Controlling Dominance Area of Solutions and Its Impact on the Performance of MOEAs

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Abstract. This work proposes a method to control the dominance area of solutions in order to induce appropriate ranking of solutions for the problem at hand, enhance selection, and improve the performance of MOEAs on combinatorial optimization problems. The proposed method can control the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter S. Modifying the dominance area of solutions changes their dominance relation inducing a ranking of solutions that is different to conventional dominance. In this work we use 0/1 multiobjective knapsack problems to analyze the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer when the number of objectives, the size of the search space, and the complexity of the problems vary. We show that either convergence or diversity can be emphasized by contracting or expanding the dominance area. Also, we show that the optimal value of the area of dominance depends strongly on all factors analyzed here: number of objectives, size of the search space, and complexity of the problems.

1 Introduction

Multiobjective evolutionary algorithms (MOEAs) [1,2] are being increasingly investigated for solving multiobjective optimization problems. MOEAs are particularly suitable for this task because they evolve simultaneously a population of potential solutions to the problem in hand, which allows us to search a set of Pareto non-dominated solutions in a single run of the algorithm.

Some important features of the latest generation MOEAs are that selection incorporates elitism and it is biased by Pareto dominance and a diversity preserving strategy in objective space. Pareto dominance based selection is thought to be effective for problems with convex and non-convex fronts and has been successfully applied, especially in two and three objective problems. However, some current research reveals that ranking by Pareto dominance on problems with an increased number of objectives might not longer be effective [3,4,5]. It has been shown that the characteristics of multiobjective landscapes viewed in terms of non-dominated fronts (that are found in the process of non-domination sorting) can change drastically as the number of objectives increases, i.e. the

S. Obayashi et al. (Eds.): EMO 2007, LNCS 4403, pp. 5–20, 2007.

number of fronts reduces substantially and become denser (more solutions per front) just by increasing the number of objectives [5]. In this case, most sampled solutions at a given time turn to be non-dominated. That is, most solutions are assigned the same rank of non-dominance and Pareto selection weakens since it has to discriminate mostly based on diversity of solutions. Another factor that affects the density of the fronts is the complexity of the individual single objective landscapes. It has been shown that the top non-dominated fronts become denser as the complexity of the landscapes reduces, and vice-versa [5]. This has been observed for multiple and many objectives landscapes and affects the behavior and effectiveness of Pareto selection in two ways. First, although the effect of the landscapes complexity on front density is not as strong as the effect of increasing the number of objectives, in practice the increased density of the top non-dominated fronts combined with elitism could make the instantaneous elite-population to be mostly composed of individuals with the same non-domination rank since early generations. In this case, again, selection has to rely mostly on diversity rather than on Pareto dominance ranking. Second, on problems of increased complexity could happen that there are too many but sparse fronts, in which case Pareto selection could become too strong increasing the likelihood that the algorithm gets trapped in local fronts. These studies suggest that for selection to be effective a more careful analysis of Pareto dominance relation is required when dealing with problems that have more than three objectives. In addition, for any number of objectives, the dominance relation should be appropriately revised according to the characteristics of the multi-objective landscape.

There are a few works on relaxed forms of Pareto dominance, such as ϵ-dominance [6] and α-domination [7]. ϵ-dominance acts as an archiving strategy and was proposed as a way of regulating convergence of a MOEA. The algorithm maintains a finite-size archive of non-dominated solutions, in which new points are only accepted if they are not ϵ-dominated by any other point of the current archive. ϵ-dominance strengthens selection during the archiving process. On the other hand, α-domination permits a solution $\boldsymbol{x}$ to dominate a solution $\boldsymbol{y}$ if $\boldsymbol{x}$ is slightly inferior to $\boldsymbol{y}$ in an objective but largely superior to $\boldsymbol{y}$ in some other objectives. α-domination was tried on an ad hoc continuous problem created specifically to illustrate a potential problem that Pareto selection could face. In addition, α-domination only introduces a method to strengthen selection and its effects have not been explained nor tested on standard test suit problems.

In this work, we propose a method to control the dominance area of solutions in order to induce appropriate ranking of solutions for the problem at hand, enhance selection, and improve the performance of MOEAs on combinatorial optimization problems. The proposed method can control the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter S. Modifying the dominance area of solutions changes their dominance relation inducing a ranking of solutions that is different to conventional dominance. Contrary to ϵ-dominance and α-domination, the proposed method can strengthen or weaken selection by expanding or contracting the area of dominance and conceptually can be considered as a generalization of Pareto dominance. In addition, the motivation and method itself of the proposed approach is different to ϵ-dominance and α-domination. See **3** and **4** for a detailed explanation about ϵ-dominance, α-domination, and the proposed method.

In this work we analyze the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer when the number of objectives, the size of the search space, and the complexity of the problems vary. We chose NSGA-II as a representative elitist algorithm that uses dominance [8] and compare its performance with NSGA-II enhanced by the proposed method. We conduct our study on 0/1 multiobjective knapsack problems with $m = \{2, 3, 4, 5\}$ objectives varying the number of items n (size of search space is given by 2^n) and the feasibility ratio ϕ of the search space, which is a good indicator of the complexity of the landscapes in this kind of problems. This work clearly shows that either convergence or diversity can be emphasized by contracting or expanding the dominance area. Also, this work shows that the optimal value of S^* that controls the area of dominance depends strongly on all factors analyzed here: number of objectives, size of the search space, and complexity of the problems.

