Soft Comput
DOI 10.1007/s00500-015-1789-z

METHODOLOGIES AND APPLICATION

MOEA/D with biased weight adjustment inspired by user preference and its application on multi-objective reservoir flood control problem

Xiaoliang Ma[1,2] · Fang Liu[1,2] · Yutao Qi[1,2] · Lingling Li[2] · Licheng Jiao[2] · Xiaozheng Deng[2] · Xiaodong Wang[1,2] · Bei Dong[2] · Zhanting Hou[1,2] · Yongxiao Zhang[1,2] · Jianshe Wu[2]

Abstract Related to the safety of public lives and property in the lower area of reservoirs, flood control is a priority for most large reservoirs. Considering both dam safety and downstream flood control, reservoir flood control is a multi-objective problem (MOP). To meet the needs of irrigation and generating electricity after the flood, the decision maker usually has his/her preferred final scheduling water level. To deal with this kind of MOP with user-preference information, we incorporate user-preference information into the framework of MOEA/D (multi-objective evolutionary algorithm-based decomposition). The widely used preference information is mainly composed of reference points and preference directions. Compared with the Pareto dominance-based multi-objective evolutionary algorithms (MOEAs), MOEA/D can naturally include two kinds of preference information since MOEA/D is directly based on the reference point and the preference direction. The weight vector of a subproblem in MOEA/D is just its preference. Aiming to obtain uniformly distributed solutions on the objective space, one of innovation points in this paper is using modified Tchebycheff decomposition instead of Tchebycheff decomposition as the decomposition method. To focus the search on the interesting regions of decision maker, the other innovation point in this paper is to integrate biased subproblem (weight vector) adjustment into the framework of MOEA/D. The distribution of subproblems (weight vectors) are adjusted periodically so that the subproblems are re-distributed adaptively to search the interesting regions. Some subproblems, which are far away from the preference regions, are deleted. And then some new subproblems, which are expected to search the preference regions, are added into the current evolutionary population. The efficiency and the effectiveness of the proposed algorithm are assessed through multi-objective reservoir flood control problem and two- to ten-objective test problems.

Keywords Multi-objective optimization · Evolutionary algorithm · Decomposition · Preference · Biased weight vector adjustment · Reservoir flood control

Communicated by V. Loia.

✉ Fang Liu
f63liu@163.com

Xiaoliang Ma
maxiaoliang@yeah.net

Yutao Qi
qi_yutao@163.com

Lingling Li
linglingxidian@gmail.com

Licheng Jiao
lchjiao@mail.xidian.edu.cn

Xiaozheng Deng
635542976@qq.com

Xiaodong Wang
570629903@qq.com

Zhanting Hou
157543744@qq.com

Yongxiao Zhang
hbjzzhyx@163.com

Jianshe Wu
jshwu@mail.xidian.edu.cn

1 School of Computer Science and Technology, Xidian University, Xi'an 710071, China

2 Key Laboratory of Intelligent Perception and Image Understanding of Ministry of Education of China, Xidian University, Xi'an 710071, China

Published online: 18 July 2015

1 Introduction

Most optimization problems in the real-world involve two or more objectives which are measured in different units and in conflict with each other. These problems are often called multi-objective optimization problems (MOPs). MOPs are more complex than single objective optimization problems. They need to handle multiple conflicting objectives simultaneously and have a group of compromise solutions which are termed as Pareto optimal solutions (Deb 2001; Coello Coello et al. 2007). Due to its high frequency and enormous destruction strength of flood disaster, reservoir flood control (RFC) is an interesting problem for further research. During the flood season, RFC problem aims to reduce the flood peak discharge and keep the highest water level of the reservoir as low as possible at the same time.

Due to the fact that the RFC is usually a nonlinear and nonconvex problem, the linear programming (Wurbs 1993) does not work and conventional nonlinear programming methods have the problem of easily getting trapped into local optima and also face the huge computational requirements (Yeh 1985; Mantawy and Soliman 2003; Catal and Mariano 2006). The widely used dynamic programming methods (Little 1955) face high-dimensional problem.

Having the ability to handle complex problems involving features such as discontinuities, multi-modality, disjoint feasible spaces, and noisy function evaluations, evolutionary algorithms (EAs) have been popular to solve MOPs. Multi-objective evolutionary algorithms (MOEAs) have been well investigated in recent years. MOEAs have shown their efficiency (Deb 2001) and have been used to solve RFC problems (Qin et al. 2009; Wang et al. 2012). MOEAs try to obtain a good representation of the Pareto front (PF) to be present to the decision maker (DM). Then the DM chooses the most preferred solution among the candidate solutions provided by MOEAs based on his/her preference information. This is called posterior approach which searches the whole Pareto optimal solutions before multi-criterion decision (Horn 1997; Miettinen 1999). However, if DM can provide some preference information about the MOPs, MOEAs are expected to converge to the preferred regions and provide more detailed sampling over these areas (Wierzbicki 1977; Fonseca and Fleming 1993; Tan et al. 1999; Coello Coello 2000; Deb and Kumar 2007; Branke 2008; Miettinen et al. 2008; Thiele et al. 2009). Compared with posterior approaches, preference-based MOEAs can be more efficient under the condition of limited computing resources. For the multi-objective RFC problem, the DM usually has his/her preferred final scheduling water level to meet the needs of irrigation and generating electricity after the flood.

The most important way to provide preference information is the usage of reference point (Wierzbicki 1977). A reference point consists of aspiration levels reflecting preference values for the objective functions. This is a natural way of expressing the preference information of DM as a target that the user is hoping to obtain. Moreover, with the help of a preference direction, the reference point can be easily projected on to the Pareto optimal front (PF) to find its closest point as shown in Fig. 2. Reference point and preference direction play important roles in the most used approaches (Nakayama and Sawaragi 1984; Jaszkiewicz and Slowinski 1999; Miettinen and Makela 2006).

Recently, Zhang and Li (2007) and Li and Zhang (2009) proposed MOEA/D which decomposes an MOP into a number of scalar optimization problems using uniformly distributed preference directions with the same reference point. As the weight vector of subproblem is just the preference of MOEA/D, one can expect that MOEA/D is more suitable to handle MOPs with preference information than Pareto dominance-based MOEAs. It is easy to imagine that we can make MOEA/D search in predefined areas of the objective space by selecting the subproblems whose optimal solutions are located in the given preferred regions. This is the motivation of this work. Before using MOEA/D to solve MOPs with preference information, there are two key issues that must be addressed. The first one is how to find the preferred subproblems whose optimal solutions lie in the preferred regions. The other is how to adjust the distribution of subproblems to focus the search on the interesting regions.

Aiming to obtain uniformly distributed solutions on the objective space, one of innovation points in this paper is using modified Tchebycheff decomposition instead of Tchebycheff decomposition as the decomposition method. To focus the search on the interesting regions of decision maker, the second innovation point in this paper is to integrate biased subproblem (weight vector) adjustment into the framework of MOEA/D. The distribution of subproblems (weight vectors) is adjusted periodically so that the subproblems are re-distributed adaptively to search the interesting regions. Some subproblems, which are far away from the preference regions, are deleted. And some new subproblems, which are expected to search the preference regions, are added into the evolutionary population. Only a few MOEAs based on preference information make a comparative study with other MOEAs based on preference information. Therefore, the third innovation point of this paper is that we compare the proposed algorithm with LBS-NSGA-II (Deb and Kumar 2007) which is another MOEAs based on preference information. It takes only a few researches (Gong et al. 2011; Mohammadi et al. 2012) to integrate the preference information into the framework of MOEA/D. We also use the proposed algorithm to solve the multi-objective RFC problem.

This paper is organized as follows: Sect. 2 introduces the related backgrounds. Section 3 analyzes the characteristics of the modified Tchebycheff decomposition approach. In Sect. 4, the proposed pMOEA/D algorithm is presented. Section

5 shows the comparison results between the developed algorithm and light beam search-based NSGA-II (Deb and Kumar 2007). Section 6 concludes the paper.

2 Related backgrounds

2.1 Multi-objective optimization

A multi-objective optimization problem can be stated as follows:

$$\begin{cases} \min \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \\ \text{subject to} : \mathbf{x} \in \Omega \end{cases} \tag{1}$$

where $\Omega \subset \mathbf{R}^n$ is the decision space and $\mathbf{x} = (x_1, \ldots, x_n) \in \Omega$ is a decision vector for the MOP. $\mathbf{F}(\mathbf{x}) : \Omega \to \mathbf{R}^m$ denotes the objective vector of the solution $\mathbf{x}$.

Suppose that $\mathbf{x}_A, \mathbf{x}_B \in \Omega$ are two solutions of an MOP, we say that $\mathbf{x}_A$ dominates $\mathbf{x}_B$ (recorded as $\mathbf{x}_A \prec \mathbf{x}_B$), if and only if $f_i(\mathbf{x}_A) \le f_i(\mathbf{x}_B), \forall i \in \{1, \ldots, m\}$ and there exists at least a $j \in \{1, \ldots, m\}$ such that $f_j(\mathbf{x}_A) < f_j(\mathbf{x}_B)$. A solution $\mathbf{x}^* \in \Omega$ is a Pareto optimal solution if and only if there is no solution that dominates it. The collection of all Pareto optimal solutions, i.e. $PS = \{\mathbf{x}^* \mid \neg\exists \mathbf{x} \in \Omega, \mathbf{x} \prec \mathbf{x}^*\}$ is defined as the Pareto optimal set. The Pareto optimal front (PF) are the corresponding objective vectors of the Pareto optimal set, i.e. $PF = \{\mathbf{F}(\mathbf{x}) \mid \mathbf{x} \in PS\}$. Without the priori information from the decision maker, solving an MOP is to find a representative set of PF.

2.2 Multi-objective reservoir flood control problem

Due to its high frequency and enormous destruction strength, flood disaster is one of the most damaging natural disasters. Therefore, RFC is worthy of intensive research. RFC is a complex multi-objective optimization problem with constraints, and it can be modeled as follows (Qi et al. 2012). Let $\mathbf{Q} = (Q_1, \ldots, Q_T)$ be the decision variable vector, and then the multi-objective optimization problem is:

$$\begin{cases} \min \mathbf{F}(\mathbf{Q}) = \min \left(\max\limits_{1 \le t \le T} (Z_t), \quad \max\limits_{1 \le t \le T} (Q_t) \right) \\ \text{subject to} : Z_{\min} \le Z_t \le Z_{\max}, 0 \le Q_t \le Q_{\max}, \\ V_t = V_{t-1} + I_t - Q_t, t = 1, \ldots, T \end{cases} \tag{2}$$

where T is the total number of scheduling times, Z_t is the upstream water level of the tth scheduling time, and Q_t is the discharge volume of the tth scheduling time. $Z_{\min}$ and $Z_{\max}$ are the minimum and maximum limit of upstream water level of the tth scheduling time, respectively. $Q_{\max}$ is the maximal discharge volume of all scheduling times. $V_t = V_{t-1} + I_t - Q_t$ is the water balance equation, in which V_t and V_{t-1} are the reservoir storages of the tth and the $(t-1)$th scheduling time, and I_t is the reservoir inflow volume of the tth scheduling time. The discharging downstream flow is used as the decision variable to encode the individuals. Every individual vector can be expressed as a series of discharge volumes during T scheduling times, that is $\mathbf{Q} = (Q_1, \ldots, Q_T)$, where $Q_t, t = 1, \ldots, T$ is the discharging downstream flow of the individual in the tth scheduling time.

To meet the needs of irrigation and generating electricity after the flood, the DM usually selects the flood control water level (FCL) of reservoir as his/her preferred final scheduling level, i.e. $Z_T \approx Z_{FCL}$.

2.3 Multi-objective optimization evolutionary algorithm based on decomposition (MOEA/D)

Under smooth conditions, a Pareto optimal solution of MOP (1) can be an optimal solution of a scalar subproblem. The objective of a subproblem is a (nonlinearly or linearly) weighted aggregation of all the objective functions $f_i(\mathbf{x}), i = 1, \ldots, m$. Thereby, approximation of the PF can be converted into a set of subproblems.

A multi-objective optimization evolutionary algorithm based on decomposition (MOEA/D) (Zhang and Li 2007) is a popular method using decomposition method to approximate the PF. It firstly decomposes a complicated MOP (1) into a number of simply scalar subproblems. Then an evolutionary algorithm (EA) is applied to solve these scalar subproblems together. The neighborhood relations among these scalar subproblems are based on the distances between their aggregation weight vectors. MOEA/D optimizes each subproblem mainly using the current solutions of its neighboring subproblems. An MOP in MOEA/D is decomposed by Tchebycheff approach due to its ability to handle MOPs with non-convex PF. A subproblem in Tchebycheff approach can be defined as follows:

$$\min_{\mathbf{x} \in \Omega} g^{tch}(\mathbf{x}|\mathbf{w}, \mathbf{z}^*) = \min_{\mathbf{x} \in \Omega} \max_{1 \le i \le m} \left\{ w_i \times |f_i(\mathbf{x}) - z_i^*| \right\} \tag{3}$$

where $\mathbf{w} = (w_1, \ldots, w_m)$ is the weight vector of this subproblem, and $\mathbf{z}^*$ is the ideal objective vector (i.e. $z_i^* = \min\{f_i(\mathbf{x}) | \mathbf{x} \in \Omega\}$). Under mild conditions, for each Pareto optimal solution $\mathbf{x}^*$ there exists a weight vector such that $\mathbf{x}^*$ is the optimal solution of subproblem (3). Moreover, the optimal solution of subproblem (3) is a Pareto optimal solution of MOP (1). Thereby, changing the weight vector can obtain different Pareto optimal solutions. Figure 1 illustrates the Tchebycheff approach. From the Fig. 1, we can see that Tchebycheff approach is like a boundary intersection method.

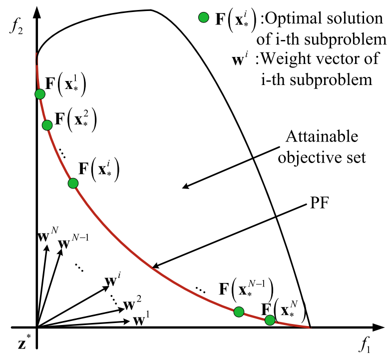

Fig. 1 Plot of the Tchebycheff approach

2.4 Overview of some techniques on incorporating preferences into an MOEA

There are many techniques to incorporate preference information into MOEAs. According to the recent surveys (Coello Coello 2000; Branke 2008; Miettinen et al. 2008), different approaches are classified based on the type of preference information that they ask from the DM.

Firstly, one of the main tools to express preference information is the usage of reference points (Wierzbicki 1977). Fonseca and Fleming (1993) firstly included the DM's preference into MOEAs to find the preferred solutions. The interesting idea in reference (Tan et al. 1999) was to rank the solutions independently with respect to all reference points. Using aspiration points and reservation points to guide the search, Deb and Kumar (2007) proposed light beam search-based NSGA-II (LBS-NSGA-II). Thiele et al. (2009) interactively used the reference points as DM's preferences. The preference degree of the solution combined its fitness function with its value of achievement scalarization function.

Secondly, if DM has no prior information about reachable specific solutions, it can specify suitable trade-offs instead, i.e., the amount of improvement in one objective is worth of the loss in the other objective. Greenwood et al. (1997) suggested a procedure which asks the user to rank a few alternatives to infer the relative importance of objectives. In the guided MOEA proposed by Branke et al. (2001), the user was allowed to offer preference information such as "how much improvement in one objective is necessary to balance one unit in the other objectives". Accordingly, it changed the dominance criterion which reflects the specified maximally acceptable trade-offs to focus the search.

