

Many-Objective Evolutionary Algorithms: A Survey

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Multi-Objective Evolutionary Algorithms (MOEAs) have been widely used in real-world applications. However, most MOEAs based on Pareto-dominance handle Many-Objective Problems (MaOPs) poorly due to a high proportion of incomparable and thus mutually non-dominated solutions. Recently, a number of Many-Objective Evolutionary Algorithms (MaOEAs) have been proposed to deal with this scalability issue. In this article, a survey of MaOEAs is reported. According to the key ideas used, MaOEAs are categorized into seven classes: relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches. Several future research directions in this field are also discussed.

CCS Concepts: • **Computing methodologies** → **Randomized search**; • **Mathematics of computing** → *Bio-inspired optimization*; Evolutionary algorithms;

General Terms: Algorithms, Optimization

Additional Key Words and Phrases: many-objective optimization, evolutionary algorithm, scalability

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1. INTRODUCTION

In the real world, we are often faced with problems with multiple objectives, which are called Multi-Objective Problems (MOPs)¹. MOPs with at least four objectives are informally known as Many-Objective Problems (MaOPs) [Farina and Amato 2002], although some papers specified problems with three or more objectives as MaOPs [Wang et al. 2014]. As an important class of MOPs, MaOPs appear widely in many real-world applications, such as engineering design [Fleming et al. 2005], air traffic control [Herrero et al. 2009], nurse rostering [Sülflow et al. 2007], car controller optimization [Narukawa and Rodemann 2012], water supply portfolio planning [Kasprzyk et al. 2012], with up to hundreds of objectives². Recently, MaOPs have drawn steady

¹Without loss of generality, we consider minimization problems where all the objectives are to be minimized.

²For more information about real-world applications and synthetic problems with at least four objectives, please refer to the online appendix of this paper, which can be found on the author's homepage (<http://home.ustc.edu.cn/~lbngd17/files/OnlineAppendixToSurvey.pdf>).

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attention (238 related papers with 8359 citations) in the Evolutionary Multi-objective Optimization (EMO) community, as can be seen from Table I. It appears that there was a recent jump (2013) of the interest in this topic, including two papers in generic artificial intelligence journals, rather than just papers in specialist evolutionary computation journals/conferences.

Table I. Number of papers tackling problems with at least 4 objectives and corresponding citations (in the parentheses), in major conferences and journals in recent years.

year	2007	2008	2009	2010	2011	2012	2013
CEC	17(666)	11(435)	13(276)	14(148)	9(94)	15(102)	16(37)
GECCO	3(358)	3(140)	8(146)	6(115)	8(66)	4(46)	19(58)
EMO	9(970)	-	8(183)	-	9(136)	-	13(102)
PPSN	-	6(118)	-	7(57)	-	2(8)	-
TEC	3(1306)	2(615)	4(239)	4(312)	3(187)	3(185)	4(146)
ECJ	2(335)	1(31)	5(281)	1(28)	1(263)	2(81)	1(63)
AIJ	0	0	0	0	1(12)	0	2(14)
total	34(3635)	23(1339)	38(1125)	32(660)	31(758)	26(422)	54(420)

Source: We conducted a comprehensive literature survey using the following databases: Google Scholar, IEEE Xplore Digital Library, ACM Digital Library, and SpringerLink, using “many-objective” as the search keyword. References that address many-objective optimization but don’t explicitly use the term “many-objective” are also included. The citation number is counted until April 5th, 2015.

A multi-objective optimization problem can be stated as follows [Miettinen 1999]:

$$\begin{aligned} & \text{minimize } \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x}))^T \\ & \text{subject to } \mathbf{x} \in \Omega \end{aligned} \quad (1)$$

where the decision vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$ belongs to the (nonempty) *decision space* Ω , the objective function vector $\mathbf{F}: \Omega \rightarrow \Lambda$ consists of m ($m \geq 2$) objectives, and the *objective space* Λ usually equals to $S \subset R^m$. For MaOPs, the number of objectives m is larger than 3. Since different solutions may have an advantage over each other on different objectives, the concept of Pareto dominance is critical when comparing solutions of a MOP. The goal of multi-objective optimization is to approximate the Pareto Front (PF) in the objective space so that no further improvement on any objective can be achieved without harming the rest of objectives.

It should be noted that many-objective optimization is closely related to Multiple Criteria Decision Making (MCDM). When dealing with MaOPs, researchers from MCDM usually use different preference models to compare non-dominated solutions [Figueira et al. 2005]. The preference models used by the MCDM methods can be divided into three categories [Løken 2007]: value measurement models which assign a numerical score to each solution³, goal, aspiration and reference level models which prefer solutions closest to the a determined goal or aspiration level, and outranking models which use pair-wise comparisons to select solutions⁴.

Evolutionary Algorithms (EAs) are population-based, black-box search/optimization methods and don’t need particular assumptions like continuity or differentiability. They are very suitable for dealing with MOPs [Yang et al. 2013]. In the past few decades, researchers have proposed plenty of Multi-Objective Evolutionary Algorithms (MOEAs), which can be categorized into four classes: Pareto-based approach, aggregation-based approach, indicator-based approach, and preference-based

³According to [Velasquez and Hester 2013], the MCDM methods based on value measurement models, such as Multi-Attribute Utility Theory (MAUT) and Analytic Network Process (ANP), are very popular recently.

⁴Extensively reviewing these methods is out of the scope of this paper. For more information, please refer to [Løken 2007; Velasquez and Hester 2013; Purshouse et al. 2014].

approach [Zhou et al. 2011]. However, when dealing with MaOPs, traditional MOEAs are more likely to fail to converge to the PF since the problem of deterioration becomes more prevalent with an increasing number of objectives [Wang et al. 2014]. Thus researchers have designed a number of algorithms to overcome the obstacles.

Although numerous studies on MaOPs have been conducted, few surveys on MaOEAs are available. Some of recent studies focus on certain sub-areas of multi-objective optimization: multi-objective algorithms for data mining ([Mukhopadhyay et al. 2014a; 2014b]) and decomposition-based multi-objective algorithms ([Santiago et al. 2014]), while some are more closely related to many-objective optimization. Ishibuchi et al. demonstrated the difficulties of MaOPs through experiments and reviewed a number of techniques for improving scalability of MOEAs [Ishibuchi et al. 2008]. Hughes reviewed different fitness assignment methods for many-objective optimization [Hughes 2008]. Purshouse et al. focused on hybrid methods which combine MCDM and multi-objective optimization [Purshouse et al. 2014]. von Lücken et al. analyzed different methods, main findings, and open questions in many-objective optimization [von Lücken et al. 2014]. Compared with it, our paper has carried out a more detailed investigation on the following aspects:

- Completeness: [von Lücken et al. 2014] conducted a good review over 112 references. Our paper provides a comprehensive survey based on 200+ related papers.
- Taxonomy: We provide a different and more comprehensive categorization of different algorithms and techniques for MaOPs. First, our categorization analyzes key techniques in algorithms in order to describe the algorithms in more details. Second, the reference set based methods are included in our categorization, but not in previous surveys. Third, for each category and sub-category, we include important details, such as methods and test problems, in our survey, supported by summary tables.
- Test Problems: In order to understand what algorithms were used to solve what problems, it is important to include problems in a comprehensive survey. That is what we have done in our survey, including the real world problems listed in the appendix.

Based on the key idea used, we categorize MaOEAs into seven classes: the relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches, summarised in Fig. 1⁵. In this paper, we conduct an extensive survey on MaOEAs and discuss the characteristics of each class of approaches.

The rest of the paper is organized as follows: Section 2 introduces the background of many-objective optimization. Sections 3 and 4 reviews the relaxed dominance based and the diversity-based approaches, respectively. The aggregation-based approach is presented in Section 5. Section 6 is devoted to the indicator-based approach. Section 7 presents the reference set based approach. The preference-based and dimensionality reduction approaches are explained in Sections 8 and 9, respectively. The last section summaries the development of MaOEAs and future research directions.

2. BACKGROUND OF MANY-OBJECTIVE OPTIMIZATION

2.1. Problem Definition

There is no generally agreed definition of MaOPs. The primary motivation behind MaOPs is to highlight the challenges posed by a large number of objectives to existing MOEAs. We therefore define a MaOP as a MOP with $m \geq 4$. Although we

⁵The first and second classes here are related to the Pareto-based methods in [Zhou et al. 2011]. The aggregation-based, indicator-based, and preference-based methods are similar to those in [Zhou et al. 2011]. The remaining two classes are not covered by [Zhou et al. 2011]. It should be noted that Zhou et al. focused on introducing algorithms for MOPs instead of MaOPs [Zhou et al. 2011].

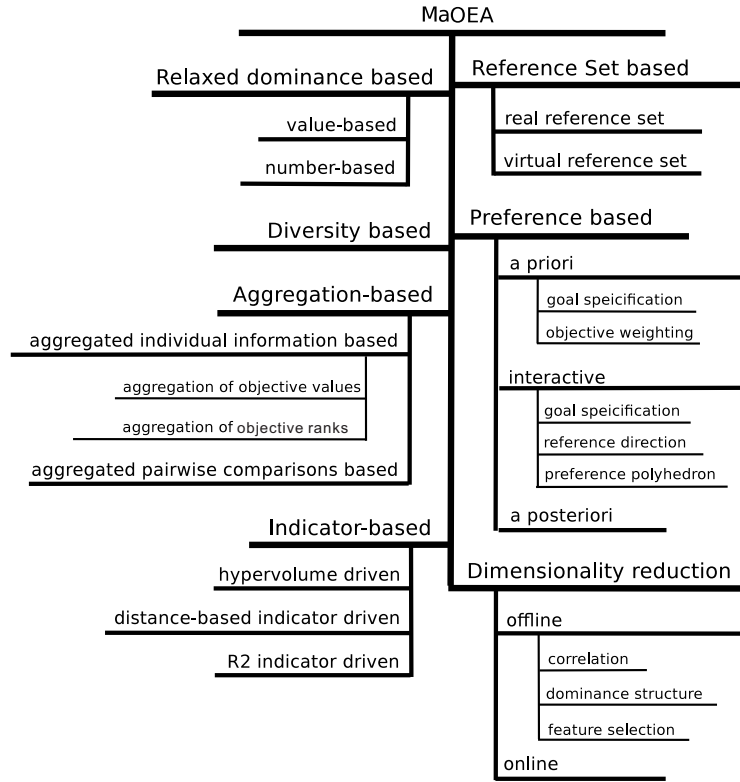


Fig. 1. Classification of Many-objective Evolutionary Algorithms

use four objectives as the dividing line between MOPs and MaOPs now, the number would increase as stronger MaOEAs are developed in the future. Hence the definition presented here is an evolving one and serves a more practical than theoretical purpose.

The Pareto dominance relation is widely used to compare solutions of a MaOP. Based on the Pareto dominance relation, Pareto Optimal Solution, Pareto Set (PS), and Pareto Front (PF) are further defined. These terms are defined as follows:

Definition 2.1 (Pareto dominance [Yu 1974]). Given two solutions $\mathbf{x}, \mathbf{y} \in \Omega_f$ and their corresponding objective vectors $\mathbf{F}(\mathbf{x}), \mathbf{F}(\mathbf{y}) \in R^m$, \mathbf{x} dominates \mathbf{y} (denoted as $\mathbf{x} \prec \mathbf{y}$) if and only if $\forall i \in \{1, 2, \dots, m\}, f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ and $\exists j \in \{1, 2, \dots, m\}, f_j(\mathbf{x}) < f_j(\mathbf{y})$.

Definition 2.2 (Pareto Optimal Solution). A solution $\mathbf{x}^* \in \Omega_f$ is Pareto optimal if there does not exist another solution $\mathbf{x} \in \Omega_f$ that dominates it.