2 Multiobjective Optimization Concepts and Definitions

A multiobjective optimization problem including m kinds of objective functions is defined as follows:

$$\begin{cases} \text{Maximize } \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})) \\ \text{subject to } \boldsymbol{x} \in \mathcal{F} \end{cases} \tag{1}$$

where, $\boldsymbol{x} \in \mathcal{F}$ is a feasible solution vector in the solution space $\mathcal{S}(\mathcal{F} \subseteq \mathcal{S})$, and $f_i(i = 1, 2, \cdots, m)$ are the m objectives to be maximized. That is, we try to find a feasible solution vector $\boldsymbol{x} \in \mathcal{F}$ in the solution space maximizing each objective function $f_i(i = 1, 2, \ldots, m)$ in a vector fitness function $\boldsymbol{f}$. Important concepts used in determining a set of solutions for multiobjective optimization problems are dominance, Pareto optimality, Pareto set and Pareto front. Next we define *dominance* between solutions $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{F}$ as follows: If

$$\begin{array}{l} \forall i \in \{1, 2, \ldots, m\} : f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{y}) \;\; \wedge \\ \exists i \in \{1, 2, \ldots, m\} : f_i(\boldsymbol{x}) > f_i(\boldsymbol{y}). \end{array} \tag{2}$$

are satisfied, $\boldsymbol{x}$ dominates $\boldsymbol{y}$. In the following, $\boldsymbol{x}$ dominates $\boldsymbol{y}$ is denoted by $\boldsymbol{f}(\boldsymbol{x}) \succeq \boldsymbol{f}(\boldsymbol{y})$. A solution vector $\boldsymbol{x}$ is said to be *Pareto optimal* with respect to $\mathcal{F}$ if it is not dominated by other solution vectors in $\mathcal{F}$. The presence of multiple objective functions, usually conflicting among them, gives rise to a set of optimal solutions. The set of Pareto optimal solutions (POS) is defined as

$$\mathcal{POS} = \{\boldsymbol{x} \in \mathcal{F} \mid \neg\exists \boldsymbol{y} \in \mathcal{F} : \boldsymbol{f}(\boldsymbol{x}) \succeq \boldsymbol{f}(\boldsymbol{y})\}, \tag{3}$$

and the Pareto front is defined as

$$\mathcal{F}ront = \{\boldsymbol{f}(\boldsymbol{x}) \mid \boldsymbol{x} \in \mathcal{POS}\}. \tag{4}$$

A convenient method to assign rank to solutions is by classifying them into non-dominated fronts [8]. Let us denote $\mathcal{Z}$ the set of solution we want to classify. The first front $\mathcal{F}ront_1$ is obtained from $\mathcal{Z}$ and corresponds to the set of POS in $\mathcal{Z}$. Let us denote this set as $\mathcal{POS}_1$. The subsequent fronts $\mathcal{F}ront_j; j > 1$, contain lower level non-dominated solutions and are obtained by disregarding solutions corresponding to the previous higher non-dominated fronts, i.e. $\mathcal{F}ront_j; j > 1$, is obtained from the set $\mathcal{Z} - \bigcup_{k=1}^{j-1} \mathcal{POS}_k$.

3 Related Works

Recently, some researchers have proposed the use of relaxed forms of Pareto dominance as a way of regulating convergence of a MOEA. Laummans et al. [6] proposed a relaxed form of dominance for MOEAs called ϵ-dominance seeking to ensure both properties of convergence towards the Pareto-optimal set and properties of diversity among the solutions found. A solution $\boldsymbol{x}$ ϵ-dominates a solution $\boldsymbol{y}$ for some $\epsilon > 0$, assuming maximization in all objectives, if

$$\forall i \in \{1, 2, \ldots, m\} : (1 + \epsilon) \cdot f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{y}). \tag{5}$$

ϵ-dominance acts as an archiving strategy, where new points are only accepted if they are not ϵ-dominated by any other point of the current archive. Thus, it strengthens Pareto selection during the archiving process. In addition, ϵ-dominance uses a set of boxes to cover the Pareto front, where the size of such boxes is set by the user-defined parameter ϵ. Within each box only one non-dominated solution is retained. Thus, by using a larger value of ϵ the user can accelerate convergence, while sacrificing the quality (preciseness) of the Pareto front obtained. In contrast, if a high quality of the front is required, a small value of ϵ must be adopted. The definition of ϵ is very important. However, it is not simple to find the most appropriate value of ϵ, especially if nothing is known in advance about the shape of the Pareto front. Also, to correlate the number of desired solutions with the value of ϵ chosen is not easy. In addition, ϵ-dominance eliminates the extreme points of the Pareto front, which may be undesirable in some cases.

Another strategy that relaxes Pareto dominance is α-domination proposed by Ikeda et al. [7] to strengthen selection. The fundamental idea of α-domination is setting upper/lower bounds of trade-offs rates between two objectives. α-domination permits a solution $\boldsymbol{x}$ to dominate a solution $\boldsymbol{y}$ if $\boldsymbol{x}$ is slightly inferior to $\boldsymbol{y}$ in an objective but largely superior to $\boldsymbol{y}$ in some other objectives. To calculate α-dominance, first a relative fitness vector $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$ between two solutions must be established. The i-th component of $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$ is calculated by

$$g_i(\boldsymbol{x}, \boldsymbol{y}) = f_i(\boldsymbol{x}) - f_i(\boldsymbol{y}) + \sum_{j \neq i}^{m} \alpha_{ij}(f_i(\boldsymbol{x}) - f_i(\boldsymbol{y})) \tag{6}$$

where $f_i(\boldsymbol{x})$ is the fitness value of solution $\boldsymbol{x}$ on the i-th objective, and α_{ij} is the trade-off rate between the i-th and j-th objectives.

