

Supplementary Document for “The Importance of Diversity in the Variable Space in the Design of Multi-objective Evolutionary Algorithms”

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Abstract

Keywords: Diversity, Decomposition, Multi-objective Optimization,
Evolutionary Algorithms.

1 This document is intended to be a supplementary material of the main
2 work titled “AVSD-MOEA/D The Importance of Diversity in the Variable
3 Space in the Design of Multi-objective Evolutionary Algorithms”. Particu-
4 larly, this extension seeks to complement the results discussed in the main
5 document. First, the main section *Performance of MOEAs in long-term exe-*
6 *cutions* in terms of *The Modified Inverted Generational Distance Plus* (IGD+)
7 is presented [1]. Second, a detailed analyses of the *Test Problems with Bias*
8 *Features* is driven. The conclusions found in this document are quite similar
9 to those obtained in the main document.

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Table 1: Summary of the IGD+ attained for problems with two objectives

	AVSD-MOEA/D			MOEA/D-DE			NSGA-II			NSGA-III			R2-EMOA		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
WFG1	0.006	0.020	0.024	0.048	0.195	0.076	0.008	0.039	0.030	0.009	0.014	0.012	0.009	0.094	0.049
WFG2	0.003	0.003	0.000	0.008	0.008	0.000	0.004	0.004	0.000	0.007	0.045	0.066	0.004	0.006	0.001
WFG3	0.008	0.008	0.000	0.010	0.010	0.000	0.021	0.022	0.001	0.010	0.010	0.000	0.010	0.011	0.000
WFG4	0.006	0.006	0.000	0.009	0.009	0.000	0.014	0.016	0.001	0.009	0.010	0.001	0.008	0.014	0.003
WFG5	0.037	0.056	0.005	0.065	0.067	0.001	0.071	0.072	0.001	0.060	0.065	0.002	0.065	0.067	0.001
WFG6	0.024	0.047	0.013	0.009	0.022	0.011	0.014	0.016	0.001	0.026	0.039	0.008	0.007	0.007	0.000
WFG7	0.006	0.006	0.000	0.009	0.009	0.000	0.013	0.015	0.001	0.009	0.009	0.000	0.007	0.007	0.000
WFG8	0.034	0.048	0.005	0.110	0.115	0.002	0.119	0.124	0.002	0.116	0.117	0.001	0.116	0.118	0.001
WFG9	0.009	0.011	0.001	0.012	0.028	0.025	0.031	0.077	0.046	0.123	0.126	0.001	0.011	0.035	0.033
DTLZ1	0.001	0.001	0.000	0.001	0.001	0.000	0.002	0.002	0.000	0.001	0.001	0.000	0.002	0.002	0.000
DTLZ2	0.002	0.002	0.000	0.003	0.003	0.000	0.003	0.004	0.000	0.003	0.003	0.000	0.002	0.002	0.000
DTLZ3	0.002	0.002	0.000	0.003	0.003	0.000	0.003	0.003	0.000	0.003	0.003	0.000	0.002	0.002	0.000
DTLZ4	0.002	0.002	0.000	0.003	0.003	0.000	0.003	0.058	0.150	0.003	0.003	0.000	0.002	0.124	0.207
DTLZ5	0.002	0.002	0.000	0.003	0.003	0.000	0.003	0.004	0.000	0.003	0.003	0.000	0.002	0.002	0.000
DTLZ6	0.002	0.002	0.000	0.003	0.004	0.003	0.003	0.003	0.000	0.003	0.003	0.000	0.002	0.004	0.005
DTLZ7	0.002	0.002	0.000	0.004	0.004	0.000	0.003	0.003	0.000	0.004	0.004	0.000	0.002	0.002	0.000
UF1	0.003	0.003	0.000	0.007	0.007	0.000	0.005	0.006	0.000	0.004	0.004	0.000	0.004	0.004	0.000
UF2	0.003	0.003	0.000	0.006	0.007	0.001	0.009	0.011	0.001	0.009	0.014	0.003	0.008	0.009	0.001
UF3	0.030	0.042	0.007	0.004	0.005	0.000	0.006	0.008	0.002	0.008	0.022	0.018	0.005	0.010	0.004
UF4	0.007	0.007	0.000	0.027	0.031	0.002	0.034	0.037	0.001	0.038	0.039	0.001	0.030	0.033	0.001
UF5	0.016	0.026	0.006	0.140	0.251	0.063	0.094	0.129	0.032	0.103	0.146	0.022	0.094	0.135	0.067
UF6	0.017	0.024	0.012	0.036	0.225	0.151	0.078	0.202	0.060	0.078	0.130	0.074	0.081	0.220	0.103
UF7	0.003	0.003	0.000	0.004	0.004	0.000	0.008	0.010	0.001	0.010	0.016	0.002	0.004	0.012	0.005
Mean	0.010	0.014	0.003	0.023	0.044	0.015	0.024	0.038	0.014	0.028	0.036	0.009	0.021	0.040	0.021

