Pareto-based Multi-Objective Differential Evolution

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Abstract — Evolutionary multi-objective optimization (EMOO) finds a set of Pareto solutions rather than any single aggregated optimal solution for a multi-objective problem. The purpose of this paper is to describe a newly developed evolutionary approach --- Pareto-based multi-objective differential evolution (MODE). In this paper, the concept of differential evolution, which is well-known in the continuous single-objective domain for its fast convergence and adaptive parameter setting, is extended to the multi-objective problem domain. A Pareto-based approach is proposed to implement the differential vectors. A set of benchmark test functions is used to validate this new approach. We compare the computational results with those obtained in the literature, specifically by strength Pareto evolutionary algorithm (SPEA). It is shown that this new approach tends to be more effective in finding the Pareto front in the sense of accuracy and approximate representation of the real Pareto front with comparable efficiency.

1. Introduction

Many engineering applications involve multiple criteria, and the exploration of Evolutionary Multi-Objective Optimization (EMOO) to solve this class of problem has increased in recent year (Coello, 1999). The ideal solution for a multi-objective problem is the one that optimizes all criteria simultaneously. However, such an ideal solution can never be obtained in practical applications where outcome criteria may be fundamentally inconsistent. Optimal performance according to a single objective, if such an optimum exists, often implies unacceptably low performance in one or more of the other objective dimensions, creating the need for compromise to be reached.

Evolutionary algorithms inherently explore a set of possible solutions simultaneously. This characteristic enables the search for an entire set of Pareto optimal solutions, at least approximately, in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of traditional mathematical programming techniques. Additionally, evolutionary algorithms are less susceptible to problem dependent characteristics, such as the shape of the Pareto front (convex, concave, or even discontinuous), and the mathematical properties of the search space, whereas these issues are concerns for mathematical programming techniques for mathematical tractability.

Since the first practical approach to multi-criteria optimization using EAs, Vector Evaluated Evolutionary Algorithm (VEEA), was proposed by Schaffer (Schaffer 1985; Schaffer & Grefenstette, 1985), there have been several other versions of evolutionary algorithms that attempt to generate multiple non-dominated solutions such as Fourman (1985), Kursawe (1991), Hajela and Lin (1992). However, none of them makes direct use of the actual definition of Pareto optimality. The concept of Pareto-based fitness assignment was first proposed by Goldberg (1989), as a means of assigning equal probability of reproduction to all non-dominated individuals in the population. This method consists of assigning rank 1 to the non-dominated individuals and removing them from contention, then finding a new set of non-dominated individuals, ranked 2, and so forth.

Fonseca and Fleming (1993) have proposed a multi-objective genetic algorithm (MOGA) using a slightly different scheme, whereby an individual's rank corresponds to the number of individuals in the current population by which it is dominated. Non-dominated individuals are, therefore, all assigned the same rank, while dominated ones are penalized according to the population density in the corresponding region of the trade-off surface. Srinivas and Deb (1995) have implemented a similar sorting and fitness assignment procedure, called NSGA, but based on Goldberg's version of Pareto ranking. Horn et al. (1994) proposed niched Pareto genetic algorithm (NPGA) using a tournament selection method based on Pareto dominance. The more recent algorithms include NSGA-II (Deb and Goel, 2002), and the strength Pareto evolutionary algorithm (SPEA) algorithm (Zitzler and Thiele, 1999).

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Differential Evolution (DE) is type of single-objective evolutionary algorithm proposed by Storn and Price (1995) for optimization problems over a continuous domain. The main motivation of this approach is to adapt the search steps while the evolutionary algorithm is proceeding. It has been empirically shown that this simple strategy turns out to be effective in terms of convergence accuracy and speed. In our previous research (Xue, Sanderson, and Graves, 2003), the DE approach to single-objective problem has been extended to solve discrete problems, and further to discrete multi-objective optimization problem. Abbass el al (2001) and Madavan (2002) independently studied the extensio of differential evolution to multi-objective optimization problem in the continuous domain, though applying different approach from that described in this paper.