Definition 2.3 (Pareto Set). The union of all Pareto Optimal Solutions are called Pareto Set: $PS = \{\mathbf{x} \in \Omega_f | \nexists \mathbf{y} \in \Omega_f, \mathbf{y} \prec \mathbf{x}\}$

Definition 2.4 (Pareto Front). The corresponding objective vector set of the Pareto Set is called the Pareto Front (PF).

2.2. Quality Measurements of Many-objective Optimization Algorithms

The goal of optimizing a MaOP, descriptively defined as follows, is the same as that of optimizing a general MOP [Li et al. 2014]:

Definition 2.5 (Goal of Optimizing a MaOP). The goal of optimizing a MaOP is to obtain an approximation set A to the PF, including the following two subgoals:

- 1) All the solutions in A are as close as possible to the PF.
- 2) All the solutions in A are as diverse as possible in the objective space.

where an approximation set is defined as follows [Zitzler et al. 2003]:

Definition 2.6 (Approximation Set). Let $A \subset \Lambda$ be a set of objective vectors, denoted as $A = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{|A|}\}$. A is called an approximation set if any element of A does not dominate or is not equal to any other objective vector in A .

Corresponding to the optimization goal, the quality of an approximation set is measured in terms of two aspects: 1) convergence — the closeness to the real PF in the objective space, and 2) diversity — whether the solutions are well-distributed over the PF. Quality indicators are quantified quality measurements [Zitzler et al. 2003]:

Definition 2.7 (Quality indicator). An k -ary quality indicator I is a function $I: \Gamma^k \rightarrow R$, which assigns each vector (A_1, A_2, \dots, A_k) of k approximation sets a real value $I(A_1, A_2, \dots, A_k)$.

The hypervolume indicator (I_{HV} , aka S metric) was described as the Lebesgue measure L of the union of the hypercubes c_i corresponding to each solution $\mathbf{a}_i \in A$ [Zitzler and Thiele 1998; Emmerich et al. 2005]: $I_{HV}(\mathbf{z}^\dagger, A) = L(\bigcup_i c_i | \mathbf{a}_i \in A) = L(\bigcup_{\mathbf{a} \in A} \{\mathbf{b} | \mathbf{a} \prec \mathbf{b} \prec \mathbf{z}^\dagger\})$, where \mathbf{z}^\dagger is the worst point in the objective space. It measures both convergence and diversity of an approximation set.

Generational Distance (GD) [Van Veldhuizen and Lamont 1998] and Inverted Generational Distance (IGD) [Bosman and Thierens 2003] are defined as follows: $I_{GD} = \frac{1}{|A|} \left(\sum_{i=1}^{|A|} \text{dis}(\mathbf{a}_i, PF')^p \right)^{\frac{1}{p}}$, $I_{IGD} = \frac{1}{|PF'|} \left(\sum_{i=1}^{|PF'|} \text{dis}(\mathbf{p}_i, A)^p \right)^{\frac{1}{p}}$, where $A = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{|A|}\}$ is the approximation set, $PF' = \{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_{|PF'|}\}$ is a subset of the PF, $\text{dis}(\mathbf{a}_i, PF')$ is the distance between \mathbf{a}_i and PF' , $\text{dis}(\mathbf{p}_i, A)$ is the distance between \mathbf{p}_i and A ⁶. GD measures the convergence quality, while IGD measures both convergence and diversity of an approximation set. Similar distance-based indicators, such as the additive approximation metric⁷ [Bringmann et al. 2011], and the Δ_p indicator [Schütze et al. 2010] are proposed for performance measurement.

The Generalized Spread (I_{GS}) [Zhou et al. 2006], which measures the diversity of an approximation set is defined as follows:

$$I_{GS} = \frac{\sum_{i=1}^s d(\mathbf{e}_i, A) + \sum_{\mathbf{p} \in PF'} |d(\mathbf{p}, A) - \bar{d}|}{\sum_{i=1}^s d(\mathbf{e}_i, A) + |PF'| \bar{d}} \quad (2)$$

where $\{\mathbf{e}_1, \dots, \mathbf{e}_s\}$ are s extreme solutions in PF' , $d(\mathbf{p}, A)$ is the Euclidean distance between \mathbf{p} and A , \bar{d} is the mean of $d(\mathbf{p}, A)$ for all $\mathbf{p} \in PF'$.

The R2 indicator is a class of utility function based indicators. Given a reference set R , the R2 indicator can be defined as: $R2(A, U) = -\frac{1}{|U|} \sum_{u \in U} (\max_{\mathbf{a} \in A} \{u(\mathbf{a})\})$, where U is a utility function set [Hansen and Jaszkiewicz 1998; Brockhoff et al. 2012]. If one sets U to a series of weighted Tchebycheff functions with a set of k weighting

⁶In practice, p is usually set to 2. The distance between a solution and a solution set is the distance between the solution and the member in the solution set that is closest to the solution.

⁷It is also known as the α indicator.

vectors V and selects the utopian point z^* into R , then the R2 indicator is defined as $R2(z^*, A, V) = \frac{1}{|V|} \sum_{\lambda \in V} \min_{\mathbf{a} \in A} \{ \max_{j \in \{1, \dots, m\}} \{ \lambda_j |z_j^* - \mathbf{a}_j| \} \}$. The R2 indicator can measure both convergence and diversity of an approximation set [Brockhoff et al. 2012].

According to [Zitzler et al. 2003], there exists no unary quality measure that can indicate whether an approximation A is better than another approximation B. Instead, most quality measures indicate that A is better B actually means that A is not worse than B. Binary indicators, such as the binary ϵ -indicator of [Zitzler et al. 2003], might be able to overcome this limitation.

2.3. Key Challenges of Many-objective Optimization

When the number of objectives increases, one has to deal with the following issues:

- Dominance resistance (DR) phenomenon [Fonseca and Fleming 1998; Purshouse and Fleming 2007; Knowles and Corne 2007]: the incomparability of solutions caused by the enormously increasing proportion of non-dominated solutions.
- Limited solution set size: Under non-degenerated scenarios, the Pareto front of a m -objective problem is a $(m-1)$ -dimensional manifold [Ishibuchi et al. 2015; Jin and Sendhoff 2003]. In order to describe such a front, we have to increase the number of solutions exponentially.
- The visualization of solution set in the objective space needs special techniques, such as projection to a lower dimension space, parallel coordinates etc. [Walker et al. 2013]

It has been shown that the Pareto-based approaches, such as Non-dominated Sorting Genetic Algorithm II (NSGA-II) [Deb et al. 2002] and the improved Strength Pareto Evolutionary Algorithm (SPEA2) [Zitzler et al. 2001], may deteriorate their performance significantly on MaOPs [Knowles and Corne 2007]. This is mainly due to the DR phenomenon and the Active Diversity Promotion (ADP) mechanisms [Purshouse and Fleming 2007]. ADP refers to that when the dominance-based primary criterion fails to discriminate solutions, the density-based secondary criterion is activated to determine which solutions are able to survive the environmental selection. As a result of DR and ADP, the final solution set might not even converge to PF but stagnate far away from it [Wagner et al. 2007].

Intuitively, there are two avenues to improve the scalability of Pareto-based algorithms: the relaxed dominance based approach and the diversity-based approach, related to DR and ADP respectively. On one hand, in order to mitigate the adverse impact of DR, researchers have proposed several variants of the Pareto dominance to enhance the selection pressure towards the PF [Laumanns et al. 2002; Sato et al. 2007]. Compared to the classical Pareto dominance, these methods are more capable of discriminating solutions of MaOPs. On the other hand, several advanced diversity management strategies are proposed to deal with the ADP problem [Adra and Fleming 2011]. In general, these methods tend to pay more attention to the convergence.

Non-Pareto-based MOEAs, such as indicator-based and aggregation-based approaches, do not suffer from the selection pressure problem, since they do not rely on the Pareto-dominance to push the population towards the PF. However, they still suffer from the "curse of dimensionality", that is, to search simultaneously in an exponentially increasing number of directions. For the aggregation-based approach, the key point is the setting of weighting vectors, since it plays a vital role in maintaining the distribution of the population [Giagkiozis et al. 2013]. For the hypervolume-based MOEAs, an important class of indicator-based MOEAs, the high computational cost of the indicator is a limitation [Wagner and Neumann 2013]. In addition to the above-mentioned approaches, the reference set based approach which uses a reference set to evaluate and select solutions provides a new alternative to solving MaOPs [Deb and

Jain 2014]. Different from other approaches, the dimensionality reduction approach aims at reducing the number of objectives by analyzing the relationship among the objectives or using feature selection techniques. To incorporate user preferences into the search process, researchers designed the preference-based approach to focus on a subregion of the PF. The dimensionality reduction approach also attempts to eliminate the less important objectives in order to reduce the difficulty of the original problem [Brockhoff and Zitzler 2009].

3. RELAXED DOMINANCE BASED APPROACH

In order to differentiate non-dominated solutions and enhance the selection pressure towards the PF, many variants of dominance have been proposed. We categorize them into the following two classes: value-based and number-based dominance.

3.1. Value-based Dominance

As its name indicates, these methods modify the Pareto dominance by changing the objective values of the solutions when comparing them. In general, these modifications aim at enlarging the dominating area of the non-dominated solutions so that some of them are more likely to be dominated by others. The summary of these kind of modifications is shown in Table II. The first column is the names of relaxed forms of dominance. The second column shows the MaOEAs where the techniques are integrated. The last two columns indicate the papers and the testing problems used in the experiments. In the remaining part of this section, we denote the newly generated objective vector corresponding to the $\mathbf{F}(\mathbf{x})$ as $\mathbf{G}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_m(\mathbf{x}))^T$.

Table II. Value-based Dominance.

technique	algorithm	paper	test problems (m)
ϵ -dom	ϵ -MOEA	[Hadka et al. 2012; Hadka and Reed 2012]	UF(5) DTLZ(8)
		[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
	ϵ -NSGA-II	[Kollat and Reed 2006]	LTM test case(4)
		[Hadka et al. 2012; Hadka and Reed 2012]	UF(5) DTLZ(8)
CDAS	CDAS	[Sato et al. 2007]	0/1 MKP(5)
	PPD-MOEA	[Sato et al. 2010a]	0/1 MKP(10)
	S-CDAS	[Sato et al. 2010b]	0/1 MKP(10)
	CCG-S-CDAS	[Sato et al. 2011b]	0/1 MKP(10)
	adaptive CCG	[Sato et al. 2012]	0/1 MKP(10)
α -dom	-	-	-
grid-dom	GrEA	[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
Cone ϵ -dom	cone ϵ -MOEA	-	-
subspace ϵ -dom	ϵ R-EMO	[Aguirre and Tanaka 2009b; 2009a; 2010b]	MNK-landscape
	μ FA ϵ R E	[Aguirre and Tanaka 2010a]	MNK-landscape
	[Jaimes et al. 2011]	[Jaimes et al. 2011]	DTLZ5(I,M) (15) 0/1 knapsack(15)
	A $\epsilon_s \epsilon_h$ EMyO	[Aguirre et al. 2013b]	DTLZ(6)
Volume-dom	SEAMO2'	[Le et al. 2009]	MKP(4)
	SPEA2'		
	NSGA-II'		
multi-level grid	random search	[Laumanns and Zenklusen 2011]	-

Laumanns et al. [Laumanns et al. 2002] proposed the concept of ϵ -dominance. Given two solutions $\mathbf{x}, \mathbf{y} \in \Omega$ and $\epsilon > 0$, \mathbf{x} is said to ϵ -dominate \mathbf{y} , iff $\forall i \in \{1, \dots, m\}, (1 - \epsilon)f_i(\mathbf{x}) \leq f_i(\mathbf{y})$. In this case, $g_i(\mathbf{x}) = (1 - \epsilon)f_i(\mathbf{x}) \leq f_i(\mathbf{x}), \forall i \in \{1, \dots, m\}$. An example is shown as Fig 2.(a). The grey area is the increased part of objective space where individuals are considered ϵ -dominated by p2 while not dominated by p2. It is obvious that the dominating space of p2 is enlarged. Based on ϵ -dominance, Deb et al.