1. Comparison against State-of-the-art MOEAs in long-term executions in terms of IGD+ metric

The IGD+ indicator measures the average distance from each reference point to the nearest dominated region of the solution set. Let us denote the reference point set as $Z = \{z_1, z_2, \dots, z_{|Z|}\}$ where z_i is a point in the objective space. In this context the reference set can be seen as a discretization of the Pareto front. Let us denote a solution set A as $A = \{a_1, a_2, \dots, a_{|A|}\}$ where a_j

Table 2: Statistical Tests and Deterioration Level of the IGD+ for problems with two objectives

	\uparrow	\downarrow	\leftrightarrow	Score	Deterioration
AVSD-MOEA/D	78	13	1	65	0.085
MOEA/D-DE	36	50	6	-14	0.770
NSGA-II	21	65	6	-44	0.620
NSGA-III	39	49	4	-10	0.581
R2-EMOA	44	41	7	3	0.675

is a point in the objective space. The IGD+ indicator is defined as

$$IGD+(A) = \frac{1}{|Z|} \sum_{i=1}^{|Z|} \min_{j=1}^{|Z|} d^+(z_i, a_j) \quad (1)$$

where $d^+(z, a) = \sqrt{\max\{\}}\}$

The basic idea in the IGD+ is to calculate the distance from each reference point to the dominated region by a solution set.

One of the aims behind the design of AVSD-MOEA/D is to profit from long-term executions. Therefore, in this section we present the results attained by the different algorithms when setting the stopping criterion to 2.5×10^7 function evaluations. Table ?? shows the HV ratios obtained for the benchmark functions with two objectives. Note that the same results can be drawn with the IGD+ metric [1] and can be inspected in the supplementary material. For each method and problem, the best, mean and standard deviation of the HV ratio values are reported. Furthermore, in order to summarize the results attained by each method, the last row shows the mean for the whole set of problems. For each test problem, the method that yielded the largest

31 mean and those that were not statistically inferior to the best are shown in
 32 **boldface**. Similarly, the method that yielded the best HV value among all
 33 the runs is underlined. From here on, the methods shown in **boldface** for a
 34 given problem are referred to as the winning methods. AVSD-MOEA/D, R2-
 35 EMOA, MOEA/D-DE, NSGA-III and NSGA-II belonged to the winning methods
 36 in 17, 6, 2, 2 and 0 problems, respectively. The superiority of AVSD-MOEA/D
 37 is clear both in terms of this metric and in terms of the mean HV. Particu-
 38 larly, AVSD-MOEA/D attained a value equal to 0.976, while all the remaining
 39 methods attained values between 0.931 and 0.937. A careful inspection of the
 40 data shows that in those cases where AVSD-MOEA/D loses, the difference with
 41 respect to the best method is low. In fact, the difference between the mean
 42 HV ratio attained by the best method and by AVSD-MOEA/D is never greater
 43 than 0.1. However, in all the other methods, there were several problems
 44 where the distance with respect to the best approach was greater than 0.1.
 45 Specifically, it happened in 4, 4, 4 and 5 problems for R2-EMOA, MOEA/D-DE,
 46 NSGA-II and NSGA-III, respectively. This means that AVSD-MOEA/D wins in
 47 most cases and that when it loses, the difference is always small. Note also
 48 that in terms of standard deviation, AVSD-MOEA/D yields much lower values
 49 than all the other algorithms, meaning it is quite robust.

