MOEA/D with DE and PSO: MOEA/D-DE+PSO

Wali Khan Mashwani

Abstract: Hybridization is one of the important research area in evolutionary multi-objective optimization (EMO).It is a method that incorporate good merits of multiple techniques aim at to enhance the search ability of EMO algorithm. In this chapter, we combine two well-known search algorithms, DE and PSO, and developed algorithm known as MOEA/D-DE+PSO. We experimentally studied its performance on two types of continuous multi-objective optimization problems and found better improvement.

1 Introduction

In this paper, we interested in solving multiobjective optimization problem (MOP) of minimization type which is generally formulated as under:

$$\text{minimize } F(x) = (f_1(x), f_2(x) \ldots, f_m(x)) \tag{1}$$

Subject to $x \in \Omega$, where $\Omega \subseteq R^n$, $x = (x_1, \ldots, x_n)^T \in \Omega$ is an n-dimensional vector of the decision variables, Ω is the *decision (variable) space*, $F : \Omega \to R^m$ consist of m real-valued objectives function. Very often, the objectives of the problem (1) are in conflict with one another or incommensurable. Due this conflict among the objectives function, no solution in Ω can optimize all the objectives simultaneously. Instead, one has to find a set of good representative optimal solutions to the problem (1) in the form of best trade-off or compromises among the objectives in terms of Pareto optimality.

Wali Khan Mashwani is a PhD student in the Department of Mathematical Sciences, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, UK e-mail: wkhan@essex.ac.uk & a Faculty member of Kohat University of Science and Technology (KUST), PAKISTAN.

M. Bramer et al. (eds.), *Research and Development in Intelligent Systems XXVIII*,
DOI 10.1007/978-1-4471-2318-7_16,

Since 1970s, numerous evolutionary multiobjective optimization (EMO) algorithms have been developed aim at finding a set optimal solutions to the problem 1. Among them, MOEA/D [1] is the novel developed paradigm which combines the decomposition strategies with evolutionary algorithm for solving problem (1) differently from existing Pareto dominance based EAs. Surprisingly, MOEA/D also used single algorithm for population evolution with exception [2].

In this paper, we study the combined effect of Differential evolution (DE) [3] and particle swarm optimization (PSO) [4] and developed hybrid version of the MOEA/D [1] known as MOEA/D-DE+PSO for continuous multiobjective optimization.We performed this study inspired by some recent reported work [5, 6, 2, 7].

The rest of this paper is organized as follows: **Section 2** explains the frameworks of MOEA/D-DE+PSO and the parameter settings in carried out experiments, **Section 3** provides a brief discussion on the experimental results, and finally **Section 4** concludes the paper.

2 MOEA/D with DE and PSO: MOEA/D-DE+PSO

The algorithmic steps of MOEA/D with DE and PSO, MOEA/D-DE+PSO is hereby outlined in **Algorithm 1**.

Table 1 Parameter Setting for solving ZDT [8] and CEC'09 Test Problems [9]. Here, N is the size of population, p_m is the mutation probability, η is the distribution index and T is the neighborhood size of subproblem in MOEA/D paradigm, r is a random number. The first column of the table represent the population size for the used test problems.

ZDT	100	$p_m = \frac{1}{n}, n = 10$	$\eta = 20$	25000	$T = 0.1N$
$CEC'09$	$600(1000)$	$p_m = \frac{1}{n}, n = 30$	$\eta = 20$	$300,000$	$T = 0.1N$
$PSO\ \&\ DE$	$w = 0.3 + r/2$	$c_1 = c_2 = 0.4$	$\xi = 0.7$	$CR = 1$	$F = 0.5$