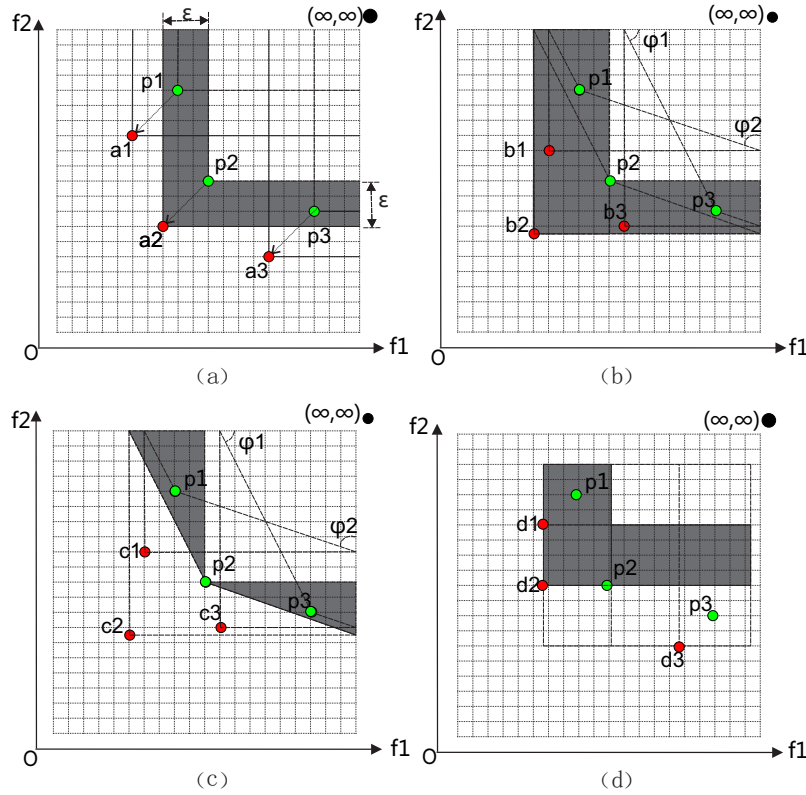


Fig. 2. Relaxed Forms of Dominance. The area on the up-right of X_i is considered relaxed-dominated by p_i , $i = 1, 2, 3$ in sub-figure (X), $X = \{a, b, c, d\}$. (a) is an example relationship of ϵ -dominance among points in the objective space. The grey area is the increased part of objective space where solutions are considered ϵ -dominated by p_2 , similarly hereinafter. (b) shows the relationship of CDAS, the shape of grey area of which is quite similar to ϵ -dominance. The angles ϕ_1 and ϕ_2 are used to control the dominance area, playing a similar role to ϵ in ϵ -dominance. (c) demonstrates an example of α -dominance. ϕ_1 and ϕ_2 are used to tune the trading ratio between objectives. (d) shows an example of grid-dominance. A 3×3 grid is created by the current population $\{p_1, p_2, p_3\}$. The shape of grey area is also similar to ϵ -dominance.

proposed a steady-state algorithm named ϵ -MOEA [Deb et al. 2005]. In ϵ -MOEA, the objective space is divided into hyperboxes, each of which is assigned at most one solution. Solutions in different hyperboxes are compared based on ϵ -dominance. Otherwise, traditional Pareto dominance is used to compare two solutions. The performance of ϵ -MOEA on MaOPs is tested [Hadka et al. 2012; Hadka and Reed 2012]. Laumanns et al. combines the multi-level grid archiving with random search [Laumanns and Zitzler 2011] and obtains similar convergence properties as hypervolume archiving. It is computationally much cheaper than hypervolume-based algorithms.

By deriving the Sine theorem, Sato et al. [Sato et al. 2007] proposed a method to control the dominance area of solutions (CDAS). It is defined as:

$$g_i(\mathbf{x}) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \quad (3)$$

where r is the norm of $F(\mathbf{x})$, ω_i is the declination angle between $F(\mathbf{x})$ and $f_i(\mathbf{y})$, S_i is a user defined parameter to control the degree of expansion or contraction, $\phi_i = S_i \cdot \pi$. As the right part of Fig 2 (b) shows, the grey area is the increased part of objective space

where individuals are considered CDAS-dominated by p_2 while not Pareto dominated by p_2 . From Fig 2, we can see that ϵ -dominance and CDAS are quite similar, despite their difference in controlling mechanisms of $g_i(x)$ s. Empirical results on 4 and 5 objective 0/1 knapsack problems have shown that, by changing the degree of expansion or contraction of the dominance area, this method can improve or reduce the selection pressure, thus emphasizing convergence or diversity of results of the algorithm. However, the setting of the angle can be a challenging task because the optimal setting differs depending on the problem at hand [Narukawa 2013]. Based on the studies on parameter settings [Sato et al. 2010a; 2010b], Narukawa proposed CDAS-D, which controls the angle dynamically during optimization [Narukawa 2013]. Specifically, the angle is replaced by a random value once the hypervolume is decreased. Experimental results on DTLZ functions with up to 9 objectives showed that CDAS-D is comparable to CDAS with the optimal ratio of the dominating space.

In [Ikeda et al. 2001], Ikeda et al. proposed another variant of dominance named α -domination. When comparing two solutions on some objective, not only the value of this objective, but all the other objective values are taken into account. The α -domination uses linear trade-off functions to define the tolerance of dominance. It allows a solution to dominate another if it's slightly inferior to the other in one objective but largely superior in other objectives. An example of α -domination is presented in Fig. 2 (c). An identical idea is used in the Guided Multi-Objective Evolutionary Algorithm (G-MOEA), although explained from the perspective of integrating the user's preference [Branke et al. 2001]. However, the experiments of G-MOEA are carried out on bi-objective problems and the α -domination has not been included into any MaOEAs to the best of our knowledge.

Yang et al. [Yang et al. 2013] proposed a grid based evolutionary algorithm (GrEA) for optimizing MaOPs. In GrEA, Yang et al. adopted the adaptive construction of grids, borrowing ideas from Knowles and Corne [Knowles and Corne 2003b]. Based on the concept of grids, the selection pressure is increased by the grid dominance and grid difference. An example of grid-dominance is depicted in Fig 2.(d). Compared to the traditional box-centered calculation of grids, individually-centered calculation of grids in [Yang et al. 2013; Knowles and Corne 2003b] can depict locations of points in the population which is changing during the evolution. The grid-based mechanism, together with a fitness adjustment strategy to avoid local overcrowding, is shown to be highly effective on 36 out of 52 test instances from three benchmark function families (i.e., 4-10 objective DTLZ, DTLZ5(I,M), and MOTSP). However, it should be noted that individually-centered calculation of grids cannot guarantee convergence and thus will suffer from potential deterioration, which is a serious drawback in the context of many-objective optimization [Knowles and Corne 2003b].

The adaptive ϵ -ranking can be seen as an extension of ϵ -dominance to some subspaces of the objective space [Aguirre and Tanaka 2009b; Batista et al. 2011b]. The ϵ -Ranking Multiobjective Optimizer (ϵ R-EMO) [Aguirre and Tanaka 2009b] applies a space partitioning strategy and the adaptive ϵ -ranking procedure. Based on the partitioning strategy, the subspace sampling schedule enables different emphasis on different subspaces at each generation. Besides, the selection pressure is enhanced using the adaptive ϵ -ranking procedure. The experiment revealed that it provided a remarkable improvement in the convergence and diversity of the results on MNK-Landscapes [Aguirre and Tanaka 2007] with up to 10 objectives. Some extensions, such as adaptive space partitioning [Aguirre and Tanaka 2010b] and ϵ -hood mating [Aguirre et al. 2013b] further improve the performance.

In addition to adaptive ϵ -ranking, another variant of ϵ -dominance, namely cone ϵ -dominance, is also proposed [Batista et al. 2011b]. The cone ϵ -dominance, can be seen as a hybridization of ϵ -dominance and α -dominance. Batista et al. [Batista et al. 2011a]

analyzed three relaxed dominance criteria, i.e., α -dominance, ϵ -dominance, and cone ϵ -dominance. The results on quadratic and DTLZ test problems with from 2 to 50 objectives demonstrated that all three relaxed forms of dominance showed better scalability to the number of objectives than Pareto dominance.

3.2. Number-based Dominance

The number-based methods try to compare a solution to another by counting the number of objectives where it is better than, the same as, or worse than the other. Only three methods are categorized into this class, as shown in Table III.

Table III. Number-based Dominance.

technique	algorithm	paper	test problems (m)
Favour	DDB ^a	[Drechsler et al. 2001]	5 benchmarks(7)
$(1 - k)$ -dom	-	[Farina and Amato 2002]	test case(12)
$(1 - k)_F$ -dominance	-	[Farina and Amato 2002]	-
$(1 - \alpha)_F$ -dominance	-	[Farina and Amato 2002]	-
L-dominance	MDMOEA	[Zou et al. 2008]	DTLZ(9)
	LD	[Garza-Fabre et al. 2010a]	DTLZ(20)

^aThe name of algorithm was not explicitly mentioned in [Drechsler et al. 2001], we name the algorithm with the initials of the authors' names.

The $(1 - k)$ -dominance proposed in [Farina and Amato 2002] is quite simple. For two solutions $\mathbf{x}, \mathbf{y} \in \Omega$, n_b, n_e, n_w are the numbers of objectives where \mathbf{x} is better than, equal to, or worse than \mathbf{y} . \mathbf{x} is said to $(1 - k)$ -dominate \mathbf{y} if and only if

$$\begin{cases} n_e < m \\ n_b \geq \frac{m - n_e}{k + 1} \end{cases} \quad (4)$$

where $0 \leq k \leq 1$. It is obvious that $k = 0$ corresponds to the classical definition of Pareto dominance. The larger k is, the more relaxed the new dominance is. In addition, two extensions are proposed by incorporating fuzzy arithmetic techniques. In [Farina and Amato 2002], no specific MaOEAs were proposed. Instead, they only tested the effects of modified dominance on the comparability of many-objective solutions. Favor relation [Drechsler et al. 2001] seems to be simpler than $(1 - k)$ -dominance. A solutions is considered to be superior than another in terms of favor relation if $n_b > n_e$.

The L-dominance (LD) [Zou et al. 2008] can be seen as an extension of the favor relation [Drechsler et al. 2001]. We say \mathbf{x} L-dominates \mathbf{y} if and only if

$$n_b - n_w = L > 0 \wedge \|\mathbf{F}(\mathbf{x})\|_p < \|\mathbf{F}(\mathbf{y})\|_p \text{ (for certain } p) \quad (5)$$

where $\|\mathbf{F}(\mathbf{x})\|_p$ is the p -norm of $\mathbf{F}(\mathbf{x})$. A solution is said to be L-optimal if there is no other solutions which can L-dominate it. L-optimal set and L-optimal front are the sets of L-optimal solutions in the decision space and objective space, respectively. From Theorems 1 and 2 in [Zou et al. 2008], we can show that Pareto-dominance implies L-dominance, but the other way around is not true. An L-optimal set is a subset of the Pareto set. An L-optimal front is a subset of the Pareto front. A dynamical MOEA based on LD, namely MDMOEA [Zou et al. 2008], can converge to the true L-optimal front and maintain a well distributed population on DTLZ problems with up to 9 objectives. In addition, experiments on 20-objective DTLZ functions showed that LD can accelerate the convergence [Garza-Fabre et al. 2010a].