50 In order to better clarify these findings, pair-wise statistical tests were ap-
 51 plied between each method tested in each test problem. For the two-objective
 52 cases, Table ?? shows the number of times that each method statistically won
 53 (column \uparrow), lost (column \downarrow) or tied (column \leftrightarrow). The **Score** column shows
 54 the difference between the number of times that each method won and the
 55 number of times that each method lost. Additionally, for each method M ,

Table 3: Summary of the IGD+ attained for problems with three objectives

	AVSD-MOEA/D			MOEA/D-DE			NSGA-II			NSGA-III			R2-EMOA		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
WFG1	0.073	0.085	0.010	0.083	0.136	0.043	0.108	0.129	0.012	0.092	0.096	0.010	0.079	0.104	0.023
WFG2	0.055	0.057	0.001	0.062	0.069	0.004	0.096	0.135	0.021	0.097	0.113	0.018	0.119	0.120	0.000
WFG3	0.026	0.027	0.000	0.032	0.032	0.000	0.047	0.095	0.030	0.084	0.098	0.012	0.033	0.034	0.000
WFG4	0.088	0.092	0.001	0.133	0.133	0.000	0.132	0.142	0.009	0.133	0.133	0.000	0.119	0.124	0.002
WFG5	0.122	0.137	0.006	0.185	0.185	0.000	0.181	0.192	0.008	0.182	0.185	0.001	0.166	0.169	0.002
WFG6	0.112	0.130	0.009	0.140	0.158	0.009	0.159	0.183	0.012	0.145	0.162	0.009	0.120	0.128	0.005
WFG7	0.089	0.091	0.001	0.133	0.133	0.000	0.130	0.158	0.010	0.133	0.133	0.000	0.117	0.119	0.001
WFG8	0.122	0.128	0.003	0.191	0.193	0.001	0.242	0.254	0.006	0.194	0.198	0.002	0.174	0.178	0.002
WFG9	0.100	0.103	0.001	0.135	0.138	0.001	0.178	0.252	0.017	0.149	0.237	0.022	0.129	0.133	0.002
DTLZ1	0.015	0.015	0.000	0.014	0.014	0.000	0.019	0.021	0.001	0.014	0.014	0.000	0.015	0.015	0.000
DTLZ2	0.023	0.024	0.000	0.029	0.029	0.000	0.033	0.037	0.002	0.029	0.029	0.000	0.026	0.027	0.000
DTLZ3	0.023	0.023	0.000	0.029	0.029	0.000	0.035	0.039	0.002	0.029	0.029	0.000	0.026	0.027	0.000
DTLZ4	0.023	0.023	0.000	0.029	0.029	0.000	0.032	0.107	0.200	0.029	0.042	0.075	0.026	0.045	0.106
DTLZ5	0.004	0.004	0.000	0.005	0.005	0.000	0.003	0.003	0.000	0.008	0.010	0.002	0.003	0.003	0.000
DTLZ6	0.004	0.004	0.000	0.005	0.009	0.007	0.003	0.010	0.029	0.010	0.013	0.002	0.003	0.003	0.001
DTLZ7	0.033	0.033	0.000	0.059	0.059	0.000	0.040	0.060	0.056	0.050	0.061	0.005	0.075	0.113	0.047
UF8	0.030	0.032	0.001	0.040	0.054	0.016	0.089	0.111	0.026	0.040	0.075	0.066	0.042	0.050	0.008
UF9	0.029	0.031	0.001	0.038	0.169	0.071	0.103	0.164	0.058	0.032	0.046	0.041	0.034	0.110	0.085
UF10	0.060	0.072	0.010	0.105	0.309	0.091	0.229	0.273	0.043	0.154	0.276	0.055	0.254	0.261	0.017
Mean	0.054	0.058	0.002	0.076	0.099	0.013	0.098	0.124	0.029	0.084	0.103	0.017	0.082	0.093	0.016

we calculated the sum of the differences between the mean HV ratio attained by the best method (the ones with the highest mean) and method M , for each problem where M was not in the group of winning methods. This value is shown in the *Deterioration* column. The data confirm that although AVSD-MOEA/D loses in some pair-wise tests, the overall numbers of wins and losses clearly favor AVSD-MOEA/D. More importantly, the total deterioration is much lower in the case of AVSD-MOEA/D, confirming that when AVSD-MOEA/D loses, the differences are low.