3 A Brief Discussion on obtained Experimental Results

We used Inverted generation distance (IGD)-metric [10] to compare and evaluate the performance of both versions of MOEA/D [1], namely, **MOEA/D-DE** and **MOEA/D-DE+PSO** on the ZDT test problems [8] and on the CEC'09 test instances [9]. To compute the IGD-metric values, we have chosen P^* of uniformly distributed points in the PF of size 1000 for all problems during 30 times simulation of each algorithm. We used the parameter setting as described in the **Table 1** in our experimental studies of both MOEA/D-DE and MOEA/D-DE+PSO to solve each test

Algorithm 1 Pseudocode of MOEA/D-DE+PSO

INPUT:

- MOP: the multiobjective optimization problem;
- N: population (i.e., the number of subproblems);
- F_{EVAL}: the maximal number of function evaluations;
- a uniform spread of N weight vectors, $\lambda_1, \ldots, \lambda_N$;
- T: the number of weight vectors in the neighborhood of each weight vector;

Output: $\{x^{(1)}, x^{(2)}, \ldots, x^{(N)}\}$ and $\{F(x^{(1)}), \ldots, F(x^{(N)})\}$;

Step 0 ***Initialization:***

0.1: Uniformly randomly generate a population of size N, $P = \{x^{(1)}, \ldots, x^{(N)}\}$ from the search space Ω;

0.2: Initialize a set of N weight vectors, $\{\lambda^1, \lambda^2, \ldots, \lambda^N\}$;

0.3: Compute the Euclidian distances between any two weight vectors and then find the T closest weight vectors to each weight vector. For the ith subproblem, set $B(i) = \{i_1, \ldots, i_T\}$, where $\lambda^{i1}, \ldots, \lambda^{iT}$ are the T closest weight vectors to λ^i;

0.4: Compute the F-function value of each member in P, $F(x^i), i = 1, 2, \ldots, N$;

0.5: Initialize $z = (z_1, \ldots, z_m)^T$ by problem-specific method;

0.6: Set $\lceil \zeta_1 = 0.5N \rceil$; $\zeta_2 = N - \zeta_1$; where ζ_1, the number of subproblems deal by search algorithm A, and ζ_2, the number of new solutions deal by search algorithm B;

0.7: Set $t = 0$;

Step 2: Updating Phase of the MOEA/D-DE+PSO:

while $t < F_{EVAL}$ **do**
  Randomly divide $I = \{1, 2, \ldots, N\}$ into two sets, I_A and I_B, such that I_A has ζ_1 indices and I_B has ζ_2 indices at t ;
  for $i = 1 : N$ **do**
    if $i \in I_A$ **then**
      Apply search operator A to generate an offspring $\tilde{x}^i$;
    else
      Apply search operator B to generate an offspring $\tilde{x}^i$;
    end if
    Mutate $\tilde{x}^i$ with probability p_m, to get $\tilde{y}^i$;
    Repair $\tilde{y}^i$ to produce an offspring y^i;
    Compute the F-function value of y^i, $F(y^i)$;
    Update of z: For each $j = 1, \ldots, m$, $f_j(y^i) < z_j$, then set $z_j = f_j(y^i)$;
    Update of Neighboring Solutions:
    For each index $k \in B(i) = \{i_1, \ldots, i_T\}$
    if $g^{te}(y^i|\lambda^k, z) \leq g(x^k|\lambda^k, z)$ **then**
      $x^k = y^i$ and $F(x^k) = F(y^i)$
    end if
  end for
  Update ζ_1 ***and*** ζ_2 ***(Detail can found in Algorithm 2.)***
  $t = t + 1$
end while

Algorithm 2 Updating Procedure of ζ_1 and ζ_2

Step 1: Compute the ζ_1, the number of subproblems deal by search algorithm A, and ζ_2 is number of subproblems deal by search algorithm B.

step 2: Compute the probability of success (i.e.,τ_1) of the search algorithm A

$$\tau_1 = \frac{\frac{\kappa_1}{\zeta_1}}{\frac{\kappa_1}{\zeta_1} + \frac{\kappa_2}{\zeta_2}} \tag{2}$$

Where κ_1 is the total successful reward of search algorithm A out of ζ_1 subproblems.