Thirdly, the DM can offer the preference information in the form of solution distribution. Deb (2003) applied a preference sharing technique to search for a biased distribution on any part of the PF. Branke and Deb (2005) proposed to prefer the solution distribution which is similar to the shape of the region of interest. Trautmann and Mehnen (2005) introduced desirability functions to guide the search to find solutions with similar desirability values.

Fourthly, the DM can formulate preference information as the attribute in the solutions. Branke et al. (2004) introduced an MOEA which focuses the search to the knee regions of PF. Zitzler and Kunzli (2004) suggested using a quality indicator as the preference of the decision maker in selecting the property of the obtained non-dominated solutions.

2.5 The used preference model—light beam search (LBS)

As stated in Sect. 2.4, there are many preference models. In this paper, we use light beam search (LBS) approach as the preference model. The reason to select LBS as preference model is that it is based on the reference point and preference direction. Therefore, it can be easier to be integrated into the framework of MOEA/D since MOEA/D is also based on reference point and preference direction.

Jaszkiewicz and Slowinski (1999) firstly proposed the light beam search (LBS) approach. In LBS-NSGA-II, Deb and Kumar (2007) incorporated the LBS method into the framework of MOEA. The preference information in LBS is given by specifying an aspiration point, corresponding reservation point and preference neighbor parameter. An aspiration point consists of aspiration levels reflecting desirable values for the objective functions. A reservation point and corresponding aspiration point determines the preference direction of the search. The preference neighbor parameter aims to determine the extent of the preferred solutions.

2.5.1 Achievement scalarizing function

To determine the closeness of a certain solution to the aspiration point, we need an achievement scalarizing function (ASF) (Wierzbicki 1980).

$$ASF(\mathbf{F}(\mathbf{x}), \mathbf{w}, \rho) = \max_{1 \le i \le m} \left\{ w_i \left(f_i(\mathbf{x}) - F_i^a \right) \right\} + \rho \sum_{i=1}^{m} \left(f_i(\mathbf{x}) - F_i^a \right) \tag{4}$$

where $\mathbf{F}^a = (F_1^a, \ldots, F_m^a)$ is an aspiration point, $\mathbf{w} = (w_1, \ldots, w_m)$ is a weighting vector, $w_i > 0, i = 1, \ldots, m$ and ρ is a relatively small positive number (fixed to be 10^{-7} here). The weighting vector can be defined by aspiration point $\mathbf{F}^a$ and reservation point $\mathbf{F}^r$ ($\mathbf{F}_j^a < \mathbf{F}_j^r, \forall j \in \{1, \ldots, m\}$) in the following manner: $w_j = \frac{1}{\mathbf{F}_j^r - \mathbf{F}_j^a}, j = 1, \ldots, m$. Firstly, the closest point, which is also called middle point marked as $\mathbf{F}^m$, to the aspiration point can be easily obtained by projecting aspiration point $\mathbf{F}^a$ onto the obtained non-dominated solutions in the direction of reservation point $\mathbf{F}^r$ as shown in Fig. 2. Secondly, an outranking relation S is used to obtain a set of the non-dominated preferred solutions closed to the middle point $\mathbf{F}^m$.

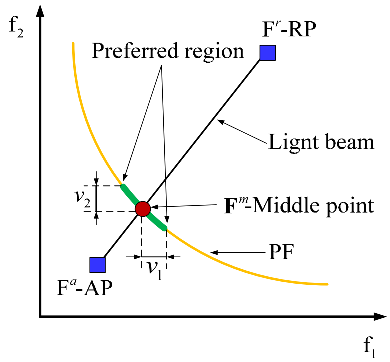

Fig. 2 The approach of LBS-NSGA-II for bi-objective optimization problem

2.5.2 Outranking relation

To find a set of non-dominated solutions in the neighborhood of the middle point $\mathbf{F}^m$, in the original LBS method (Jaszkiewicz and Slowinski 1999), the DM has to specify three preference parameters for each objective. However, this is quite demanding on the part of the DM. To reduce the requirement of input parameter, Deb and Kumar (2007) suggested using the veto threshold only:

$$\mathbf{F}S\mathbf{F}^m \Leftrightarrow \mathbf{F}_j - \mathbf{F}_j^m < v_j, \quad j = 1, \ldots, m \tag{5}$$

where $\mathbf{v} = (v_1, \ldots, v_m)$ is a veto threshold vector. This outranking relation used by LBS-NSGA-II is shown in Fig. 2.

3 Modified Tchebycheff decomposition versus Tchebycheff decomposition

There are many approaches that can convert an MOP into a number of scalar subproblems (Miettinen 1999; Zhang and Li 2007; Das and Dennis 1998). Among these decomposition techniques, the Tchebycheff approach (Bowman 1976) is popular for its ability to handle MOPs with a non-convex PF. However, Tchebycheff decomposition has a shortcoming that with uniformly distributed weight vectors, the optimal solutions of subproblems under Tchebycheff decomposition scheme are not very uniform (Liu et al. 2009; Deb and Jain 2012). To deal with this issue, we use modified Tchebycheff decomposition.

3.1 Analysis on the Modified Tchebycheff Decomposition for the MOPs

Compared with Tchebycheff decomposition, modified Tchebycheff decomposition (Deb and Jain 2012) proposed to divide the term $f_i(\mathbf{x}) - z_i^*$ by w_i, instead of multiplying by w_i in Tchebycheff decomposition approach as follows:

$$\begin{aligned}\min_{\mathbf{x}\in\Omega} g^{mtch}(\mathbf{x}|\mathbf{w}, \mathbf{z}^*) &= \min_{\mathbf{x}\in\Omega} \max_{1\le i\le m} \left\{ \frac{\left|f_i(\mathbf{x}) - z_i^*\right|}{w_i} \right\} \\ &= \min_{\mathbf{x}\in\Omega} \max_{1\le i\le m} \left\{ \frac{f_i(\mathbf{x}) - z_i^*}{w_i} \right\}\end{aligned} \tag{6}$$

where $\mathbf{w}$ is a weight vector and $\mathbf{z}^* = (z_1^*, \ldots, z_m^*)$ is the ideal objective vector (i.e. $z_i^* = \min\{f_i(\mathbf{x})|\mathbf{x} \in \Omega\}, i = 1, \ldots, m$). Due to $z_i^* \le f_i(\mathbf{x})$, the absolute value $\left|f_i(\mathbf{x}) - z_i^*\right|$ in Eq. (6) could be omitted. To avoid $w_i = 0, i = 1, \ldots, m$ as the denominator, a relatively small positive number (for example, 10^{-7}) is used to replace zero.

It can be proved that under mild conditions, for each Pareto optimal solution $\mathbf{x}^*$ there exists a weight vector $\mathbf{w} > 0$ such that $\mathbf{x}^*$ is the optimal solution of (6) and each optimal solution of (6) is a Pareto optimal solution to the MOP (1). The detailed proof can be found in Appendix A. This property allows the algorithm to obtain various Pareto optimal solutions by modifying the weight vectors. Therefore, the modified Tchebycheff decomposition approach also has the capability of solving MOPs with a non-convex PF.

Before comparing with Tchebycheff decomposition, modified Tchebycheff decomposition is analyzed firstly. In the modified Tchebycheff decomposition method, the geometric relationship between the weight vector of a subproblem and its optimal solution is described as follows.

Theorem 1 *Let us suppose that the PF of an MOP is piecewise continuous. If the straight line* $\frac{f_1 - z_1^*}{w_1} = \frac{f_2 - z_2^*}{w_2} = \cdots = \frac{f_m - z_m^*}{w_m}$ $(w_i > 0, i = 1, \ldots, m)$*, treating* $f_1, f_2, \ldots, f_m$ *as decision variables, has an intersection point with the PF, then the intersection point is also the optimal solution of the scalar subproblem with weight vector* $\mathbf{w} = (w_1, \ldots, w_m)$ $\left(\sum_{i=1}^m w_i = 1, w_i > 0, i = 1, \ldots, m\right)$ *under the modified Tchebycheff decomposition scheme, where* $\mathbf{z}^* = (z_1^*, \ldots, z_m^*)$ *is the ideal objective vector.*

The proof can be found in Appendix B. If we assume $w_i > 0, i = 1, \ldots, m$, then

$$\frac{f_1 - z_1^*}{w_1} = \cdots = \frac{f_m - z_m^*}{w_m}, \quad w_i > 0, \quad \forall i = 1, \ldots, m$$

is the straight line that passes through the ideal objective vector $\mathbf{z}^* = (z_1^*, \ldots, z_m^*)$ with the direction (weight vector) $\mathbf{w} = (w_1, \ldots, w_m)$ as shown in Fig. 3 (left). Based on the ideal objective vector $\mathbf{z}^*$, the direction of the optimal solution of subproblem is just its weight vector $\mathbf{w}$. In the following, we will use the preference direction of a subproblem to represent its weight vector.

Figure 3 takes bi-objective optimization problem as an example to illustrate contour lines of a scalar subproblem with $\mathbf{w}$ and the geometric relationship between weight vector $\mathbf{w}$ of a subproblem and the direction of its optimal solution for

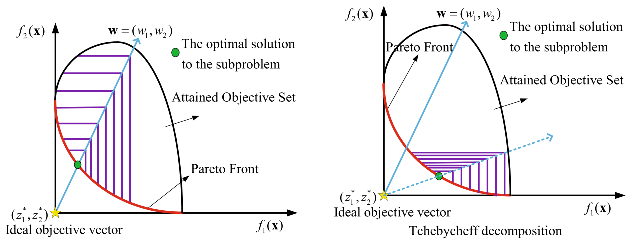

Fig. 3 Contour lines of a scalar subproblem and the geometric relationship between weight vector of subproblem and its optimal solution for modified Tchebycheff decomposition (*left*) and Tchebycheff decomposition (*right*)

modified Tchebycheff decomposition (left) and Tchebycheff decomposition (right).

From the Fig. 3, we can see that the optimal solution of the subproblem with weight vector **w** is on the lowest contour line. For Tchebycheff decomposition, the weight vector **w** of a subproblem does not agree with the direction of its optimal solution as shown in the Fig. 3 (right). But, for modified Tchebycheff decomposition, the weight vector **w** of subproblem agrees with the direction of its optimal solution as shown in the Fig. 3 (left). For the modified Tchebycheff decomposition, the optimal solution of the subproblem with weight vector **w** is just the intersection between PF and weight vector **w**. Therefore, the weight vector **w** in modified Tchebycheff decomposition also can be considered as a preference direction.

The nonlinear relationship between the weight vector of subproblem and its optimal solution under Tchebycheff decomposition has a serious effect on the uniformity of obtained optimal solutions. With uniformly distributed weight vectors, MOEA/D using Tchebycheff approach cannot find evenly scattered solution over PF (Zhang and Li 2007) for 3-objective and many-objective optimization problems. Figure 4 takes 3-objective DTLZ1 problem (Deb et al. 2002b) as an example to illustrate the effect of nonlinear relationship in Tchebycheff decomposition. From the Fig. 4, we can see the selected weight vectors presenting a shape of smile on the left. It is clearly shown that the distribution of the obtained solutions by MOEA/D with Tchebycheff decomposition (MOEA/D-TCH) is entirely different from that of selected weight vectors. The shape of obtained solutions by MOEA/D-TCH looks like a upside-down angry look. On the contrary, the distribution of the obtained solutions by MOEA/D with modified Tchebycheff decomposition (MOEA/D-MTCH) is similar to the selected weight vectors.

To obtain uniformly distributed optimal solutions over PF, we use modified Tchebycheff approach instead of the Tchebycheff approach.

3.2 Optimal choice of weight vector for a given preferred solution under the modified Tchebycheff decomposition approach

According to the Theorem 1, if the whole PF is known in advance, it is easy to obtain the optimal solution of the modified Tchebycheff approach (6) with a given reference point $\mathbf{z}^*$ and preference direction $\mathbf{w} > 0$. Combining the outranking relation (5), it is also easy to get the preferred set nearing the optimal solution. However, the PF is usually unknown beforehand. In this work, we suppose that the DM can offer the reference points and the corresponding preference directions. An MOEA is first used to find a representative approximate set of the PF. Then an outranking relation can be used to find the approximated preferred set from the obtained approximate set.

MOEA/D is selected as the basic MOEA since it is inherently based on the reference point and preference direction. The weight vector of subproblem is just the preference of MOEA/D. However, in the framework of MOEA/D, the challenging issue is how to recognize and adjust the distribution of preferred subproblems, which can guide Pareto optimal solutions within the preferred regions, under the modified Tchebycheff decomposition approach. To handle this issue, the definition of the optimal weight vector for a given pre-

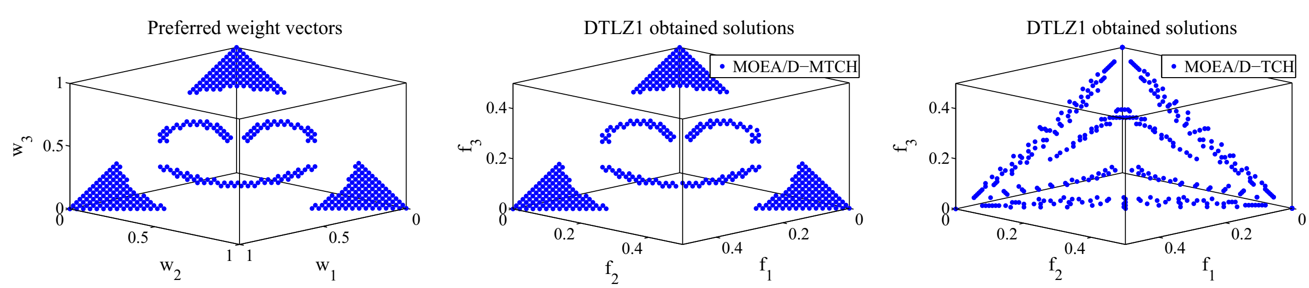

Fig. 4 Preferred weight vectors with a shape of smile are plotted on the *left*. The obtained solutions by MOEA/D with modified Tchebycheff decomposition (MOEA/D-MTCH) for 3-objective DTLZ1 problem are plotted on the *middle*. The obtained solutions by MOEA/D with Tchebycheff decomposition (MOEA/D-TCH) for 3-objective DTLZ1 problem are plotted on the *right*

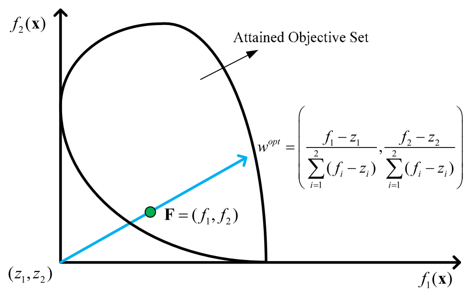

Fig. 5 Illustration of the optimal weight vector generation for a reference point **z** and a given preferred solution **F** on bi-objective problems under the modified Tchebycheff decomposition scheme

ferred solution is first stated. Then the detail of constructing the optimal subproblem for a preferred solution is given.

Definition 1 For a reference point $\mathbf{z} = (z_1, \ldots, z_m)$ and a preferred solution $\mathbf{F} = (f_1, \ldots, f_m)$ satisfying $\mathbf{z} \prec \mathbf{F}$, we mark $h(\mathbf{w} \mid \mathbf{F}, \mathbf{z}) = \max_{1 \le i \le m} \left\{ \frac{f_i - z_i}{w_i} \right\}$ and $W_m = \left\{ (w_1, \ldots, w_m) \mid \sum_{i=1}^{m} w_i = 1, w_i \geq 0, i = 1, \ldots, m \right\}$. $\mathbf{w}^{opt}$ is the optimal weight vector to the preferred solution $\mathbf{F}$, if

$$h(\mathbf{w}^{opt} \mid \mathbf{F}, \mathbf{z}) = \min_{\mathbf{w} \in W_m} h(\mathbf{w} \mid \mathbf{F}, \mathbf{z}) = \min_{\mathbf{w} \in W_m} \max_{1 \le i \le m} \left\{ \frac{f_i - z_i}{w_i} \right\} \tag{7}$$

Giagkiozis et al. (2012) suggested that problem (7) is a convex programming problem, used some interior-point methods to solve it, and obtained a unique optimal weight vector. In the following Theorem, a more simple and direct manner to generate the optimal weight vector for a preferred solution **F** is given.

