A Multi-objective Decomposition Algorithm Based in Variable Space Diversity

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***Abstract*—The abstract goes here.**

***Index Terms*—IEEE, IEEEtran, journal, LaTeX, paper, template.**

I. Introduction

MULTI-OBJECTIVE Evolutionary Algorithms (MOEAs) are one of the most popular approaches to deal with Multi-objective Optimization Problems (MOPs). MOEAs are usually employed in problems whose formulation is complicated or inaccessible. A continuous box-constrained minimization MOP involves two or more conflicting objectives and are defined in Eq. (1)

$$\begin{aligned} &min \quad F(x) = (f_1(x), ..., f_M(x)) \\ &s.t. \quad x \in \Omega. \end{aligned} \tag{1}$$

where $\Omega \subseteq \Re^D$ denotes the decision space, $F : \Omega \rightarrow Y \subseteq \Re^M$ consists of M objectives and Y is the objective space. Given two solutions $x_1, x_2 \in \Re^D$ is said that x_1 dominates x_2 denoted as $x_1 \prec x_2$ if and only if $f_i(x_1) \leq f_i(x_2)$ for all $i \in \{1, ..., M\}$ and $f_i(x_1) < f_i(x_2)$ for at least one objective. A solution $F(x^*)$ is called a Pareto-optimal solution if there does not exist $F(x) \in Y$ such that $x \prec x^*$. The set of all $x^* \in Y$ is called the Pareto-optimal solution set (PS), and their image is the Pareto Front (PF). The goal of the MOEAs is to find a set of solutions that are well-distributed and converged to the PF in the objective space [1].

The Evolutionary Algorithms (EAs) are popular metaheuristics to deal with MOPs due its capability to approximate several solutions in a single run. In the last decade, several strategies that take into account executions in long-term have been quite successfull mainly in the most complex problems [2]. These strategies explicitly preserves the diversity in the population incorporating the stopping criterion and elapsed time to attain a properly balance between exploration and exploitation [3].

The mechanisms designed to deal with diversity have turned to be essential to attain quality solutions in single-EAs. Perhaps, one of the most critical issues of promoting diversity is that it provides a way to deal with premature convergence and stagnation. Diversity can be taken into account in the design of several components such as in the variation stage [4], [5], replacement phase [3] and/or popultions models [6].

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Recently, several remarkable EAs incorporates a replacement phase, which maintains a balance between exploration and exploitation. Such transition is gradually imposed taking into account the stopping criterion and the elapsed time. Those strategies that incorporate such replacement phase has attained remarkable results, mainly in long-term executions. For instance, in combinatorial domains new best-known solutions for some well-known variants of the frequency assignment problem [2], and for a two-dimensional packing problem [7]. In addition, this principle guided the design of the winning strategy at the Second Wind Fram Layout Optimization Competition, that was held in the Genetic and Evolutionary Conference. Recently, in the case of continuous domains this replacement phase has been incorporated to Differential Evolution (DE) [8], which attained remakably superior results than the winners of the competition carried out in IEEE Congress of Evolutionary Computation (CEC) of the years 2016 and 2017. Therefore, this novel principle of incorporing the replacement phase to explicitly control the diversity has given quite good results in both discrete and continuous domains. Remarkably, the incorporation of this paradigm in EAs have allowed to discover new solutions in several NP-hard problems, which required long-term executions which tend to be even more feasible caused by the constatn growing of computational power.

The usual design of MOEAs is especifically build to attain well-spread solutions and to cover the PF (coverage), therefore some of them incorporate different mechanisms to achieve such goal. However, most of the MOEAs disregard the variable space even though those algorithms can suffer the same drawbacks raisen in single-objective space, e.g. premature convergence and stagnation. Perhaps one of the main challenges to incorporate strategies to control the diversity in variable space is that in multi-objective spaces inducing diversity in variable spaces does not guarantee diversity in the objective space. Following the goal of MOEAs where is desired to attain solutions well-spread in the objective space the MOEAs implicilty inherit some degree of diversity in the variable space, therefore convergence does not appear in the variable space. Nevetheless, the implicit diversity induced in the variable space by the objective space might not be enough an the reproduction operators could lose its exploratiory strength.

In spite of the amazing amount of MOEAs that have been developed, this paper proposes a novel and different MOEA *the Variable Space Diversity MOEA based in Decomposition* (VSD-MOEA/D), which explicitly induces diversity in the variable space through several stages to obtain a proper balance from exploration toward exploitation. Particularly, the MOEA/D-DE that attained the first place in the CEC-09 is taken into account [9]. This MOEA is transformed incorporating an original replacement phase that takes into consideration the stopping criterion and the number of function evaluations. In this way, this algorithm grants more importance to the diversity of variable space in the initial stages, and as the function evaluations evolve, it gradually grants more importance to the diversity of the objective space. Thus in the last functions evaluations the algorithm has a similar behavior than the state-of-the-art MOEAs. In addition, VSD-MOEA/D employes three populations and deals with the diveristy problems caused by the mating selection and replacement mechanisms of the MOEA/D-DE. Since that in the literature exists a broad kind of MOEAs based in decomposition the validation of our proposal is carried out taking into consideration the MOEA/D [10] and MOEA/D-DE [9] that are based in decomposition and R2-EMOA that is based in indicators. This paper clearly shows the remarkable benefits of properly taking into account the diversity of the variable space.

