypermutation with the hypermutation rate m_{H} to some individuals in the population specified by *the perturbation rate* γ.

Assuming that binary string representation and locus-wise mutation, the random immigrants is represented by a case that the perturbation signal occurs in every generation, $m_{\mathrm{H}} = 0.5$ and γ is taken as an adjustable parameter. The triggered hypermutation is a case that the perturbation signal occurs when the time-averaged best performance declines, m_{H} is adjustable and $\gamma = 1$. We call the simple genetic algorithm which uses this method *the Perturbation SGA* (PSGA).

The authors have proposed another method categorized into this approach[7]. That is, the selection operation of the GAs is designed so as to keep the diversity of the population systematically. It is called the thermodynamical genetic algorithm (TDGA)[8].

This paper proposes a modified TDGA called *the Feedback Thermodynamical Genetic Algorithm* (FTDGA) which controls the temperature, an adjustable parameter of the TDGA, so as to regulate the level of the entropy of the population.

2 Feedback Thermodynamical Genetic Algorithm

2.1 Thermodynamical Genetic Algorithm (TDGA)

In the selection operation used in the conventional GA, an individual having the larger fitness value is allowed to yield the more offspring in the next generation.

While it is a basic mechanism to find the optimal solution by focusing search on the neighborhood of good solutions, it also brings about the problem of premature convergence.

In the TDGA, the selection operation is designed to minimize the free energy of the population. The free energy F is defined by:

$$F = \langle E \rangle - HT, \tag{1}$$

where $\langle E \rangle$ is the mean energy of the system, H is the entropy and T is a parameter called the temperature. Minimization of the free energy can be interpreted as taking a balance between minimization of the energy function (the first term in the RHS of Eq. (1), or equivalently maximization of the fitness function in GAs by regarding $-E$ as the fitness function) and maintenance of the diversity measured by the entropy (the second term in the RHS of Eq. (1)).

Hence, individuals having relatively small energy (or relatively large fitness) values will be given priorities to survive in the next generation. At the same time, individuals having rare genes will also be preserved owing to their contribution to minimization of the free energy via increase in the entropy term HT of Eq. (1). Thus, the diversity of the population can be controlled by adjusting the temperature T explicitly.

2.2 Adaptation to Changing Environments by TDGA

We have proposed to utilize the TDGA to problems of adaptation to changing environments[7]. In that paper, we have set the temperature constant, and its value was adjusted by a trial-and-error manner.

However, in some cases, the TDGA with a constant temperature could not follow the environmental change well. Suppose a situation that variance of the energy function decreases as an environmental change. Then, in the definition of the free energy given by Eq. (1), the entropy term TH becomes more dominant. Consequently, the selection mechanism of the TDGA tries to make the ratio of alleles in each locus more even. However, it will be harmful in finding good solutions for the new situation if it requires a biased allele ratio.

In the previous study[7], such a situation was observed in an application of the TDGA to a time-varying constrained optimization problem. The constraint was treated by a penalty function. When the constraint became severer as an environmental change, all the individuals became infeasible solutions, and the variance of the energy function decreased remarkably due to the penalty function. Then, the TDGA faced difficulty in recovering the feasibility of populations because of the mechanism mentioned above.

2.3 Feedback Control Method of Temperature

To overcome the aforesaid difficulty of the TDGA, the temperature should be controlled adaptively. This paper proposes the following feedback control method of the temperature. The temperature at generation t, T_t is controlled by the following equation:

$$T_t = \exp(\tau(H^* - H)) \times T_{t-1} \tag{2}$$

or taking logarithm of the both sides of Eq. (2), we obtain the following equation:

$$\log T_t = \log T_{t-1} + \tau(H^* - H) \tag{3}$$

where τ is a constant representing the feedback gain, H^* is the target entropy and H is the entropy of the current population. If $H > H^*$, this rule decreases the temperature, and if $H < H^*$, it increases the temperature keeping its positiveness. With this feedback mechanism, the diversity of the population in the genotypic level will be kept well even when the energy function changes largely.

We call the TDGA which uses this feedback control method *the feedback TDGA* (FTDGA). In the FTDGA, the feedback gain τ and the target entropy H^* are adjustable parameters.