Theorem 2 *For a reference point* $\mathbf{z} = (z_1, \ldots, z_m)$ *and a preferred solution* $\mathbf{F} = (f_1, \ldots, f_m)$*, if the condition* $\mathbf{z} \prec\prec \mathbf{F}$ *(i.e.* $z_i < f_i, \forall i = 1, \ldots, m$*) is satisfied, then* $\mathbf{w}^{opt} = \left(\frac{f_1 - z_1}{\sum_{i=1}^{m}(f_i - z_i)}, \ldots, \frac{f_m - z_m}{\sum_{i=1}^{m}(f_i - z_i)} \right)$ *is the optimal weight vector to the preferred solution* **F** *based on the reference point* **z** *under the modified Tchebycheff decomposition scheme.*

The proof can be found in Appendix C. For simplicity, Fig. 5 takes a bi-objective problem as an example to describe how to generate the optimal weight vector for a preferred solution **F** and a reference point **z**. Intuitively speaking, the optimal weight vector (preference direction) is the normalization of the vector $\mathbf{F} - \mathbf{z}$ under the modified Tchebycheff decomposition scheme.

4 The Algorithm—pMOEA/D

Firstly, the preference model in pMOEA/D is introduced. Then the idea of pMOEA/D and its main framework are presented. Aiming to provide diverse preference solutions, the update of external population is proposed. Finally, to focus the search on the interesting regions of decision maker, biased subproblems adjustment is described.

4.1 The preference model

Algorithm 1 Calculate the preferred neighboring distance

Require: Pop : a set of solutions (for example the current evolutionary population EP or the external population ExP).

Ensure: pnd - the min preferred neighboring distances between the solutions in Pop and all middle points.

Step 1: For each solution of Pop, use achievement scalarizing function to calculate the distance between the solution and all aspiration points.

$$wd_{ik} = ASF(\mathbf{F}^i, \mathbf{w}^k, \rho) = \max_{1 \le j \le m} \left\{ w_j^k (f_j^i - F_j^{a_k}) \right\} + \rho \sum_{j=1}^{m} (f_j - F_j^{a_k}), i = 1, \ldots, |Pop|, k = 1, \ldots, N_a \tag{8}$$

where $\mathbf{w}^k = \left(\frac{1}{F_1^{r_k} - F_1^{a_k}}, \ldots, \frac{1}{F_m^{r_k} - F_m^{a_k}} \right)$ is the kth weighting vector.

Step 2: Find the middle points: for each aspiration point $\mathbf{F}^{a_k} = (F_1^{a_k}, \ldots, F_m^{a_k}), k = 1, \ldots, N_a$, the corresponding middle point $(\mathbf{F}^{m_k}, k = 1, \ldots, N_a)$ is the closest solution to the aspiration point. Its preferred neighboring distance is assigned as zero.

Step 3: For each solution of Pop, calculate the minimum preferred neighboring distance (Mahalanobis distance) between the solution and all middle points.

$$pnd_i = \min_{1 \le k \le N_a} \sqrt{\sum_{j=1}^{m} \left(\frac{f_j^i - F_j^{m_k}}{v_j^k}\right)^2}, \quad i = 1, \ldots, |Pop| \tag{9}$$

Formula 4, which is the same as the LBS-NSGA-II, is used as the achievement scalarizing function to find the preferred solution (middle point). With these middle points, the modified outranking relation is applied to calculate the preferred neighboring distances, which will be used as the basis to adjust the distribution of subproblems.

Formula 5 proposed by LBS-NSGA-II just gives a qualitative measure on whether a solution is preferred or not. In this paper, we design a new outranking relation which can give quantitative measure on the preference degree of solution. The newly outranking relation S is defined as follows: for arbitrary $\mathbf{F}, \mathbf{F}^m \in \mathbf{R}^m$,

$$\mathbf{F} S \mathbf{F}^{\text{mid}} \Leftrightarrow \left| \mathbf{F} - \mathbf{F}^{\text{mid}} \right| = \sqrt{\left(\mathbf{F} - \mathbf{F}^{\text{mid}}\right) \Sigma^{-1} \left(\mathbf{F} - \mathbf{F}^{\text{mid}}\right)} = \sqrt{\sum_{i=1}^{m} \left(\frac{\mathbf{F}_i - \mathbf{F}_i^{\text{mid}}}{v_i} \right)^2} \leq 1 \tag{10}$$

where the matrix Σ is a diagonal matrix with $(v_1^2, \ldots, v_m^2)$ as its main diagonal vector and $\mathbf{v} = (v_1, \ldots, v_m)$ is the semi-major axis vector. Precisely, for the middle point $\mathbf{F}^{mid}$, a set

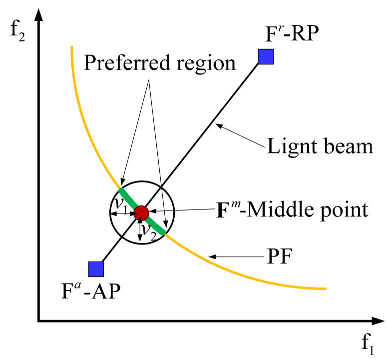

Fig. 6 Illustration of the outranking relation of pMOEA/D for bi-objective optimization

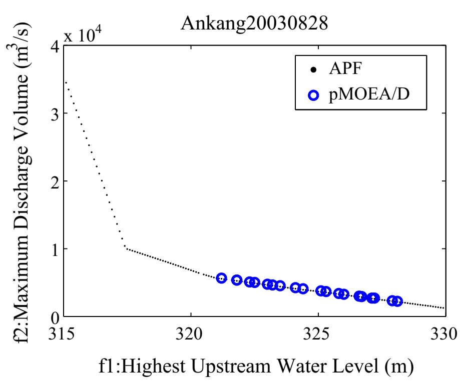

Fig. 7 Plot of a case that MOP has disparately scaled objective functions

of non-dominated set, which are in an ellipsoid centered at $\mathbf{F}^{mid}$ with semi-major axis vector $\mathbf{v} = (v_1, \ldots, v_m)$, have an outranking relation with it as shown in Fig. 6.

To decide whether the subproblem searches the preferred regions or not, we use the preferred neighboring distance as the criterion. The preferred neighboring distance of subproblem is defined by the Mahalanobis distances (10) of the solution and its closest middle point.

Compared with Euclidean distance and Manhattan distance which equally treat all the component $\mathbf{F}_i - \mathbf{F}_i^{\text{mid}}, i = 1, \ldots, m$, Mahalanobis distance introduces a factor v_i for each component $\mathbf{F}_i - \mathbf{F}_i^{\text{mid}}$. Using $v_i, i = 1, \ldots, m$, we can adjust the weights of different components. Therefore, Mahalanobis distance is better than Euclidean distance to solve MOP whose m objective functions $f_i(\mathbf{x}), i = 1, \ldots, m$ are disparately scaled. Our application problems—multi-objective reservoir flood control problems—are the problems whose objective functions are disparately scaled. Figure 7 gives this case that objective functions $f_i(\mathbf{x}), i = 1, \ldots, m$ in MOP are disparately scaled ($f_2 \in (0, 3.5 * 10^4]m^3/s$, $f_1 \in [315, 330]m$).

Algorithm 1 gives the detail of calculating the preferred neighboring distance of subproblem.

4.2 General algorithm description

Similar to MOEA/D, pMOEA/D firstly decomposes a complicated MOP (1) into a number of simply scalar subproblems (6), and uses evolutionary algorithm to optimize these subproblems together. To focus the search on the interesting region of decision maker, pMOEA/D integrates the preference information. During the evolution, pMOEA/D maintains the following items:

- population $evol\_pop = \{ind^1, \ldots, ind^N\}$, $ind^i = (\mathbf{x}^i, \mathbf{F}^i, \mathbf{w}^i)$, $i = 1, 2, \ldots, N$, where $\mathbf{x}^i$, $\mathbf{F}^i$ and $\mathbf{w}^i$ are the current solution, corresponding objective vector and weight vector to the ith subproblem, respectively;
- an external population ExP for the storage of the preferred non-dominated set.
- a reference point $\mathbf{z} = (z_1, \ldots, z_m)$, where z_i saves the best value obtained so far for the ith objective;

pMOEA/D needs a number of parameters including the population size N, the maximum number of function evaluation $FE_{\max}$, the number of the weight vectors in the neighborhood of each weight vector T, the maximal number of subproblems required to be adjusted nas in each biased weight vector adjustment, the iteration interval generation of biased weight vector adjustment $Interval$, the number of the aspiration points N_a, the aspiration points $\{\mathbf{F}^{a_k}, k = 1, \ldots, N_a\}$, the reservation points $\{\mathbf{F}^{r_k}, k = 1, \ldots, N_a\}$, the preference semi-major axis vectors $\{\mathbf{v}^k = (v_1^k, \ldots, v_m^k), k = 1, \ldots, N_a\}$. Given these parameters, pMOEA/D can be stated in Algorithm 2.

pMOEA/D combines LBS technique and MOEA/D procedure to MOP with preference information. The aim of pMOEA/D is to obtain a set of preferred solutions, neither one solution nor all solutions, on the PF. An intuitive idea is that pMOEA/D should assign more computing resources to the subproblems whose optimal solutions are located on the preferred regions. Therefore, the main issue of pMOEA/D is how to adjust the distribution of subproblems to narrow the search on the interesting regions of DM. Based on the geometric analysis in Sect. 3, some subproblems will be removed from the evolutionary population because their optimal solutions are far from the preference direction. Meanwhile, with the help of the external population, some new subproblems will be created and added into the evolutionary population since their optimal solutions are around the preference direction.

On the other hand, the basic idea of pMOEA/D is to apply a two-stage strategy to deal with MOPs with preference information. During the early stage of evolution, pMOEA/D pursues the convergence of solutions. In this stage, pMOEA/D mainly prefers the non-dominated solutions until the population is considered to converge to some extent. In the second stage, pMOEA/D prefers the solutions

Algorithm 2 pMOEA/D

Require: A stopping criterion and the parameter set.
Ensure: $\mathbf{x}^1, \ldots, \mathbf{x}^N$ and $\mathbf{F}^1, \ldots, \mathbf{F}^m$.

Step 1 : Initialization

1.1 Initialize the evenly scattered weight vectors $\{\mathbf{w}^1, \ldots, \mathbf{w}^N\}$ (Scheffe 1958, 1963) and calculate these neighbor weight vectors (Zhang et al. 2009).
1.2 Initialize the evolutionary population by generating $\mathbf{x}^1, \ldots, \mathbf{x}^N$ at random, set $\mathbf{F}^i = \mathbf{F}(\mathbf{x}^i)$; Set the reference point $\mathbf{z} = (z_1, \ldots, z_m)$ by $z_i = \min\{f_i(\mathbf{x}^1), \ldots, f_i(\mathbf{x}^N)\}$ and $gen = 1$.

Step 2: Create offspring population through applying evolutionary operators (crossover and mutation), evaluate them and update the solutions of subproblems and the reference point $\mathbf{z} = (z_1, \ldots, z_m)$ (Zhang et al. 2009). The difference is that the modified Tchebycheff decomposition approach is used as the decomposition technique.
Step 3: Use the offspring population to update the external population ExP by Algorithm 3.
Step 4: Perform biased weight vector adjustment inspired by preference information **if** $|ExP| \geq N$ and gen **mod** $internal = 0$, then adjust the weight vectors as follows:

4.1 Delete subproblems based on their preferred neighboring distances in EP by Algorithm 4.
4.2 Add new subproblems to the sparse regions based on their preferred neighboring distances in ExP by Algorithm 5.
4.3 Update the neighbor subproblems of all subproblems. **end if**

Step 5: If the stopping criterion is met, stop; else set $gen = gen + 1$, go to Step 2.

in the interesting regions of decision maker. It adjusts the distribution of subproblems to make their optimal solutions located on the interesting regions. Specially, for the second stage, some subproblems which are far away from the preference regions will be removed, and some new subproblems will be created and added into the preference regions with the help of the external elite population.

Biased weight vector adjustment inspired by preference information in step 4 is used periodically (gen **mod** $internal = 0$). It works when the evolutionary population has converged to some extent (for example $|ExP| \geq N$). The external population is aimed to provide diverse preference solutions for adding subproblems.

4.3 Updating of external population

The external population is introduced to keep a set of preferred non-dominated solutions rather than the solutions trying to cover the whole PF. Intuitively, the role of external population in pMOEA/D is to offer non-dominated solutions which approximate the interesting regions. Algorithm 3 summarizes the process of the external population updating.

4.4 Biased weight vector adjustment inspired by preference information

One of innovation points in this paper is to adjust the distribution of subproblems to focus the search on the interesting regions of decision maker. To deal with this issue, we propose biased subproblems adjustment which is composed of two strategies. One is deleting subproblems whose optimal solutions are far away from preferred areas. The other is adding new sparse subproblems whose optimal solutions are within the preferred regions. Algorithms 4 and 5, respectively, summarize the process of deleting subproblems and adding new subproblems.

Specially, in the strategy of adding subproblems, we firstly select nas solutions with small preferred neighboring distances in the external population. For each selected preferred

Algorithm 3 Update of the external population

Step 1: Get non-dominated solutions NS from the unified set of external population ExP and offspring population OP.
Step 2: Select solutions based on the preference information:

if $|NS| \leq N_{ex}$, **then**

Use the non-dominated solutions NS as new external population;

else

- Calculate the preferred neighboring distance of solutions in ExP using Algorithm 1.
- Choose the solutions, whose preferred neighboring distances are first N_{ex} small numbers, in NS as the new external population ExP.

end if

Algorithm 4 Deleting subproblems

Require: nas : the maximal number of subproblems adjusted in biased weight vector adjustment.
Ensure: the adjusted evolutionary population EP^-.