The rest of this paper is organized as follows. Section II provides a widely review of decomposition algorithms, diversity in EAs and related works. The VSD-MOEA/D proposal is detailed in section III. Section IV is devoted to the experimental validation of the novel proposal. Finally, conclusions and some lines of future work are given in section V

II. Literature Review

A. Decomposition-Based MOEAs

B. Diversity in Evolutionary Algorithms

The proper balance between exploration and exploitation is one of the keys to designing a successful EAs. In the single-objective domain, it is known that properly managing the diversity of the variable space is a way to achieve this balance, and as a consequence, a large number of diversity management techniques have been devised [**?**]. Specifically, these methods are classified depending on the component(s) of the EA that is modified to alter how much diversity is maintained. A popular taxonomy identifies the following groups [**?**]: *selection-based*, *population-based*, *crossover/mutation-based*, *fitness-based*, and *replacement-based*. Additionally, the methods are referred to as *uniprocess-driven* when a single component is altered, whereas the term *multiprocess-driven* is used to refer to those methods that act on more than one component.

Among the previous proposals, the replacement-based methods have yielded very high-quality results in recent years [**?**], so this alternative was selected with the aim of designing a novel MOEA that explicitly incorporates a way to control the diversity of the variable space. The basic principle of these methods is to bias the level of exploration in successive generations by controlling the diversity of the survivors [**?**]. Since premature convergence is one of the most common drawbacks in the application of EAs, modifications are usually performed with the aim of slowing down the convergence. One of the most popular proposals belonging to this group is the *crowding* method, which is based on the principle that offspring should replace similar individuals from the previous generation [**?**]. Several replacement strategies that do not rely on crowding have also been devised. In some methods, diversity is considered as an objective. For instance, in the hybrid genetic search with adaptive diversity control (HGSADC) [**?**], individuals are sorted by their contribution to diversity and by their original cost. Then, the rankings of the individuals are used in the fitness assignment phase. A more recent proposal [**?**] incorporates a penalty approach to gradually alter the amount of diversity maintained in the population. Specifically, the initial phases preserve a higher amount of diversity than the final phases of the optimization. This last method has inspired the design of the novel proposal put forth in this paper for multi-objective optimization.

It is important to remark that in the case of multi-objective optimization, little work related to maintaining the diversity of the variable space has been done. The following section reviews some of the most important MOEAs and introduces some of the works that consider the maintenance of diversity of the variable space.

C. Related Works

In the last decade few MOEAs were specifically designed to address the diversity in the decision variables space. Although that the diversity in single-objectives EAs is a matter of importance refs, in multi-objective optimization usually the diversity in the variable space is ignored. This might occurs, since the objectives are usually in conflict, therefore often is maintained a diversity level in the decision space. Also, the decision space is disregarded, since that at the end of the optimization process the quality of the solutions relies only in the objective space. In single-objective optimization, high quality solutions have been provided since that a balance between exploration and exploitation is reached through the optimization process refs. In this way, the premature convergence, which is considered as a drawback, can be avoided. Some strategies in single-objective optimization to avoid this drawback is explicitly induce a the diversity considering the criteria stop, thus at first stages the exploration levels are promoted and at the end the exploitation of the promising regions is induced. A similar issue is addressed in multi-objective problems, in such a way that the evolutionary search is stagnated, and only are explored the same region. Particularly, the idea to integrate decision space diversity into the optimization has been proposed in 1994 with the first NSGA work REF. In this last work the decision vectors are considered into the fitness sharing procedure. Thereafter, the most algorithms concentrated in the objectives space only. Alternatively, several approaches with MOPs related directly in decision space has arisen. These approaches, further as the usual MOPs, also aims to provide diverse solutions in the decision space. Principally, based in that there exists a variety of problems where the image of the Pareto Front corresponds to several distributions in the Pareto Set.

In 2003 GDEA [11] integrated diversity into the search as an additional objective. This MOEA introduced by Toffolo and Benini invoked two selection criteria, non-dominated sorting as the primary one and a metric for decision space diversity as the secondary one. In 2005, Chan and Ray [12] suggested to use two selection operators in MOEAs; one encourages the diversity in the objective space and the other does so in the decision space. They implemented KP1 and KP2, two algorithms using these two selection operators. After that, in 2008, the Omni-optimizer [13] was developed, which extends the original idea of the NSGA. Particularly, the diversity measure take both the decision and the objective space diversity into account, Omni-optimizer first uses a rank procedure, were the objective space measure is always considered first, and only if there are ties the diversity in decision space is taken into consideration. However, the drawback of this approach is that the diversity plays an inferior role and there is no possibility to change the tradeoff between the diversity measures. In 2009, were proposed the CMA-ES niching framework [14] , and the probabilistic Model-based Multi-objective Evolutionary Algorithm (MMEA)[15] . The first, extend a niching framework to include the diversity in the space diversity. The second, applies a clustering procedure in objective space and then builds a model from the solutions in these clusters. In 2010, was proposed the Diversity Integrating Hypervolume-based Search Algorithm (DIVA) [16], this algorithm introduces a method to integrate decision space diversity into the hypervolume indicator, such that these two set measures can be optimized simultaneously.

III. Proposal

IV. Experimental Validation

A. Comparison Against State-of-the-art

B. Effect of the Initial Distance Factor

V. Conclusion

Appendix A
Proof of the First Zonklar Equation

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Appendix B

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