2.4 Algorithm of FTDGA

The algorithm of FTDGA is as follows:

1. Select appropriate values for
   N_{p}: the population size,
   N_{g}: the maximum number of generations,
   T_{init}: the initial temperature,
   τ: the feedback gain, and
   H^*: the target entropy.
2. Let $t = 0$, $T_{-1} = T_{\mathrm{init}}$, and construct the initial population $\mathcal{P}(0)$ randomly.
3. Observe the entropy H of $\mathcal{P}(t)$, and set T_t by the following equation:

$$T_t = \exp(\tau(H^* - H)) \times T_{t-1}$$

4. Preserve the individual having the minimum energy function value as an elite.
5. Pair all the individuals in $\mathcal{P}(t)$ randomly. Apply the crossover operator to all the pairs, and obtain N_{p} offsprings. Then, apply the mutation operator to all the N_{p} parents and N_{p} offsprings. Let $\mathcal{P}'(t)$ be the population consisting of the above $2N_{\mathrm{p}}$ individuals and the elite preserved in Step 4.
6. Let $i = 1$, and make the population $\mathcal{P}(t+1)$ at the next generation empty.
7. We refer to an individual in $\mathcal{P}'(t)$ by its number $h = 1, \cdots, 2N_{\mathrm{p}} + 1$. Let $\mathcal{P}(t+1, i, h)$ be the population which consists of already selected $i - 1$ individuals for $\mathcal{P}(t+1)$ and the h-th individual of $\mathcal{P}'(t)$. Calculate the free energy of $\mathcal{P}(t+1, i, h)$ for all individuals $h = 1, \cdots, 2N_{\mathrm{p}} + 1$:

$$F = \langle E \rangle - T_t \sum_k H_k = \frac{\sum_{l=1}^{i-1} E_l + E'_h}{i} - T_t \sum_{k=1}^{M} H_k(i, h), \tag{4}$$

where $H_k(i,h)$ is the entropy evaluated as follows[1]:

$$H_k(i,h) = -\sum_{j\in\{0,1\}} P_j^k(i,h)\log P_j^k(i,h), \tag{5}$$

E_l is the energy of the l-th individual of $\mathcal{P}(t+1)$, E'_h is the energy of the h-th individual of $\mathcal{P}'(t)$, $H_k(i,h)$ is the entropy of the k-th locus of $\mathcal{P}(t+1,i,h)$, and $P_j^k(i,h)$ is the ratio of gene j on the locus k of $\mathcal{P}(t+1,i,h)$. Find the individual h that minimizes the free energy F given by Eq. (4) from $\mathcal{P}'(t)$, and add it to $\mathcal{P}(t+1)$ as the i-th individual.
Repeated selection of the same individual in $\mathcal{P}'(t)$ is allowed.

8. Let $i=i+1$. If $i \le N_{\rm p}$, go to Step 7.
9. Let $t=t+1$. If $t < N_{\rm g}$, go to Step 3, otherwise terminate the algorithm.

3 Computer Simulation

3.1 Time-varying Knapsack Problem

We use the following time-varying knapsack problem[7] (TVKP) as an example:

$$\max_{x_i(t)\in\{0,1\}} \quad z=\sum_{i=1}^{N} c_i(t)x_i(t), \quad \text{sub. to} \sum_{i=1}^{N} a_i(t)x_i(t) \le b(t), \quad t=1,2,\cdots, \tag{6}$$

where N is the number of items, a_i and c_i are the weight and the value of item i, respectively, b is the weight limit and t is discrete time. In this paper, the following TVKP is used to examine the adaptation performance of FTDGA.

The number of items N is 30. The weight a_i and the value c_i are set randomly in the range $1 \sim 500$. These parameters are kept constant. The weight limit $b(t)$ is always reduced. Since the optimal solution is usually located near the boundary of the constraint, this environmental change makes the good solution infeasible suddenly. Thus the adaptation problem becomes difficult.

Following three reduction patterns of $b(t)$ are used.

Case 1 $b(t)$ is reduced from 90% of $b_{\rm sum}$ to 50% of $b_{\rm sum}$ by 10% of $b_{\rm sum}$,
Case 2 $b(t)$ is reduced from 50% of $b_{\rm sum}$ to 10% of $b_{\rm sum}$ by 10% of $b_{\rm sum}$,
Case 3 $b(t)$ is reduced from 90% of $b_{\rm sum}$ to 10% of $b_{\rm sum}$ by 20% of $b_{\rm sum}$,

where $b_{\rm sum}=\sum_{i=1}^{N} a_i$. Fig. 1 shows the variation of the weight limit $b(t)$ and the optimal value in each generation. In the used TVKP, a_i and c_i are chosen so that the optimal solution is not found by a greedy algorithm in all the generations.

[1] The entropy is evaluated in a locus-wise manner to cope with the problem that the population size is much smaller than the number of the possible species[8].