This paper is organized as follows: section 2 is a simple introduction about multi-objective problem and the Pareto optimal solutions; section 3 briefly introduces the differential concept and one particular implementation for single-objective problem; section 4 elaborates the main idea of the Pareto-based differential evolution for multi-objective optimization and its operators; section 5 describes the measures we used to evaluate the algorithm performance; section 6 provides a briefing to the set of benchmark functions and the experimental results using the newly developed approach; and section 7 is the conclusion.

2. Multi-Objective Optimization

In mathematical notation, a multi-objective optimization problem (MOOP) can be loosely posed as (without loss of any generality, minimization of all objectives is assumed):

$$\min Z(\mathbf{x}) = \begin{bmatrix} z_1(\mathbf{x}) \\ z_2(\mathbf{x}) \\ \vdots \\ z_k(\mathbf{x}) \end{bmatrix}, \quad \mathbf{x} \in \Omega \tag{1}$$

where $\Omega = \{\mathbf{x} \mid h(\mathbf{x}) = 0, g(\mathbf{x}) \le 0\}$, and $\mathbf{x}$ is the decision variable in $\Re^n$; Z, h, g are vector functions among which $Z: \Re^n \to \Re^k$, $h: \Re^n \to \Re^{m1}$, and $g: \Re^n \to \Re^{m2}$; k is the number of objectives, $m1$ and $m2$ are the number of equality and inequality constraints, respectively.

In practical applications, there is no solution that can minimize all of the k objectives simultaneously. As a result, multi-objective optimization problems tend to be characterized by a family of alternatives that must be considered equivalent in the absence of information concerning the relevance of each objective relative to the others. These alternatives are referred to as Pareto optimal solutions (as shown in Figure 1), which have the same meaning with efficient or non-inferior solutions in decision space. The corresponding mapped points in objective space are usually referred to as non-dominated solutions. A Pareto optimal solution is defined as follows:

Definition: The vector $Z(\hat{\mathbf{x}})$ is said to dominate another vector $Z(\bar{\mathbf{x}})$, denoted by $Z(\hat{\mathbf{x}}) \prec Z(\bar{\mathbf{x}})$, if and only if $z_i(\hat{\mathbf{x}}) \le z_i(\bar{\mathbf{x}})$ for all $i \in \{1,2,\cdots,k\}$ and $z_j(\hat{\mathbf{x}}) < z_j(\bar{\mathbf{x}})$ for some $j \in \{1,2,\cdots,k\}$. A solution $\mathbf{x}^* \in \Omega$ is said to be Pareto optimal solution for MOOP if and only if there does not exist $\mathbf{x} \in \Omega$ that satisfies $Z(\mathbf{x}) \prec Z(\mathbf{x}^*)$.

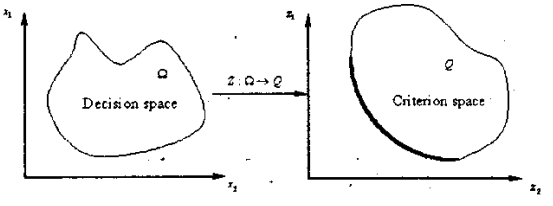

Figure 1: Illustrative example of a multi-objective minimization problem with two objectives, z_1 and z_2, that are plotted in the criterion space mapped from the decision space. The bold curve indicates the Pareto front. In this case, the Pareto front is convex.

3. Differential Evolution

Differential Evolution (DE) is a type of evolutionary algorithm proposed by Storn and Price (1995) for optimization problems over a continuous domain. DE is similar to (μ, λ) evolution strategy in which mutation plays the key role. There are several variants of the original differential evolution. The particular one described below follows Joshi and Sanderson (1999a, 1999b). The main operators that control the evolutionary process are the mutation and selection operators. The following paragraph provides a brief description.