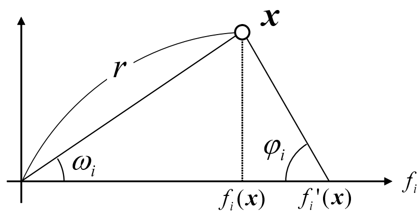


Fig. 1. Fitness modification to change the covered area of dominance

A solution $\boldsymbol{x}$ α-dominates a solution $\boldsymbol{y}$, assuming maximization in all objectives, if

$$\begin{aligned}&\forall i \in \{1, 2, \ldots, m\} : g_i(\boldsymbol{x}, \boldsymbol{y}) \geq 0 \ \wedge \\ &\exists i \in \{1, 2, \ldots, m\} : g_i(\boldsymbol{x}, \boldsymbol{y}) > 0.\end{aligned} \tag{7}$$

To calculate α-domination, α_{ij} trade-off rates must be properly set for each pair of objectives. Assessing the appropriate trade-offs between objectives could be a difficult problem, especially if nothing is known in advance about the landscape and shape of Pareto front. In addition, note that α-domination strengthens selection only.

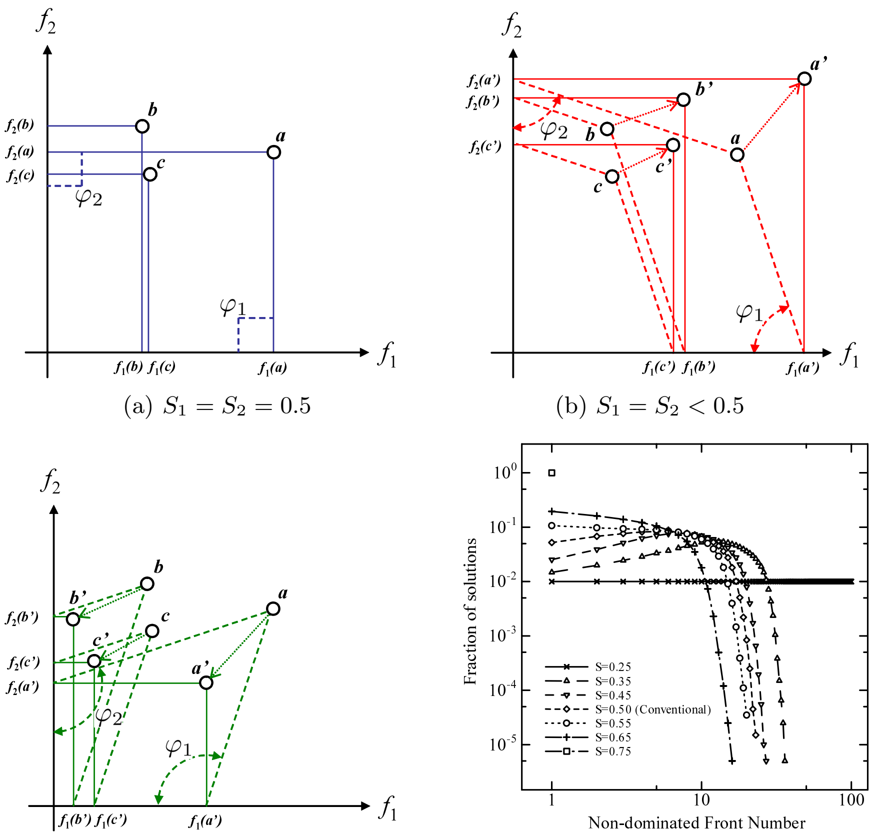
4 Proposed Method

4.1 Contraction and Expansion of Dominance Area

In this work, we try to control the covered area of dominance. Normally, the dominance area is uniquely determined with a fitness vector $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x}))$ in the objective space when a solution $\boldsymbol{x}$ is given. To contract and expand the dominance area of solutions, we modify fitness value for each objective function by changing the user defined parameter S_i in the following equation

$$f_i'(x) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \qquad (i = 1, 2, \cdots, m) \tag{8}$$

where $\varphi_i = S_i \cdot \pi$. This equation is derived from the Sine theorem. We illustrate the fitness modification in **Fig. 1**, where r is the norm of $\boldsymbol{f}(\boldsymbol{x})$, $f_i(\boldsymbol{x})$ is the fitness value in the i-th objective, and ω_i is the declination angle between $\boldsymbol{f}(\boldsymbol{x})$ and $f_i(\boldsymbol{x})$. In this example, the i-th fitness value $f_i(\boldsymbol{x})$ is increased to $f_i'(\boldsymbol{x}) > f_i(\boldsymbol{x})$ by using $\varphi_i < \pi/2$ ($S_i < 0.5$). In case of $\varphi_i = \pi/2$ ($S_i = 0.5$), $f_i(\boldsymbol{x})$ does not change and $f_i'(\boldsymbol{x}) = f_i(\boldsymbol{x})$. Thus, this case is equivalent to the conventional dominance. On the other hand, in case of $\varphi_i > \pi/2$ ($S_i > 0.5$), $f_i(\boldsymbol{x})$ is decreased so $f_i'(\boldsymbol{x}) < f_i(\boldsymbol{x})$. Such fitness modification changes the dominance area of solutions. We show an example in **Fig. 2** (a)-(c), where three solutions $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are distributed in 2-dimensional objective space. In **Fig. 2** (a), $\boldsymbol{a}$ dominates $\boldsymbol{c}$, but