Tables ?? and ?? shows the same information for the problems with three objectives. In this case, the number of times that each method be-

Table 4: Statistical Tests and Deterioration Level of the IGD+ for problems with three objectives

	\uparrow	\downarrow	\leftrightarrow	Score	Deterioration
AVSD-MOEA/D	69	5	2	64	0.005
MOEA/D-DE	35	34	7	1	0.774
NSGA-II	6	65	5	-59	1.260
NSGA-III	22	48	6	-26	0.844
R2-EMOA	46	26	4	20	0.656

66 longed to the winning groups were 17, 2, 0, 0 and 0 for AVSD-MOEA/D,
 67 R2-EMOA, MOEA/D-DE, NSGA-III and NSGA-II, respectively. Thus, AVSD-
 68 MOEA/D yielded quite superior results. Considering the whole set of prob-
 69 lems, AVSD-MOEA/D obtained a much larger mean HV ratio than the other
 70 ones. Moreover, the difference between the mean HV ratio obtained by the
 71 best method and by AVSD-MOEA/D was never greater than 0.1. However, all
 72 the other methods exhibited a deterioration in excess of 0.1 in several cases.
 73 In particular, this happened in 2, 2, 2 and 6 problems for MOEA/D-DE,
 74 R2-EMOA, NSGA-III and NSGA-II respectively. Remarkably, AVSD-MOEA/D is
 75 quite superior in both the total deterioration and in the score generated from
 76 the pair-wise statistical tests. In fact, its deterioration for the entire problem
 77 set is just 0.006. Beating all the state-of-the-art algorithms in such a large
 78 number of problem benchmarks is a quite significant achievement, and shows
 79 the robustness of AVSD-MOEA/D. Our results show that the superiority of
 80 AVSD-MOEA/D persists, and even increases, when problems with three ob-
 81 jective functions are considered. For a better comprehension of the strenghts
 82 and weakness of the algorithms, in the Figure ?? is shown the 50% attain-
 83 ment surfaces for WFG8 and UF5. An attainment surface approximation can

84 be interpreted as the spatial region that is statistically attained among all
 85 the runs that were carried out by an algorithm [2, 3]. In other words, it can
 86 be understood as *the spatial region that is achieved by the $k\%$ among all the*
 87 *runs by one algorithm.* The most challenging characteristic of these problems
 88 are that WFG8 has strong dependencies among all the parameters, and UF5
 89 is a multi-modal biased problem whose Pareto optimal front is discrete and
 90 consists of 21 points. In both problems AVSD-MOEAD was the only one that
 91 converged adequately to the Pareto front at least 50% among all the runs.
 92 Even more, given that the standard deviation is too low it can be though
 93 that all the runs converged similarly well.

94 We can better understand the reasons behind the benefits of AVSD-MOEAD/D
 95 against the state-of-the-art MOEAs by analyzing the evolution of the HV val-
 96 ues and the diversity. Note that in some MOPs, variables can be classified
 97 into two types: distance variables and position variables. A variable x_i is
 98 a distance variable when for all x , modifying x_i results in a new solution
 99 that dominates x , is equivalent to x , or is dominated by x . Differently, if
 100 x_i is a position variable, modifying x_i in x always results in a vector that is
 101 incomparable or equivalent to x [4]. This is important because in some cases,
 102 MOEAs do not maintain a large enough diversity in the distance variables [5],
 103 so analyzing the diversity trend for these kinds of variables provides an useful
 104 insight into the dynamics of the population.

105 In order to show the behavior of the different schemes, we selected WFG5
 106 and UF5. They are complementary in the sense that in WFG5, all the Pareto
 107 solutions exhibit constant values for the distant variables, which is not the
 108 case in UF5. Moreover, in UF5, the optimal regions are isolated in the vari-

109 able space, meaning that more diversity is required. For each algorithm, the
 110 diversity is calculated as the average Euclidean distance between individuals
 111 (ADI) in the population by considering only the distance variables. Figures ??
 112 and ?? show the evolution of the ADI (top) and the mean of HV (bottom) for
 113 WFG5 and UF5, respectively. In the WFG5 problem, the distance variables
 114 quickly converged to a small region in state-of-the-art MOEAs. Thus, the
 115 differential evolution operator loses its exploring power and as a result, those
 116 MOEAs were unable to significantly improve the quality of the approxima-
 117 tions as the evolution progresses. By contrast, in the case of AVSD-MOEA/D,
 118 the decrease in ADI is quite linear until the midpoint of the execution, and
 119 the increase in HV is gradual. The final HV attained by AVSD-MOEA/D is the
 120 largest one, which shows the important benefit of gradually decreasing the
 121 diversity.