The algorithm follows the general procedure of an evolutionary algorithm: an initial population is created by random selection and evaluated; then the algorithm enters a loop of generating offspring, evaluating offspring, and selecting individuals to create the next generation. In DE, for a particular individual p_i in the parent population, the following mutation operator is used to create its offspring:

$$p_i' = \gamma \cdot p_{best} + (1-\gamma) p_i + F \cdot \sum_{k=1}^{K} \left(p_{i_a^k} - p_{i_b^k} \right) \tag{2}$$

where p_{best} is the best individual in the parent population, $\gamma \in [0,1]$ represents greediness of the operator, and K is the number of perturbation vectors, F is the scale factor of

the perturbation, $p_{i_2^k}$ and $p_{i_3^k}$ are randomly selected mutually distinct individuals in the parent population, and p_i' is the offspring that is generated; γ, K, and F are the parameters associated with the algorithm. The DE approach is illustrated schematically in Figure 2.

The basic idea of DE is to adapt the search step inherently along the evolutionary process in a manner that trades exploitation off exploration. The mutation operator consists of two components: the differential and the perturbation. The differential part makes usage of the information contained in the best solution and lead other solutions in the population toward the best one. On the other hand, the perturbation part introduces random variation though such variation adapts in the evolutionary process. At the beginning of the evolution, the perturbation is large since parent individuals are far away to each other. As the evolutionary process proceeds to the final stage, the population converges to a small region and the perturbation becomes small. As a result, the adaptive search step enables the evolution algorithm to perform global search with a large search step at the beginning and refine the population with a small search step at the end. The selection operator in DE selects the better one between the parent and its offspring by comparing their fitness value:

$$p_i^{(t+1)} = \begin{cases} p_i'^{(t)} & \text{if } \Phi\left(p_i'^{(t)}\right) > \Phi\left(p_i^{(t)}\right) \\ p_i^{(t)} & \text{otherwise} \end{cases} \quad (3)$$

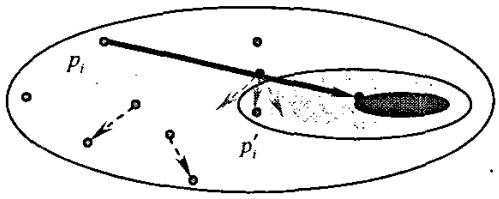

Figure 2: Illustrative example of differential evolution for single objective optimization in a 2-dimention decision space. The darker area indicates better fitness value. The thick solid arrow represents the differential vector, and dashed arrows represent the perturbation vectors. The individual p_i creates its offspring p_i' after the operation.

4. Multi-Objective Differential Evolution

As a general evolutionary algorithm, this MODE approach has three major components, those are mutation, Pareto-based evaluation, and selection. These components will be described in details in this section.

4.1 Mutation operator

In order to mimic the mutation operator in the DE approach described in Section 3, we need to define two kinds of vectors: the differential vector and the perturbation vectors. In the DE for single-objective problem, the differential vector is defined as the vector between the best individual and the individual under operation. This best individual is usually the individual with highest fitness value in the population. However, in a multi-objective domain, the purpose of evolutionary algorithm is to identify a set of solutions, the so called Pareto optimal solutions. Thus, the efforts to adapt individuals toward a single solution is contradictive to the original desire of finding the whole set of Pareto solutions. In the proposed MODE, a Pareto-based approach is introduced to implement the selection of the best individual for the mutation operation of an individual. As a by-product of the evaluation and selection operation, the non-dominated solutions (Pareto optimal solutions) D in the population are identified at each generation of the evolutionary process. In order to apply the mutation operation to an individual, p_i, we need to examine whether the individual is dominated or not. If this is a dominated individual, a set of non-dominated individuals, D_i, that dominates this individual can be identified. A "best" solution, p_{best}, is chosen randomly from the set D_i. The vector defined between p_{best} and p_i becomes the differential vector for the mutation operation. If the individual is already a non-dominated individual, the p_{best} will be the individual itself. In this case, the differential vector becomes 0 and only perturbation vectors play effect. The major difference from single-objective DE is that the best individual is varying rather than fixed for the reproduction of all individuals in the population. This is in accordance with the ultimate purpose of finding the whole Pareto optimal set.