Step 1: Calculate the preferred neighboring distances of the solutions in EP using Algorithm 1.
Step 2: Record the solutions, whose preferred neighboring distances are greater than 1, in EP as P_{dd}.
Step 3: Preferentially delete the subproblems whose solutions are far from the preference regions and then delete the crowding preferred subproblems

if $|P_{dd}| \geq nas$

delete the solutions whose preferred neighboring distances are top nas large numbers from the EP. Record the result evolutionary population as EP^-.

else

- Delete the $|P_{dd}|$ solutions whose preferred neighboring distances are more than 1 in the current EP. Record the result evolutionary population as EP^-.
- **while** the number of deleted solutions does not reach the required number nas **do**
  - Calculate vicinity distance for each solution $ind^i (i = 1, \ldots, |EP^-|)$ in EP^- among the population EP^- by Eq. (11).
  - delete the solution from EP^- whose solution has the minimum vicinity distance;
- **end while**

end if

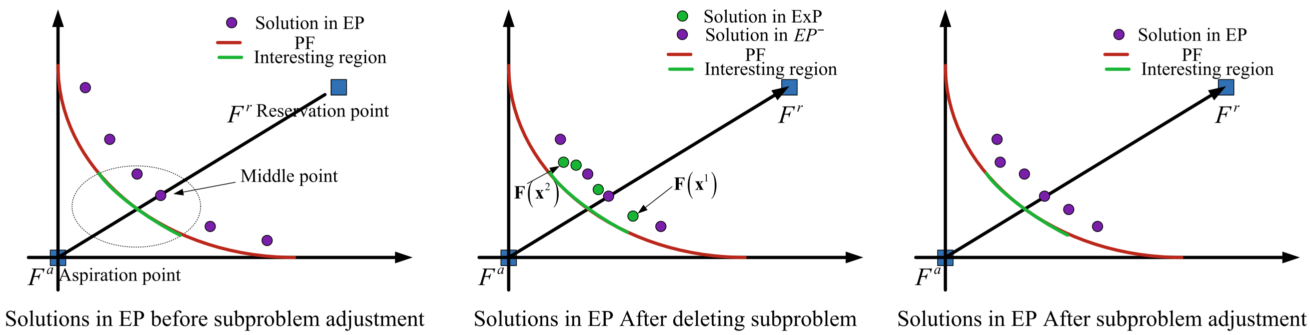

Fig. 8 Illustration of the process of subproblem adjustment, where $nas = 2$

solution, we use the Theorem 2 to construct its optimal subproblem. Then, the newly constructed subproblem will be added into the evolutionary population. Figure 8 gives an example to illustrate the process of deleting subproblems and adding new subproblems. Figure 8 (left) plots the current solutions in evolutionary population before subproblem adjustment. Figure 8 (middle) illustrates the process to delete $nas = 2$ solutions which are far from the middle point. Figure 8 (middle) also presents the process of adding subproblem. Firstly, $\mathbf{F}(\mathbf{x}^1)$ has the maximum vicinity distance (11) from external population ExP to evolutionary population EP^-. Therefore, $\mathbf{F}(\mathbf{x}^1)$ has been added into the evolutionary population. Secondly, $\mathbf{F}(\mathbf{x}^2)$ has been added into the evolutionary population since it has the maximum vicinity distance from the external population ExP to evolutionary population. Therefore, the adjusted evolutionary population (subproblems) are plotted on the Fig. 8 (right).

To keep the diversity of the solutions in the preferred regions, pMOEA/D uses the vicinity distance (Kukkonen and Deb 2006). The vicinity distance of solution ind among population $population$ can be defined as:

$$VD(ind, population) = \prod_{i=1}^{m} d_2^{NN_i} \tag{11}$$

where $d_2^{NN_i}$ is the Euclidean distance from the solution ind to its ith nearest neighbor in the population $population$. It is important to note that the solution ind is not always the member of the population $population$.

Algorithm 5 Adding new subproblems to the sparse regions

Require: EP^- : the resultant population after subproblems deletion. nas : the maximal number of subproblems adjusted in biased weight vector adjustment.

Ensure: the adjusted evolutionary population.

Step 1: Calculate the preferred neighboring distance of solutions in ExP using Algorithm 1.

Step 2: Record the solutions, whose preferred neighboring distances are less than or equal to 1, in ExP as P_{nn}.

Step 3: Select nas solutions from ExP to add into the evolutionary population EP^-.

if $|P_{nn}| \geq nas$, **then**

For $i = 1 : nas$

- Calculate the vicinity distances from the preferred solutions in P_{nn} to the EP^- using Eq. (11); Mark the solution which has the maximum vicinity distance as $(\mathbf{x}^p, \mathbf{F}^p)$;
- Generate a new subproblem for the preferred selected individual $(\mathbf{x}^p, \mathbf{F}^p)$. Based on the Theorem 2, the optimal preference direction (weight vector) $\mathbf{w}^p$ of the newly constructed subproblem can be calculated as follows, in which $\mathbf{F}^p = (f_1^p, \ldots, f_m^p)$,

$$\mathbf{w}^p = \left(\frac{f_1^p - z_1}{\sum_{i=1}^{m}(f_i^p - z_i)}, \ldots, \frac{f_m^p - z_m}{\sum_{i=1}^{m}(f_i^p - z_i)} \right) \tag{12}$$

- Add the newly constructed subproblem $\mathbf{ind}^p = (\mathbf{x}^p, \mathbf{F}^p, \mathbf{w}^p)$ into the evolutionary population;

end for

else

Add nas solutions, whose preferred neighboring distances are top nas small in external population ExP, into the evolutionary population. The weight vectors of these newly added subproblems are constructed by Eq. (12).

end if

5 Experimental study

In this section, we use LBS-NSGA-II (Deb and Kumar 2007) and LBS-NSGA-III as the comparison algorithms. LBS-NSGA-II is classical preference-based MOEA using reference point and preference direction information. NSGA-III is recently proposed by Deb and Jain (2012). To study the efficiency and effectiveness of the proposed algorithm, we use the ZDT problems, DTLZ problems, UF problems, two many-objective problems and a real engineer problem. The simulation codes of the compared approaches are developed using visual C++ 6.0 and run on a workstation with Inter Core6 2.8 GHz CPU and 32 GB RAM.

5.1 Multi-objective test problems

In the experimental study, we select 5 bi-objective ZDT test instances (Zitzler et al. 2000) and 5 tri-objective DTLZ problems (Deb et al. 2002b) to investigate the ability of pMOEA/D. To study the capability of pMOEA/D in solving MOPs with complicated PSs, 10 UF test problems (Zhang et al. 2008) are selected as test problems. Two many-objective DTLZ problems are selected to observe the effectiveness of pMOEA/D for many-objective problems. Finally, to investigate the ability of pMOEA/D to solve the real-world engineering problem, two flood control problems for reservoir are tested.

5.2 Performance metric

The inverted generational distance (IGD), additive ε-indicator ($I_{\varepsilon+}$) and Hypervolume metrics are used to evaluate the performance of the compared algorithms.

Inverted generational distance metric is a comprehensive index of convergence and diversity (Zitzler et al. 2003). Let P^* be a set of uniform distributed solutions along the true preferred Pareto optimal frontier (in the objective space). Let $\bar{P}$ be the approximate solutions of the preferred PF, the average distance from P^* to $\bar{P}$ is defined as:

$$\mathrm{IGD}(P^*, \bar{P}) = \frac{\sum\limits_{\mathbf{F}\in P^*} d(\mathbf{F}, \bar{P})}{|P^*|}$$

where $d(\mathbf{F}, \bar{P})$ is the minimum Euclidean distance between preferred Pareto solution $\mathbf{F}$ and the points in approximate solutions $\bar{P}$. If P^* is large enough to represent the preferred PF very well, $\mathrm{IGD}(P^*, \bar{P})$ can measure both the uniformity and convergence of $\bar{P}$ in a sense. To have a low value of $\mathrm{IGD}(P^*, \bar{P})$, the set $\bar{P}$ should be very near to the PF and cannot lose any part of the preferred PF.

Additive ε-indicator ($I_{\varepsilon+}$) metric (Zitzler et al. 2003) is also used in this paper. The additive ε-indicator ($I_{\varepsilon+}$) is defined as follows:

$$I_{\varepsilon+}(P^*, \bar{P}) = \inf_{\varepsilon\geq 0}\left\{\forall\, \mathbf{F}^2 \in P^*,\ \exists\, \mathbf{F}^1 \in \bar{P}\ :\ \mathbf{F}^1 \prec_{\varepsilon+} \mathbf{F}^2\right\}$$

where $\mathbf{F}^1 \prec_{\varepsilon+} \mathbf{F}^2$ is established, if and only if $\forall 1 \leq i \leq m, \mathbf{F}^1_i \leq \varepsilon + \mathbf{F}^2_i$. Accordingly, the $I_{\varepsilon+}$-indicator gives an addition item by which the preferred Pareto solutions P^* is worse than the approximation set $\bar{P}$ with respect to all objectives.

When the PF is unknown, Hypervolume metric is used to evaluate the performance of the compared algorithms. Let $\{\mathbf{p}^1, \ldots, \mathbf{p}^N\}$ be the approximate non-dominated solutions of the preferred PF and $\mathbf{r}$ be the reference point such that $\forall 1 \leq i \leq m, \mathbf{p}^i \prec \mathbf{r}$. Then the Hypervolume of $\{\mathbf{p}^1, \ldots, \mathbf{p}^N\}$ can be defined as follows:

$$\mathrm{HV} = volume\left(\bigcup_{i=1}^{N} v_i\right)$$

where v_i indicates a hyper-rectangle formed by the reference point $\mathbf{r}$ and the ith non-dominated solution $\mathbf{p}^i$.

The number of the solutions in preferred PF P^* used to calculate the value of IGD and $I_{\varepsilon+}$ is set to be 500 for 2-objective problems, 1000 for 3-objective problems, 2000 for 4-objective problems and 4000 for 10-objective problems. All compared algorithms are independently run 30 times on all test problems.

For many-objective optimization problems, it is difficult to define preferred Pareto solutions. We take 3-objective problem as an example to illustrate how to generate uniformly preferred solutions as shown in Fig. 9. Firstly, we use simplex lattice design (Scheffe 1958, 1963) to generate uniformly distributed solutions S^1. Secondly, scale and translate S^1 as S^2 to approach preferred region. The scaling factor is different from problem to problem. Thirdly, project S^2 to the PF as S^3. A worth remembering thing is to ensure the preferred solutions locating in the scope of S^3. In general, parameter H in simplex lattice design is set as a big value and scaling factor from S^1 to S^2 should be set suitably. Finally, we use preference information to select preferred solutions S^4 on the PF. $P^* = S^4$ in calculating IGD and $I_{\varepsilon+}$ metrics. If the

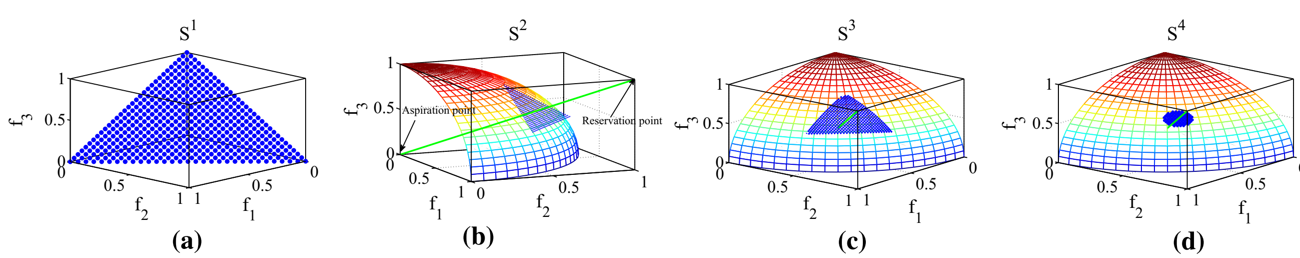

Fig. 9 The process of generate uniformly preferred solutions. **a** Plots of the uniformly distributed solutions S^1 generated by simplex lattice method. **b** Illustration of the solution S^2 by translation and scaling on S^1. **c** shows the projection S^3 from S^2 to the PF of DTLZ3–DTLZ4. **d** Plots of the selected preferred solutions S^4 on the PF

number of obtained preferred solutions in S^4 is less than the need number, you can adjust two parameters: (1) increase the value H; (2) adjust the value of scaling factor from S^1 to S^2.

5.3 The compared algorithms: light beam search-based NSGA-II (LBS-NSGA-II) and LBS-NSGA-III

The main difference between LBS-NSGA-II and NSGA-II is the definition of crowding distance. In original LBS-NSGA-II, the crowding distance of solution is defined as the maximum difference in objective value between solution and its nearest middle point: $CD_i = \min_{1\le k\le N_a} \max_{1\le j\le m} \{f_j^i - F_j^{m_k}\}, i = 1, \ldots, N$. In this paper, we use preferred neighboring distance (9) to define the crowding distance. Algorithm 6 summarizes the process of LBS-NSGA-II used in this paper. The main differences between the proposed pMOEA/D and LBS-NSGA-II are listed as follows:

1. LBS-NSGA-II applies Pareto dominance to help the evolution of a population, while pMOEA/D uses decomposition technique to guide the search.
2. LBS-NSGA-II uses the preference information to focus the search in each generation, while pMOEA/D does not employ the preference information to guide the evolution until the evolutionary population has converged to some extent.
3. LBS-NSGA-II employs the crowding distance (Deb et al. 2002a) to keep the diversity of the preferred non-dominated solutions, while pMOEA/D uses the vicinity distance to maintain the the diversity of the preferred non-dominated solutions.

The only difference between LBS-NSGA-II and LBS-NSGA-III are the diversity maintaining technique. LBS-NSGA-II uses the crowd distance to keep the diversity of preference solutions, while LBS-NSGA-III uses the supplied reference points (Deb and Jain 2012) to keep the diversity of preference solutions.

Algorithm 6 LBS-NSGA-II

Step 1: Initialize evolutionary population randomly and use the non-dominated solutions in EP as the initial external population ExP.
Step 2: Create offspring population OP by applying evolutionary operators (crossover and mutation) and evaluate them.
Step 3: Non-domination ranking is done for $ExP \bigcup OP$.
Step 4: For each front $\mathbf{Front}^i$, use Algorithm 1 to calculate their preferred neighboring distances.
Step 5: mating selection: use 2-tournament selection to choose $2N$ parent solutions based on their preferred neighboring distances.
Step 6: Use OP to update the external population ExP by Algorithm 3.
Step 7: If the stopping criterion is met, stop; else go to Step 2.

5.4 Parameter settings

Parameters of all compared algorithms are set as follows: the simulated binary crossover (SBX) operator and polynomial mutation (Deb and Beyer 2001) are employed in pMOEA/D and LBS-NSGA-II for solving bi-objective ZDT problems and DTLZ problems. The differential evolution (DE) operator and polynomial mutation (Li and Zhang 2009) are employed in pMOEA/D and LBS-NSGA-II for solving UF problems and the real-world problems. The parameter values are listed in Table 1, where n is the number of variables and $rand$ is a uniform random value in [0,1].

Population size N is set to be 50 for ZDT problems, 100 for the 3-objective DTLZ problems, 200 for the 4-objective DTLZ problems, 400 for the 10-objective DTLZ problems, 200 and 300 for the bi-objective UF1–UF7 problems with one and two preferred regions, respectively, 300 and 400 for the tri-objective UF8-UF10 problems with one and two preferred regions, respectively. The external elite size is set as 1.5 N.

All the compared algorithms stop when their function evaluation costs reach the maximum number. $FE_{\max}$ is set to be 30,000 for ZDT problems, 60,000 for tri-objective DTLZ problems, 120,000 for 4-objective DTLZ problems, 400,000 for 10-objective DTLZ problems, 150,000 and 250,000 for bi-objective UF1-UF7 problems with one and two preferred regions, respectively, 300,000 for tri-objective UF8-UF10 problems.

For pMOEA/D, the size of neighborhood list T is set to be 0.1 N. The probability of choosing mate subproblem from its neighborhood is set to be 0.9, nas is set to be 0.1 N and $Interval$ is set to be $\frac{FE_{\max}}{30\ N}$. The preference information of the test problems are listed in Table 2. For simplicity, all the aspiration points uses $[0, \ldots, 0]$. It is easy to change the aspiration point. For UF problems, the reservation points are selected for finding the difficult region(s) of PF.