(a) $S_1 = S_2 = 0.5$

(b) $S_1 = S_2 < 0.5$

(c) $S_1 = S_2 > 0.5$

(d) Solutions per front

Fig. 2. Conventional dominance (a), examples of expanding (b) and contracting (c) the dominance area of solutions, and solutions per front varying the parameter S (d)

$\boldsymbol{a}$ and $\boldsymbol{b}$, and $\boldsymbol{b}$ and $\boldsymbol{c}$ do not dominate each other. However, if we modify fitness values for each solution by using **Eq. (8)**, the location of each solution moves in the objective space, and consequently the dominance relationship among solutions changes. For example, if we use $S_1 = S_2 < 0.5$ as shown in **Fig. 2** (b), the dominance area of solutions $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ is expanded from the original one of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. This causes that $\boldsymbol{a}'$ dominates $\boldsymbol{b}'$ and $\boldsymbol{c}'$, and $\boldsymbol{b}'$ dominates $\boldsymbol{c}'$. That is, expansion of dominance area by smaller $S_i (< 0.5)$ works to produce a more fine grained ranking of solutions and would strengthen selection. On the other hand, if we use $S_1 = S_2 > 0.5$ as shown in **Fig. 2** (c), the dominance area of solutions $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ is contracted from the original one of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. This causes that $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ do not dominate each other. That is, contracting the area of dominance by larger $S_i (> 0.5)$ works to produce a coarser ranking of solutions and would weaken selection.

4.2 Effects of Controlling Dominance Area

As indicated above, expanding or contracting the dominance area of solutions change the dominance relation of some solutions and therefore modify the distribution of the fronts (number of fronts and solutions per front). Since front distribution significantly relates to selection, we verify and illustrate the effect of expanding or contracting the dominance area on the distribution of the fronts changing the parameter S_i in **Eq. (8)**. Here, we randomly generate 100 solutions in the 2-dimensional objective space of $[0,1]^2$, calculate dominance among them after recalculating fitness with **Eq. (8)**, and perform a non-domination sorting to obtain the fronts. We repeat the above steps a 1000 times and calculate the average number of fronts and solutions per front, for each value of S_i. In this work, we use a common parameter $S = S_i (i = 1, 2, \cdots, m)$ for all objective functions, because we assume that all objective functions are normalized. **Fig. 2** (d) shows the fraction of number of solutions per front varying S in the range $[0.25, 0.75]$ in intervals of 0.1 along with results for conventional dominance ($S = 0.5$).

From this figure, note that if we gradually expand the area of dominance by decreasing S below 0.5, the number of fronts increases and the ranking of solutions by non-dominance can be fine grained. Note that for maximum expansion of the dominance area $S = 0.25$ there is one solution per front. On the other hand, if we gradually contract the area of dominance by increasing S above 0.5, the number of fronts decreases and ranking of solutions by non-dominance becomes coarser. Note that for maximum contraction of the dominance area $S = 0.75$ there is only one front that contains all solutions. Since different rankings can be produced, we can expect that the optimum parameter S^* that yields maximum search performance exists for a given kind of problem.

5 Benchmark Problems, Metrics, and Parameters

In this paper we use multiobjective 0/1 knapsack problems [9] as benchmark problems to study and compare the effects on search performance of ranking solutions by expanding or contracting their dominance area. The problem (KPn-m) is formulated to maximize the function

$$f_j(\boldsymbol{x}) = \sum_{i=1}^{n} x_i \cdot p_{i,j} \tag{9}$$

subject to

$$g_j(\boldsymbol{x}) = \sum_{i=1}^{n} x_i \cdot w_{i,j} \leq W_j \tag{10}$$

where $x_i \in \{0, 1\}$ $(i = 1, 2, \cdots, n)$ are elements of solution vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$, which gives a combination of items. Thus, we use binary representation in this work. Note that here we are interested in finding a set of non-dominated Pareto solutions. Also, $p_{i,j}$ and $w_{i,j} (j = 1, 2, \cdots, m)$ denote profit and weight of item i according to knapsack (objective) j. W_j is the capacity of knapsack j, and solutions not satisfying this condition are considered as infeasible solutions $\bar{\mathcal{F}} = (\mathcal{S} - \mathcal{F})$. In this paper, we use benchmark problems with $m = \{2, 3, 4, 5\}$ objectives, $n = \{100, 250, 500, 750\}$ items and feasibility ratio $\phi = \{0.75, 0.5, 0.25\}$ downloaded from [10], for which we know the true Pareto non-dominated set only in case of two objectives $m = 2$. In these particular problems, we use a constant S for all objectives because the scale of each objective function is similar.

The hypervolume is used as a metric to evaluate sets of non-dominated solutions obtained by MOEAs. The hypervolume measures the m-dimensional volume of the region in objective space enclosed by the obtained non-dominated solutions and a dominated reference point [11]. Here we use $(f_1, f_2, \cdots, f_m)$=$(0, 0, \cdots, 0)$ as the reference point to calculate the hypervolume. A set of non-dominated solutions showing higher value of hypervolume can be considered as a better set of solutions from both convergence and diversity viewpoints. The hypervolume metric is a reliable metric and it is among the few recommended metrics to compare non-dominated sets [12]. To provide additional information separately on convergence and diversity of the obtained solutions in this work we also use Inverse Generational Distance (IGD) [13] and Spread (SP) [1], respectively. IGD takes the average distance for all members in the true Pareto front to their nearest solutions in the obtained set of non-dominated solutions (exactly the inverse process followed by Generational Distance GD [14]).

