A Distance Metric for Evolutionary Many-Objective Optimization Algorithms Using User-Preferences

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Abstract. In this paper we propose to use a distance metric based on user-preferences to efficiently find solutions for many-objective problems. In a user-preference based algorithm a decision maker indicates regions of the objective-space of interest, the algorithm then concentrates only on those regions to find solutions. Existing user-preference based evolutionary many-objective algorithms rely on the use of dominance comparisons to explore the search-space. Unfortunately, this is ineffective and computationally expensive for many-objective problems. The proposed distance metric allows an evolutionary many-objective algorithm's search to be focused on the preferred regions, saving substantial computational cost. We demonstrate how to incorporate the proposed distance metric with a user-preference based genetic algorithm, which implements the reference point and light beam search methods. Experimental results suggest that the distance metric based algorithm is effective and efficient, especially for difficult many-objective problems.

Keywords: Distance metric, User-preference, Many-objective optimization, Multi-objective optimization, Reference point, Light beam search.

1 Introduction

The use of Evolutionary Multi-objective Optimization (EMO) algorithms to find solutions for problems with two or three objectives have been very popular in the recent times [1]. In these studies, the concept of *dominance* plays a major role in the functionality of the algorithms. In many-objective optimization problems (where the number of objectives are greater than three), comparing individuals using dominance becomes less effective [2,3]. Theoretical results in [2] shows that in many-objective search-spaces the number of non-dominated individuals increases to a point where the entire population becomes non-dominated to each other. This severely limits an algorithm's ability to compare and search for solutions in many-objective problems.

A different approach than modifying the dominance concept [3,4] that has been gathering popularity recently in EMO algorithms is *user-preferences* [5,6,7].

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These preference mechanisms were seen originally in Multi-Criteria Decision Making (MCDM) literature [8]. In user-preference based algorithms, a Decision Maker (DM) is required to first indicate *preferred regions* of the objective-space for an algorithm to find solutions in. This information is extremely valuable and can be used to guide the algorithm to further explore the search-space.

The user-preference based EMO algorithms seen in the literature all use *dominance comparisons* to select their candidate solutions [5,6,7]. Unfortunately, these algorithms suffer from the problem of not being able to distinguish solutions effectively for problems with a large number of objectives, where most solutions are non-dominated to each other. Consequently these algorithms are less effective in search, and inclined to converge prematurely to local Pareto-fronts. To address this issue, we introduce a *distance metric* utilizing the user-preference information which is provided by the DM. This method removes the need to use dominance comparisons. We have presented an EMO algorithm using this distance metric which is capable of handling problems of large number of objectives. This paper focuses on EMO algorithms for many-objective problems, therefore we use the term EMO here onwards to refer to Evolutionary Many-objective Optimization.

This paper is organized as follows. Section 2 briefly describes the user-preference methods used in this study. These include the classical definitions of the *reference point method* and *light beam search.* Section 3 presents the distance metric and an implementation of an EMO algorithm using this metric. The experiments used to evaluate the EMO algorithm are provided in section 4. Finally, in section 5, we present our conclusions and avenues for future research.

2 Background

To better understand the distance metric we first describe user-preference mechanisms used in this study.

2.1 Reference Point Method

The classical reference point method was first described by Wierzbicki [6,9]. It has been included successfully in several EMO algorithms [6,7]. A reference point $\overline{\mathbf{z}}$ for a many-objective problem consists of *aspiration values* for each objective. In the classical MCDM literature this reference point is used to construct a single objective function (given by (1)), which is to be minimized over the entire search-space. If $\mathbf{x} = [x_0, \ldots, x_{n-1}]$ is a solution in the search-space of n dimensions,

$$minimize \quad \max_{i=0,\ldots,M-1} \{w_i(f_i(\mathbf{x}) - \overline{z}_i)\} \tag{1}$$

where $\overline{\mathbf{z}} = [\overline{z}_0, \ldots, \overline{z}_{M-1}]$ is the reference point and $\mathbf{w} = [w_0, \ldots, w_{M-1}]$ is a set of weights. f_i is the i^{th} objective function, while M denotes the number of objectives. The DM can assign values for weights, which represents any bias toward an objective.

