Supplementary Document for "The Importance of the Diversity on Variable Space in the Design of Multi-objective Evolutionary Algorithms"

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***Index Terms*—Diversity, Decomposition, Multi-objective Optimization, Evolutionary Algorithms.**

This document contains supplementary material to provide a clearer understanding of the specifics of VSD-MOEA. First, a multimedia material that provides a visualization of the internal behavior of VSD-MOEA in comparison to other state-of-the-art schemes is provided. Additionally, some of the results described in the main document are analyzed in terms of the Modified Inverted Generational Distance (IGD+) [1], with the conclusions being quite similar to those obtained in the main document with the hypervolume.

I. Comparison against State-of-the-art MOEAs in long-term executions

References

[1] H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima, "Modified distance calculation in generational distance and inverted generational distance," in *International conference on evolutionary multi-criterion optimization*. Springer, 2015, pp. 110–125.

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TABLE I
SUMMARY OF THE HYPERVOLUME RATIO RESULTS ATTAINED FOR PROBLEMS WITH TWO OBJECTIVES

	AVSD-MOEA/D			**MOEA/D-DE**			**NSGA-II**			**NSGA-III**			**R2-EMOA**		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
WFG1	0.995	**0.982**	0.020	0.957	0.842	0.058	0.984	0.888	0.053	0.993	0.919	0.051	0.762	0.628	0.077
WFG2	0.999	**0.999**	0.000	0.996	0.996	0.000	0.998	**0.998**	0.000	0.997	0.996	0.000	0.998	0.998	0.000
WFG3	0.993	**0.993**	0.000	0.992	0.992	0.000	0.984	0.982	0.001	0.992	0.992	0.000	0.992	0.991	0.000
WFG4	0.991	**0.991**	0.000	0.988	0.988	0.000	0.985	0.983	0.001	0.988	0.988	0.000	0.991	0.987	0.006
WFG5	0.933	**0.905**	0.008	0.891	0.882	0.004	0.892	0.883	0.003	0.892	0.890	0.001	0.888	0.885	0.002
WFG6	0.959	0.922	0.020	0.988	0.963	0.019	0.980	0.978	0.001	0.980	0.959	0.010	0.991	**0.990**	0.001
WFG7	0.991	**0.991**	0.000	0.988	0.988	0.000	0.984	0.982	0.001	0.988	0.988	0.000	0.991	0.990	0.000
WFG8	0.963	**0.954**	0.004	0.846	0.833	0.004	0.821	0.815	0.003	0.832	0.829	0.001	0.837	0.834	0.001
WFG9	0.978	**0.976**	0.002	0.974	0.954	0.039	0.941	0.853	0.071	0.799	0.798	0.001	0.975	0.936	0.063
DTLZ1	0.993	**0.993**	0.000	0.993	**0.993**	0.000	0.992	0.991	0.000	0.993	**0.993**	0.000	0.992	0.992	0.000
DTLZ2	0.991	0.991	0.000	0.989	0.989	0.000	0.989	0.988	0.001	0.989	0.989	0.000	0.992	**0.992**	0.000
DTLZ3	0.991	0.991	0.000	0.989	0.989	0.000	0.989	0.932	0.229	0.989	0.989	0.000	0.992	**0.992**	0.000
DTLZ4	0.991	0.991	0.000	0.989	0.989	0.000	0.990	0.926	0.204	0.989	0.989	0.000	0.992	0.740	0.348
DTLZ5	0.991	0.991	0.000	0.989	0.989	0.000	0.989	0.988	0.001	0.989	0.989	0.000	0.992	**0.992**	0.000
DTLZ6	0.991	**0.991**	0.000	0.989	0.986	0.014	0.989	0.984	0.024	0.989	0.989	0.000	0.992	0.456	0.366
DTLZ7	0.997	**0.997**	0.000	0.996	0.996	0.000	0.997	**0.997**	0.000	0.996	0.996	0.000	0.997	**0.997**	0.000
UF1	0.995	**0.995**	0.000	0.987	0.986	0.001	0.990	0.989	0.001	0.992	0.989	0.002	0.993	0.992	0.000
UF2	0.995	**0.995**	0.000	0.990	0.988	0.001	0.984	0.982	0.001	0.989	0.985	0.002	0.988	0.987	0.001
UF3	0.938	0.906	0.016	0.991	**0.990**	0.001	0.975	0.967	0.008	0.935	0.781	0.097	0.984	0.974	0.006
UF4	0.979	**0.977**	0.001	0.914	0.904	0.006	0.898	0.888	0.006	0.889	0.885	0.002	0.908	0.898	0.005
UF5	0.990	**0.975**	0.009	0.715	0.439	0.137	0.785	0.598	0.173	0.690	0.409	0.144	0.803	0.679	0.160
UF6	0.962	**0.938**	0.013	0.928	0.748	0.175	0.819	0.752	0.030	0.743	0.526	0.177	0.897	0.732	0.049
UF7	0.993	**0.993**	0.000	0.991	0.990	0.001	0.981	0.978	0.002	0.968	0.956	0.023	0.988	0.977	0.004
Mean	0.983	**0.976**	0.004	0.960	**0.931**	0.020	0.954	**0.927**	0.035	0.939	**0.905**	0.022	0.954	**0.897**	0.047