In our study we compare the performance of a conventional NSGA-II [8] with NSGA-II enhanced by the proposed method. We adopt two-point crossover with a crossover rate $p_c = 1.0$ for recombination, and apply bit-flipping mutation with a mutation rate $p_m = 1/n$. In the following experiments, we show the average performance with 30 runs, each of which spent 2,000 generations. Population size is set to $|P| = 200$ and the parent and offspring population sizes $|Q|$ and $|R|$ are set to half the population size $|P|$, i.e. $|Q| = |R| = 100$.

6 Experimental Results and Discussion

6.1 Performance Varying the Number of Objectives

In the following sections we observe the effects of varying the parameter S that controls the area of dominance of the solutions on the performance of the algorithm measured by the hypervolume. Recall that $S = 0.5$ indicates conventional dominance, values of $S > 0.5$ indicate contraction of the dominance area of the solutions, and values of $S < 0.5$ indicate expansion of the dominance area of the solutions.

First, we observe the effect of varying S on problems with different number of objectives. **Fig. 3** shows the values of the hypervolume achieved varying S in the range $[0.25, 0.75]$ in intervals of 0.05 on problems with $m = \{2, 3, 4, 5\}$ objectives, $n = 500$ items, and feasibility ratio $\phi = 0.50$. From this figure important observations are as follow. First, there is an optimum value S^* for each number of objectives that maximizes the hypervolume. Note however that the maximum value of hypervolume is not achieved by conventional dominance ($S = 0.5$) for any number of objectives. Second, to achieve the maximum value

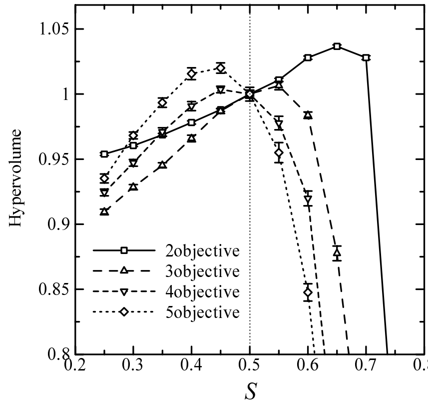
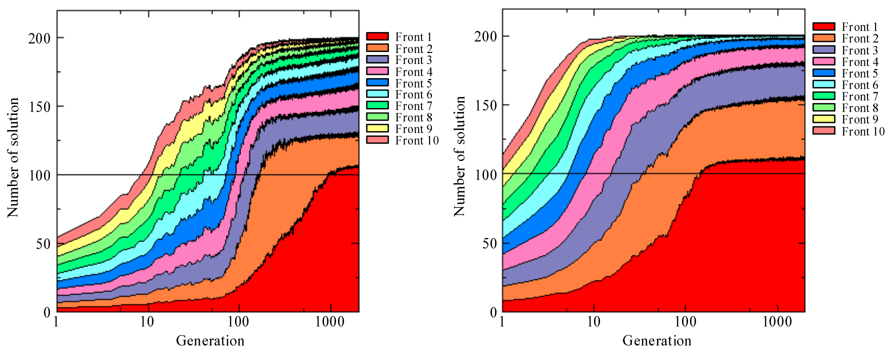
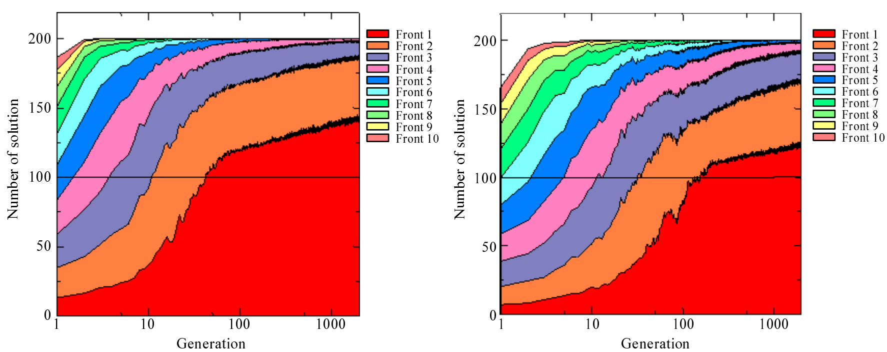


Fig. 3. Hypervolume as we increase the number of objectives m for problems with $n = 500$ items and $\phi = 0.5$ feasibility ratio



(a) $S = 0.5$, conventional dominance (b) $S^* = 0.65$, best by contracting dominance

Fig. 4. Front distribution over generation $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio



(a) $S = 0.5$, conventional dominance (b) $S^* = 0.45$, best by expanding dominance