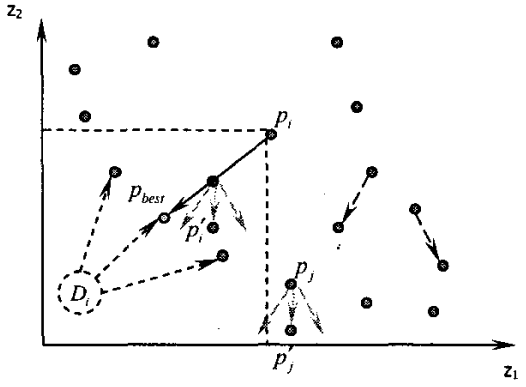

Figure 3: In order to realize the mutation operator of a dominated individual in current generation, those individuals in the first rank that dominate this individual are identified and the differential vector is defined; a non-dominated individual employs only the perturbation part of the mutation operator.

The perturbation vectors are defined by randomly chosen individual pairs from the parent population. Once the differential vector and the perturbation vectors are defined, the mutation operation can be formulated in the similar way as in the single-objective DE. Assuming the natural chromosome representation is applied, that is, each allele represents one decision variable. We use this kind of representation in our implementation of MODE and conduct the experiments in this paper. Thus, each allele of an individual will undertake the following procedure with certain mutation probability p_m (reader can regard an individual as a vector, in which the operation applies to each component of the vector):

$$p_i' = \begin{cases} p_i + F \cdot \sum_{k=1}^{K} \left(p_{i_a^k} - p_{i_b^k} \right) & \text{if } p_i \text{ is non-dominated} \\ \gamma \cdot p_{best} + (1-\gamma) p_i + F \cdot \sum_{k=1}^{K} \left(p_{i_a^k} - p_{i_b^k} \right) & \text{otherwise} \end{cases} \quad (4)$$

where p_{best} is the best individual in the Pareto sense chosen from the parent population, $\gamma \in [0,1]$ represents greediness of the operator, and K is the number of perturbation vectors, F is the factor the scale the perturbation, $p_{i_a^k}$ and $p_{i_b^k}$ are randomly selected mutually distinct individuals in the parent population, and p_i' is the offspring. This approach is illustrated in the objective space for a bi-objective problem as shown in Figure 3. In this particular case, the dominant set, D_i, of individual p_i contains three individuals, one of which would be the p_{best} for the mutation operator of individual p_i, and the two dashed arrows are the perturbation vectors. For individual p_j, which is a non-dominated solution, only the perturbation part is applied. These perturbation vectors are possibly different from the ones applied on individual p_i since they are defined independently for each individual. It should be noted that the operation is applied to decision variables in the decision space though this concept is shown in the objective space in Figure 3 for the convenience of illustration.

4.2 Pareto-based evaluation

The Pareto-based rank assignment (Goldberg, 1989) is employed as the way of evaluating individuals. Rank 1 is assigned to the non-dominated individuals and thus indicating highest fitness values in the population and these assigned individuals are removed from contention. A new set of non-dominated individuals in the rest of the population are ranked as 2 with next highest fitness values, and so forth until all of the individuals in the population are assigned a rank. In order to maintain multiple optimal solutions, the fitness sharing technique (Goldberg, 1989) is usually employed in MOEA in the literature. In this method, distance (usually Euclidean distance in either objective space or decision space) is calculated between an individual to all other individuals in the same rank. A predefined sharing radius is used as the boundary to count the number of individuals within the boundary. The more individuals counted in the boundary for an individual, the heavier the fitness value of this individual is panelized. The difficulty with this approach is to determine the value of the fitness sharing radius, which is dependent to the nature of the objective or the decision space of the problem.