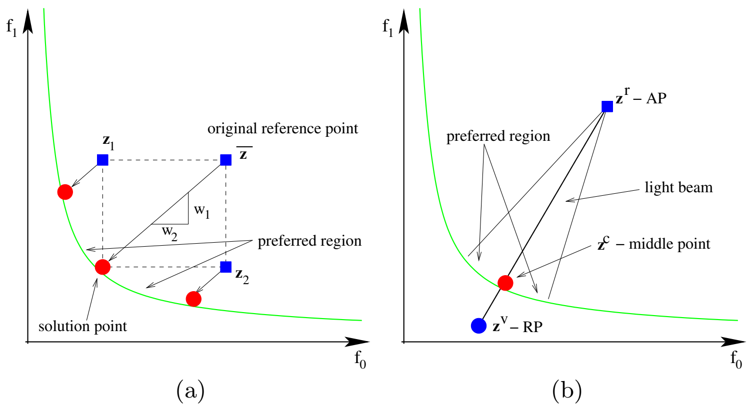


Fig. 1. (a) Classical reference point method (b) Classical light beam search method

Figure 1(a) illustrates the classical reference point method in a two-objective space. The classical MCDM literature [6] shows that several other reference points ($\mathbf{z_1}$ and $\mathbf{z_2}$) can be derived using the original reference point ($\overline{\mathbf{z}}$) and the solution point. In a recursive manner these new reference points can be used to derive more solution points. This traditional approach can be used to define preferred regions in the objective-space.

Using a reference point approach within EMO algorithms is efficient because the entire population can concentrate on finding solutions within this preferred region in a single execution run. We illustrate how preferred regions can be defined with the notion of *outranking* later.

2.2 Light Beam Search Method

The light beam search was first introduced by Jaszkiewicz and Slowinski [10]. The DM first needs to indicate two points in the objective-space, the Aspiration Point (AP), denoted by $\mathbf{z}^r$ and the Reservation Point (RP), denoted by $\mathbf{z}^v$. In situations where the AP and RP are not given, some other points like the *nadir point* and *ideal point* can be used instead. The search direction is given from AP to RP. Metaphorically, this illustrates a light beam originating from AP in the direction of RP. Figure 1(b) illustrates the classical light beam search setup in a two-objective space.

In the classical MCDM literature the light beam search method uses an *achievement scalarizing function* (given by (2)), which is to be minimized. If $\mathbf{x}$ is a solution in the search-space,

$$minimize \quad \max_{i=0,\ldots,M-1} \{\lambda_i \left(f_i(\mathbf{x}) - z_i^r\right)\} + \rho \sum_{i=0}^{M-1} \left(f_i(\mathbf{x}) - z_i^r\right) \tag{2}$$

where, $\mathbf{z}^r = [z_0^r, \ldots, z_{M-1}^r]$ and $\mathbf{z}^v = [z_0^v, \ldots, z_{M-1}^v]$. ρ is a sufficiently small positive number called the *augmentation coefficient* usually set to 10^{-6}.

$\boldsymbol{\lambda} = [\lambda_0, \ldots, \lambda_{M-1}]$, where $\lambda_i > 0$ is a weighted vector. This weighted vector is derived from (3).

$$\lambda_i = \frac{1}{|z_i^r - z_i^v|} \tag{3}$$

The projection of the AP in the direction of the RP will result in a middle point on the non-dominated solution front. In the usual notation, a middle point is given by $\mathbf{z}^c = [z_0^c, \ldots, z_{M-1}^c]$. The DM can then decide on a region surrounding this middle point, which gives the preferred region. This region is obtained by the notion of outranking (S) [10]. **a** outranks **b** (denoted by $\mathbf{a}S\mathbf{b}$) if **a** is considered to be at least as good as **b** within some *threshold* value. Here, the term *better* can be defined according to the used algorithms and problems. *Fitness* and *dominance* are some such definitions. In this study we use the distance metric to define the outranking criteria.