One obvious advantage of the number-based dominance is that the normalization of objectives is done automatically by comparing elements with each other. This may result in significant speed-ups and simplify MOEAs [Drechsler et al. 2001].

4. DIVERSITY-BASED APPROACH

Although most of recent work focuses on enhancing the selection pressure towards the PF, another avenue to adjusting Pareto-based algorithms for MaOPs is to apply a customized diversity-based approach. In general, these methods (as shown in Table IV) intend to improve the performance by reducing the adverse impact of diversity maintaining.

Table IV. Diversity-based Ranking Methods

technique	algorithm	paper	test problems (m)
grid + adaptive neighborhood	layering selection	[Li et al. 2010]	DTLZ(15)
grid crowding distance, adaptive neighborhood, fitness adjustment	GrEA	[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
DM1	NSGA-II/DM1	[Adra and Fleming 2011]	DTLZ(20)
DM2	NSGA-II/DM2		
SDE	SPEA2+SDE NSGA-II+SDE PESA-II+SDE	[Li et al. 2013]	DLTZ(10) TSP(10)

Adra et al. [Adra and Fleming 2011] introduced a diversity management mechanisms named DM1. DM1 determines whether or not to activate diversity promotion according to the spread of the population. The diversity promotion mechanism in the selection is deactivated once the population is excessively diverse. On a set of test functions with up to 20 objectives, NSGA-II/DM1 repeatedly outperformed NSGA-II in terms of convergence and distribution.

In addition to using the grid-dominance to enhance the selection pressure, three grid-based criteria, i.e. grid ranking, grid crowding distance, and grid coordinate point distance are introduced into GrEA [Yang et al. 2013] to help maintain the distribution among solutions, since it is important to balance convergence and diversity [Črepinšek et al. 2013]. In order to avoid local overcrowding, they used an adaptive neighborhood to estimate the density of individuals, which is similar to [Li et al. 2010]. The fitness of individuals is adjusted in the selection process considering both neighborhood and grid dominance relations. The experimental results showed GrEA can achieve a better coverage of the PF than the other six state-of-the-art algorithms on 36 out of 52 test instances. Similar grid-based diversity management is also used in [Li et al. 2010] for the Average Ranking (AR) algorithm and has been shown to have a good balance among convergence, uniformity and spread.

Recently, a shift-based density estimation (SDE) strategy is proposed in [Li et al. 2013]. Specifically, when measuring the density of a solutions neighborhood, SDE shifts the position of other solutions according to their convergence comparison with the current solution, thus both the distribution and the convergence information⁸ of the solutions are taken into account. As a result, it can reduce the detrimental impact of dominance resistant solutions on the convergence of Pareto-based MOEAs. Another advantage of SDE is that it can be easily adopted by traditional Pareto-based algorithms such as NSGA-II [Deb et al. 2002] and SPEA2 [Zitzler et al. 2001]. Its effectiveness in balancing convergence and diversity is demonstrated by experimental results on DTLZ and TSP test functions with up to 10 objectives.

Pasia et al. [Pasia et al. 2011] examined multi-objective random one-bit climbers (moRBCs) with tabu moves and adaptive ϵ -ranking on MNK-landscapes [Aguirre and

⁸Convergence information of a solution means the relative proximity of the solution to the Pareto front. Given two solutions, SDE estimates the density of one solution's neighborhood by considering only distance of the better objective dimensions compared with the other one. By ignoring the distance of the worse dimensions of the first solution, its convergence information is reflected during the density estimation.

Tanaka 2009b]. In tabu moves, each solution is associated with a tabu list of moves to avoid re-visiting previous solutions. Experimental results indicated that tabu moves can greatly improve the spread quality of moRBCs while the adaptive ϵ -ranking contributed mostly to the convergence improvement. Tabu moves and adaptive ϵ -ranking complement each other and lead to improvement in terms of both convergence and diversity performance of moRBCs on MNK-landscapes.

5. AGGREGATION-BASED METHODS

Using aggregation functions is another way to differentiate many-objective solutions. According to the information aggregated, the methods are categorized into two classes: aggregation of individual information and aggregation of pairwise comparisons.

5.1. Aggregated Individual Information based Methods

This kind of methods usually use the aggregated individual information to compare solutions. The methods can be divided into two sub-classes: aggregation of objective values and aggregation of objective ranks, as shown in Tables V and VI respectively.

Table V. Aggregation of Objective Values

algorithm	variant	paper	test problems (m)
MSOPS	MSOPS	[Hughes 2003]	hypersphere (20) [Hughes 2011]
	MSPOS-II	[Hughes 2007a]	test functions(5) [Hughes 2007a]
	MODELS	[Hughes 2011]	hypersphere (20) [Hughes 2011]
MOEA/D	MOEA/D-PBI	[Zhang and Li 2007]	DTLZ(3)
	W-MOEA/D	[Zhang and Li 2007]	DTLZ(3)
	T-MOEA/D	[Zhang and Li 2007]	DTLZ(3)
	MOEA/D-DRA	[Zhang et al. 2009]	CEC09(5)[Zhang et al. 2008]
			DTLZ(5)
	UMOEAD/D	[Tan et al. 2013]	F1(5),F2(5)[Tan et al. 2013] MKP(4)[Zitzler and Thiele 1999]
	MOEA/D-b	[Wang et al. 2013]	WFG(7)
DBEA-Eps	DBEA-Eps	[Asafuddoula et al. 2013]	DTLZ(15)
			CSIP(3) [Jain and Deb 2014] WRMP(5) [Ray et al. 2001] GAADP(10) [Hadka et al. 2012]
DQGA	DQGA	[Ray et al. 2013]	DTLZ(8) WROP(5) [Ray et al. 2001]
WS	WS	[Garza-Fabre et al. 2009]	DTLZ(50)
MDFA	MDFA	[Garza-Fabre et al. 2010b]	DTLZ(50)
PGA	PGA	[Garza-Fabre et al. 2011]	DTLZ(50)

5.1.1. Aggregation of Objective Values. Weighted sum, weighted Tchebycheff (aka weighted min-max methods), vector angle distance scaling, and the boundary intersection methods are commonly used aggregation functions [Hughes 2011; Zhang and Li 2007]. Garza-Fabre et al. [Garza-Fabre et al. 2009] tested the weighted sum as a ranking method for MaOPs, where the weighting coefficients are used to specify importance of objectives. By employing a series of weighting vectors, the researchers are able to decompose a MaOP into many single-objective sub-problems, each of which corresponds to a weighting vector [Ray et al. 2013].⁹ Since these MOEAs do not rely on the Pareto dominance when conducting the selection, they can be applied directly to MaOPs without any additional change. Three aspects are critical to these algorithms: what aggregation functions are used, how to set the weighting vectors for sub-problems, and how to update the best-known solution for each sub-problem.

⁹The aggregation-based approach, aka compromise programming, can be dated back to the 1970s [Miettinen 1999].

In Multiple Single Objective Pareto Sampling (MSOPS), population members are ranked according to both the vector angle distance scaling and weighted Tchebycheff methods [Hughes 2003]. MSOPS enables users to analyse a MaOP at hand, especially in terms of bounds and discontinuities of the PF¹⁰. However, this feature of MSOPS is only demonstrated by examples with 2 and 3 objectives. When dealing with MaOPs, it is highly possible that the large angle is due to a limited solution set size (a sample size on the Pareto front) compared to the high Pareto front dimensionality. The Many-Objective Directed Evolutionary Line Search (MODELS) algorithm [Hughes 2011] combines the aggregation-based method from MSOPS and the directed line search based on approximated local gradient. Both exploration and exploitation within the search landscape are taken into account in MODELS.

Zhang and Li [Zhang and Li 2007] proposed a Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D). Unlike MSOPS where an individual is evaluated with a number of weighting vectors, MOEA/D specifies an solution with only one vector [Yang et al. 2013]. In MOEA/D, each subproblem has a neighborhood defined by their weighting vectors and local information is shared by neighboring sub-problems. This scheme has two benefits: lowering the computational complexity and improving results using local neighborhood information sharing. Ishibuchi et al. [Ishibuchi et al. 2013a] studied the relation between the neighborhood size for local mating and local replacement and the performance of MOEA/D on MaOPs.

Asafuddoula et al. [Asafuddoula et al. 2013] introduced a Decomposition Based Evolutionary Algorithm called DBEA-Eps with systematic sampling [Deb and Jain 2012a] for generating reference directions and adaptive epsilon control to balance convergence and diversity. The experimental results on DTLZ functions (with up to 15 objectives) and several engineering problems demonstrated that it outperformed M-NSGA-II [Deb and Jain 2012a] and MOEA/D-PBI when tackling unconstrained and constrained MaOPs [Deb and Jain 2014].

In order to deal with the diversity loss due to the high selection pressure, Fabre et al. proposed Parallel Genetic Algorithm (PGA) which combines effective ranking with speciation [Garza-Fabre et al. 2011]. Different from the above mentioned approaches, PGA uses a subpopulation for each subproblem derived by aggregation functions, guided by the belief that isolated subpopulation leads to speciation and might help improve the performance of the algorithm. The experimental results demonstrated that using multiple subpopulations outperformed the conventional method in terms of convergence and isolation improves the diversity in the final approximation set.

How to set the weighting vectors is a key issue which determines the searching direction of these aggregation-based algorithms. Yet the selection of weighting vectors is still an open question and ad-hoc methods are widely used [Giagkiozis et al. 2013].

Some methods are designed to generate a restricted number of weighting vectors. This kind of generating methods are based on the normal-boundary insertion method [Das and Dennis 1998]), and the number of weighting vectors (H) for a m -objective MaOP satisfies the following equation: $H = \binom{m+p-1}{p}$, where p is the number of divisions along each objective. However, this may cause problems when the objective number increases. For example, in order to obtain at least one intermediate reference point for a 8-objective problem, $p = 8$ is the least number of divisions we can set. This results in 5,040 reference points in total. In order to avoid this situation, Deb et al. [Deb and Jain 2014] proposed a two-layer generation method where the division num-

¹⁰In MSOPS, users know the investigated area of the Pareto front by setting the weighting vectors. They can compute the angle between the weighting vector and the solution whose direction is closest to the weighting vector. If the angle corresponding to a certain weighting vector is extremely large, then one might say that there exists discontinuity in the neighboring area of the weighting vector direction.

bers of the boundary and inside layer are set to (3,2) or (2,1) so that the total number of reference points is acceptable.

Several methods are also proposed to generate an arbitrary number of weighting vectors. Hughes [Hughes 2007a] used an on-the-fly weighting vector generating method where the weighting vectors are updated using current population members in each generation. Thus they relieved the algorithm users of the burden of setting a-priori the target vector and enabled a more general algorithm for handling MaOPs. Zhang et al. [Zhang et al. 2009] tested MOEA/D on CEC09 unconstrained MOP test instances. They suggested an iterative way for setting weighting vectors with random sampling combined with iteratively selection. Tan et al. [Tan et al. 2013] adopted an uniform design method to set the aggregation coefficient vectors, thus the distribution of the coefficient vectors is more uniform over the objective space. The number of the coefficient vectors is unrestricted. Wang et al. [Wang et al. 2013] argued that an uniformly distributed searching direction, instead of weighting vectors, could produce an uniformly distributed population. They set the weight values to the normalized reciprocal of the corresponding value in the original vector (a small positive is added to avoid division by zero). The method is evaluated by experimental results on DTLZ and WFG test suites with 2 and 7 objectives. Giagkiozis et al. proposed the generalized Decomposition (gD) which can achieve the optimal distribution of solutions once the geometry of the front is known a-priori [Giagkiozis et al. 2013]. The generated distribution can be adapted with the intervention of user's preferences. If no *a priori* knowledge is given, setting the vectors with a linear PF assumption is suggested. Experimental results demonstrated that it performed best in comparison with evenly distributed weighting vectors from [Das and Dennis 1998; Zhang and Li 2007] and the method suggested by Jaszkievicz [Jaszkievicz 2002] to generate a set of weighting vectors uniformly distributed on the probability simplex.