In NSGA-II (Non-dominated Sorting Genetic Algorithm II), Deb et al. (2002) introduced another way to realize the fitness sharing idea. The approach has similar initiatives to the previous fitness sharing techniques. The individuals within each rank that reside in the least crowded region in that rank are less penalized. Certain crowding distance metric is calculated to determine the within-rank solution density. A crowding distance metric for a particular individual is obtained by calculating the summation of normalized distance along each objective dimensions between the two individuals within the same rank that define the smallest interval of this individual in that objective dimension. Let $B_j (j = 1, 2 \cdots k)$ be the boundary for objective j, which can be roughly estimated by examining the solutions in the first rank of the population, specifically the difference between maximum and minimum along dimension j in the first rank defines the corresponding boundary for that objective dimension. The crowd distance metric can be formulated as:

$$Cr_i = \sum_{j=1}^{k} \left(\frac{\left| z_{i_l}^j - z_{i_u}^j \right|}{B_j} \right) \quad (5)$$

where $z_{i_l}^j$ is the j^{th} objective value of the individual i_l, this value is the next smaller value to individual i for the j^{th} objective; similarly, $z_{i_u}^j$ is the j^{th} objective value of the individual i_u, this value is the next greater value to individual i for the j^{th} objective; k is the total objectives of the problem. Such a crowding distance metric is used to estimate the density of solutions around such particular individual. Smaller Cr_i indicates individual i is located in a dense region. This crowd distance metric is used to penalized the fitness of the individual, the denser area that the individual is located, the more penalty is applied.

4.3 Selection operator

The original NSGA-II applies a $(\mu + \lambda)$ selection strategy, that is, both parents and offspring are put together to compete for entering into the next generation. In the selection procedure, this approach incorporates an elite-preserving strategy. The individuals are first compared using their Pareto ranks. The individuals with higher rank are selected into the next population. If the Pareto ranks ties, the crowd distance metric is compared to fill the population of next generation. As pointed elsewhere (Xue et al., 2003), this strong elitism strategy does not produce

good results. Using this fitness evaluation and selection strategy, any solution in a higher rank would be selected for sure whatever its within-rank density is. The author (Deb and Goel, 2001) also points out the importance to keep diversity among ranks to allow individuals in lower rank to enter the next generation.

In the MODE, there is another parameter σ_{crowd} to specify how close the solution is to its surrounding solutions in objective space in order to reduce its fitness to a very small value. This strategy could prevent very similar individuals from entering the next generation, which might lead to premature convergence. In shortly, the MODE based only the fitness ranking to select the best N individuals as the next generation from both parents and offspring produced by the reproduction operator.

Comparing to the fitness sharing techniques proposed by Goldberg (1989), the crowd distance metric can be calculated without specifying any parameters in ahead. Although a new parameter σ_{crowd} is introduced in the proposed MODE, this is not that sensitive as the sharing radius to the performance of MOEA since the purpose is to prevent very similar individuals. In our experiments, a small value of σ_{crowd} is good for different situations.

5. Performance measure of MOEA

In recent years, there has been research works on MOEA and the comparisons of quantitative performance measures on a common comparative basis. (Fonseca and Fleming , 1996; Zitzler, et al., 2000). There is no general agreement on the criteria to evaluate performance of MOEA, commonly used criteria include (1) the distance of the computed Pareto set to the theoretical Pareto set; (2) the uniform spread of solutions over the Pareto front; (3) the ratio of the Pareto solutions obtained using one algorithm to those obtained using another algorithm for relative performance between two algorithms.