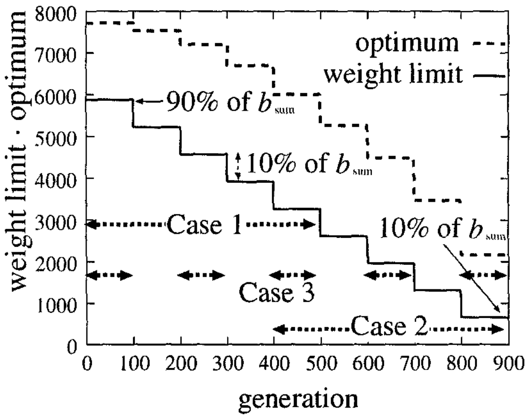

Fig. 1. The variation of TVKP

3.2 Setup of GAs

The genetic representation for the problems is a 30 bit binary code. The fitness of an individual which satisfies the constraint is the value of the objective function itself. The fitness of an infeasible individual is given by:

$$f = \left(b_{\text{sum}} - \sum_{i=1}^{N} a_i(t)x_i(t) \right) \times 0.01, \tag{7}$$

which is much smaller than the fitness of feasible individuals because of the factor 0.01. We use the following index I to measure adaptation performance:

$$I = \frac{1}{T_{\max}} \sum_{t=1}^{T_{\max}} \alpha \times \frac{f_{\text{best}}(t)}{f_{\text{opt}}(t)}, \quad \alpha = \begin{cases} 1, & f_{\text{best}}(t) = f_{\text{opt}}(t) \\ 0.5, & f_{\text{best}}(t) < f_{\text{opt}}(t) \end{cases} \tag{8}$$

where $T_{\max}$ is the maximum number of generations, $f_{\text{best}}(t)$ is the best objective function value in the population in the generation t, and $f_{\text{opt}}(t)$ is the value of the optimal solution in generation t. We introduce a weight parameter α so as to enhance finding of the optimal solution.

TVKP is solved by FTDGA, TDGA and PSGA. Setup of PSGA is the same as the triggered hypermutation except the trigger mechanism. The perturbation signal of PSGA is given whenever an environmental change occurs. The PSGA uses hypermutation in only one generation after an environmental change. Thus, the PSGA is given advantage of knowing that the environment is changed while the FTDGA and the TDGA receive no direct information about the environmental change.

In PSGA, the following liner scaling of the fitness function is used:

Table 1. Parameter List

Case	FTDGA	TDGA	PSGA
1	$H^* = 7,\ \tau = 0.1,\ m = 0.02$	$T = 40,\ m = 0.05$	$m = 0.02,\ m_{\rm H} = 0.02,\ A = 3$
2	$H^* = 7,\ \tau = 0.2,\ m = 0.02$	$T = 50,\ m = 0.02$	$m = 0.02,\ m_{\rm H} = 0.05,\ A = 3$
3	$H^* = 7,\ \tau = 0.5,\ m = 0.02$	$T = 30,\ m = 0.05$	$m = 0.01,\ m_{\rm H} = 0.05,\ A = 3$

m: mutation rate, H^*: target entropy, τ: feedback gain, T: temperature
$m_{\rm H}$: hypermutation rate ($m_{\rm H} \geq m$), A: scaling parameter of Eq. (9)

$$f_{\rm new} = \frac{A - B}{f_{\max} - \bar{f}} f_{\rm old} + \frac{B \cdot f_{\max} - A \cdot \bar{f}}{f_{\max} - \bar{f}} \tag{9}$$

where $f_{\rm old}$ and $f_{\rm new}$ are the fitness values before and after scaling, respectively, $f_{\max}$ and $\bar{f}$ are the maximum and the mean fitness values, respectively, and A and B are constant parameters. In the following simulation, we set $B = 1$ and adjust A. If $f_{\rm new} < 0$, we set $f_{\rm new} = 0$. In the TDGA, the temperature is fixed. The uniform crossover is used in all the algorithms with unity crossover rate. In the PSGA, we set the population size to 100, i.e., twice as large as those in the TDGA and the FTDGA where the population size is 50 in order to make the number of fitness evaluation equal. We set the maximum number of generations $N_{\rm g}$ to 500 and fluctuation interval to 100. We have applied elitism to all the algorithms.

3.3 Simulation Results

The adjustable parameters of FTDGA, TDGA and PSGA have been set to the best values after preliminary experiments. These parameters are shown in Table 1. The mutation and hypermutation rates represent the probabilities of flip in each locus.