5.1.2. Aggregation of Objective Ranks. Instead of objective values, Ranking Dominance [Kukkonen and Lampinen 2007] aggregates the ranks of each objective values of a non-dominated solution. A solution is said to ranking-dominates another if its aggregation value is smaller than the other one. In [Kukkonen and Lampinen 2007], two aggregation functions: sum of elements and minimum of elements, are introduced to RD. Experimental results demonstrated that RD showed a better coverage of PF than Pareto dominance. However, in some cases, it might mislead the search since it allows objective deterioration. Similar methods are also examined in [Garza-Fabre et al. 2009].

Table VI. Aggregation of Objective Ranks.

algorithm	variant	paper	test problems (m)
MR	MR	[Garza-Fabre et al. 2009]	DTLZ(50)
AR	AR	[Garza-Fabre et al. 2009]	DTLZ(50)
RD	GDE3-R _{sum}	[Kukkonen and Lampinen 2007]	DTLZ(50)
	GDE3-R _{min}	[Kukkonen and Lampinen 2007]	DTLZ(50)

5.2. Aggregated Pairwise Comparisons (APC) based Methods

In addition to the individual information, pairwise comparison results with other solutions in the population can be used in aggregation-based methods to evaluate many-objective solutions. A summary can be seen in Table VII. For a solution \mathbf{x} , its fitness value is the aggregation of the comparisons of \mathbf{x} and all the remaining solutions.

A general description of these methods is as follows: $APC(\mathbf{x}) = \text{agg}(\text{comp}(\mathbf{x}, \mathbf{y}))$, where $\text{agg}()$ is an aggregation function. For Global Detriment (GD) [Garza-Fabre et al.

Table VII. Aggregation of Pairwise Comparisons

technique	algorithm	paper	test problems (m)
maxmin fitness	MCDE	[Mendez and Coello 2012]	DTLZ(5)
	MC-MOEA	[Mendez and Coello 2013]	DTLZ(8)
SV-DOM	SV-DOM	[Köppen and Yoshida 2007]	DTLZ(15)
$-\epsilon$ -DOM	$-\epsilon$ -DOM		
FPD	FPD		
SOD-CNT	SV-DOM		

2009], $comp(\mathbf{x}, \mathbf{y})$ is defined as $\sum_{k=1}^m \max(f_k(\mathbf{x}) - f_k(\mathbf{y}), 0)$. A similar mechanism is used in Profit(PF) [Garza-Fabre et al. 2009], except that the quality of a solution is expressed according to their profit other than detriment. The results on DTLZ functions with up to 50 objectives showed that these approaches behaved better than six state-of-the-art fitness assignment methods [Bentley and Wakefield 1998] in terms of convergence quality.

For the maxmin fitness function [Balling and Wilson 2001], agg is the maximum function while $comp(\mathbf{x}, \mathbf{y})$ is defined as $\min_k (f_k(\mathbf{x}) - f_k(\mathbf{y}))$. Thus a solution is dominated(non-dominated, weakly dominated) if its maxmin fitness value is greater than (less than, equal to) zero. In [Mendez and Coello 2013], the properties of maxmin fitness function are analyzed and the Maximin-Clustering Multi-Objective Evolutionary Algorithm (MC-MOEA) which encompasses three selection operators is proposed to overcome the disadvantages of maxmin fitness function. The experimental results showed that MC-MOEA is a good choice to solve MOPs having both low dimensionality (two or three) and high dimensionality (more than three) in the objective space.

As shown in [Mendez and Coello 2013], these methods have a great advantage: good results in less time when compared to the hypervolume-based approach, which we will discuss later. Jaimes et al. [Jaimes and Coello 2009] presented a quantitative analysis of different preference relations for MaOPs. The study revealed that average ranking and preference order relation stress the solutions far from the knee region, which might limit the applicability of them, since it is commonly assumed that the user prefers solutions in the knee region in general.

5.3. Discussion

Although aggregation-based methods do not rely on Pareto dominance to push the population towards the Pareto front, these methods face their own challenges which point to potential future research directions:

- To configure the weighting vectors: For aggregated individual information based methods, the weighting vectors dramatically affect the diversity performance. How to deal with the dilemma between limited computational resources and the exponentially increasing amount of weighting vectors is still an open question.
- To choose the aggregation functions: In [Yang et al. 2013], the experimental studies showed that the solution set of MOEA/D is close to the real PF for some test cases, while the coverage of the PF is not as good. As indicated by [Yang et al. 2013; Deb and Jain 2014; Ishibuchi et al. 2010], it might do harm to the diversity maintenance when the Tchebycheff function is chosen as the aggregation method. It is possible that some weighting vectors may correspond to only one Pareto-optimal point by the Tchebycheff function [Deb and Jain 2014; Ishibuchi et al. 2010]. As for the weighted sum aggregation methods, their incapability of dealing with problems with non-convex Pareto fronts was identified [Zhang and Li 2007] and needs to be improved.

6. INDICATOR-BASED APPROACH

Using indicator values to guide the search process seems to be a direct way to solve MaOPs, since the approximation set is evaluated according to the indicator. According to the indicator applied, we categorize these methods into three classes: hypervolume-driven, distance-based indicator driven, and R2 indicator driven algorithms.

6.1. Hypervolume-driven Algorithms

The hypervolume indicator is the only metric known to be strictly monotonic to the Pareto dominance [Bader and Zitzler 2011]. This property gives rise to a series of hypervolume-based algorithms, as shown in Table VIII.

Table VIII. Hypervolume-driven Methods.

indicator	algorithm	paper	test problems (m)
hypervolume	SMS-EMOA	[Wagner and Neumann 2013]	DTLZ(20) WFG, LZ
	HypE	[Bader and Zitzler 2011]	DTLZ(50) MKP(50) WFG(50)
	HypE _{uni}	[Auger et al. 2009]	DTLZ(25)
	HypE _p		

In order to maximize the hypervolume value of the solution set, Emmerich et al. proposed the S-metric Selection Evolutionary Multi-Objective Algorithm (SMS-EMOA) [Emmerich et al. 2005]. SMS-EMOA is a $(\mu + 1)$ evolutionary algorithm¹¹, guided by the gradient of the hypervolume metric [Bosman 2012], where each solution $\mathbf{a} \in A$ is evaluated by its hypervolume contribution: $Con_{HV}(\mathbf{a}, A) = I_{HV}(A) - I_{HV}(A \setminus \{\mathbf{a}\})$. The convergence and diversity of the resulting population is quite promising on low-dimensional MOPs. However, SMS-EMOA suffers from the exponentially increasing computational load in terms of the dimension of the objective space. As described by Beume and Rudolph [Beume and Rudolph 2006], the runtime of a generation of SMS-EMOA is $O(\mu^{m/2+1})$, where μ is the population size, m is the number of objectives.

To deal with the high computational time of calculating hypervolume values, Bader et al. proposed a Hypervolume Estimation algorithm (HypE) [Bader and Zitzler 2011]. They used Monte Carlo sampling to approximate the exact hypervolume values, allowing a tradeoff between accuracy and time. In HypE, the non-dominated solutions are compared according to their hypervolume-based fitness values. Specifically, the fitness value of a solution \mathbf{a} , denoted as $I_h^k(\mathbf{a}, A)$, is defined as the expected hypervolume loss that can be attributed to \mathbf{a} when it is removed from the population A along with $k - 1$ other uniformly randomly chosen solutions. During selection, k equals to the number of solutions to be removed from the non-dominated solution set Q , which is the latest front of the combined population to be added to the next generation. The experimental results showed that HypE achieved competitive performance in terms of the average hypervolume on the DTLZ, WFG, and MKP problem suites with up to 50 objectives.

6.2. Distance-based Indicator driven Algorithms

Since computing the hypervolume indicator is too time-consuming, several distance-based indicator driven algorithms are proposed (Table IX). Among them, the Indicator-Based Evolutionary Algorithm (IBEA) uses a binary additive ϵ -indicator $I_{\epsilon+}$, the Δ_p Differential Evolution (DDE) incorporates the Δ_p indicator into the selection mechanism, the approximation-guided evolutionary (AGE) algorithm aims at minimizing the

¹¹In each generation, the current population of a $(\mu + 1)$ evolutionary algorithm, aka a steady-state evolutionary algorithm, includes μ individuals while the number of offspring is 1.

α indicator, while the Many-Objective Differential Evolution with Mutation Restriction (MyO-DEMR) takes advantage of the IGD values [Villalobos and Coello 2012; Bringmann et al. 2011]. The performance of these algorithms is quite competitive according to experimental studies. For example, the experimental results on DTLZ functions with up to 20 objectives indicated that AGE could achieve better performance in terms of the α and hypervolume indicator when compared with IBEA, NSGA-II, SMS-EMOA and SPEA2 [Bringmann et al. 2011].

Table IX. Distance-based Indicator driven Algorithms.

indicator	algorithm	paper	test problems (m)
α indicator	AGE	[Bringmann et al. 2011]	DTLZ(20)
	AGE-II	[Wagner and Neumann 2013]	DTLZ(20)
IGD	MyO-DEMR	[Denysiuk et al. 2013]	DTLZ(20)
Δ_p indicator	DDE	[Villalobos and Coello 2012]	DTLZ(10)
IBEA	IBEA	[Zitzler and Künzli 2004]	EXPO(4)

6.3. R2 indicator driven algorithms

R2-MOGA and R2MODE integrate the $R2$ indicator into a modified version of Goldberg's non-dominated sorting method [Manriquez et al. 2013]. The empirical results on DTLZ with up to 10 objectives indicate that these algorithm can outperform SMS-EMOA by using much less computational time. Another algorithm based on the $R2$ indicator, namely Many Objective Metaheuristic Based on $R2$ indicator (MOMBI), achieved similar performance [Gómez and Coello 2013].

Table X. R2 Indicator based Approaches.

indicator	algorithm	paper	test problems (m)
$R2$ indicator	MOMBI	[Gómez and Coello 2013]	DTLZ(8) WFG(8)
	R2-MOGA	[Manriquez et al. 2013]	DTLZ(10)
	R2-MODE		WFG(3)

6.4. Discussion

When it comes to the indicator-based algorithms, their difficulties are different from Pareto-based approaches since they do not rely on Pareto dominance to push the solutions towards the PF. However, the indicator-based MaOEAs do have their own issues.

First, the computational cost of the hypervolume value is high. As the experimental results indicated [Bringmann et al. 2011], the running time of SMS-EMOA has severely limited its application to MaOPs with more than 8 objectives. While other indicators cost much less time, they are not strictly monotonic with the Pareto dominance and might lead to performance degradation. There has been some recent work on faster computation of the hypervolume indicator, such as Hypervolume by slicing objectives (HSO) with worst-case complexity being $O(N^{M-1})$ [While et al. 2006], the WFG algorithm with worst-case complexity being $O(2^{N-1})$ [While et al. 2012], etc¹².