The distance measurement can be formulated as follows: let Z be the computed Pareto set and $\bar{Z}$ be the theoretical Pareto set in the objective space, the average distance of the computed Pareto set to the real Pareto set can defined as:

$$D := \frac{1}{|Z|}\sum_{z \in Z} \min\{\|z - \bar{z}\|, \bar{z} \in \bar{Z}\} \qquad (6)$$

where $\|\cdot\|$ defines the 2-norm of a vector, and $|\cdot|$ represents the cardinality of a set.

The performance measure of the MOEA itself has multiple criteria: how close the computed Pareto front is to the real Pareto front; how well the Pareto solutions are evenly distributed along the Pareto front. The overall purpose of MOEA is to obtain a set of discrete solutions (individuals in the population of MOEA) to best represent the real Pareto front. The distribution measure proposed by Zitzler, et al (2000) basically considers the proportion of the computed Pareto solutions that have smaller distance metric than a pre-defined distance metric σ. However, this measure cannot capture the situations when solutions are too far from each other resulting in information loss (loss of large potions of tradeoff solutions of the problem). In this research, we have used the distance criterion to evaluate the performance of a MODE.

We use the quantitative measure (the distance metric) as the first criterion to evaluate the performance of the algorithm. The second and the third criteria are visually evaluated by looking at the plotting of the computed Pareto front compared to the real Pareto font. The comparison with the results obtained by SPEA is based on both of the distance metric and the visual plotting.

6. Experimental Results

The proposed MODE is applied to solving a suite of benchmark functions proposed in Zitzler et al. (2000). These benchmark functions are carefully designed to represent different families of difficulties to multi-objective evolutionary algorithms. This suite of functions consists of six functions, one of which is discrete and represents a deceptive problem. The other 5 continuous functions are chosen as the test bed for the MODE and the computed results are compared to those obtain by SPEA (Zitzler et al., 2000). These test functions are briefly described in below following Zitzler et al. (2000).

For all the test function, the objective $f_2 = h \cdot g$, and function h and g are defined in each test function. The dimension of decision space is $m = 30$, and the interval for each dimension is $x_i \in [0,1], i = 1 \cdots m$ for all of the test functions except function $T4$, where $m = 10$ and $x_i \in [0,1], x_i \in [-5,5], i = 2 \cdots m$. These test functions are list as in Table 1.

Test function $T1$ has a convex Pareto-optimal front, while $T2$ has the non-convex counterpart of the function $T1$. Test function $T3$ represents the discreteness features, whose Pareto-optimal front consists of several disjointed continuous convex parts. The test function $T4$ contains 21^9 local Pareto-optimal fronts. This function is very hard for MOEA to find the global Pareto-optimal front. The test function $T5$ includes two difficulties caused by the non-uniformity of the search space: firstly, the Pareto-optimal solutions are non-uniformly distributed along the global Pareto-optimal front (the front is biased for solutions for which f_1 is near to 1); secondly, the density of the solutions is least near the global Pareto-optimal and highest away from the front.

$T1$	$f_1(x_1) = x_1; g(x_2 \cdots x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i / (m-1);$ $h(f_1, g) = 1 - \sqrt{(f_1 / g)};$
$T2$	$f_1(x_1) = x_1; g(x_2 \cdots x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i / (m-1);$ $h(f_1, g) = 1 - (f_1 / g)^2;$
$T3$	$f_1(x_1) = x_1; g(x_2 \cdots x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i / (m-1);$ $h(f_1, g) = 1 - \sqrt{f_1 / g} - (f_1 / g)\sin(10\pi f_1);$
$T4$	$f_1(x_1) = x_1;$ $g(x_2 \cdots x_m) = 1 + 10 \cdot (m-1) + \sum_{i=2}^{m} \left(x_i^2 - 10 \cdot \cos(4\pi x_i)\right);$ $h(f_1, g) = 1 - \sqrt{f_1 / g};$
$T5$	$f_1(x_1) = 1 - \exp(-4x_1) \cdot \sin^6(6\pi x_1);$ $g(x_2 \cdots x_m) = 1 + 9 \cdot \left(\sum_{i=2}^{m} x_i \Big/ (m-1) \right)^{0.25};$ $h(f_1, g) = 1 - (f_1 / g)^2;$