In the FTDGA, the best target entropy H^* is same in all the cases. It shows that suitable diversity is almost constant in these environmental changes. The best feedback gain τ becomes larger in the order of Case 1, 2 and 3. Since the weight limits in Case 2 are smaller than those in Case 1, and the change rate of $b(t)$ in Case 3 is larger than that of the other cases, large τ may be needed in these cases to achieve quick adaptation. It also should be noted that we have observed that instability such as oscillation with too large τ.

In the TDGA, the performance of $T = 40$ is the second highest value in Case 2 and Case 3 and these second values are almost the same as the best values in each case. Therefore the best temperature T is almost the same in all the cases like H^*. This result shows that the characteristic of T is similar to that of H^*.

In the PSGA, hypermutation works well in Case 2 and Case 3 because the hypermutation rate $m_{\rm H}$ is larger than the mutation rate m. However, the best

m_{H} is not much larger than m. In Case 1, m_{H} is equal to m, which means that the hypermutation is ineffective. These results show that large perturbation worsens the search performance.

Figs. 2 (a) $\sim$ (f) show the search process of FTDGA, TDGA and PSGA in Case 2 respectively. The upper panels show the evolution of the fitness values, and the lower panels show the entropy variation.

It is shown in Fig. 2 (a) that the search by FTDGA follows the environmental change well. The search by TDGA shown in Fig. 2 (b) can not follow the environmental change around generation 300 because all individuals become infeasible and the problem discussed in 2.2 occurs. Fig. 2 (c) shows that the search by PSGA almost follows the environmental change. However the PSGA fails in finding the optimal solution in many periods, especially in generation 400 $\sim$ 500.

Figs. 2 (d) $\sim$ (f) show that the entropy values of the search are almost same in the three algorithms, which is the result of the parameter tuning for each algorithm. Fig. 2 (d) shows that the entropy of the FTDGA is controlled well to the target entropy value. When the environmental change occurs, the entropy value deviates from the target value, but it returns quickly. Fig. 2 (e) shows that the entropy of TDGA is more disordered than that of FTDGA. Fig. 2 (f) shows that the entropy of PSGA gets down in the generation 400 $\sim$ 500. This decline means that convergence of the population occurs in this period, where the PSGA failed in finding the optimal solution.

Looking at the fitness distribution shown in Figs. 2 (a) $\sim$ (c), it may seem that the PSGA maintains the diversity more than the FTDGA and the TDGA because the mean fitness value is much closer to the best one in the FTDGA and the TDGA than in the PSGA. However, looking at the entropy shown in Figs. 2 (d) $\sim$ (f), the FTDGA and the TDGA maintain the diversity well. This result is explained as follows. The fitness distribution indicates *the phenotypic diversity*, while, the entropy indicates *the genotypic diversity*. In the knapsack problem taken as an example, there exist many suboptimal solutions. Hence, by careful selection operation such as FTDGA and TDGA, maintenance of the diversity can be achieved keeping the fitness value high. It ensures the good adaptation by crossover operation to the environmental change.

Fig. 3 shows performance variation of FTDGA with various target entropy H^*. It is shown in Fig. 3 that the performance of FTDGA has a peak at $H^* = 7$. At a low H^*, the FTDGA puts more emphasis on selecting good individuals than maintaining diversity which is important to follow the environmental change. On the other hand, at a high H^*, the FTDGA maintains diversity well, but the search performance in stationary periods becomes worse.

Fig. 4 shows performance comparison of FTDGA, TDGA and PSGA. The abscissa indicates the case number, and the ordinate indicates the performance index I, which is the mean value of 50 trials. Fig. 4 shows that the best performance is obtained by FTDGA in all the cases. The performance of TDGA falls down remarkably in Case 2 and Case 3 because the environmental change that makes all individuals infeasible occurs many times in these Cases. These results show the effectiveness of the feedback control of the temperature. The performance of PSGA is the lowest in all the cases, although the PSGA is given the advantage of detecting the environmental changes.

4 Conclusion

In this paper, the authors propose a modified thermodynamical genetic algorithm called the *Feedback Thermodynamical Genetic Algorithm* (FTDGA), in which the feedback control method of the temperature is introduced into the TDGA. The comparative study of FTDGA with TDGA and the triggered hypermutation through computer simulation shows that a satisfactory performance is obtained by FTDGA.

To combine memory-based approach[4] with the FTDGA and analysis of the stability of the feedback control are subjects of further study.

Finally, the authors would like to acknowledge helpful discussions by Prof. Keinosuke Matsumoto of Osaka Prefecture University. This research was supported by "The Research for the Future" program, 'Biologically Inspired Adaptive Systems (JSPS-RFTF96I00105)' of The Japan Society for the Promotion of Science.