Second, the hypervolume might not be appropriate when the decision maker aims to find a uniform spread optimal set. For SMS-EMOA, which aims at obtaining the maximal hypervolume value, the algorithm prefers knee points and returns a uniformly distributed population only when the PF is linear [Emmerich et al. 2005]. Although these observations are based on the results on 2-objective problems, it can be inferred that SMS-EMOA also has difficulties in generating a uniformly spreading solution set

¹²Here N is the population size and M is the number of objectives

when applied to MaOPs. Additional archiving strategies or niching methods might be needed to improve the distribution quality.

Third, in order to compute the indicator values, a reference point(s) is needed by the hypervolume and R2 indicator. While for other indicators such as IGD, GD, and Δ_p , a subset of the real Pareto-front is needed. However, to get such an appropriate subset *a priori* is a challenging task.

7. REFERENCE SET BASED APPROACH

In recent years, several reference set based MaOEAs are proposed. These methods use a set of reference solutions to measure the quality of solutions. Thus the search process is guided by the solutions in the reference solution set. There are two key points about the reference set based approach: (a) how to construct the reference set and (b) how to measure the quality of the populations members using the reference set. As shown in Table XI, there are three reference set based MaOEAs: the Two Archive Algorithms (TAAs) [Praditwong and Yao 2006; Wang et al. 2014], Many Objective NSGA-II (NSGA-III) [Deb and Jain 2014; Jain and Deb 2014], and the Taxi-Cab Surface Evolutionary Algorithm (TC-SEA) [Moen et al. 2013]. TAA selects solutions from historical or current populations to construct the reference set. NSGA-III and TC-SEA, in contrast, just use a set of virtual solutions to form an ideal solution front as the search goal. For virtual solutions, we only know the objective values without having any idea about their corresponding decision vectors.

Table XI. Reference Set based Approach.

algorithm	variants	paper	test problems (m)
TAA	TAA	[Praditwong and Yao 2006]	DTLZ(8)
	TAA	[Praditwong et al. 2011]	SMCP(5)
	Two_Arch2	[Wang et al. 2014]	DTLZ(20),WFG(20)
TC-SEA	TC-SEA	[Moen et al. 2013]	DTLZ(20)
NSGA-III	NSGA-III	[Deb and Jain 2012a]	DTLZ(10)
	ANSGA-III	[Jain and Deb 2013]	DTLZ(8)
	A ² NSGA-III	[Jain and Deb 2013]	DTLZ(8)

7.1. Real Reference Set based Approach

In [Praditwong and Yao 2006; Wang et al. 2014], the authors proposed a novel two archive algorithm (TAA) and its improved version (Two_Arch2), which separate non-dominated solutions of each generation into two archives, namely Convergence Archive (CA) and Diversity Archive (DA). CA can be seen as an online-updated real reference set and contains only non-dominated solutions that once dominated some existing archive members. When the total solutions in the union of CA and DA overflow, the solution in DA with the shortest distance to CA is removed iteratively until the archives meet the capacity constraint. Experimental results [Wang et al. 2014] demonstrate that TAA and its latest variant (Two_Arch2) outperform other state-of-the-art MaOEAs significantly in terms of convergence with comparable diversity quality. This approach was also applied to real-world problems in software engineering [Praditwong et al. 2011] and in sensornet protocol optimization [Tate et al. 2012].

7.2. Virtual Reference Set based Approach

Different from TAA, NSGA-III and TC-SEA use virtual points in the objective space to constitute the reference set. The two algorithms use similar methods to construct the reference set¹³, while there are differences between their selection mechanisms.

¹³Both of them incorporate the normal-boundary insertion method from [Das and Dennis 1998].

In order to reduce the deterioration of NSGA-II on MaOPs, NSGA-III replaces the crowding distance based diversity-preservation operator in NSGA-II by a reference set based niche-preservation strategy [Deb and Jain 2014]. After determining the reference solutions on a hyper-plane, the population members are normalized and then associated to a reference solution according to the perpendicular distances to the reference lines defined by joining the reference solution with the origin. Solutions associated with the reference solution, which is associated with less population members, are preferred. Thus the search power of all the reference directions is balanced. NSGA-III achieves desirable performance on a number of problems with up to 10 objectives [Deb and Jain 2014]. Some adaptive variants of NSGA-III are also proposed and obtain better distributions of solutions [Jain and Deb 2013; 2014].

The Taxi-Cab Surface Evolutionary Algorithm (TC-SEA) [Moen et al. 2013] differs from NSGA-III in three aspects. First, TC-SEA uses an online constructed reference set while NSGA-III uses a predefined structured reference set. Second, TC-SEA associates solutions to reference solutions according to the TC value instead of the orthogonal distance as in NSGA-III. Third, TC-SEA uses the rank and the value of TC metric as two sorting criteria for selection. The algorithm is tested on the DTLZ benchmark problems with up to 20 objectives.

7.3. Discussion

Although TAA was originally proposed several years ago, the reference set based approach is still in its infancy and there are many topics which deserve more investigation. First, whether the real reference set is better than the virtual one for guiding the search process is still an open question. The real reference set seems to contain more information of the evolution process while the virtual reference set is better-organized. Second, TAA, NSGA-III, and TC-SEA use Euclidean distance, orthogonal distance and TC value to associate solutions with reference solutions. One might wonder which one is the most appropriate for the association. Third, both NSGA-III and TC-SEA seem to follow a convergence first, divergence second pattern, while TAA manages two archives focusing on convergence and diversity simultaneously. Like other evolutionary algorithms, the balance between convergence and diversity is an important research topic in the reference set based approach.

8. PREFERENCE-BASED APPROACH

In order to approximate the entire PF using MaOEAs, we need to increase the population size exponentially in terms of the objective space dimension [Ishibuchi et al. 2008]. However, in most real-world MaOPs, the population size is often too limited compared to the large objective space to produce any results in a meaningful way [Rachmawati and Srinivasan 2006]. Thus it seems to be a good idea to focus on a subset of the PF according to the users' preference. This kind of approach has been discussed in the literature [Ishibuchi et al. 2008].

There are two key points in the preference-based approach: what preference models are chosen and when to integrate the preference information. There are a wide variety of preference models, such as goal specification, trade-off between objectives, objective rankings, and so on [Rachmawati and Srinivasan 2006]. According to the timing of integrating the preference information into the optimizing process, three classes of algorithms can be defined [Evans 1984; Jaimes et al. 2011]:

- *A priori* algorithms (selection before search): The preference information is set up before the search and it will guide the population to converge to a subset of the PF [Auger et al. 2009; Qiu et al. 2012; Kim et al. 2012; Becerra et al. 2013].

- Interactive algorithms (selection during search): The optimization process asks the Decision Makers (DMs) for the preference information interactively to direct the search to the region of interest, which is a subset of the PF [Deb and Chaudhuri 2005; Deb et al. 2006; Deb and Kumar 2007; Thiele et al. 2009; Jaimes et al. 2011; Gong et al. 2013a].
- *A posteriori* algorithms (selection after search): The preference information is introduced after running MaOEAs and obtaining a solution set which approximates the real PF [Purshouse et al. 2011; Wang et al. 2013c; 2012; 2013a].

8.1. A priori Preference-based Algorithms

Although sharing the same timing of integrating preferences, these algorithms use quite different methods for modeling the preference information. As shown in Table XII, there are eight preference models, which can be categorized into two classes: goal specification, and objective weighting.

Table XII. A priori Preference-based Algorithms.

algorithm	paper	test problems	preference model
ϵ -Constraint+CDE	[Becerra et al. 2013]	WFG(10)	goal-constraint
2p-NSGA-II	[Qiu et al. 2012]	DTLZ(15)	distances to goal points
PBEA	[Thiele et al. 2009]	LPMS(5)	goal point
MQEA-PS	[Kim et al. 2012]	DTLZ(7)	objective weighting
MQEA-PS2	[Ryu et al. 2012]	DTLZ(7)	objective weighting
SBGA	[Gong et al. 2013b]	DTLZ(20)	preferred region
Reference point (RP) Guided dominance PROMETHEE II (P2)	[Epe et al. 2011]	mTSP(4)	distances to one point objective weighting objective weighting
MOPSO-PS	[Lee and Kim 2011]	DTLZ(7)	objective weighting
HypE _{uni}	[Auger et al. 2009]	DTLZ(25)	weight distribution functions

8.1.1. Goal Specification. Goal specification allows a user to provide one solution, a set of solutions, or a subregion of the solution space that they are most interested in as preference information. The Preference-Based Evolutionary Algorithm (PBEA) [Thiele et al. 2009] takes into account the preference information by integrating an achievement function that relies on the reference point. It is evaluated experimentally on a 5-objective problem from [Miettinen 2003]. In [Becerra et al. 2013], a goal vector is provided as the preference information and the goal-constraint method is used to embed it into many-objective optimization. In [Qiu et al. 2012], a bipolar preference dominance is proposed, where two points is required from the Decision Maker (DM): the ideal point and the nadir point. The bipolar preference pushes the population towards the ideal point and away from the nadir point. Gong et al. used preferred regions to describe the DM's preferences [Gong et al. 2013b]. Solutions are compared according to their ranks and distances from the DM's preferred regions.

8.1.2. Objective Weighting. In objective weighting models, a user will assign each objective some sort of weighting in order to aggregate them. Kim et al. used objective weighting to represent a user's preference in MQEA-PS [Kim et al. 2012]. λ -fuzzy measures are used to integrate the objective weighting and the objectives of the solutions and to measure the quality of the solutions in terms of user's preference. The results on 7-objective DTLZ problems and the 3-objective fuzzy path planner optimization problem demonstrated that MQEA-PS improves the performance for both problems compared to MQEA with dominance-based selection and other MOEAs like NSGA-II and MOPBIL. An improved version, MQEA-PS2, which uses multiple populations, is further developed in [Ryu et al. 2012]. Guided dominance [Branke et al. 2001] and PROMETHEE II (P2) [Brans and Mareschal 2005] are two similar methods, except

that guided dominance incorporates linear trade-off functions while PROMETHEE II uses aggregated preference functions. Auger et al. used weight distribution functions in HypE_{uni} to emphasize specific solutions or subregions in the objective space [Auger et al. 2009]. The efficiency of HypE_{uni} was tested on test problems with up to 25 objectives.

8.2. Interactive Preference-based Algorithms

In interactive algorithms (selection during search, shown in Table XIII), the decision makers need to provide preference information to the algorithms interactively. These algorithms can reduce the computational load online and introduce the preference information progressively [Fonseca and Fleming 1998]. There are several different preference models such as goal point(s), reference direction, and preference polyhedron.

Table XIII. Interactive Preference-based Algorithms.

algorithm	paper	test problems	preference model
P-MOET	[Kalboussi et al. 2013]	MOAATP(7)	goal point
ITuCPR	[Jaimes et al. 2011]	ADP(6)	goal point
R-NSGA-II	[Deb et al. 2006]	DTLZ(10)	goal points
RD-NSGA-II	[Deb and Kumar 2007]	DTLZ(10)	reference direction
IEA with preference polyhedron	[Gong et al. 2013a]	DTLZ(5)	preference polyhedron

8.2.1. Goal Specification. Reference Point based NSGA-II (R-NSGA-II) introduced a preference operator which prefers solutions closer to the goal points and de-emphasizes solutions within a ϵ -neighborhood of a goal point [Deb et al. 2006]. In [Kalboussi et al. 2013], R-NSGA-II was integrated into a Preference-based Many-Objective Evolutionary Testing (P-MOET) method to solve a 7-objective software engineering problem.