Someone may wonder that if the aspiration point and reservation point is difficult to obtain in advance, can the proposed algorithm still work? We can use interactive approaches (Deb et al. 2010; Said et al. 2010) to interleave the optimization with a progressive elicitation of user preferences. In the initial stage, the DM has a rough idea about the preferred solutions

Table 1 Parameter settings for SBX, DE and PM

Parameter	pMOEA/D	LBS-NSGA-II
Crossover probability (p_c)	1	1
Distribution index for crossover	20	20
Mutation probability (p_m)	1/n	1/n
Distribution index for mutation	20	20
DE parameter (CR)	1	1
DE parameter (F)	rand	rand

Table 2 Preference information in experimental studies

Problem	Single preference region			Multiple preference regions		
	AP	RP	**v**	APs	RPs	**v**
ZDT1	[0 0]	[1 1]	[0.05 0.05]	[0 0]	[1 0.5]	[0.05 0.05]
				[0 0]	[0.5 1]	[0.05 0.05]
ZDT2	[0 0]	[1 1]	[0.05 0.05]	[0 0]	[1 0.5]	[0.05 0.05]
				[0 0]	[0.5 1]	[0.05 0.05]
ZDT3	[0 0]	[1 1]	[0.05 0.05]	[0 0]	[1 0.5]	[0.05 0.05]
				[0 0]	[0.5 1]	[0.05 0.05]
ZDT4	[0 0]	[1 1]	[0.05 0.05]	[0 0]	[1 0.5]	[0.05 0.05]
				[0 0]	[0.5 1]	[0.05 0.05]
ZDT6	[0 0]	[1 1]	[0.05 0.05]	[0 0]	[1 0.5]	[0.05 0.05]
				[0 0]	[0.5 1]	[0.05 0.05]
DTLZ1	[0 0 0]	[0.5 0.5 0.5]	[0.05 0.05 0.05]	[0 0 0]	[0.5 0.1 0.3]	[0.05 0.05 0.05]
				[0 0 0]	[0.3 0.5 0.1]	[0.05 0.05 0.05]
DTLZ2	[0 0 0]	[1 1 1]	[0.05 0.05 0.05]	[0 0 0]	[1 0.2 0.6]	[0.05 0.05 0.05]
				[0 0 0]	[0.6 1 0.2]	[0.05 0.05 0.05]
DTLZ3	[0 0 0]	[1 1 1]	[0.05 0.05 0.05]	[0 0 0]	[1 0.2 0.6]	[0.05 0.05 0.05]
				[0 0 0]	[0.6 1 0.2]	[0.05 0.05 0.05]
DTLZ4	[0 0 0]	[1 1 1]	[0.05 0.05 0.05]	[0 0 0]	[1 0.2 0.6]	[0.05 0.05 0.05]
				[0 0 0]	[0.6 1 0.2]	[0.05 0.05 0.05]
DTLZ6	[0 0 0]	[0.75 0.75 5]	[0.05 0.05 0.3]	[0 0 0]	[0.75 0.75 5]	[0.05 0.05 0.3]
				[0 0 0]	[0.75 0.13 5.3]	[0.05 0.05 0.3]
UF1	[0 0]	[1 0.1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.1 1]	[0.05 0.05]
UF2	[0 0]	[1 0.1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.1 1]	[0.05 0.05]
UF3	[0 0]	[0.1 1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.1 1]	[0.05 0.05]
UF4	[0 0]	[0.95 0.05]	[0.05 0.05]	[0 0]	[0.95 0.05]	[0.05 0.05]
				[0 0]	[0.05 0.95]	[0.05 0.05]
UF5	[0 0]	[1 0.1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.1 1]	[0.05 0.05]
UF6	[0 0]	[1 0.1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.4 0.6]	[0.05 0.05]
UF7	[0 0]	[1 0.1]	[0.05 0.05]	[0 0]	[1 0.1]	[0.05 0.05]
				[0 0]	[0.1 1]	[0.05 0.05]
UF8	[0 0 0]	[0.75 0.2 0.75]	[0.05 0.05 0.05]	[0 0 0]	[0.75 0.2 0.75]	[0.05 0.05 0.05]
				[0 0 0]	[0.75 0.75 0.2]	[0.05 0.05 0.05]
UF9	[0 0 0]	[0.1 0.6 0.8]	[0.05 0.05 0.05]	[0 0 0]	[0.1 0.6 0.8]	[0.05 0.05 0.05]
				[0 0 0]	[0.6 0.1 0.8]	[0.05 0.05 0.05]
UF10	[0 0 0]	[0.5 0.111 1]	[0.05 0.05 0.05]	[0 0 0]	[1 0.2 0.6]	[0.05 0.05 0.05]
				[0 0 0]	[0.6 1 0.2]	[0.05 0.05 0.05]
DTLZ3(4)	[0, 0, 0, 0]	[1, 1, 1, 1]	[0.05, 0.05, 0.05, 0.05]			
DTLZ3(10)	[0, . . . , 0]	[1, . . . , 1]	[0.05, . . . , 0.05]			
DTLZ4(4)	[0, 0, 0, 0]	[1, 1, 1, 1]	[0.05, 0.05, 0.05, 0.05]			
DTLZ3(10)	[0, . . . , 0]	[1, . . . , 1]	[0.05, . . . , 0.05]			

AP aspiration point, *RP* reservation point at least. After the DM specify partial preferences, the algorithm search the biasing solutions that are considered as the interesting solutions of the DM. The goal is to generate a good approximation to a small set of solutions that contain the DM's preferred solution with the highest probability. The DM can modify his/her preference information in the interactive process and refine the knowledge of his/her preference. Therefore, we can say our algorithm can still work in the case that the aspiration point and the reservation point is difficult to obtain in advance.

5.5 Experimental studies on pMOEA/D, LBS-NSGA-II and LBS-NSGA-III

This part of experiments is designed to study the effectiveness of pMOEA/D on different types of MOPs. At first, the classical ZDT and DTLZ problems with simple PSs are investigated. Performances of pMOEA/D on MOPs with complicated PSs are studied afterwards.

5.5.1 Experimental results on MOPs with simple PSs : ZDT and DTLZ Problems

ZDT and the DTLZ problems are bi-objective and tri-objective problems, respectively. The mathematical descriptions and the ideal PFs of the ZDT and DTLZ problems can be found in Zitzler et al. (2000) and Deb et al. (2002b), respectively.

Tables 3 and 4 present the mean and standard deviation of the IGD-metric and $I_{\varepsilon+}$ metric values of the final solutions obtained by each algorithm for five 30-dimensional ZDT problems and five tri-objective 10-dimensional DTLZ problems with one and two preference regions, respectively.

For one preference region, Table 3 reveals that in terms of IGD-metric and $I_{\varepsilon+}$ metric, the final solutions obtained by pMOEA/D perform a little better than LBS-NSGA-II and LBS-NSGA-III on six out of ten problems, except for ZDT1-ZDT4 problems. LBS-NSGA-II performs the best on ZDT1 and ZDT2 problems, while LBS-NSGA-III does the best on ZDT3 and ZDT4 problems. Compared with LBS-NSGA-II, LBS-NSGA-III preforms better on eight out of ten problems including ZDT3-ZDT4, ZDT6 and five 3-objective DTLZ problems. For two preference regions, it is evident from the Table 4 that the final solutions obtained by pMOEA/D do a little better than LBS-NSGA-II on eight out of ten problems, except for the simple problems ZDT1 and ZDT3, as far as the IGD and $I_{\varepsilon+}$ metrics are considered. Especially for all tri-objective DTLZ problems, Tables 3 and 4 show that pMOEA/D performs better than LBS-NSGA-II for both one and two preference regions. The reason may be that in pMOEA/D, it uses the vicinity distance (11) rather than the crowd distance in LBS-NSGA-II to maintain the diversity of the preferred non-dominated solutions.

Table 3 The values of the IGD and $I_{\varepsilon+}$ metrics between the algorithm pMOEA/D, LBS-NSGA-II and LBS-NSGA-III for ZDT and DTLZ problems with one preference region

Problem	pMOEA/D				LBS-NSGA-II				LBS-NSGA-III			
	IGD		$I_{\varepsilon+}$		IGD		$I_{\varepsilon+}$		IGD		$I_{\varepsilon+}$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
ZDT1	9.979e−3	5.814e−5	6.906e−3	3.730e−4	**6.654e−4**	3.719e−5	**6.610e−3**	3.446e−4	1.014e−3	7.884e−5	7.318e−3	7.268e−4
ZDT2	8.771e−4	8.033e−5	1.539e−3	2.295e−4	**3.646e−4**	2.009e−5	**1.215e−3**	3.231e−4	6.687e−4	5.472e−5	1.924e−3	4.167e−4
ZDT3	3.314e−4	3.165e−5	5.936e−4	6.201e−5	2.754e−4	3.301e−5	6.917e−4	2.003e−4	**2.632e−4**	6.816e−5	**5.210e−4**	3.661e−4
ZDT4	3.266e−3	4.083e−3	8.489e−3	3.780e−3	4.344e−3	3.892e−3	1.036e−2	4.142e−3	**2.649e−3**	1.286e−1	**7.088e−3**	1.485e−3
ZDT6	**1.602e−3**	3.518e−4	**2.341e−3**	4.871e−4	4.077e−3	7.680e−4	5.097e−3	9.834e−4	1.863e−3	1.074e−3	3.484e−3	2.518e−3
DTLZ1	**4.073e−3**	2.254e−3	**7.776e−3**	3.859e−3	5.341e−3	1.048e−3	9.927e−3	1.351e−3	5.079e−3	6.799e−4	9.603e−3	1.459e−3
DTLZ2	**3.287e−3**	3.190e−5	**5.533e−3**	5.037e−4	4.042e−3	1.280e−4	8.810e−3	9.974e−4	3.820e−3	2.483e−4	7.535e−3	1.229e−3
DTLZ3	**3.577e−3**	2.313e−4	**6.893e−3**	8.479e−4	7.312e−3	2.331e−3	1.229e−2	1.985e−3	7.050e−3	2.241e−3	1.001e−2	2.005e−3
DTLZ4	**3.306e−3**	4.341e−5	**5.938e−3**	7.984e−4	2.054e−1	3.211e−1	7.304e−2	1.222e−4	6.650e−2	2.933e−2	5.646e−2	4.086e−2
DTLZ6	**7.701e−3**	1.725e−4	**1.460e−2**	6.625e−3	3.609e−2	1.051e−1	3.009e−2	4.655e−2	1.644e−2	1.348e−2	2.323e−2	1.180e−2

The value of metric with bold is the best among the three compared algorithms

Table 4 The values of the IGD and $I_{\varepsilon+}$ metrics between the Algorithm pMOEA/D and LBS-NSGA-II for ZDT and DTLZ problems with two preference regions

Problem	pMOEA/D				LBS-NSGA-II			
	IGD		$I_{\varepsilon+}$		IGD		$I_{\varepsilon+}$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
ZDT1	1.3182e−3	8.7235e−5	2.7310e−3	5.8467e−4	**6.4174e−4**	2.3079e−5	**2.2902e−3**	4.8006e−4
ZDT2	**1.5280e−3**	1.0519e−4	**3.0050e−3**	6.4661e−4	7.5988e−3	2.0440e−2	1.7229e−2	4.3988e−2
ZDT3	7.7453e−4	4.8486e−5	1.4217e−3	7.4054e−4	**5.7756e−4**	1.4281e−5	**1.2766e−3**	6.2713e−4
ZDT4	**9.0108e−3**	2.2549e−2	**1.5208e−2**	2.5773e−2	1.2917e−2	2.5559e−2	2.2927e−2	3.1944e−2
ZDT6	**2.4976e−3**	4.0988e−4	**4.5765e−3**	1.4324e−3	5.7268e−3	1.0567e−3	7.5022e−3	1.3691e−3
DTLZ1	**5.4056e−3**	5.5265e−4	**1.1168e−2**	1.9491e−3	7.3133e−3	3.9853e−3	1.7452e−2	1.1705e−2
DTLZ2	**4.9105e−3**	2.4763e−4	**6.0501e−3**	6.4984e−4	6.7502e−3	4.3149e−3	1.0009e−2	7.3812e−3
DTLZ3	**5.8906e−3**	2.5540e−3	**8.2448e−3**	5.5990e−3	2.7901e−2	2.9502e−2	3.5264e−2	3.1256e−2
DTLZ4	**5.0460e−3**	4.6261e−4	**6.4410e−3**	9.8866e−4	1.5165e−1	2.4632e−1	9.1447e−2	1.5749e−1
DTLZ6	**1.1383e−2**	3.6125e−4	**3.2029e−2**	8.5751e−3	5.5526e−2	1.0838e−1	7.6517e−2	1.0424e−1

The value of metric with bold is the best between both compared algorithms

5.5.2 *Experimental results on MOPs with complicated PSs: UF test problems*

In this part of experiments, the UF problems whose Pareto sets are complicated include seven bi-objective problems and three tri-objective problems. The mathematical descriptions and the ideal PFs of UF problems can be found in Zhang et al. (2008).

Tables 5 and 6 present the mean and standard deviation of the IGD-metric and $I_{\varepsilon+}$ metric values of the final solutions obtained by each algorithm for ten 30-dimensional UF problems with one and two preference regions, respectively. Figures 10 and 11, respectively, show the distribution of the final solutions obtained with the lowest IGD value by each algorithm for ten UF problems with one and two preference regions in the objective space.

For one preference region, Table 5 reveals that in terms of IGD-metric, the final solutions obtained by pMOEA/D are better than LBS-NSGA-II and LBS-NSGA-III on UF2–UF6, UF8 and UF10 problems, while the final solutions obtained by LBS-NSGA-III are best on UF1, UF7 and UF9 problems. LBS-NSGA-III is better than LBS-NSGA-II on eight out of ten UF problems. As far as the $I_{\varepsilon+}$ metric is concerned, pMOEA/D performs better than LBS-NSGA-II on eight out of ten UF problems, while LBS-NSGA-II does better than pMOEA/D on UF1 and UF4. $I_{\varepsilon+}$-indicator denotes an addition item by which the preferred PF is worse than the approximation set with respect to all objectives. Taking UF3 as an example, on average, the condition that the preferred PF is worse than the approximation set with respect to all objectives are added 0.062634 for pMOEA/D, while 0.28216 for LBS-NSGA-II.

For one preference region, it is very clear in Fig. 10 that as to the convergence and coverage of final solutions, pMOEA/D performs better than LBS-NSGA-II on UF1, UF2, UF3, UF6, UF7 and UF10. The final non-dominated solutions found by LBS-NSGA-II miss some parts of the preferred PF especially for the boundary of the preferred PF.

For two preference regions, it can be seen from Table 6 that the proposed pMOEA/D does better than LBS-NSGA-II on nine out of ten UF problems, except UF5 in terms of IGD-metric and $I_{\varepsilon+}$ metric. Taking UF2 as an example, the condition that an addition item by which the preferred PF is worse than the approximation set with respect to all objectives are added 0.04459 for pMOEA/D, while 0.17756 for LBS-NSGA-II averagely. It is visually evident in Fig. 11 that pMOEA/D performs much better than LBS-NSGA-II on UF1, UF2, UF3, UF4, UF6, UF7 and UF10 as far as the uniformity and convergence are concerned. It should be pointed that the final non-dominated solutions found by LBS-NSGA-II almost miss one preferred region for UF2, UF3 and UF7.