Solutions are obtained in this preferred region *illuminated* by the light beam. We next illustrate how a distance metric is derived from these classical user-preference approaches and how it is used in an EMO algorithm.

3 The Distance Metric

The classical user-preference methods were used to find a single solution on the Pareto front. This single solution would be the closest point to a reference point on the Pareto front or the middle point derived from the light beam search. We introduce a distance metric based on the process of obtaining this solution point.

We define distance of an individual $\mathbf{x}$, to a reference point $\overline{\mathbf{z}}$ using (1) as:

$$dist(\mathbf{x}) = \max_{i=0,\ldots,M-1} \{w_i(f_i(\mathbf{x}) - \overline{z}_i)\} \tag{4}$$

Similarly in the light beam search for any individual $\mathbf{x}$, its distance is defined using (2) as:

$$dist(\mathbf{x}) = \max_{i=0,\ldots,M-1} \{\lambda_i \left(f_i(\mathbf{x}) - z_i^r\right)\} + \rho \sum_{i=0}^{M-1} \left(f_i(\mathbf{x}) - z_i^r\right) \tag{5}$$

We consider **a** to be better than **b** if $dist(\mathbf{a}) < dist(\mathbf{b})$. This distance metric will guide the EMO algorithm towards the Pareto front as illustrated in Figure 2(a). We have incorporated both features of the reference point method and light beam search in Figure 2(a) for brevity. Here, it is important to note that RP is defined only if AP is given (for the light beam search). The term *intersection point* ($\mathbf{u}$) is used to identify the closest solution point to the reference point or the middle point of the light beam. It is useful to realize that the many-objective problem is not converted to a single-objective problem with the use of scalarizing functions as seen in traditional MCDM literature. Although (4) and (5) provide a scalar value, the target is not to optimize that value, but to use the value as a metric to guide the population (the closer the individual is to the preferred region, the better it is).

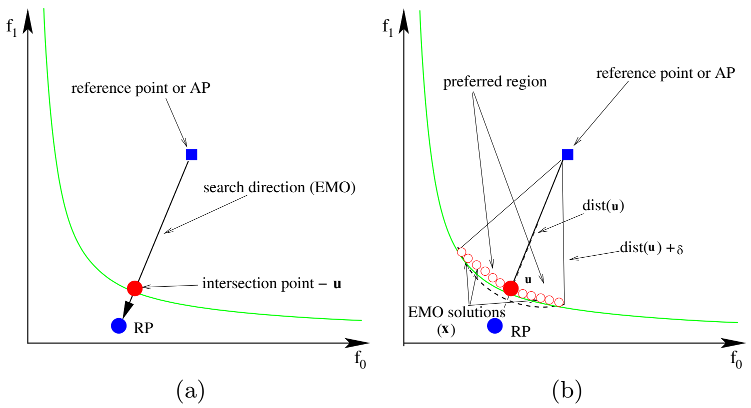


Fig. 2. (a) EMO search yielding a solution point (b) EMO using outranking to define preferred regions

3.1 Controlling the Spread of Solutions

As seen in Figure 2(a) if no control of the spread of solutions is present, the EMO algorithm will explore the search-space along the given direction and converge to the intersection point $\mathbf{u}$. To have a control over the spread of solutions we define a threshold value (δ) for the distance metric using the notion of outranking. Here, the aim is to obtain a set of solutions around the intersection point $\mathbf{u}$. More specifically, to allow the EMO algorithm to converge to an area of solution points rather than a single solution. If $\mathbf{x}$ is any solution point we can define $\mathbf{x}S\mathbf{u}$ as:

$$\mathbf{x}S\mathbf{u} \Leftrightarrow dist(\mathbf{x}) < dist(\mathbf{u}) + \delta \tag{6}$$

This relation allows the EMO algorithm to converge not only to the solution point $\mathbf{u}$, but to any other solution $\mathbf{x}$ around $\mathbf{u}$ as long as $\mathbf{x}S\mathbf{u}$. All the solutions outranking $\mathbf{u}$ by a given δ defines a preferred region (Figure 2(b)). It is important to note that a preferred region is defined by the search direction, governed by the indicated points, and the spread of solutions. A larger value of δ provides a larger region and a smaller value provides a smaller region. It is also clear that $\delta = 0$ gives $\mathbf{u}$. With this δ threshold value the EMO algorithm can have a control of the spread of solutions as required by the DM.