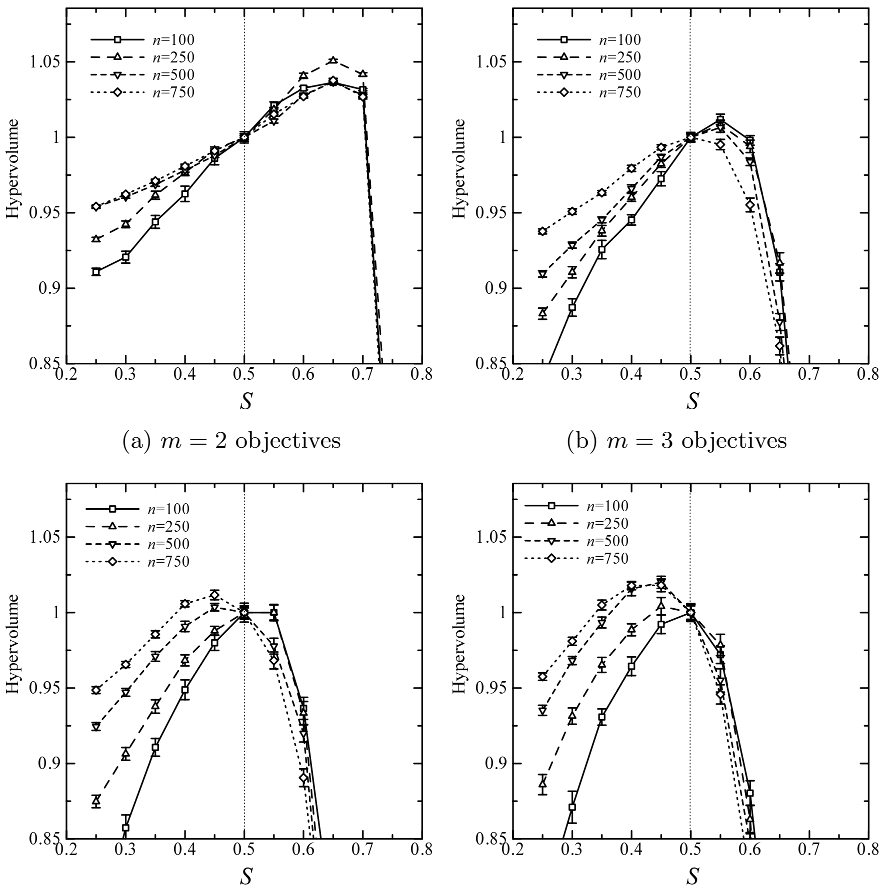
Fig. 5. Front distribution over generation $m = 4$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio

of hypervolume, the degree of expansion or contraction of dominance area of solutions should be adjusted accordingly to the number of objectives. Note that maximum values of the hypervolume are achieved for two and three objectives by contracting the dominance area of the solutions ($S > 0.5$), whereas for four and five objectives the maximum hypervolume values are achieved by expanding the dominance area of the solutions ($S < 0.5$). Third, as a general trend in problems with $n = 500$ items and feasibility ratio $\phi = 0.50$, we observe that the optimum value S^* reduces as we increase the number of objectives. That is, increasing the number of objectives the area of dominance should be expanded by using smaller values of S^* to achieve maximum hypervolume.

Fig. 4 (a) and (b) show the front distribution over generation by conventional dominance ($S = 0.5$) and by contracting dominance with the optimum parameter ($S^* = 0.65$), respectively, on $m = 2$ objectives, $n = 500$ items and feasibility ratio $\phi = 0.5$. Similarly, **Fig. 5** (a) and (b) show on $m = 4$ objectives the front distributions by conventional dominance ($S = 0.5$) and by expanding dominance with the optimum parameter ($S^* = 0.45$), respectively. Results are presented for the ten top fronts obtained from the combined population of parents and offspring before truncation. The horizontal line indicates the truncation point after front non-domination sorting. These figures illustrate and corroborate our expectation that contraction or expansion of area of dominance changes the ranking of solutions. Remember that contraction of the area of dominance weakens selection and induces a coarse ranking of solution, as illustrated in **Fig. 4** (a) and (b), which works better for two and three objectives. Also, remember that an expansion of the area of dominance strengthen selection and induces a fine grained ranking of solutions, as illustrated in **Fig. 5** (a) and (b), which works better for four and five objectives.

6.2 Performance Varying the Size of the Search Space

Second, we observe the effects of varying S on problems with different number of items n. Note that the size of the search space is given by 2^n. **Fig. 6** shows the hypervolume varying S on problems with $n = \{100, 250, 500, 750\}$ items and feasibility ratio $\phi = 0.5$ for $m = \{2, 3, 4, 5\}$ objectives. From **Fig. 6** (a) we can see that in the case of $m = 2$ objectives the optimum S^* is similar for all n, around 0.65. However, from **Fig. 6** (b),(c), and (d) we observe that increasing the number of items n produces a clear shift of the optimum S^* towards smaller values (greater expansion of area of dominance), especially in the case of $m = 4$ and $m = 5$ objectives. For example, for $m = 4$, note the optimal $S^* = \{0.55, 0.5, 0.45, 0.45\}$ on $n = \{100, 250, 500, 750\}$, respectively. In the previous section, fixing the number of items to $n = 500$, results suggested that the degree of expansion or contraction of dominance area of solutions should be adjusted according to the number of objectives. The results presented in this section suggest that the degree of expansion or contraction of dominance area of solutions should also be adjusted according to the size of the search space, especially for an increased number of objectives.