Jaimes et al. used the Chebyshev preference relation to compare solutions [Jaimes et al. 2011; Jaimes et al. 2013]. The Region of Interest (ROI) is defined as a neighborhood region near the goal point. For two solutions in the ROI, the Chebyshev preference becomes classical Pareto-dominance. Otherwise, the one nearer to the goal point is preferred. The experimental results showed that the Chebyshev relation improves the convergence of NSGA-II without sacrificing the distribution [Jaimes et al. 2013].

8.2.2. Reference Direction. In Reference Direction based NSGA-II (RD-NSGA-II) [Deb and Kumar 2007], the decision makers are supposed to supply a reference direction in the objective space in each iteration. Then the solutions are ranked with the aid of an achievement scalarizing function and the crowding distance value. The algorithm is tested on DTLZ functions with up to 10 objectives.

8.2.3. Preference polyhedron. Gong et al. proposed an Interactive Evolutionary Algorithm (IEA) based on the preference polyhedron theory [Gong et al. 2013a]. During the search process, the decision maker periodically obtains a non-dominated solution set and chooses the most preferred one.

8.3. *A posteriori* Preference-based Algorithms

The *a posteriori* preference-based algorithms (selection after search) try to avoid the intervention of the decision makers before or during the optimization process. Users do not need to participate until the solution set is obtained. As shown in Table XIV, there are two main algorithms, namely PICEA-g and MUF-EA.

In preference-inspired coevolutionary algorithms (PICEAs), preferences are modeled as a set of solutions which co-evolve along with the population [Wang et al. 2013b; 2013a]. It should be noted that these algorithms aim at approximating the whole

Table XIV. *A posteriori* Preference-based Algorithms.

algorithm	paper	test problems	keywords
a-PICEA-g	[Wang et al. 2013a]	WFG(7)	preference solutions
PICEA-w	[Wang et al. 2013b]	WFG(7)	
PICEA-g	[Wang et al. 2013c]	WFG(10)	
	[Purshouse et al. 2011]	WFG(13)	
LPICEA-g	[Wang et al. 2012]	WFG(4)	marginal utility function
MUF-EA	[Li and Liu 2012]	DTLZ(10)	

Pareto front, different from the above-mentioned preference-based algorithms. By systematically comparing a variant of PICEA-g [Purshouse et al. 2011] with representative methods of other classes, Wang et al. explored the potential of PICEAs [Wang et al. 2013c]. The empirical results on the WFG test problems with up to 10 objectives indicated that PICEA-g was always the 1st or 2nd ranked method in terms of convergence and spread when compared to several state-of-the-art methods.

Different from PICEAs, Li et al. [Li and Liu 2012] used a marginal utility function to describe the user's preferred range for each objective. In that way, the algorithm can supply the user with a set of solutions in the region that he/she is interested in. The algorithm is tested on DTLZ test problems with up to 10 objectives.

8.4. Discussion

Since the search direction is biased towards the ROI, the *a priori* and interactive algorithms can reduce the computational load during the search process and pay more attention to preferred solutions. The *a posteriori* preference-based algorithms are inferior to the other two classes since they might obtain a large number of solutions that the decision maker is not interested in. One problem about the interactive algorithms lies in that the algorithms need to interact with the decision maker frequently, who are prone to fatigue [Gong et al. 2013b]. In some occasions, the preference information provided by a tired user might be very misleading¹⁴.

9. DIMENSIONALITY REDUCTION APPROACH

While some algorithms try to tackle the difficulties of MaOPs directly, a series of dimensionality reduction algorithms are proposed to circumvent the roadblock of MaOPs [Van der Maaten et al. 2009; Saxena et al. 2013]. The dimensionality reduction approach aims to deal with MaOPs with redundant objectives. Ishibuchi et al. [Ishibuchi et al. 2011] showed that the performance of NSGA-II was not degraded by the increased objectives when they are highly correlated or dependent. When a high-dimensional MaOP has a similar Pareto front to another problem with less objectives, we can try to optimize the lower-dimensional problem instead of the original one. According to the timing of incorporating the dimensionality reduction techniques into a MaOEA, this approach can be categorized into two classes: offline and online methods.

9.1. Offline Dimensionality Reduction

For offline methods (summarized in Table XV), dimensionality reduction is carried out after obtaining a set of Pareto optimal solutions. According to the key techniques used, they can be further divided into three sub-classes [Singh et al. 2011]: correlation-based, dominance structure based, and feature selection based methods.

9.1.1. Correlation-based Methods. One direction for dimensionality reduction is to examine the correlation among objectives. Saxena et al. proposed L-PCA based on Principal Component Analysis and NL-MVU-PCA based on Maximum Variance Unfolding for

¹⁴For more information about preference incorporation in MOEA, please refer to [Deb 2001; Rachmawati and Srinivasan 2006; Bechikh 2013].

Table XV. Offline Dimensionality Reduction Methods.

algorithm	paper	test problems (m)
L-PCA	[Saxena et al. 2013]	DTLZ(25), WFG(25)
NL-MVU-PCA		DTLZ5(I,M)(50)
exact- δ -MOSS/ k -EMOSS	[Brockhoff and Zitzler 2009]	-
greedy- δ -MOSS	[Brockhoff and Zitzler 2006a; 2006b; 2009]	DTLZ(25), MKP(25)
greedy- k -EMOSS	[Brockhoff and Zitzler 2006a; 2006b; 2009]	DTLZ(25), MKP(25)
greedy- k -EMOSS-omission	[Brockhoff and Zitzler 2009]	-
greedy-OA _{max/avg}	[Brockhoff and Zitzler 2010]	DTLZ(15), WFG(15)
		DTLZ2 _{BZ} (15) DTLZ3 _{BZ} (15) MKP(15), EXPO(4)
PCSEA	[Singh et al. 2011]	DTLZ2(20) DTLZ5(I,M)(100), WFG(3)
ϵ -optimization	[Lindroth et al. 2010]	truck model(12)
algorithm1	[Jaimes et al. 2008]	DTLZ5(I,M)(10), MKP(20)
algorithm2		DTLZ2 _{BZ} (20)

linear and nonlinear objective reduction, respectively [Saxena et al. 2013]. The performance of these algorithms has been studied on 30 test instances and two real-world problem instances. When minimizing the differences between the respective Pareto optimal sets, the ϵ -optimization from [Lindroth et al. 2010] is an alternative method based on the correlations between objectives.

9.1.2. Dominance Structure Based Methods. The dominance structure based methods aim to reduce the number of objectives by considering the dominance relationships among the solutions obtained by an entire run of a MaOEA. The more dominance structure the population keeps, the less side effects the information loss coming with the dimensionality reduction will cause.

By measuring the degree of conflict of objective sets based on ϵ -dominance, Brockhoff et al. proposed both exact and heuristic algorithms to reduce the number of objectives while minimizing the harm to the dominance structure [Brockhoff and Zitzler 2009]. They [Brockhoff and Zitzler 2010] proposed a heuristic which iteratively merges two objectives into a new one using a weighted combination method. The method is demonstrated to be highly useful for reducing the information loss.

Singh et al. [Singh et al. 2011] proposed the Pareto corner search evolutionary algorithm (PCSEA). Instead of search for the whole Pareto front, PCSEA searches corners of the PF to obtain individuals to identify the relevant objectives. After obtaining a solution set from PCSEA, a heuristic technique based on the change of number of non-dominated solutions is used to iteratively reduce the objectives. The method is evaluated by experiments on objectives with up to 100 objectives.

9.1.3. Feature Selection Based Method. Jaimes et al. [Jaimes et al. 2008] proposed two algorithms to reduce objective numbers based on a feature selection technique from [Mitra et al. 2002]. Both methods select essential objectives according to the correlation between each pair of objectives so that the least conflicting objectives are discarded. Experimental results showed that they could reduce the computational cost compared to algorithms from [Brockhoff and Zitzler 2006a] and [Deb and Saxena 2006].

9.2. Online Dimensionality Reduction

By iteratively obtaining solution sets and invoking the dimensionality reduction technique, the number of objectives can be reduced gradually during the search process. As shown in Table XVI, there are seven online dimensionality reduction algorithms.

As far as we know, all the online algorithms follow correlation-based pattern. C-PCA-NSGA-II, MVU-PCA-NSGA-II, and PCA-NSGA-II use the same framework of iteratively obtaining solution sets and reducing the objectives using information of

Table XVI. Online Dimensionality Reduction Methods

algorithm	variants	paper	test problems (m)
SIBEA	SIBEA _{fixed,kEMOSS}	[Brockhoff and Zitzler 2007]	DTLZ(9)
	SIBEA _{fixed,random}		DTLZ2 _{BZ} (9)
	SIBEA _{online} SIBEA _{random}	[Brockhoff and Zitzler 2009]	DTLZ(9) DTLZ2 _{BZ} (9)
C-PCA-NSGA-II	C-PCA-NSGA-II	[Saxena and Deb 2007]	DTLZ5(I,M)(50)
MVU-PCA-NSGA-II	MVU-PCA-NSGA-II		DTLZ2(5)
PCA-NSGA-II	PCA-NSGA-II	[Brockhoff et al. 2008]	DTLZ5(I,M)(50)
REDGA	REDGA-S-1	[Jaimes et al. 2009]	MKP(20), DTLZ2 _{BZ} (20)
	REDGA-S-m		
	REDGA-X-m		
MICA-NORMOEA	MICA-NORMOEA	[Guo et al. 2012]	DTLZ2(M)(5) DTLZ5(I,M)(20)
OC-ORA	OC-ORA	[Guo et al. 2013]	DTLZ(20)
			DTLZ5(I,M)(20) SDSSP(5) ^a

^aSDSSP is short for Storm Drainage Systems Planning Problem.

correlations among the objectives [Saxena and Deb 2007; Brockhoff et al. 2008]. Using the inter-dependence coefficient to represent relations among objectives, Guo et al. proposed an objective reduction algorithm named MICA-NORMOEA [Guo et al. 2012]. In MICA-NORMOEA, the Partitioning Around Medoids (PAM) [Kaufman and Rousseeuw 2009] clustering algorithm and NSGA-II are invoked iteratively to reduce the objective numbers until the stop criterion is met. Jaimes et al. [Jaimes et al. 2009] considered two schemes to embed objective reduction into a MaOEA: one scheme continuously reduces the objective dimension and only uses the original objective set in the last generation, the other uses reduced and original objective sets in turn to guide the search process. Results on 10 objective DTLZ and MKP problems showed that both schemes improve the convergence of a MaOEA.

9.3. Discussion

The dimensionality reduction approach has three main advantages. First, it can reduce the computational load of a MaOEA. Second, it can help decision makers understand the MaOP better by pointing out the redundant objectives. Third, it is consistent with other approaches and is easy to be combined other approaches. For example, Sinha et al. [Sinha et al. 2013] proposed a framework named PI-EMO-VF, which combines an objective reduction based problem simplification procedure with an interaction based solution procedure. Brockhoff and Zitzler [Brockhoff and Zitzler 2007] integrated the objective reduction technique into hypervolume-based algorithms to save the time of computing hypervolumes.

The dimensionality reduction approach assumes that the MaOP at hand has redundant objectives. This assumption may limit the application of the approach. When faced with MaOPs without redundant objectives, these algorithms may fail to reduce the number of objectives or return a solution set which does not cover the complete Pareto front [Singh et al. 2011]. Moreover, the aim of saving computational time is achieved at the cost of losing information of the dominance structure of the original problem which is related to all the objectives [Brockhoff and Zitzler 2010]. Whether the information loss might cause problems is still an open question.

10. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, we have conducted a comprehensive survey of different MaOEAs. They are categorized into seven classes: relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches.

The relaxed dominance based approach tries to relax the definition of dominance and increase the selection pressure towards the Pareto Front. To this end, a series of methods have been proposed, such as value-based dominance modifications (ϵ -dominance, CDAS, grid dominance, and so on), and number-based methods which evaluate a solution according to the number of objectives that it is better than, the same as, or worse than the other. However, the drive toward a more aggressive selection pressure could make diversity maintenance more difficult in a relaxed dominance based MaOEA.