3.2 A Distance Metric Based EMO Algorithm

We now outline how the distance metric can be integrated to an EMO algorithm. Here, a Genetic Algorithm (GA) is used as the search strategy. We have used the original NSGA-II [1] algorithm and replaced the non-dominated sorting procedure by integrating the proposed distance metric approach. Other than GA, the distance metric can be easily integrated into Particle Swarm Optimization (PSO) or Differential Evolution (DE) based EMO algorithms.

- **Step 1: Obtain preferences from the DM**
  The DM will first choose a preference method; either the reference point or light beam search. Depending on the preference method the DM will provide aspiration values to indicate the reference points or the APs and RPs. The DM will next provide a δ value indicating the spread of solutions. The DM has the freedom to indicate any points on the objective-space without considering them to be *feasible* or *infeasible* points.
- **Step 2: Initialize the population**
  A population of size N is first initialized. After initialization, each individual's distance to the preferred regions are calculated using (4) or (5) depending on the preference method. The individuals are then evaluated with the objective functions and fitness is assigned.
- **Step 3: Select parents and reproduce**
  Parents are obtained using tournament selection. Here, individuals closest to the preferred regions are given priority. The parents will crossover and mutate to produce offspring. Here we used the SBX crossover [1] and Polynomial Mutation [1]. The parent population of size N will create N number of offspring.
- **Step 4: Select survivors**
  The parent population of size N is combined with the offspring population of size N to create a population of size $2N$. From this combined population, N number of individuals are selected to move to the next iteration. More specifically, first, the $2N$ population is sorted according to the distance metric. The individuals closest to the preferred regions are selected as the *intersection points*. Next the individuals which outrank these intersection points are selected. If the total number of such selected points are less than N, random individuals are selected from the population to make a final population of size N. If the number of outranked individuals and intersection points are greater than N, the individuals furthest from the preferred regions are removed.

The steps 3 and 4 are repeated until the maximum number of iterations is reached. The distance metric approach guides the population towards the preferred regions such that solutions are found on the Pareto front (Figure 2). At the end of the execution the first non-dominated front is extracted from the population, giving the final solution set.

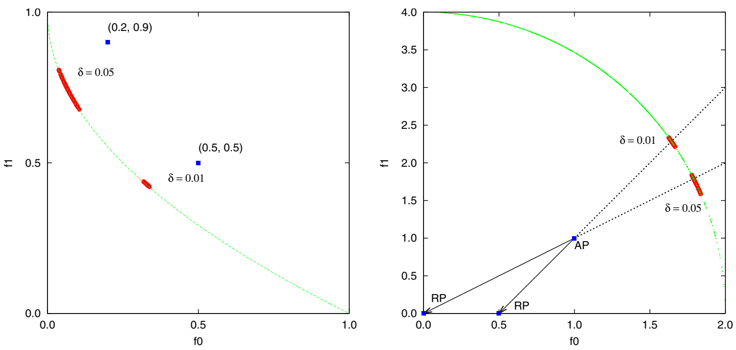
A dominance comparison based EMO algorithm normally has a computational complexity of $O(MN^2)$ because of the use of the non-dominated sorting procedure [1]. However, the proposed distance metric approach only depends on the sorting procedure using the distance metric. As a result, the computational complexity of using the distance metric (for the entire population) is $O(NlogN)$.

4 Experiments

To evaluate the performance of the EMO algorithm using the distance metric, we used following test problem suits; ZDT [11] for two-objective problems, WFG [12] and DTLZ [13] for two and up to ten objective problems. These test problem suites contain many varieties of multi-objective problems including some with many local optima fronts (multi-modal). The parameter settings were constant throughout the experimentation process because the algorithm was robust, not requiring tweaking depending on the type of problem. The population size was 200 and the maximum number of iterations was 500. The SBX crossover probability was set to 0.9 and the mutation probability was $1/n$, where n was the number of decision variables of each problem. The algorithm executed 50 runs on each problem instance. The proposed algorithm was always able to converge on the global Pareto fronts on the simpler problems and frequently on the more difficult problems. In this section, we only illustrate some of the best results from the more interesting (and difficult) problems from the test problem suites because of the lack of space.