122 As expected, explicitly promoting diversity is also beneficial for problems
 123 with disconnected optimal regions. As the data in Figure ?? show, the ad-
 124 vantage of promoting diversity in the UF5 test problem is clear. In this case,
 125 state-of-the-art algorithms maintain some degree of diversity in the distance
 126 variables for the entire search. However, a large degree of diversity is re-
 127 quired to obtain the 21 optimal solutions, and these MOEAs do not maintain
 128 the required amount of diversity, and as a result, they miss many of the so-
 129 lutions. In the case of AVSD-MOEA/D, enforcing a large degree of diversity
 130 in the initial phases promotes more exploration, which makes it possible to
 131 find additional optimal regions. Once these regions are located, they are not
 132 discarded, meaning that a larger level of diversity is maintained throughout
 133 the execution. This way, AVSD-MOEA/D not only attained better HV values

134 for the first 10% of the total function evaluations, but it also kept looking
 135 for promising regions. In fact, its HV values improved significantly until the
 136 midpoint of the execution period i.e., the final moment when diversity was
 137 explicitly promoted. Then, an additional increase was obtained due to in-
 138 tensification in the regions identified. This analysis shows that the dynamic
 139 of the population depends on the problem at hand. The behavior of AVSD-
 140 MOEA/D with all the problems tested was similar to those already presented.
 141 Scenarios where the optimal regions consists of constant values for the dis-
 142 tance variables behave like WFG5, whereas the behavior in those cases where
 143 the optimal regions consist of non-constant values for the distance variables
 144 is more similar to the UF5 case. Note, however, that in these cases, different
 145 levels of diversity are required, so the behavior is not as homogeneous.

146 In order to better understand the importance of D_I , the entire set of
 147 benchmark problems was tested with different values of D_I . As in previous
 148 experiments, the stopping criterion was set to 2.5×10^7 function evaluations.
 149 Since normalized distances are used, the maximum attainable distance be-
 150 tween pairs of individuals is 1.0. Also note that setting D_I to 0 implies not
 151 promoting diversity in the variable space. Thus, several values in this range
 152 were considered. Specifically, the values $D_I = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$
 153 were tested. Figure ?? shows the mean HV ratio obtained for both the two-
 154 objective and the three-objective case with the D_I values tested. The AVSD-
 155 MOEA/D performed worst when D_I was set to 0. The HV ratio quickly in-
 156 creased as higher D_I values up to 0.2 were used. Larger values yielded quite
 157 similar performances. Thus, a wide range of values (from 0.2 to 1.0) exhib-
 158 ited very good performance, meaning that the behavior of AVSD-MOEA/D is

Performance of the BT Problems Taking Into Consideration Several Biases

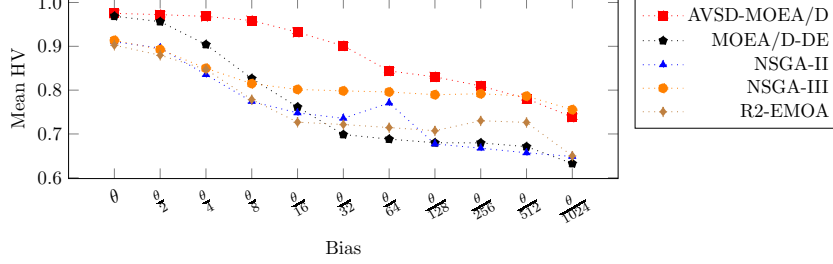


Figure 1: Mean of HV values for eight BTs problems (y-axis) against several biases ratios (x-axis). The BT2 problem is not taken into consideration due that it suffers of numerical stability.

quite robust. Thus, properly setting this parameter is not a complex task.