The reason for these results can be accounted for that in pMOEA/D, the basic algorithm MOEA/D is performed until the evolutionary population has converged to some extent (for example $|ExP| \geq N$). Since MOEA/D decomposes an MOP into a number of scalar optimization problems using uniformly distributed weight vectors in advance. Generally, the diversity of final obtained solutions can be maintained by subproblems with uniform weight vectors in the framework of MOEA/D. Therefore, in a sense, pMOEA/D can find an approximate solution set of the PF and cover the whole PF well before it includes the preference information to guide the search. While, in each generation, LBS-NSGA-II uses

Table 5 The values of the IGD and I_{ε^+} metrics between the Algorithm pMOEA/D, LBS-NSGA-II and LBS-NSGA-III for UF problems with one preference region

Problem	pMOEA/D				LBS-NSGA-II				LBS-NSGA-III			
	IGD		I_{ε^+}		IGD		I_{ε^+}		IGD		I_{ε^+}	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
UF1	1.156e−2	1.298e−2	3.378e−2	2.174e−2	1.293e−2	2.591e−2	2.423e−2	3.9749e−2	**5.232e−3**	6.390e−4	**7.643e−3**	2.504e−3
UF2	**3.691e−2**	3.841e−2	**5.204e−2**	4.568e−2	3.976e−2	3.125e−2	6.113e−2	3.6959e−2	3.817e−2	3.472e−2	5.923e−2	4.430e−2
UF3	**4.748e−2**	4.423e−2	**6.263e−2**	5.104e−2	2.514e−1	7.073e−2	2.821e−1	7.555e−2	6.943e−2	1.085e−3	7.300e−2	8.629e−2
UF4	**5.127e−2**	1.033e−2	6.135e−2	1.176e−2	5.156e−2	7.560e−3	**6.109e−2**	1.461e−2	5.408e−2	4.507e−3	6.250e−2	8.455e−3
UF5	**3.291e−1**	2.150e−1	**2.933e−1**	1.726e−1	4.313e−1	4.675e−1	3.605e−1	3.777e−1	3.958e−1	3.617e−1	3.475e−1	3.316e−1
UF6	**4.428e−4**	4.755e−4	**1.351e−3**	2.245e−3	9.718e−3	7.114e−3	2.174e−2	1.038e−2	6.145e−4	6.568e−5	3.368e−3	5.087e−4
UF7	1.328e−2	3.195e−3	3.050e−2	8.954e−3	4.559e−2	3.124e−2	6.877e−2	3.163e−2	**9.204e−3**	3.599e−3	**2.803e−2**	8.106e−3
UF8	**2.603e−3**	2.786e−3	**4.134e−3**	2.331e−3	3.701e−3	2.138e−4	5.483e−3	4.607e−4	4.615e−3	5.494e−4	6.896e−3	1.073e−3
UF9	1.055e−1	1.712e−1	1.119e−1	1.513e−1	1.252e−1	1.753e−1	1.153e−1	1.518e−1	**9.404e−2**	1.395e−1	**9.207e−2**	1.275e−1
UF10	**2.143e−1**	1.616e−1	**2.170e−1**	1.462e−1	4.504e−1	1.573e−1	3.594e−1	1.155e−1	4.338e−1	1.394e−1	3.453e−1	9.846e−2

The value of metric with bold is the best among the three compared algorithms

the preference information to focus the search. When the MOP is difficult, just as UF test problems, using the preference information will often make LBS-NSGA-II premature convergence.

Figure 12 plots the solutions found by two compared algorithms in the process of evolution on UF2 problem. This MOP is difficult problem. The main differences between our proposed algorithm and LBS-NSGA-II are listed as follows:

1. At each generation, LBS-NSGA-II uses the preference information to select preferred solutions.
2. Our proposed algorithm uses the preference information to adjust the distribution of subproblems once every several generations. For UF2 problem, pMOEA/D only performs subproblems adjustment once every 20 generations in the late stage of evolution (where population size $N = 200$, maximum function evaluations $FE_{\max} =$ 150,000).

When the number of non-dominated solutions in parent population and offspring population is greater than the population size N, the selection based on preference information will make LBS-NSGA-II converge rapidly to a part of PF and the diversity of evolution population will quickly reduce. If the MOP is difficult, evolution population of LBS-NSGA-II may trap into a part of PF but not the interesting region of decision maker as shown in Fig. 12. When the diversity of evolution population is reduced, LBS-NSGA-II may fail to go towards the preferred solutions. When evolution population goes towards the preferred solution, pMOEA/D performs subproblem adjustment once every several generations while LBS-NSGA-II uses the preference information to select preferred solutions at each generation. Therefore, the diversity of evolution population in pMOEA/D is better than that in LBS-NSGA-II when evolution population goes towards the preferred solution as shown in Fig. 12. Compared with LBS-NSGA-II, pMOEA/D is expected to have higher probability to find the interesting regions of decision maker. pMOEA/D may have better robustness than LBS-NSGA-II to find the preferred solutions in most of selected MOPs.

5.5.3 Experimental results on many-objective problems

In this section, two popular many-objective test problems DTLZ3 and DTLZ4 are used to study the ability of the pMOEA/D on many-objective problems.

Table 7 demonstrates the mean and standard deviations of the IGD-metric values found by the compared algorithms for many-objective test problems DTLZ3 and DTLZ4. Figure 13 shows the evolution of the average IGD-metric values of the non-dominated solutions found by pMOEA/D and LBS-NSGA-II in the current populations for 4-objective and 10-objective DTLZ3 and DTLZ4 problems.

Table 6 The values of the IGD and $I_{\varepsilon+}$ metrics between the algorithm pMOEA/D and LBS-NSGA-II for UF problems with two preference regions

Problem	pMOEA/D				LBS-NSGA-II			
	IGD		$I_{\varepsilon+}$		IGD		$I_{\varepsilon+}$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SDSD
UF1	**4.7817e−3**	1.4490e−2	**1.5774e−2**	4.0701e−2	2.2469e−2	1.6550e−2	7.5025e−2	4.2417e−2
UF2	**1.0775e−2**	1.4596e−2	**4.0459e−2**	4.1729e−2	6.7901e−2	2.2458e−2	1.7756e−1	4.4861e−2
UF3	**7.2471e−2**	1.0656e−1	**1.4921e−1**	1.9133e−1	1.8479e−1	3.4570e−2	2.9886e−1	1.4255e−2
UF4	**5.1185e−2**	8.0837e−3	**6.0989e−2**	7.0015e−3	5.3054e−2	6.4893e−3	6.2067e−2	1.1101e−2
UF5	2.1567e−1	9.3111e−2	2.6871e−1	1.1701e−1	**1.6119e−1**	2.9861e−1	**1.5123e−1**	2.5057e−1
UF6	**3.2744e−3**	2.4699e−4	**9.7887e−3**	2.7574e−3	1.2801e−2	1.0344e−2	3.9434e−2	2.1384e−2
UF7	**1.7938e−2**	1.6437e−2	**5.6681e−2**	3.4208e−2	1.8652e−2	1.2779e−2	5.7107e−2	2.5083e−2
UF8	**2.5025e−2**	1.7364e−2	**1.6249e−2**	4.2327e−3	1.1655e−1	5.2041e−2	2.1912e−1	1.0079e−1
UF9	**8.0539e−2**	8.9702e−2	**1.3907e−1**	1.5258e−1	1.4778e−1	7.9579e−2	2.4370e−1	1.1709e−1
UF10	**4.3226e−1**	1.4735e−1	**5.0201e−1**	1.3699e−1	1.0691	1.5940e−1	9.1036e−1	1.2012e−1

The value of metric with bold is the best between the both compared algorithms

From Table 7, we can see that in terms of IGD-metric, the final solutions obtained by pMOEA/D are better than LBS-NSGA-II for 4-objective and 10-objective DTLZ3 and DTLZ4 problems. Figure 13 gives us a clear view that pMOEA/D is better than LBS-NSGA-II in solving 4-objective and 10-objective DTLZ3 and DTLZ4 problems as far as the IGD-metric is concerned.

Overall, considering the experimental results, we can conclude that

1. For the MOPs with simple PSs, three compared algorithms are capable of approximating the true preferred Pareto optimal fronts for ZDT and DTLZ problems.
2. For the UF problems with single preference region, pMOEA/D and LBS-NSGA-III can produce better approximations than LBS-NSGA-II in terms of IGD-metric and $I_{\varepsilon+}$ metric on most UF test problems. For the UF problems with two preference regions, pMOEA/D can produce better solutions than LBS-NSGA-II in terms of IGD-metric and $I_{\varepsilon+}$ metric on most UF test problems, especially for the case with two preferred regions.
3. For many-objective test problems, pMOEA/D is better than LBS-NSGA-II on 4-objective and 10-objective DTLZ3 and DTLZ4 problems.

5.6 Experimental results on real-world problem

In this part, a real-world problem of flood control for reservoir is investigated to show the superiority of the proposed approach. The RFC problem is defined in Sect. 2.2. To meet the needs of irrigation and generate electricity after the flood, the most preferred upstream water level is 325 m, which is equal to flood control water level Z_{FCL}. The preferred upstream water level region [320, 330] m near the flood control water level is used. Therefore, the aspiration point is set as [0, 325]. The reservation point is set as [20,000, 325]. The veto threshold vector is set as $\mathbf{v} = [5000, 5]$.

In the part of experiments, two typical reservoir flood control problems, namely Ankang20030828 and Ankang 20051001, of the Ankang reservoir in Shanxi province of China on August 28, 2003 and October 1, 2005 are applied to study the ability of the proposed approach. The left panel of Fig. 14 illustrates the variation of water level to volume curve of the Ankang reservoir. The middle and right panels of Fig. 14 plot reservoir inflow volume information of two representative floods on August 28, 2003 and October 1, 2005. It can be seen that the two floods are different in characters. The first flood has two smaller flow peaks while the second has one larger flow peak.

The initial water level is set as 317 m. The limits of upstream water level $Z_{\min}$ and $Z_{\max}$ are set to be 300 and 330 m, respectively. The maximal discharge volume $Q_{\max}$ is set to be 37,474 m^3/s. The flood restricted water level is set as 325m. Figure 15 gives a schematic diagram of Ankang reservoir. The interval for scheduling time is set as 3 and 4 h for two reservoir flood control problems Ankang20030828 and Ankang20051001, respectively. The parameters setting of the compared algorithms are the same as described in Sect. 5.4.

Table 8 demonstrates the mean and standard deviations of the Hypervolume-metric values found by the compared algorithms for two multi-objective reservoir flood dispatching problems of the Ankang reservoir on August 28, 2003 and October 1, 2005. From this table we can see that in terms of Hypervolume-metric, the final solutions obtained by pMOEA/D are better than LBS-NSGA-II for two multi-objective reservoir flood dispatching problems.

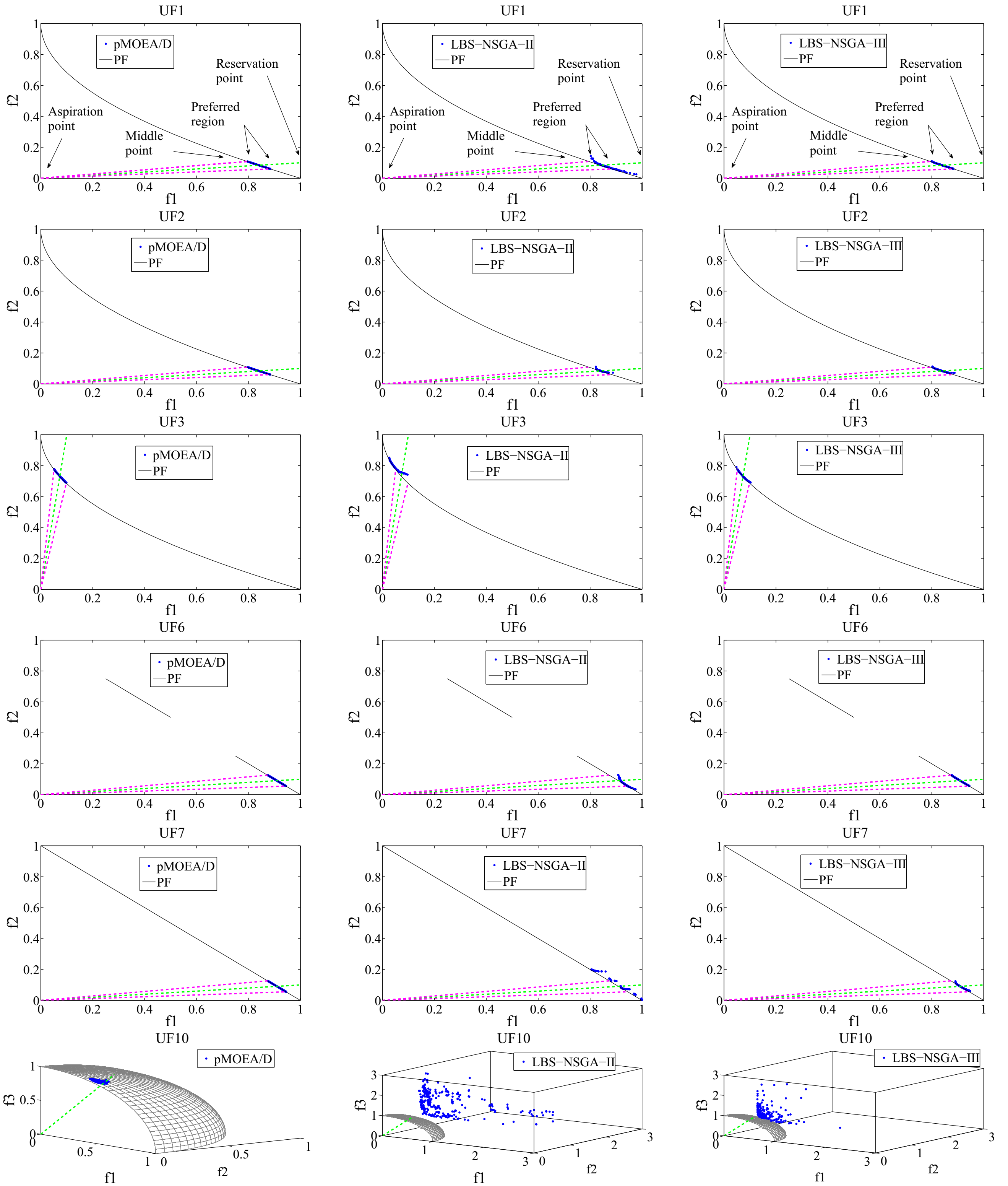

Fig. 10 The distribution of the final non-dominated solutions with the lowest IGD values found by pMOEA/D, LBS-NSGA-II and LBS-NSGA-III in solving the 30-dimensional UF problems with one preference region

The references for calculating the Hypervolume are set as [330.0,7826.21] and [330.0,17000.0] for Ankang20030828 and Ankang20051001, respectively. Before we calculate the Hypervolume of the approximate solution set, it is necessary to remove the approximate solutions which are not in preferred region for MOPs with preference information. For the

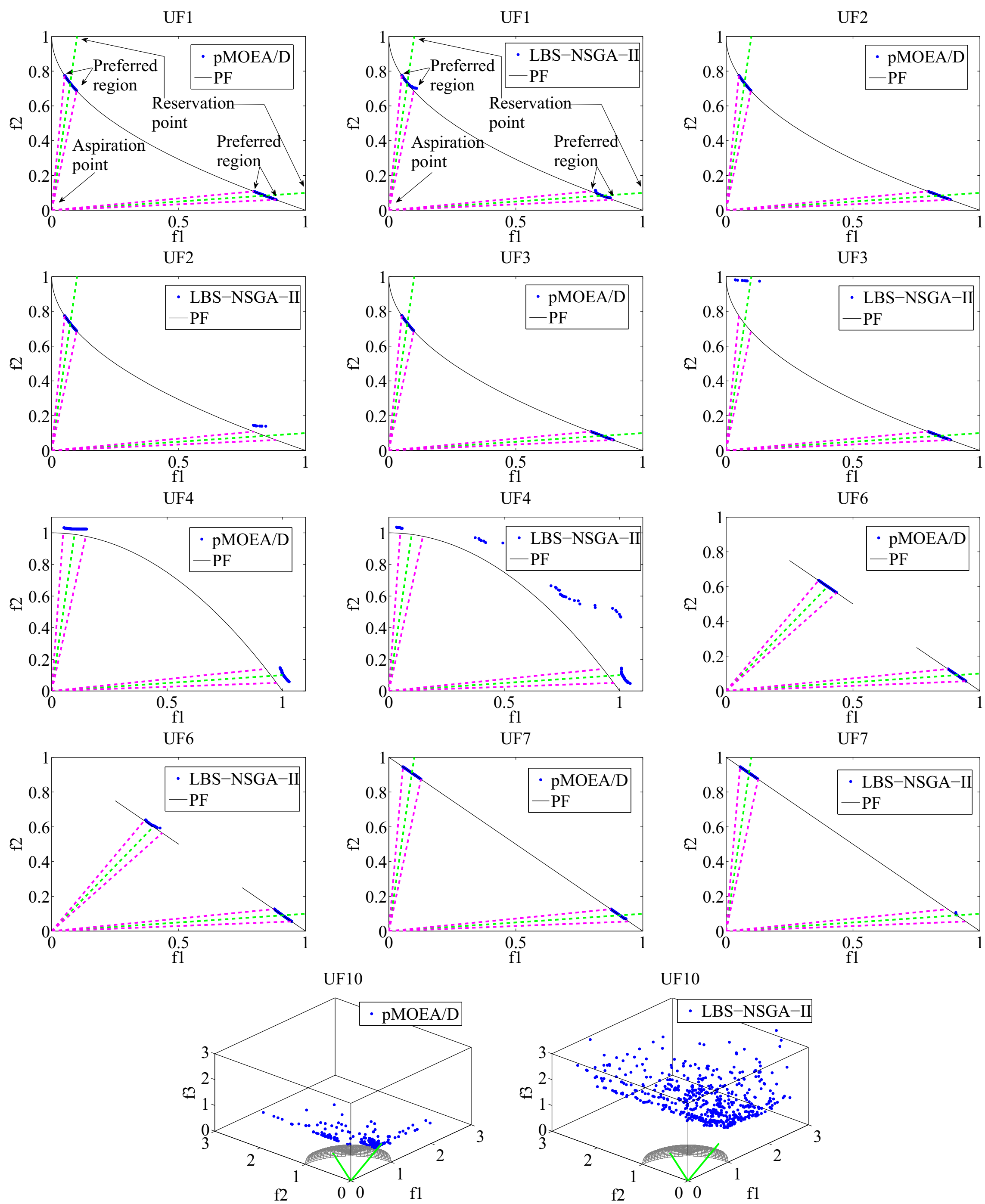

Fig. 11 The distribution of the final non-dominated solutions with the lowest IGD values found by pMOEA/D and LBS-NSGA-II in solving the first 30-dimensional UF problems with two preference regions

Ankang20030828 and Ankang20051001 problems, it is reasonable to remove the approximate solutions whose upstream water level are less than 320 or greater than 330 before we calculate the Hypervolume of the approximate solution set. The reason is that the preferred upstream water level region is set to be [320,330] m in the Ankang reservoir in Shanxi province of China.

