# A Multi-objective Decomposition Algorithm Based in Variable Space Diversity

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***Abstract*—The abstract goes here.**

***Index Terms*—IEEE, IEEEtran, journal, LATEX, paper, template.**

## I. Introduction

MULTI-OBJECTIVE Optimization Algorithms (MOEAs) have being converted in one of the most popular approaches to deal with Multi-objective Optimization Problems (MOPs). Perhaps, one of the most importants reasons of its applicability is the information required to solve a problem, which is the usual situation in practice with real problems REF-implementaciones. Particularly, a continuous box-constrained minimization MOP involves two or more conflicting objectives that can be defined in Eq. (1)

$$\begin{aligned} &min \quad F(x) = (f_1(x), ..., f_M(x)) \\ &s.t. \quad x \in \Omega, \end{aligned} \tag{1}$$

where $\Omega \subseteq \Re^D$ denotes the decision space, $F : \Omega \rightarrow Y \subseteq \Re^M$ consists of $M$ objectives and $Y$ is the objective space. Given two solutions $x_1, x_2 \in \Re^D$ is said that $x_1$ dominates $x_2$ denoted as $x_1 \prec x_2$ if and only if $f_i(x_1) \leq f_i(x_2)$ for all $i \in \{1, ..., M\}$ and $f_i(x_1) < f_i(x_2)$ for at least one objective. Therefore, $f_i(x_1)$ should be better or equal to $f_i(x_2)$ and $f_i(x_1)$ should be better for at least one objective. A solution $F(x^*)$ is called a Pareto-optimal solution if there does not exist $x \in Y$ such that $x \prec x^*$. The set of all $x^* \in Y$ is called the Pareto-optimal solution set (PS), and their image is the Pareto Front (PF). The goal of the MOEAs is to find a set of solutions that are well-distributed and converged to the PF in the objective space REFERENCIA.

In the last decade, several strategies that takes into account executions in long-term have been quite successfull. The latter strategies are designed with the aim of balancing the search process between exploration and intensification. This balance is reached by incorporing the stopping criterion and the elapsed time with an explicit preservation of diversity in the decision variable space.

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## II. Literature Review

### A. Decomposition-Based MOEAs

### B. Diversity in Evolutionary Algorithms

### C. Related Works

In the last decade few MOEAs were specifically designed to address the diversity in the decision variables space. Although that the diversity in single-objectives EAs is a matter of importance refs, in multi-objective optimization usually the diversity in the variable space is ignored. This might occurs, since the objectives are usually in conflict, therefore often is maintained a diversity level in the decision space. Also, the decision space is disregarded, since that at the end of the optimization process the quality of the solutions relies only in the objective space. In single-objective optimization, high quality solutions have been provided since that a balance between exploration and exploitation is reached through the optimization process refs. In this way, the premature convergence, which is considered as a drawback, can be avoided. Some strategies in single-objective optimization to avoid this drawback is explicitly induce a the diversity considering the criteria stop, thus at first stages the exploration levels are promoted and at the end the exploitation of the promising regions is induced. A similar issue is addressed in multi-objective problems, in such a way that the evolutionary search is stagnated, and only are explored the same region. Particularly, the idea to integrate decision space diversity into the optimization has been proposed in 1994 with the first NSGA work REF. In this last work the decision vectors are considered into the fitness sharing procedure. Thereafter, the most algorithms concentrated in the objectives space only. Alternatively, several approaches with MOPs related directly in decision space has arisen. These approaches, further as the usual MOPs, also aims to provide diverse solutions in the decision space. Principally, based in that there exists a variety of problems where the image of the Pareto Front corresponds to several distributions in the Pareto Set.

In 2003 GDEA [1] integrated diversity into the search as an additional objective. This MOEA introduced by Toffolo and Benini invoked two selection criteria, non-dominated sorting as the primary one and a metric for decision space diversity as the secondary one. In 2005, Chan and Ray [2] suggested to use two selection operators in MOEAs; one encourages the diversity in the objective space and the other does so in the decision space. They implemented KP1 and KP2, two algorithms using these two selection operators. After that, in 2008, the Omni-optimizer [3] was developed, which extends the original idea of the NSGA. Particularly, the diversity measure take both the decision and the objective space diversity into account, Omni-optimizer first uses a rank procedure, were the objective space measure is always considered first, and only if there are ties the diversity in decision space is taken into consideration. However, the drawback of this approach is that the diversity plays an inferior role and there is no possibility to change the tradeoff between the diversity measures. In 2009, were proposed the CMA-ES niching framework [4] , and the probabilistic Model-based Multi-objective Evolutionary Algorithm (MMEA)[5] . The first, extend a niching framework to include the diversity in the space diversity. The second, applies a clustering procedure in objective space and then builds a model from the solutions in these clusters. In 2010, was proposed the Diversity Integrating Hypervolume-based Search Algorithm (DIVA) [6], this algorithm introduces a method to integrate decision space diversity into the hypervolume indicator, such that these two set measures can be optimized simultaneously.

## III. Proposal

## IV. Experimental Validation

### A. Comparison Against State-of-the-art

### B. Effect of the Initial Distance Factor

## V. Conclusion

## Appendix A
## Proof of the First Zonklar Equation

Appendix one text goes here.

## Appendix B

Appendix two text goes here.

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**Michael Shell** Biography text here.

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