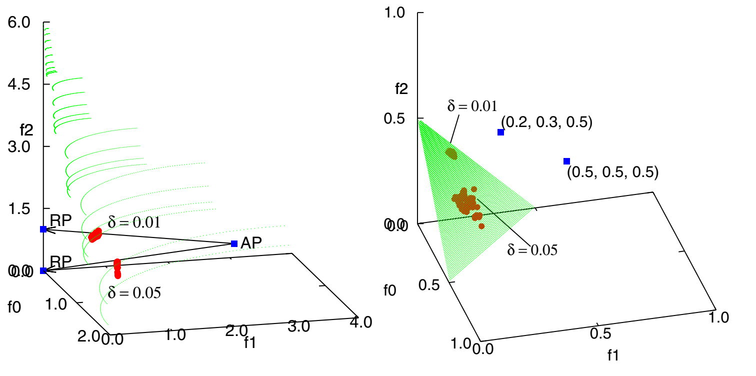
4.1 Two-Objective Problems

Figure 3 shows the solutions fronts obtained for the two-objective multi-modal ZDT4 ($n = 10$) and WFG4 ($n = 6$) test problems. The two preferred regions have spread values of $\delta = 0.01$ and $\delta = 0.05$. Figure 3(a) shows two reference points in the feasible region. ZDT4 is a very challenging problem for EMO algorithms because of its modality (219 local optima fronts). However, the EMO algorithm using reference points is still able to converge onto the global Pareto front. A very interesting result can be seen for the two-objective WFG4 in Figure 3(b). Here, the light beams are located in the infeasible region of the objective-space, because both AP and RPs are infeasible. However, the EMO algorithm with the light beam search still managed to guide the population in the direction of the light beams until solutions are located on the global Pareto front.



(a) (b)

Fig. 3. (a) Two-objective ZDT4 with two reference points (b) Two-objective WFG4 with two light beams



(a) (b)

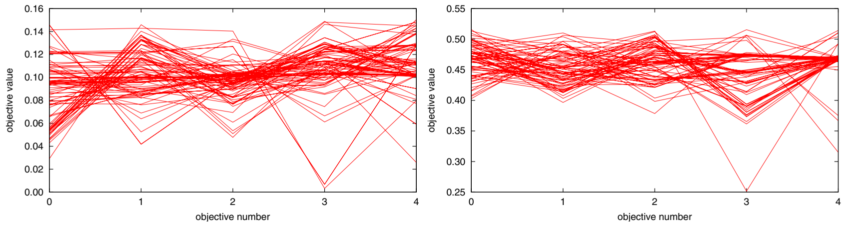
Fig. 4. (a) Three-objective WFG2 with two light beams (b) Three-objective DTLZ1 with two reference points

4.2 Three-Objective Problems

Figure 4 shows the solutions fronts obtained for three-objective WFG2 ($n = 6$) and DTLZ1 ($n = 7$) test problems. Here, WFG2 has disjointed Pareto fronts and DTLZ1 is multi-modal (having $11^5 - 1$ local optima). It is interesting to note that in Figure 4(a) the light beam (with AP $(2.0, 2.0, 2.0)$ and RP $(0.0, 0.0, 0.0)$) goes through the disjoint Pareto front, but the algorithm was still able to locate solutions on the region of the Pareto front which is closest to this light beam. The distance metric guides individuals in the direction given by the vector from AP to RP. This is possible because the algorithm concentrates its search in the direction of this vector. With the population the algorithm has the ability to move in parallel along the direction of this vector until a middle point is found on the Pareto front. Figure 4(b) shows that regardless of the modality, the EMO algorithm was able to converge on the true Pareto with spread values of $\delta = 0.01$ and $\delta = 0.05$.