In order to better understand the implications of D_I on the dynamics of the population, Figure ?? shows, for AVSD-MOEA/D, the evolution of diversity in the distance variables in the WFG9 case for three different values of D_I . When setting $D_I = 0$, the diversity is reduced quite quickly, which results in premature convergence. The result is a hypervolume that is not too high. However, when $D_I = 0.4$ and $D_I = 1$ are used, the loss of diversity is slowed down, and the resulting hypervolume is quite large. Note that setting $D_I = 1$ promotes greater diversity, so the hypervolume increases slower than when $D_I = 0.4$. However, the degree of exploration in both cases is enough to yield high-quality solutions. The behavior is quite similar in every problem, which explains the stability of the algorithms for different values of D_I . Note that for shorter periods, setting a proper D_I value is probably much more important. However, for long-term executions at least, practically any value higher than 0.2 yields similar solutions, which we regard as a highly positive feature.

175 2. On the Convergence of MOEAs in Test Problems with Bias Fea- 176 tures

177 As pointed out in [6, 7, 4], the bias feature is one of the most challenging
178 difficulties that MOEAs might face. Recently, the BTs test problems were
179 proposed to facilitate the study of the ability of MOEAs for dealing with biases.
180 In this context bias means that small variations in the decision space around
181 the Pareto set cause significant changes in vicinities of some Pareto front
182 solutions [4]. Particularly, those problems are built with transformations
183 that induce position-related bias and distance-related bias. While the former
184 means that a small change on the position-related variables of one solution
185 in the Pareto set projects a significant change along the Pareto front. The
186 later imposes that a small variation on the distance-related variables of one
187 solution in the Pareto set causes a significant deterioration on the convergence
188 towards the Pareto front.

189 In order, to analyze the capability of the MOEAs to deal with bias features
190 the BTs problems are taken into account. Specifically, this section analyses
191 the sensitivity of the algorithms imposing several levels of bias in the distance-
192 related variables. Initially, for each problem the position-related bias and
193 distance-related bias (θ) are kept exactly as the one proposed in the original
194 work [6]. Then, for each problem its initial distance-related bias value (θ)
195 is iteratively decreased by a factor of two. Specifically, the distance-related
196 bias taken into account are $\{\theta, \frac{\theta}{2}, \frac{\theta}{4}, \frac{\theta}{8}, \frac{\theta}{16}, \frac{\theta}{32}, \frac{\theta}{64}, \frac{\theta}{128}, \frac{\theta}{256}, \frac{\theta}{512}, \frac{\theta}{1028}\}$. Figure 1
197 shows the mean HV ratio obtained with several distance-related biases. Also
198 note that the BT2 problem is not taken into consideration due that increas-
199 ing its bias values provokes numerical instability since that it incorporates

200 a different bias transformation, nevertheless all the results can be consulted
 201 in the supplementary document. Taking exactly the original configuration
 202 (bias of θ) [6] AVSD-MOEA/D is slightly better than MOEA/D-DE, but as soon
 203 as the bias is decreased to $\frac{\theta}{32}$ the performance of MOEA/D-DE decays ag-
 204 gressively. Furthermore, the performance of AVSD-MOEA/D is superior than
 205 0.9 with biases values upper or equal to $\frac{\theta}{256}$ which is quite superior than
 206 the state-of-the-art MOEAs whose values at that point are approximately
 207 of 0.75. Figure ?? shows the 50% of attainment surface of BT6, BT7 and
 208 BT8 with a bias of $\frac{\theta}{32}$. BT6 and BT8 have simple nonlinear Pareto set while
 209 BT7 has a complicated nonlinear Pareto set. BT8 is multimodal. Although
 210 that MOEA/D-DE converged to a region of the Pareto front with BT6 AVSD-
 211 MOEA/D covered a huge region of the Pareto front, in fact this shows that
 212 for this problem promoting diversity in the decision space results in diversity
 213 in the objective space. In addition, AVSD-MOEA/D converges quite well in
 214 complicates nonlinear Pareto sets shown in the 50% attained surface of BT7
 215 (Figure ??). Finally but not less important AVSD-MOEA/D shows a superior
 216 behaviour with biased and multimodal problems as is the case of BT8 whose
 217 attainment surfaces have converged much better to the Pareto front.

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