The diversity-based approach tries to improve the performance of MaOEAs by reducing the adverse impact of diversity maintaining. To this end, several customized diversity maintaining strategies are proposed, such as activation/de-activation of the diversity promotion, grid-based neighborhood niching, shift-based density estimation, and tabu moves, which are demonstrated to be highly effective.

The aggregation-based approach optimizes MaOPs using a series of scalarizing functions of the individual information. Using scalarizing functions to rank the solutions overcomes the incomparability of the non-dominated solutions. Both the weighting vectors and the scalarizing functions could influence the performance of these algorithms. Compared to the exponentially increasing objective space, the number of scalarizing functions are usually very limited. This situation might cause difficulties for diversity maintenance of the solution set.

The indicator-based approach optimizes MaOPs under the guidance of an indicator value of the solution set, since a better indicator value indicates that the solution set is a better approximation to the true PF. The hypervolume metric has gained much attention because of its inherent consistency with the definition of Pareto-dominance. However, the computation cost has hindered the application of hypervolume-based methods. Having this in mind, some researchers use other indicators with less computational costs to steer the optimizing process.

The reference set based methods use a set of reference solutions to estimate the quality of solutions and guide the search process. There are two key points about the reference set based approach: how to manage the reference set and how to use the reference set to estimate the quality of the populations members. Both of the two points are worth further investigation for developing new algorithms.

The preference-based approach is originally designed to bias the search to a subregion of the PF with the aid of user's preference. According to the timing when the user's preference is incorporated into the search process, the preference-based approach can be categorized into three classes: *a priori*, interactive, and *a posteriori* methods, which take advantage of the user's preference before, during, and after optimization.

Different from other classes of approaches, the dimensionality reduction approach tries to reduce the number of objectives and change the MaOP at hand into another one with less objectives with a similar Pareto set. According to the dimensionality reduction techniques used, we categorize these algorithms into three classes: correlation-based, dominance structure based, and feature selection based methods. These methods can reduce the computational load but potentially losing some information as a result of the reduced objectives.

In addition to the above-mentioned approaches, there have been some other topics closely related to MaOPs, such as a large population size [Kawatari et al. 2012; Aguirre et al. 2013a], customized mutation and crossover operators [Sato et al. 2011a], non-dominated sorting methods [Wang and Yao 2014], better crowding distance measures [Wang et al. 2010; Minku and Yao 2013], and so on.

Even though a number of MaOEAs have been proposed in recent years, there are still many open problems that need to be solved. On one hand, it is useful to improve the current approaches by developing a better understanding of their behaviors and mechanisms and overcoming their drawbacks. For example, for the relaxed dominance

based approach, more adaptive, flexible and robust selection pressure enhancement methods need to be studied. More effective diversity maintaining strategies need to be designed for the aggregation-based approach. In the indicator-based approach, it might be an interesting idea to combine computationally costly indicator (i.e., hypervolume) with computationally cheap indicators (i.e., GD, IGD, et al.) to guide the search process. For the reference set based approach, the updating strategy of reference sets is worth studying. On the other hand, it would be very interesting to combine two or more approaches together. The resulting hybrid algorithms (such as [Aguirre and Tanaka 2009b; Brockhoff and Zitzler 2007]) might show better performance on MaOPs. For example, the relaxed dominance based methods can be combined with the dimensionality reduction methods to further increase the selection pressure towards the PF.

APPENDIX

“Online Appendix to: Many-Objective Evolutionary Algorithms: A Survey” can be found from <http://home.ustc.edu.cn/~lbngd17/files/OnlineAppendixToSurvey.pdf>.

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Online Appendix to: Many-Objective Evolutionary Algorithms: A Survey

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A. REAL-WORLD AND SYNTHETIC MANY-OBJECTIVE PROBLEMS

This section is devoted to a survey of real-world applications and synthetic benchmark problems with at least four objectives, presented in Tables XVII and XVIII.

Table XVII. Real-world Applications of MaOEAs.

paper	year	journal/conf.	Name(objectives)	remark
[Simpson et al. 1996]	1996	AMAOS ^a	GAADP(10) ^b	General Aviation Aircraft Design Problem
[Ray et al. 2001]	2001	Eng. Optim	WRMP(5)	Water Resource Management Problem
[Characklis et al. 2006]	2006	WRR ^c	LRGV	Lower Rio Grande Valley
[Wickramasinghe et al. 2010]	2010	CEC	ADP(6)	Airfoil Designing Problem
[Ursem 2010]	2010	TechR ^d	CPDP(8)	Centrifugal Pump Designing Problem
[Ishibuchi et al. 2011]	2011	GECCO	MBTP	MaOP-based Test Problem
[Deb and Jain 2012b]	2012	TechR	CSIP(3)	Car Side Impact Problem
[Lygoe et al. 2013]	2013	EMO	AECP(10)	automotive engine calibration problem
[Jaimes et al. 2013]	2013	CEC	TDMOP	trajectory design MOP, NSGA-Ch
[Kudikala et al. 2013]	2013	GECCO	EHM(6)	aero engine health management (EHM) system
[Narukawa and Rodemann 2012]	2012	ICGEC	HCCOP(6)	hybrid car controller optimization problem
[Tagawa and Imamura 2013]	2013	GECCO	SDDP(7)	SAW Duplexer Designing Problem
[DeRonne and Karypis 2013]	2013	TCBB	SAP(11)	Sequence alignment of proteins
[Musselman and Talavage 1980]	1980	Operations Research	SDSPP(5)	storm drainage systems planning problem
[Kalbousi et al. 2013]	2013	SBSE	MOAATP(7)	Many-Objective Autonomous Agent Testing Problem
[Hughes 2007b]	2007	EMO	RWOP(9)	Radar Waveform Optimization Problem
[Deb et al. 2009]	2009	TEC	CSIP	Car Side-Impact Problem
[Kostin et al. 2012]	2012	IECR	MODIBSSC(5)	Multi-objective Design of Integrated Bioethanol/Sugar SCs
[Reed and Kollat 2013]	2013	AWR	GMDP(4)	Groundwater Monitoring Design Problem
[Tang et al. 2012]	2012	CEC	DASP(4)	Dynamic Airspace Sectorization Problem
[Cotton et al. 2003]	2003	MaOP	MeteoAG	workflow designed for meteorological simulations
[Güneş and Tokan 2010]	2010	ESA	OPSLAA(5)	Optimizing the Pattern of Symmetrical Linear Antenna Array
[Venkatarayalu and Ray 2004]	2004	TAP	ODYUAA(4)	Optimizing the Design of a YagiUda Antenna Array
[Narukawa and Rodemann 2012]	2012	ICGEC	HCCOP(6)	Hybrid Car Controller Optimization Problem
[Fu et al. 2012]	2012	JWRPM	WDS(6)	Water Distribution System Design
[Reed and Kollat 2012]	2012	AWR	LTGMND(6)	Long-Term Groundwater Monitoring Network Design
[Stouch et al. 2011]	2011	GECCO	CMPCUAV(10)	coordinated mission plans for constellations of unmanned aerial vehicles
[Palmer et al. 2011]	2011	GCMSC	OSPS(7)	optimization of a submarine propulsion system
[Miettinen 2003]	2003	MOPGP/TA	LPMS(5)	Locating a Pollution Monitoring Station
[Praditwong et al. 2011]	2011	TSE	SMCP(5)	Software Module Clustering Problem
[Tate et al. 2012]	2012	TSMCB	SPOP(5)	Sensornet Protocol Optimisation Problem

^aProceedings of the 6th AIAA/NASA/USAF Multidisciplinary Analysis and Optimization Symposium.

^bThe number in the braces indicates the number of objectives of the MaOP, hereinafter.

^cWater Resources Research.

^d“TechR” is short for “Technical Report”, hereinafter.

Table XVIII. Synthetic Benchmark Problems tackled by MaOEAs.

paper	year	journal/conf.	Name(objectives)	remark
[Zitzler and Thiele 1999]	1999	TEC	MKP	Multiobjective 0 / 1 Knapsack Problem
[Zitzler and Thiele 1998]	1998	PPSN		
[Deb et al. 2002]	2002	CEC	DTLZ	scalable
[Köppen et al. 2005]	2005	EMO	PBP	scalable, Pareto-Box Problem
[Huband et al. 2006]	2006	TEC	WFG	scalable
[Corne and Knowles 2007]	2007	GECCO	MOTSP MOSMJSP	multiobjective traveling salesperson problem multiobjective single-machine job-shop problem
[Zhang et al. 2008]	2008	TechR	CEC09(5)	Extended Rotated DTLZ and WFG, scalable
[Ishibuchi et al. 2010]	2010	PPSN	PBTP	Polygon-based Test Problem
[Ishibuchi et al. 2013a]	2013	CEC	PBTP ^a	visual examination
[Knowles and Corne 2003a]	2003	EMO	QAP	Quadratic Assignment Problem
[Kusuno et al. 2013]	2013	GECCO	EDTLZ	Extend DTLZ
[Reeves 1995]	1995	COR ^b	MOFSSP _{Reeves}	MO Flow Shop Scheduling Problem
[Saxena et al. 2013]	2010	TEC	DTLZ5(LM)	scalable
[Ishibuchi et al. 2013b]	2013	CIMCDM ^c	MKPDO(5)	MKP with Duplicated Objectives, SMS-EMOA
[Ishibuchi et al. 2013b]	2013	LION	MaKPO(10)	Many-Objective Knapsack Problem
[Sayyad et al. 2013]	2013	ICSE	SPLEP(5)	Software Product Line Engineering Problem
[Deb and Jain 2014]	2013	TEC	SDTLZ	Scaled DTLZ
[Deb and Jain 2014]	2013	TEC	CDTLZ2	Convex DTLZ2
[Jain and Deb 2014]	2013	TEC	C1(C2/C3)-DTLZ IDTLZ1	Type-1(Type-2/Type-3) constrained DTLZ2 Problem Inverted DTLZ1 Problem
[Deb and Jain 2012a]	2012	CEC	SDTLZ(10) Constr1 Constr2	Scaled DTLZ Constr1(10) Constr2(10)
[Ishibuchi et al. 2011]	2011	CEC	MKP-DRS	MKP with Different Relationship Strength
[Tan et al. 2013]	2012	COR	F1(5), F2(5)	-
[Hamada et al. 2011a]	2011	CEC	skew-MED	MOP without proper local Pareto solutions
[Johnson et al. 2000]	2001	TechR	mTSP	multi-objective Traveling Salesman Problem
[Hamada et al. 2011b]	2011	CEC	MED	similar with skew-MED
[Aguirre and Tanaka 2004]	2004	CEC	MNK-landscapes	MNK-landscapes
[Saxena et al. 2011]	2011	EMO	TPSCPS	Test Problems with Simple and Complicated Pareto-Set Shapes
[Takahashi et al. 2011]	2011	EMO	TCW	Takahashi1, Carrano and Wanner
[Hughes 2011]	2011	GECCO	hypersphere (20)	-
[Verel et al. 2011]	2011	GECCO	Set-based moFiL	set-based multi-objective fitness landscape
[Hirano and Yoshikawa 2012]	2013	CEC	realMKP	real-coded Multi-objective Knapsack Problem

^aThe PBTP was first proposed in [Ishibuchi et al. 2010] and then mentioned in [Ishibuchi et al. 2011; Ishibuchi et al. 2013a].

^bComputers and Operations Research.

^cComputational Intelligence in Multi-Criteria Decision-Making (MCDM).