Figure 16 illustrates the non-dominated solutions with highest hypervolume values of the two compared algorithms in solving the two reservoir flood dispatching problems. Figure 17 illustrates the discharging downstream flow of the scheduling solutions with highest Hypervolume values found by pMOEA/D and LBS-NSGA-II over 30 independent runs in solving two problems with 200,000 function evaluations.

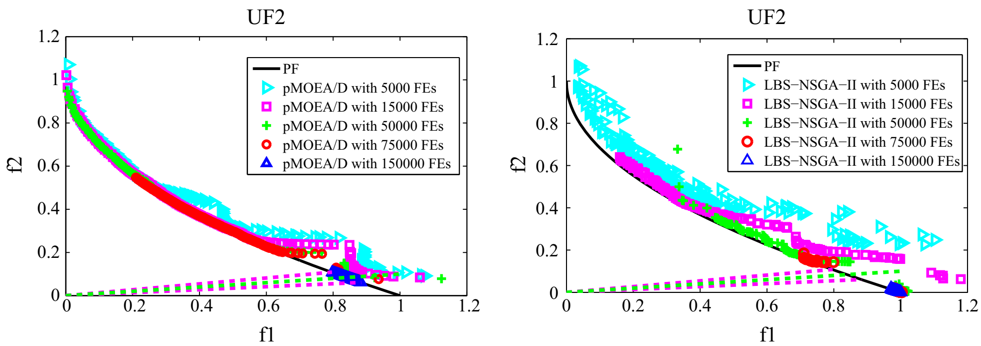

Fig. 12 Plot of the solutions found by two compared algorithms in the process of evolution on UF2 problem

Table 7 Statistic IGD-metric values of the solutions founded by the two compared algorithms on many-objective DTLZ3 and DTLZ4 problems. The numbers in parentheses present the standard deviation

IGD	*m*	pMOEA/D	LBS-NSGA-II
DTLZ3	4	**1.0685e−2**	1.3167e−2
		(2.2270e−3)	(4.9635e−3)
	10	**8.0721e−2**	8.6714e−2
		(6.8609e−2)	(6.9944e−2)
DTLZ4	4	**8.3175e−3**	4.9858e−1
		(5.6714e−4)	(2.5459e−1)
	10	**4.7664e−1**	5.7776e−1
		(1.0156e−1)	(1.2202e−1)

The value of metric with bold is the best between both compared algorithms

Figure 18 shows the highest upstream water level of the scheduling solutions of with highest Hypervolume values by the two compared algorithms in solving the two problems with 200,000 function evaluations. In Fig. 16, the data APF is approximation solution set of PF. It uses the non-dominated set of union the approximate solution sets which are obtained by running MOEAs (include NSGA-II and MOEA/D) with 2,000,000 function evaluations.

From Fig. 16, it can be seen that the proposed pMOEA/D obtains a better preferred solution set, whose water levels are in the preferred region [320,330] m, than LBS-NSGAII in terms of coverage, especially for the region near the flood restricted water level 325 m. For the Ankang20030828 problem with two flood peaks, LBS-NSGA-II obtains a set of solution set which near the upstream water level 320 and miss the part of [323,330] m. For the Ankang20051001 problem with a big flood, LBS-NSGA-II only finds few preferred solutions and misses most part in the preferred upstream water level region [320,330] m.

Figure 17 illustrates that, for two different types of floods, the stability of discharging downstream flow in each scheduling time found by pMOEA/D is much better than LBS-NSGA-II. It is evident that, in reducing the effect of flood peak, pMOEA/D is better than LBS-NSGA-II in finding stable discharging downstream flow solutions in each scheduling time. Moreover, Fig. 17 also shows that, for two different types of floods, the diversity of scheduling solutions in discharging downstream flows found by pMOEA/D is much better than LBS-NSGA-II, especially for the scheduling solutions with low maximum discharging downstream flow in [2000,4000] and [11,000,14,000] m^3/s for Ankang20030828 and Ankang20051001, respectively. It suggests that, in ensuring the safety of the people on the downstream of reservoir, pMOEA/D is better than LBS-NSGA-II in find scheduling solutions with low maximum discharging downstream flow in the flood.

We can see From Fig. 18 that, for two different types of floods, the diversity of final upstream water levels found by pMOEA/D is much better than LBS-NSGA-II. It is evident that, to cope with the next flood, pMOEA/D can provide a more diversity of decision supporting information for decision makers to control final upstream water level flexibly. What is more, it can be seen from Fig. 18 that pMOEA/D obtains more solutions than LBS-NSGA-II near the flood restricted water level 325 m. It suggests that, in meeting the needs of irrigation after the flood, pMOEA/D is better than LBS-NSGA-II in finding the solutions near the flood restricted water level 325m especially for the Ankang20051001 problem with a large flood.

6 Concluding remarks

Related to the safety of public lives and property in the lower area of reservoirs, flood control is a priority for most large reservoirs. Considering both dam safety and downstream flood control, reservoir flood control (RFC) is a multi-objective problem (MOP). To meet the needs of irrigation and generating electricity after the flood, the DM usually has his/her preferred final scheduling water level. To deal with this kind of MOP with user-preference infor-

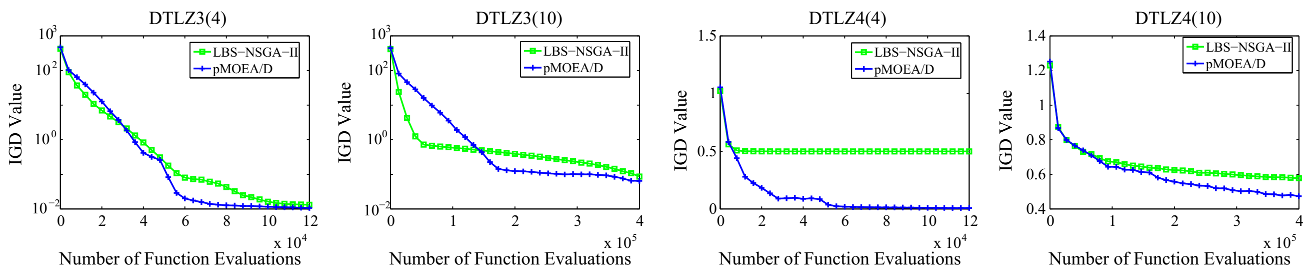

Fig. 13 The evolution of the average IGD-metric values of the non-dominated solutions found by pMOEA/D and LBS-NSGA-II in the current populations for 4-objective and 10-objective DTLZ3 and DTLZ4 problems

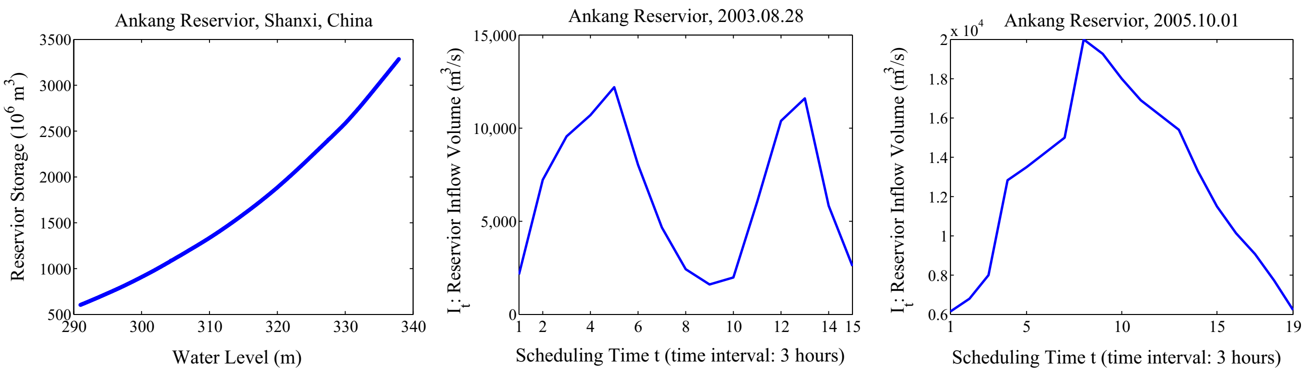

Fig. 14 Plot of the water level to volume curve of the Ankang reservoir on the *left* and the Ankang reservoir inflow volume information on August 28, 2003 and October 1, 2005 on the *middle* and *right*

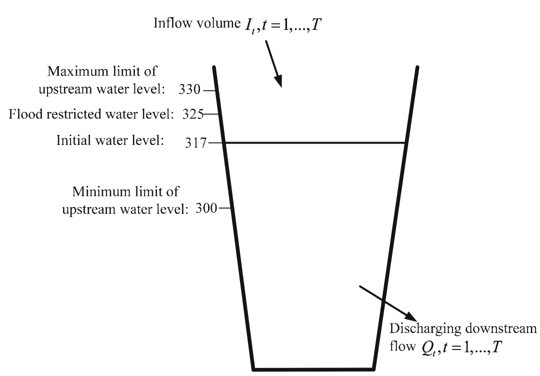

Fig. 15 Plot of a schematic diagram of Ankang reservoir

Table 8 Statistic Hypervolume-metric values of the solutions founded by pMOEA/D and LBS-NSGA-II on the two reservoir flood control problems Ankang20030828 and Ankang20051001, the numbers in parentheses present the standard deviation

Hypervolume	pMOEA/D	LBS-NSGA-II
Ankang20030828	**3.5424e4**	2.3504e4
	(3.2675e2)	(4.9054e2)
Ankang20051001	**3.6913e4**	5.5229e3
	(1.1789e3)	(1.0192e4)

The value of metric with bold is the best between both compared algorithms

mation, in this paper, we have proposed an MOEA/D-based biased weight vector adjustment (pMOEA/D). The proposed pMOEA/D is an enhanced MOEA/D with a biased weight vector adjustment method based on the geometric analysis of the modified Tchebycheff decomposition approach for handling the MOPs with preference. We compare pMOEA/D with LBS-NSGA-II. Experimental studies are executed on ten well-known ZDT and DTLZ instances with simple PSs, ten UF instances with complicated PSs, two many-objective problems and a real-world problem of flood control for reservoir.

Experimental results have indicated that the proposed pMOEA/D approach is able to successfully obtain a well-converged and well diversified set of non-dominated solutions on the preference regions. A detailed study also have indicates that with the help of the proposed biased weight vector adjustment method, pMOEA/D outperforms LBS-NSGA-II in terms of both uniformity and convergence especially for the case of multiple preference regions. As for solving many-objective problems, pMOEA/D performs significantly better than LBS-NSGA-II in terms of both uniformity and convergence for the problem of selection pressure. As for flood control for reservoir problem, We have shown that pMOEA/D can obtain a good diversity of solutions on the preference region.

The future research topics may focus on the following aspects: (1) incorporate new forms of preference informa-

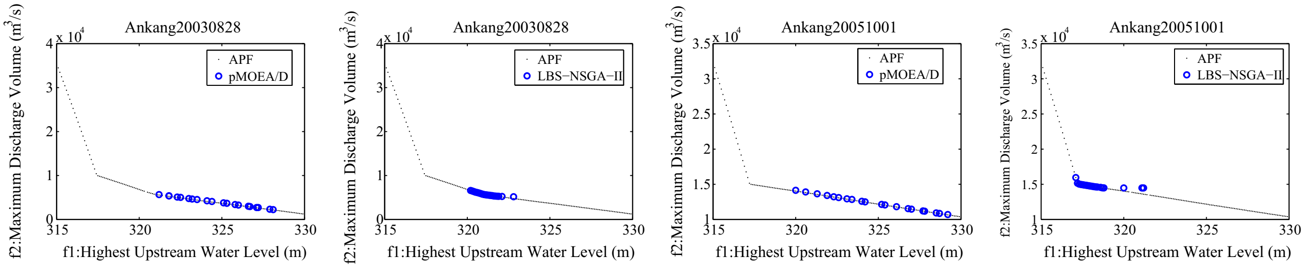

Fig. 16 The final non-dominated fronts with highest Hypervolume values by pMOEA/D and LBS-NSGA-II over 30 independent runs in solving two reservoir flood control problems Ankang20030828 and Ankang20051001 with 200,000 function evaluations

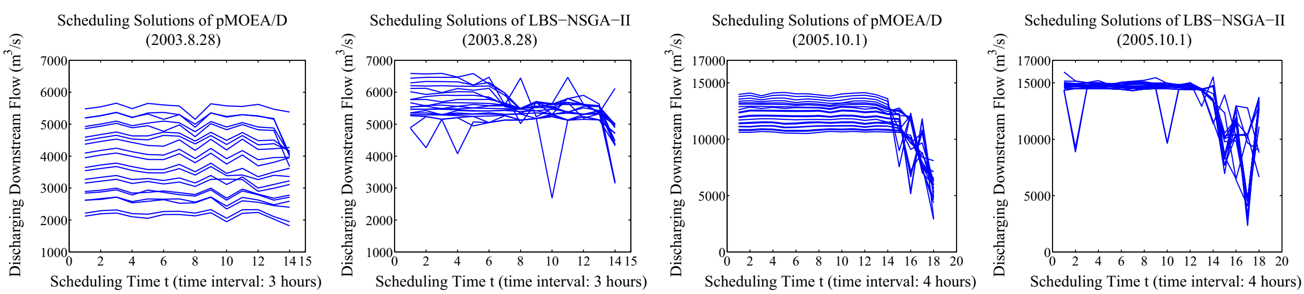

Fig. 17 The discharging downstream flow of the scheduling solutions with highest hypervolume values found by pMOEA/D and LBS-NSGA-II over 30 independent runs in solving two reservoir flood control problems Ankang20030828 and Ankang20051001 with 200,000 function evaluations

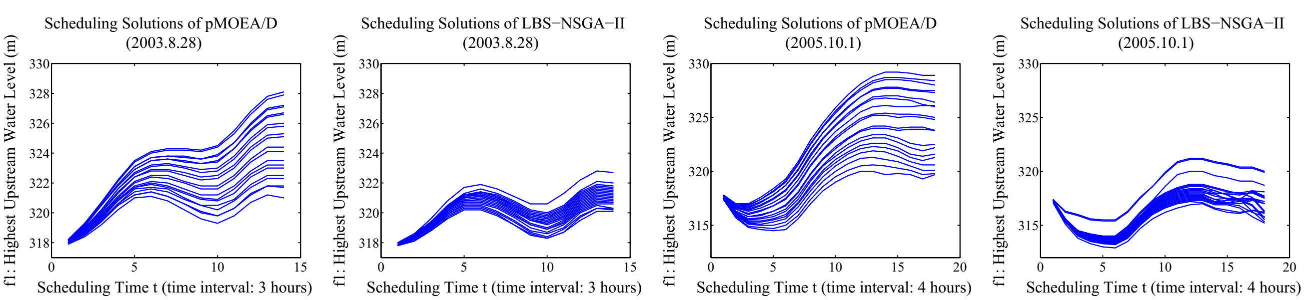

Fig. 18 The highest upstream water level of the scheduling solutions of with highest hypervolume values by pMOEA/D and LBS-NSGA-II over 30 independent runs in solving two reservoir flood control problems Ankang20030828 and Ankang20051001 with 200,000 function evaluations

tion, such as preference point alone, into the framework of MOEA/D. (2) Extend the use of pMOEA/D to many-objective problems.