4.3 Five-Objective Problems

Figure 5 illustrates the solution obtained by each preference mechanism with $\delta = 0.05$. Figure 5(a) shows the result obtained for a five-objective DTLZ1 ($n = 9$) instance. Here, the reference point was at 0.5 for all of the objectives in the objective-space. The sum of the objective values of each individual was found to be in the range $[0.5039, 0.5373]$ for DTLZ1. This suggests that the individuals are very close to the true Pareto front of DTLZ1, since it holds the condition $\sum_{i=0}^{M-1} f_i(\mathbf{x}) = 0.5$ for every $\mathbf{x}$ on the true Pareto optimal front. Figure 5(b) shows the five-objective DTLZ3 ($n = 14$) instances where the AP was set to be the nadir point having the value of 1.0 for all objectives and the RP to be the ideal point having 0.0 for all objectives. DTLZ3 is one of the more difficult multi-modal problems having close to 3^{10} number of local Pareto fronts and one global



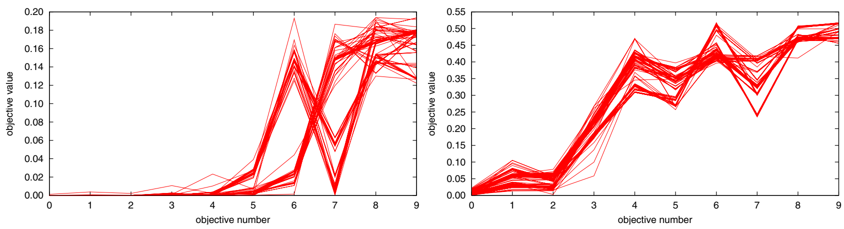
(a) (b)

Fig. 5. (a) Five-objective DTLZ1 with one reference point (b) Five-objective DTLZ3 with one light beam (each line represents a solution point, where the intersection at the objectives axis represents the value for that objective)

Pareto front. The solutions given for DTLZ3 showed that for each individual $\mathbf{x}$, the sum of its squared objective values were in $[1.0475, 1.0671]$. This shows that the solutions are very close to the true Pareto front, because for DTLZ3 a solution $\mathbf{x}$ is on the true Pareto front if $\sum_{i=0}^{M-1} (f_i(\mathbf{x}))^2 = 1$.

4.4 Ten-Objective Problems

Figure 6 illustrates the solution obtained by each preference mechanism with $\delta = 0.05$. Figure 6(a) shows the result obtained for a ten-objective DTLZ1 ($n = 14$) instance. Here, the reference point was at 0.5 for all of the objectives in the objective-space. The sum of the objective values of each individual was found to be in the range $[0.5084, 0.5662]$ for DTLZ1. Figure 6(b) shows the ten-objective DTLZ3 ($n = 19$) instances where the AP was the nadir point and the RP was the ideal point. The solutions obtained for the DTLZ3 instance indicated that the sum of the squared objective values were in $[1.0837, 1.1322]$, showing that the individuals were very close to the global Pareto front.



(a) (b)

Fig. 6. (a) Ten-objective DTLZ1 with one light beam (b) Ten-objective DTLZ3 with one reference point

5 Conclusion and Future Work

In this paper we have proposed a distance metric for EMO algorithms which does not rely on dominance comparisons to find solutions. The proposed distance metric obtained by utilizing user-preferences, either by the reference point or light beam search method, has been integrated into a GA based EMO algorithm. The resulting user-preference based EMO algorithm is shown to provide good performances especially for problems characterized by a high number of objectives and multiple local Pareto-fronts.

Interesting results can be also observed in the behaviour of the proposed EMO algorithm when the preferred regions specified by the DM are in the infeasible regions. In such cases the EMO algorithms are still able to converge to the Pareto front near those specified preferred regions. This property provides an advantage to the DM, since the DM does not have to have the knowledge of where the actual true Pareto optimal front is.

In future we will carry out more comprehensive studies on the distance metric and variations of it. We are also interested in applying EMO algorithms based on this distance metric to solving real world problems.

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