(a) $m = 2$ objectives

(b) $m = 3$ objectives

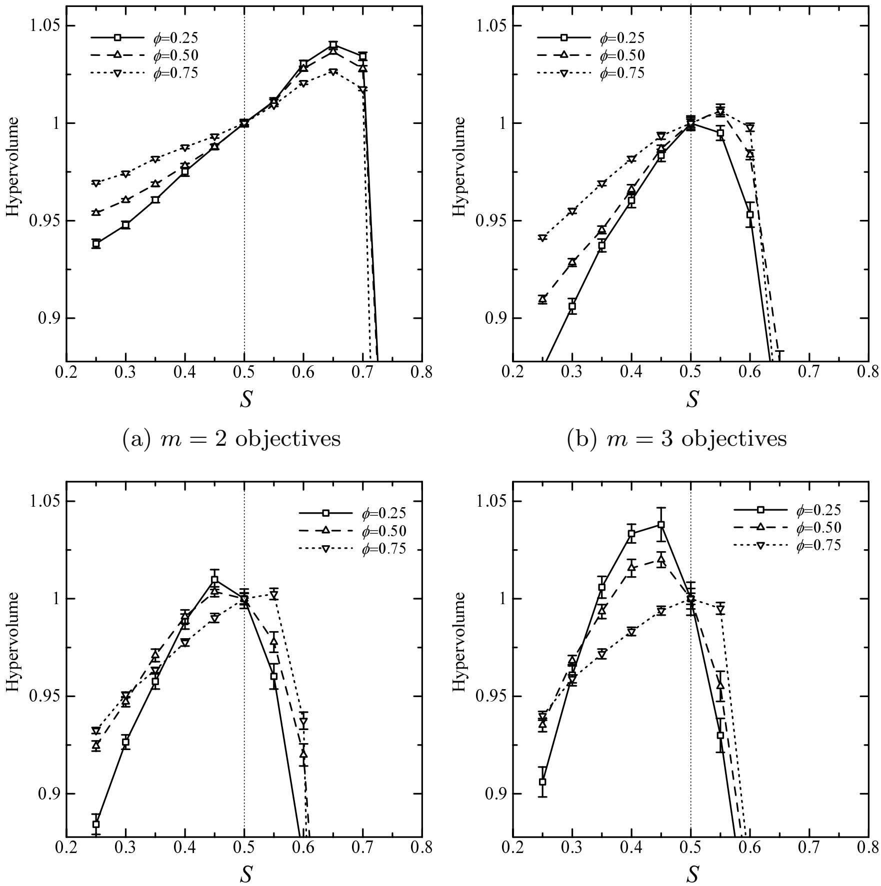
(c) $m = 4$ objectives

(d) $m = 5$ objectives

Fig. 6. Hypervolume as we increase the number of items n for problems with $m = \{2, 3, 4, 5\}$ objectives and $\phi = 0.5$ feasibility ratio

6.3 Performance Varying the Search Space Feasibility Ratio ϕ

Third, we observe the effects of varying S on problems with different feasibility ratio ϕ. **Fig. 7** shows the hypervolume varying S on problems with feasibility ratio $\phi = \{0.75, 0.5, 0.25\}$ and $n = 500$ items for $m = \{2, 3, 4, 5\}$ objectives. From **Fig. 7** (a)-(d) note that the effects on problems with different feasibility ratio ϕ resemble those observed on problems with different number of items. That is, in $m = 2$ objectives the optimum S^* is the same for all ϕ. However, reducing the feasibility ratio ϕ from 0.75 to 0.25, there is a shift of the optimum S^* towards smaller values, which becomes more notorious for $m = 4$ and $m = 5$ objectives. For example, for $m = 4$, note the optimal $S^* = \{0.55, 0.45, 0.4\}$ on $\phi = \{0.75, 0.5, 0.25\}$, respectively. These results suggest that the optimum degree of expansion or contraction of dominance area of solutions also depends on the



(a) $m = 2$ objectives

(b) $m = 3$ objectives

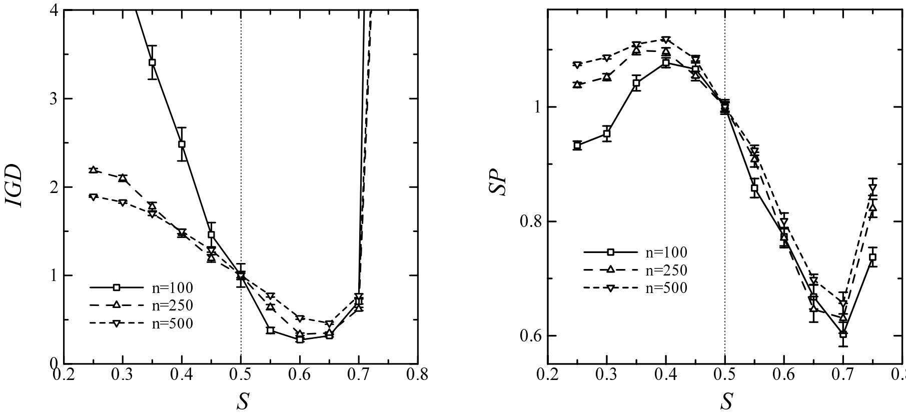
(c) $m = 4$ objectives

(d) $m = 5$ objectives

Fig. 7. Hypervolume as we decrease feasibility ratio for problems with $m = \{2, 3, 4, 5\}$ objectives and $n = 500$ items

feasibility ratio of the search space (complexity of the landscapes), especially for an increased number of objectives.