Acknowledgments This work has been supported by the National Basic Research Program (973 Program) of China No. 2013CB329402, the National Natural Science Foundation of China under Grant Nos. 61173090, 61273317, 61271301, 61272279, 61001202, 61072106, 61072139, 61203303 and 61003199, the Fund for Foreign Scholars in University Research and Teaching Programsthe 111 ProjectNo. B07048, the National Research Foundation for the Doctoral Program of Higher Education of China No. 20100203120008, 20110203110006, the Program for Cheung Kong Scholars and Innovative Research Team in University No. IRT1170, the Fundamental Research Funds for the Central Universities under Grant Nos. K5051203007, K5051203002.

7 Appendix A: Analysis of the ability of modified Tchebycheff approach

Theorem 3 *Let* $\mathbf{x}^* \in \Omega$ *be a Pareto optimal solution of an MOP (1). Then there exists a weighting vector* $0 < \mathbf{w} \in \mathbf{R}^m$ *such that* $\mathbf{x}^*$ *is a optimal solution of modified Tchebycheff problem (6), where the ideal objective vector* $\mathbf{z}^*$ *is replaced by the utopian objective vector* $\mathbf{z}^{**} = \mathbf{z}^* - \varepsilon$, $\varepsilon > \mathbf{0}$ *is a relatively small but computationally significant vector. (This Theorem is similar to the Theorem 3.4.5 in Miettinen (1999).)*

Proof Let us suppose that there does not exist a weight vector $\mathbf{w} > \mathbf{0}$ such that $\mathbf{x}^*$ is a solution of modified Tchebycheff problem (6). Due to $f_i(\mathbf{x}) > z_i^{**}, i = 1, \ldots, m, \forall\, \mathbf{x} \in \Omega$, we mark $w_i^* = \frac{f_i(\mathbf{x}^*) - z_i^{**}}{\alpha}, i = 1, \ldots, m$ where $\alpha > 0$ is some normalizing factor. If $\mathbf{x}^*$ is not a optimal solution of subproblem with weight vector $\mathbf{w}^* = (w_1^*, \ldots, w_m^*)$, there exists another solution $\bar{\mathbf{x}} \in \Omega$ that is a optimal solution of this subproblem, meaning that $\max_{1\le i\le m}\left\{\frac{f_i(\bar{\mathbf{x}}) - z_i^{**}}{w_i}\right\} < \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}^*) - z_i^{**}}{w_i}\right\} = \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}^*) - z_i^{**}}{\frac{f_i(\mathbf{x}) - z^{**}}{\alpha}}\right\} = \alpha$. Thus $\frac{f_i(\mathbf{x}^*) - z_i^{**}}{w_i} < \alpha$. For each $i = 1, \ldots, m$, we have that $\frac{f_i(\bar{\mathbf{x}}) - z_i^{**}}{w_i} = \frac{f_i(\mathbf{x}^*) - z_i^{**}}{\frac{f_i(\mathbf{x}) - z^{**}}{\alpha}} = \alpha \times \frac{f_i(\bar{\mathbf{x}}) - z_i^{**}}{f_i(\mathbf{x}) - z^{**}} < \alpha$. Therefore, we can obtain $f_i(\bar{\mathbf{x}}) < f_i(\mathbf{x}^*)$. Here we have a contradiction with the Pareto optimality of $\mathbf{x}^*$, which completes the proof. □

Theorem 4 *The optimal solution of modified Tchebycheff approach* (6) *with* $\mathbf{w} > \mathbf{0}$ *is a Pareto optimal solution of an MOP* (1)*, where* $g^{mtch}(\mathbf{x} \mid \mathbf{w}, \mathbf{z}^*) = \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}) - z_i^*}{w_i}\right\}$ *is replaced by* $g^{mtch}(\mathbf{x} \mid \mathbf{w}, \mathbf{z}^*) = \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}) - z_i^*}{w_i}\right\} + \rho\sum_{i=1}^{m}\left(\frac{f_i(\mathbf{x}) - z_i^*}{w_i}\right)$*,* $\rho > 0$ *is a relatively small but computationally significant scalar.*

Proof Let $\mathbf{x}^* \in \Omega$ be a solution to modified Tchebycheff problem (6). Let us assume that $\mathbf{x}^*$ is not Pareto optimal solution of MOP (1). Then there exists a point $\bar{\mathbf{x}} \in \Omega$ such that $\bar{\mathbf{x}}$ dominates $\mathbf{x}^*$ (i.e. $f_i(\bar{\mathbf{x}}) \le f_i(\mathbf{x}^*), i = 1, \ldots, m$ and $f_j(\bar{\mathbf{x}}) < f_j(\mathbf{x}^*)$ for at least one $j \in \{1, 2, \ldots, m\}$). Now we have $f_i(\bar{\mathbf{x}}) - z_i^* \le f_i(\mathbf{x}^*) - z_i^*, i = 1, \ldots, m$ and $f_j(\bar{\mathbf{x}}) - z_j^* < f_j(\mathbf{x}^*) - z_j^*$ for at least one j. Due to $\mathbf{w} > \mathbf{0}$, we obtain $\frac{f_i(\bar{\mathbf{x}}) - z_i^*}{w_i} \le \frac{f_i(\mathbf{x}^*) - z_i^*}{w_i}, i = 1, \ldots, m$ and $\frac{f_j(\bar{\mathbf{x}}) - z_j^*}{w_j} \le \frac{f_j(\mathbf{x}^*) - z_j^*}{w_j}$, at least one $j \in \{1, \ldots, m\}$. Then we have $\max_{1\le i\le m}\left\{\frac{f_i(\bar{\mathbf{x}}) - z_i^*}{w_i}\right\} \le \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}^*) - z_i^*}{w_i}\right\}$ and $\sum_{1\le i\le m}\left\{\frac{f_i(\bar{\mathbf{x}}) - z_i^*}{w_i}\right\} < \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}^*) - z_i^*}{w_i}\right\}$. So we can get $g^{mtch}(\bar{\mathbf{x}} \mid \mathbf{w}, \mathbf{z}^*) = \max_{1\le i\le m}\left\{\frac{f_i(\bar{\mathbf{x}}) - z_i^*}{w_i}\right\} + \rho\sum_{i=1}^{m}\left(\frac{f_i(\bar{\mathbf{x}}) - z_i^*}{w_i}\right) < \max_{1\le i\le m}\left\{\frac{f_i(\mathbf{x}^*) - z_i^*}{w_i}\right\} + \rho\sum_{i=1}^{m}\left(\frac{f_i(\mathbf{x}^*) - z_i^*}{w_i}\right) = g^{mtch}(\mathbf{x}^* \mid \mathbf{w}, \mathbf{z}^*)$. Here we have a contradiction with the assumption that $\mathbf{x}^* \in \Omega$ is a solution of modified Tchebycheff problem (6), which completes the proof. □

8 Appendix B: Proof of Theorem 1

Proof Using reduction to absurdity, we will prove that the intersection point of the line $\frac{f_1 - z_1^*}{w_1} = \frac{f_2 - z_2^*}{w_2} = \cdots = \frac{f_m - z_m^*}{w_m}(w_i \ne 0, i = 1, 2, \ldots, m)$ and the PF is the optimal solution of the subproblem with preference direction (weight vector) $\mathbf{w} = (w_1, w_2, \ldots, w_m)\,(\sum_{i=1}^{m} w_i = 1, \lambda_i > 0, i = 1, 2, \ldots, m)$. Let us suppose that the optimal solution of the resultant subproblem with preference direction (weight vector) $\mathbf{w} = (w_1, w_2, \ldots, w_m)$ is $\bar{\mathbf{f}} = (\bar{f}_1, \ldots, \bar{f}_m)$ and $\bar{\mathbf{f}} = (\bar{f}_1, \ldots, \bar{f}_m)$ is a Pareto optimal solution but not in the line $\frac{f_1 - z_1^*}{w_1} = \frac{f_2 - z_2^*}{w_2} = \ldots = \frac{f_m - z_m^*}{w_m}(w_i \ne 0, i = 1, 2, \ldots, m)$. Then we have the two following non-empty sets $\bar{L} = \left\{l \mid \frac{\bar{f}_l - z_l^*}{w_l} < \max_{1\le i\le m}\left\{\frac{\bar{f}_i - z_i^*}{w_i}\right\}, l = 1, \ldots, m\right\}$ and $\bar{G} = \left\{g \mid \frac{\bar{f}_g - z_g^*}{w_g} = \max_{1\le i\le m}\left\{\frac{\bar{f}_i - z_i^*}{w_i}\right\}, g = 1, \ldots, m\right\}$. Therefore, we have $\frac{\bar{f}_l - z_l^*}{w_l} < \max_{1\le i\le m}\left\{\frac{\bar{f}_i - z_i^*}{w_i}\right\} = \frac{\bar{f}_g - z_g^*}{w_g} = g^{mtch}\left(\bar{\mathbf{f}} \mid \mathbf{w}, \mathbf{z}^*\right), \forall l \in \bar{L}, g \in \bar{G}$.

Let us suppose that $\bar{\mathbf{f}} = (\bar{f}_1, \ldots, \bar{f}_m)$ is an internal point of the PF without loss of generality. Because of the PF is piecewise continuous, we can find the point $\hat{f} = (\hat{f}_1, \ldots, \hat{f}_m)$ in the δ-neighborhood of $\bar{\mathbf{f}} = (\bar{f}_1, \ldots, \bar{f}_m)$ meeting the two following conditions:

- $\hat{f} = (\hat{f}_1, \ldots, \hat{f}_m)$ is a Pareto optimal solution.
- $\hat{f}_l > \bar{f}_l, l \in \bar{L}$; $\hat{f}_g > \bar{f}_g, \forall g \in \bar{G}$

As $\hat{f} = (\hat{f}_1, \ldots, \hat{f}_m)$ is a δ-neighbor to $\bar{\mathbf{f}} = (\bar{f}_1, \ldots, \bar{f}_m)$, this implies that $\frac{\bar{f}_l - z_l^*}{w_l} < \frac{\hat{f}_l - z_l^*}{w_l} < \frac{\hat{f}_g - z_g^*}{w_g} < \frac{\bar{f}_g - z_g^*}{w_g}, \forall l \in \bar{L}, g \in \bar{G}$. Therefore, $g^{mtch}\left(\hat{\mathbf{f}} | \mathbf{w}, \mathbf{z}^*\right) = \max_{g\in\bar{G}}\left\{\frac{\hat{f}_g - z_g^*}{w_g}\right\} < \max_{g\in\bar{G}}\left\{\frac{\bar{f}_g - z_g^*}{w_i}\right\} = g^{mtch}(\bar{\mathbf{f}} \mid \mathbf{w}, \mathbf{z}^*)$. This conclusion is clearly inconsistent with the assumption, so the Theorem 1 is proofed. □

9 Appendix C: Proof of Theorem 2

Proof Let us assume that $\acute{\mathbf{w}} = (\acute{w}_1, \ldots, \acute{w}_m)$, rather than $\mathbf{w}^{opt}$, is the optimal weight vector to the preferred solution $\mathbf{F} = (f_1, \ldots, f_m)$ based on the reference point $\mathbf{z} = (z_1, \ldots, z_m)$, i.e. $h(\acute{\mathbf{w}} \mid \mathbf{F}, \mathbf{z}) < h(\mathbf{w}^{opt} \mid \mathbf{F}, \mathbf{z})$ and $\acute{\mathbf{w}} \ne \mathbf{w}^{opt}$. If we note that $L = \left\{l \mid \acute{w}_l < w_l^{opt}\right\}$, $E = \left\{e \mid \acute{w}_e = w_e^{opt}\right\}$, $G = \left\{g \mid \acute{w}_g > w_g^{opt}\right\}$. Let $W_m = \{\mathbf{w} = (w_1, \ldots, w_m) | \sum_{i=1}^{m} w_i = 1, w_i \ge 0\}$, then there exist $G \ne \emptyset$ and $L \ne \emptyset$ for $\acute{\mathbf{w}} \ne \mathbf{w}^{opt}$ and $\acute{\mathbf{w}}, \mathbf{w}^{opt} \in W_m$, we have $0 < w_i^{opt} < 1$ for each $i = 1, \ldots, m$.

Due to $\acute{w}_l < w_l^{opt}, \forall l \in L$, we have $\frac{f_l - z_l}{\acute{w}_l} > \frac{f_l - z_l}{w_l^{opt}} = \sum_{i=1}^{m}(f_i - z_i), \forall l \in L$. Because $\acute{w}_e = w_e^{opt}, \forall e \in E$, we have $\frac{f_e - z_e}{\acute{w}_e} = \frac{f_e - z_e}{w_e^{opt}} = \sum_{i=1}^{m}(f_i - z_i), \forall e \in E$. Owing to $0 < w_g^{opt} < \acute{w}_g, \forall g \in G$, we have $\frac{f_g - z_g}{\acute{w}_g} < \frac{f_g - z_g}{w_g^{opt}} = \sum_{i=1}^{m}(f_i - z_i), \forall g \in G$. Therefore, $\frac{f_l - z_l}{\acute{w}_l} > \sum_{i=1}^{m}(f_i - z_i) = \frac{f_e - z_e}{\acute{w}_e} > \frac{f_g - z_g}{\acute{w}_g}, \forall l \in L, \forall e \in E, \forall g \in G$. Because of $L \ne \emptyset$, we can get

$h(\acute{\mathbf{w}} \mid \mathbf{F}, \mathbf{z}) = \max_{1 \le i \le m} \left\{ \frac{f_i - z_i}{\acute{w}_i} \right\} > \sum_{i=1}^{m} (f_i - z_i) = \max_{1 \le i \le m} \left\{ \frac{f_i - z_i}{w_i^{opt}} \right\} = h(\mathbf{w}^{opt} \mid \mathbf{F}, \mathbf{z})$. But this conclusion is inconsistent with the assumption, so Theorem 2 is proved. □

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