Table 1: Formulation of the five test functions

For these test evaluation tests, the MODE is performed with a population size N = 100, mutation probability $p_m = 0.3$, maximal generations $G_{\max} = 250$, crowd distance $\sigma_{crowd} = 0.001$, number of perturbation vectors $K = 2$, greediness $\gamma = 0.7$ and perturbation factor $F = 0.5$. The population size and the maximal generations are set the same as that used in Zitzler's (2000) experiments for valid comparisons. We run 30 independent trials with random seeds for each of these five test functions. The distance of the computed Pareto-optimal front to the real Pareto-optimal front is calculated for each run. The mean distance and the variance of the 30 runs for each function are summarized in Table 2. Also, the mean and variance of distances to real Pareto front obtained using SPEA (results of 30 runs, obtained from http://www.tik.ee.ethz.ch/~zitzler/testdata.html) are also summarized in the same table.

Function	SPEA		MODE	
	Mean	Variance	Mean	Variance
$T1$	0.04	2.41E-05	0.0058	2.7907E-07
$T2$	0.07	2.6E-04	0.0055	3.1446E-07
$T3$	0.03	7.3E-06	0.02156	4.3944E-07
$T4$	4.28	3.62	0.63895	0.5002
$T5$	0.48	0.02	0.02623	8.6126E-04

Table 2: Mean distance and the variance for each of the five test functions obtained using SPEA and MODE

The experimental results show that there is a consistent improvement for all of the five test functions with the comparable setting, i.e., same population size and generations, between MODE and SPEA. MODE obtained smaller distance metric with very small variance than those obtained using SPEA. As described in the extensive comparative study conducted by Zitzler et al. (2000), test function $T4$ is the most difficult one to a variety of multi-objective methodologies. This is also true to the proposed MODE. However, there is an even larger improvement by using the proposed MODE than SPEA. In order to evaluate the performance of the proposed MODE in the Pareto dominance sense compared with the SPEA, the results obtained from a typical single run of MODE and SPEA are plotted in the same graph along the theoretical Pareto front for each of the five test functions (Figure 4). It is visually shown that the results obtained using MODE clearly dominate those obtained using SPEA. In addition, the results obtained by MODE provide better representation of the real Pareto front for each of the test functions.

In all of the experiments, the maximal generation and population size are set the same as those in the experiments in Zitzler et al.(2000). Thus the number of objective function evaluation is roughly the same, which indicates that the experiments conducted in this paper are based on comparable computational resources.

7. Conclusions

Multi-objective optimization is of increasing importance in various fields and has very broad applications. Powerful and efficient multi-objective algorithms can advance both scientific research and engineering applications in different areas. In this paper, the differential evolution approach in single-objective problem domain is extended to the multi-objective domain by incorporating the Pareto optimal concept into the mutation operator. This proposed multi-objective differential evolution (MODE) approach is tested using a set of benchmark functions from the literature. The obtained experimental results are compared to those obtained by a recent multi-objective evolutionary approach, strength Pareto evolutionary algorithm (SPEA). Three evaluation criteria, including both quantitative and qualitative ones, are used to evaluate the performance of the proposed evolutionary approach. For these test examples, it has been found that the proposed MODE outperforms the SPEA in all of these criteria with comparable computational efficiency.