Summarizing, the optimum degree of expansion or contraction of the dominance area depends on the three aspects investigated in this work; that is, number of objectives, size of the search space, and feasibility ratio of the search space. For most real world combinatorial problems we can know in advance the number of objectives and size of the search space. Based on these information, we can use the results presented here as a good initial guidelines to properly set the degree of expansion or contraction of the area of dominance in order to achieve higher performance. However, the feasibility ratio (or complexity of the single objective landscapes) is usually unknown. It would be interesting to find adaptive ways to fine tune the parameter S for problems of different complexity.



(a) IGD (b) SP

Fig. 8. Inverse generational distance IGD and Spread SP varying the number of items n on problems with $m = 2$ objectives and $\phi = 0.5$ feasibility ratio

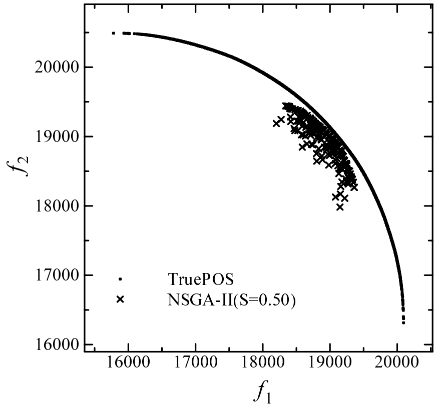
6.4 Results on Complementary Metrics and Obtained Solutions

Fig. 8 (a) and (b) show the Inverse Generational Distance IGD and Spread SP, respectively, varying the number of items on problems with $m = 2$ objectives and feasibility ratio $\phi = 0.5$. From these figures note that optimum IGD and SP (smaller values) are achieved when the dominance area of solutions is contracted ($S > 0.5$) rather than by conventional dominance ($S = 0.5$), similar to the results shown in **Fig. 6** (a). The values S achieving minimum IGD are almost coincident with $S^* = 0.65$ achieving maximum hypervolume. However, in case of SP the values S are slightly shifted towards larger values. Also, note that the graph of SP shows a maximum peak in the area of $S < 0.5$ and a minimum peak in the area of $S > 0.5$.

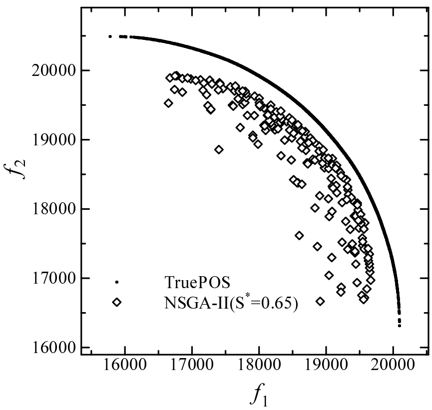
To analyze the above observations further, **Fig. 9** illustrates the obtained solutions in the final generation for all 30 simulations by conventional dominance $S = 0.5$, contracting dominance $S^* = 0.65$, and expanding dominance $S = 0.4$ for $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio. Note that solutions obtained by conventional dominance are close to the true Pareto front but are clustered in a limited region of objective space. By contracting dominance with the optimum parameter $S^* = 0.65$, we can spread the obtained solutions showing the maximum hypervolume, although convergence of some of them seems to deteriorate. On the other hand, by expanding dominance with $S = 0.4$ showing the maximum SP (worst spread), we can further enhance convergence of the solutions within a narrower region of objective space.

7 Conclusions

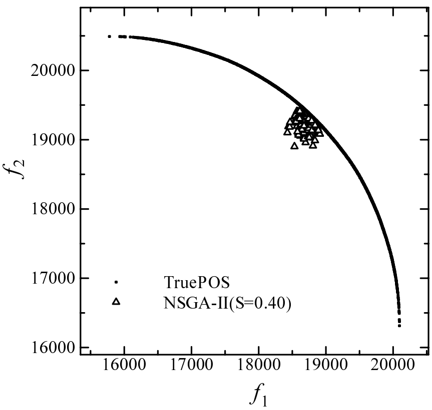
In this work we have proposed a method that can control dominance area of solutions by a user defined parameter S. We showed that contracting or expanding



(a) Conventional dominance ($S = 0.5$)



(b) Contracting dominance ($S^* = 0.65$)



(c) Expanding dominance ($S = 0.40$)

Fig. 9. Obtained solutions by conventional dominance $S = 0.5$, contracting dominance $S^* = 0.65$, and expanding dominance $S = 0.4$ for $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio

the dominance area of solutions changes their dominance relation, modifying the distribution of solutions (number of fronts and number of solutions per front) in the multiobjective landscape. Since front distribution significantly relates to selection, we analyzed the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer. We used 0/1 multiobjective knapsack problems as benchmark problems and showed that the optimum value of S^* depends strongly on number of objectives, size of the search space, and feasibility ratio of the search space (complexity). In addition, we showed that significantly better performance can be achieved either on convergence or diversity of obtained solutions by contracting or expanding the dominance area rather than by using conventional dominance.

In this work, we have assumed a constant parameter $S_i = S$ on all objectives ($i = 1, 2, \cdots, m$) to control the expansion or contraction of dominance area. It would be interesting in the future to investigate the effect of varying S_i for each objective and control S adaptively, especially for problems of unknown characteristics. In addition, it could be valuable to combine this approach with the inclusion of preferences to guide the search towards a particular region of objective space. With the proposed method, we can improve either convergence or diversity of solutions but not simultaneously both. Therefore, we would like to combine the proposed method with other selection methods to achieve higher convergence while covering the whole true Pareto front. Furthermore, we should try our method on other kind of problems with more objectives and compare our method with other approaches that aim to solve many objective optimization problems.

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