TABLE II
SUMMARY OF THE HYPERVOLUME RATIO RESULTS ATTAINED FOR PROBLEMS WITH THREE OBJECTIVES

	AVSD-MOEA/D			**MOEA/D-DE**			**NSGA-II**			**NSGA-III**			**R2-EMOA**		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
WFG1	0.985	**0.982**	0.007	0.972	0.937	0.030	0.960	0.899	0.042	0.971	0.966	0.011	0.976	0.939	0.028
WFG2	0.991	**0.991**	0.000	0.981	0.979	0.001	0.951	0.922	0.027	0.973	0.970	0.002	0.963	0.962	0.000
WFG3	0.995	**0.994**	0.000	0.990	0.990	0.000	0.983	0.974	0.005	0.929	0.915	0.008	0.992	0.992	0.000
WFG4	0.943	**0.941**	0.001	0.899	0.898	0.001	0.898	0.879	0.008	0.885	0.881	0.002	0.915	0.909	0.002
WFG5	0.901	**0.872**	0.011	0.831	0.831	0.000	0.832	0.812	0.012	0.830	0.828	0.001	0.848	0.846	0.001
WFG6	0.912	0.888	0.011	0.887	0.862	0.013	0.861	0.838	0.013	0.897	0.880	0.030	0.904	**0.893**	0.005
WFG7	0.943	**0.942**	0.001	0.899	0.898	0.001	0.892	0.874	0.009	0.897	0.897	0.000	0.912	0.904	0.002
WFG8	0.910	**0.902**	0.003	0.816	0.812	0.003	0.765	0.752	0.007	0.807	0.806	0.001	0.826	0.824	0.001
WFG9	0.910	**0.894**	0.006	0.875	0.862	0.005	0.822	0.721	0.027	0.747	0.741	0.002	0.884	0.881	0.003
DTLZ1	0.967	**0.967**	0.000	0.953	0.953	0.000	0.953	0.795	0.312	0.953	0.953	0.000	0.942	0.941	0.001
DTLZ2	0.945	**0.944**	0.000	0.914	0.914	0.000	0.894	0.879	0.009	0.913	0.913	0.000	0.916	0.915	0.001
DTLZ3	0.945	**0.944**	0.000	0.914	0.914	0.000	0.892	0.395	0.432	0.913	0.913	0.000	0.916	0.915	0.001
DTLZ4	0.945	**0.944**	0.000	0.914	0.914	0.000	0.900	0.731	0.269	0.913	0.913	0.000	0.916	0.850	0.214
DTLZ5	0.985	0.985	0.000	0.979	0.979	0.000	0.981	0.979	0.001	0.978	0.971	0.003	0.986	**0.986**	0.000
DTLZ6	0.985	**0.985**	0.000	0.979	0.959	0.038	0.982	0.932	0.177	0.974	0.968	0.003	0.986	0.551	0.355
DTLZ7	0.970	**0.968**	0.001	0.922	0.922	0.000	0.946	0.926	0.029	0.950	0.939	0.006	0.889	0.850	0.019
UF8	0.922	**0.916**	0.003	0.891	0.862	0.032	0.861	0.832	0.057	0.553	0.550	0.001	0.903	0.885	0.007
UF9	0.957	**0.951**	0.003	0.947	0.813	0.071	0.937	0.879	0.066	0.871	0.815	0.041	0.953	0.846	0.080
UF10	0.831	**0.787**	0.041	0.681	0.435	0.147	0.629	0.295	0.171	0.553	0.539	0.066	0.579	0.566	0.056
Mean	0.944	**0.937**	0.005	0.908	**0.881**	0.018	0.891	**0.806**	0.088	0.869	**0.861**	0.009	0.906	**0.866**	0.041

TABLE III
STATISTICAL TESTS AND DETERIORATION LEVEL OF THE HV RATIO FOR PROBLEMS WITH TWO OBJECTIVES

	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Score**	**Deterioration**
AVSD-MOEA/D	80	12	0	68	0.153
MOEA/D-DE	39	47	6	-8	1.175
NSGA-II	21	66	5	-45	1.270
NSGA-III	33	55	4	-22	1.765
R2-EMOA	48	41	3	7	1.951

TABLE IV
STATISTICAL TESTS AND DETERIORATION LEVEL OF THE HV RATIO FOR PROBLEMS WITH THREE OBJECTIVES

	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Score**	**Deterioration**
AVSD-MOEA/D	73	2	1	71	0.006
MOEA/D-DE	34	39	3	-5	1.075
NSGA-II	7	65	4	-58	2.492
NSGA-III	26	47	3	-21	1.447
R2-EMOA	43	30	3	13	1.351