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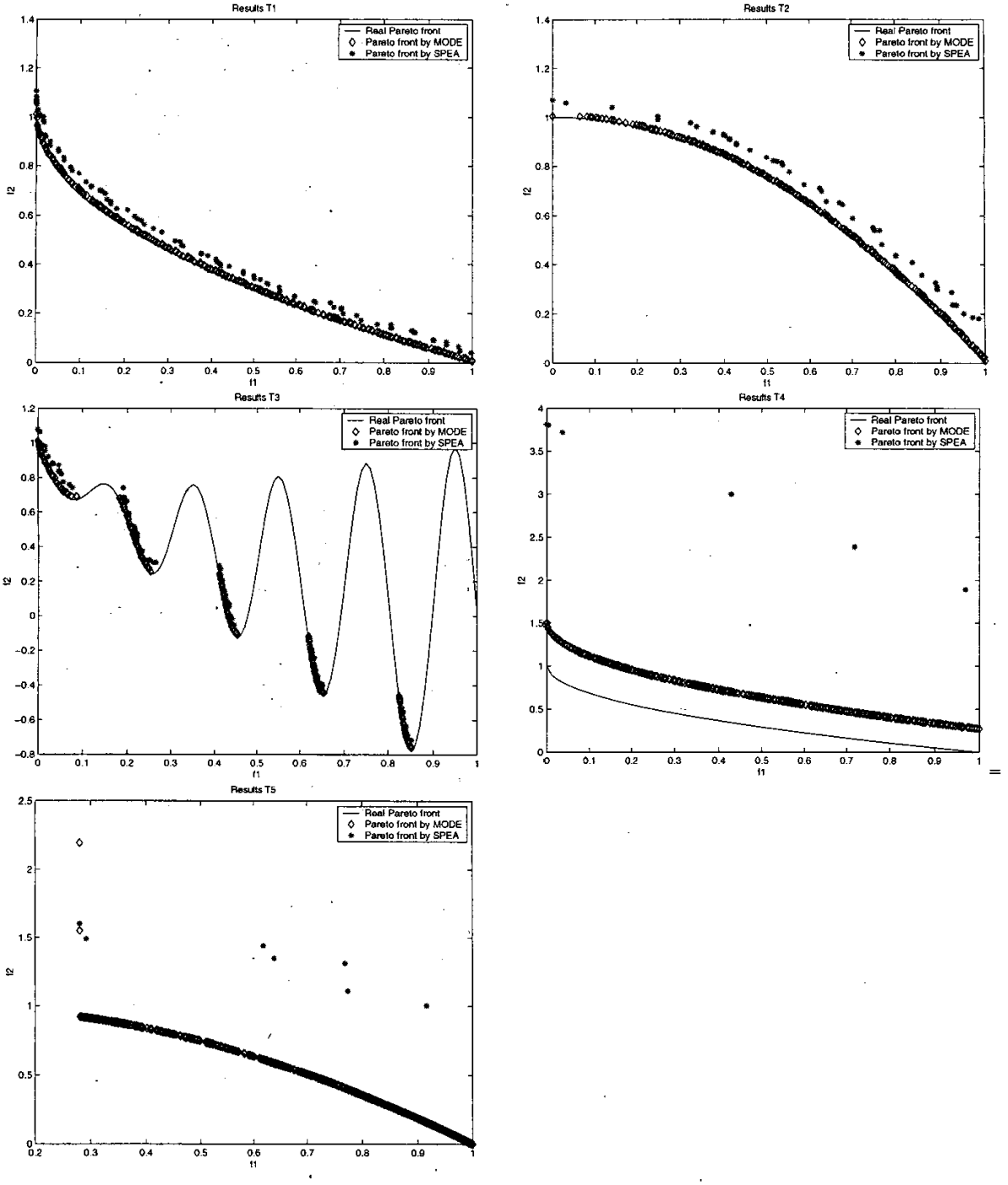

Figure 4: Typical computed Pareto front obtained from a single run of MODE and SPEA for each of the five test functions versus the real Pareto front in objective space. The curve in each plot (part of the curve for test function $T3$) is the real Pareto front, whilst the diamonds are those obtained using MODE and the stars are those obtained using SPEA.

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