References

1. J. H. Holland: Adaptation in Natural and Artificial Systems, The University of Michigan (1975).
2. D. E. Goldberg: Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley (1989).
3. D. Dasgupta and D. R. McGregor: Nonstationary function optimization using the structured genetic algorithm, *Proc. of 2nd PPSN*, Vol. 2, pp. 145-154, (1992).
4. N. Mori, S. Imanishi, H. Kita, Y. Nishikawa: Adaptation to Changing Environments by Means of the Memory Based Thermodynamical Genetic Algorithm, *Proc. of 7th ICGA*, pp. 299-306 (1997).
5. J. J. Grefenstette : Genetic algorithms for changing environments, *Proc. of 2nd PPSN*, Vol. 2, pp.137-144 (1992).
6. H. G. Cobb : An investigation into the use of hypermutation as an adaptive operator in genetic algorithms having continuous, time-dependent nonstationary environments, *NRL Memorandum Report 6760* 523-529 (1990).
7. N. Mori, H. Kita and Y. Nishikawa: Adaptation to a Changing Environment by Means of the Thermodynamical Genetic Algorithm, *Proc. of 4th PPSN*, Vol. 4, pp. 513-522 (1996).
8. N. Mori, J. Yoshida, H. Tamaki, H. Kita and Y. Nishikawa: A Thermodynamical Selection Rule for the Genetic Algorithm, *Proc. of 2nd IEEE Conference on Evolutionary Computation*, pp. 188-192 (1995).

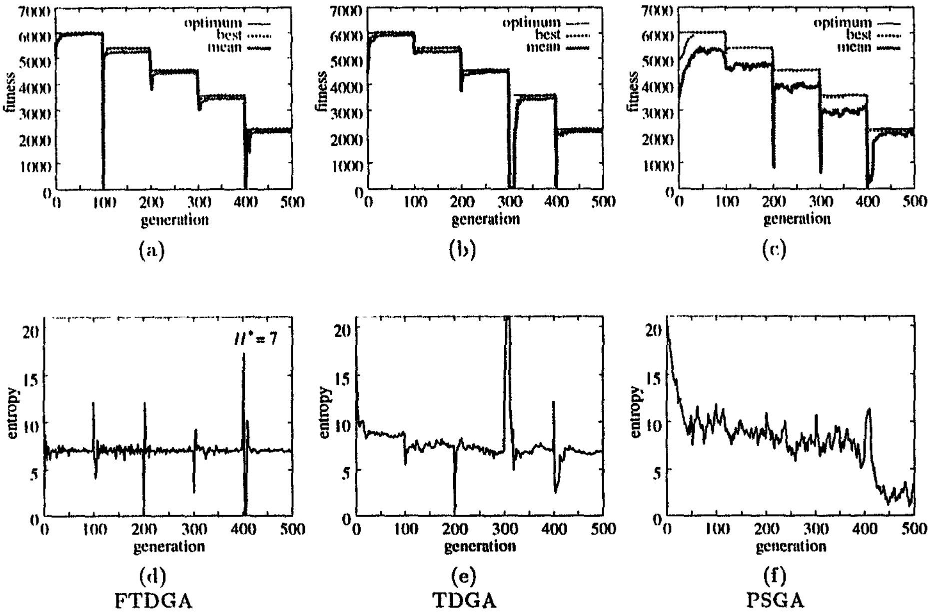

Fig. 2. Search processes of FTDGA, TDGA and PSGA in Case 2.

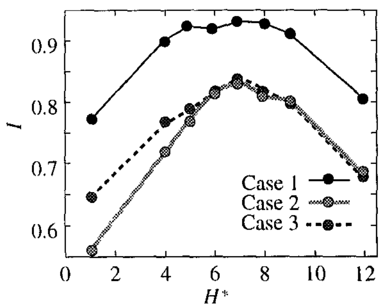

Fig. 3. Variation of performance index I w.r.t target entropy H^*.

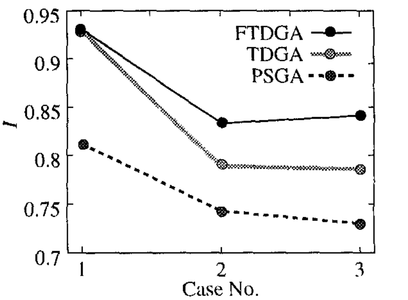

Fig. 4. Performance comparison of FTDGA, TDGA and PSGA.