Step 3: Replace $\zeta_1 = \lceil N \times \tau_1 \rceil$ and $\zeta_2 = N - \zeta_1$.

problem.

Table 2 and **Table 3** summarizes the IGD-metric values in terms of best (minimum), median, mean, standard deviation (std), worst (maximum), which are found by MOEA/D-DE and MOEA/D-DE+PSO, respectively. A bold data represent the best results for the corresponding algorithm on respective problem. This results show that MOEA/D-DE+PSO is more effective than MOEA/D-DE on most test problems. We didn't includes the plots of the Pareto front (PF) of the used test problems found by MOEA/D-DE and MOEA/D-DE+PSO due page limitation.

Table 2 ***The IGD-metric value statistics found by MOEA/D-DE, MOEA/D-DE+PSO during*** 30***times independent runs for ZDT test problems [8], ZDT1-ZDT4 and ZDT6***.

MOEA/D, MOEA/D-DE+PSO						
ZDT problems	min	median	mean	std	max	*Algorithms*
ZDT1	0.00401	0.00419	0.00421	0.00018	0.00455	MOEA/D-DE
	0.00402	**0.00414**	**0.00415**	**0.000071**	**0.00436**	**MOEA/D-DE+PSO**
ZDT2	0.00383	0.00387	0.00388	0.000042	0.00403	MOEA/D-DE
	0.00381	**0.00385**	**0.00387**	**0.000052**	**0.0039**	**MOEA/D-DE+PSO**
ZDT3	**0.00848**	**0.00906**	**0.00914**	**0.000662**	**0.01252**	**MOEA/D-DE**
	0.00860	0.00908	0.0202	0.03753	0.19706	MOEA/D-DE+PSO
ZDT4	0.01196	0.03056	0.04258	0.03342	0.15796	MOEA/D-DE
	0.00414	**0.00709**	**0.0075**	**0.0019**	**0.01158**	**MOEA/D-DE+PSO**
ZDT6	0.00856	0.0151	0.01478	0.00405	0.0235	MOEA/D-DE
	0.0069	**0.0146**	**0.0145**	**0.0045**	**0.0246**	**MOEA/D-DE+PSO**

4 Conclusion

Different search algorithms suit different problems. It is natural way to use the desirable properties of multiple algorithms for population evolution self-adaptively

Table 3 ***The IGD-metric value statistics found by MOEA/D-DE, MOEA/D-DE+PSO after*** 30***times independent runs for the CEC,09 unconstrained test problems [9], UF1-UF10***.

IGD Statistical Results found by *MOEA/D-DE andMOEA/D-DE+PSO*						
CEC'09	min	median	mean	std	max	*Approach-Name*
UF1	0.004499	0.073061	0.078707	0.051734	0.193602	MOEA/D-DE
	0.004466	**0.018066**	**0.030952**	**0.023114**	**0.063822**	***MOEA/D-DE+PSO***
UF2	0.018233	0.057095	0.062814	0.032670	0.142833	*MOEA/D-DE*
	0.010218	**0.011671**	**0.011737**	**0.000756**	**0.013261**	***MOEA/D-DE+PSO***
UF3	0.027292	0.238281	0.220884	0.084442	0.319024	*MOEA/D-DE*
	0.003966	**0.006314**	**0.006529**	**0.002050**	**0.011023**	***MOEA/D-DE+PSO***
UF4	0.062797	0.070280	0.070560	0.004064	0.077784	*MOEA/D-DE*
	0.062322	0.071080	0.070883	0.003396	0.076828	*MOEA/D-DE+PSO*
UF5	**0.210113**	**0.428818**	**0.426645**	**0.131033**	**0.707106**	***MOEA/D-DE***
	0.282111	0.454364	0.490882	0.118561	0.708999	*MOEA/D-DE+PSO*
UF6	0.244165	**0.456803**	**0.508164**	**0.142289**	**0.798462**	***MOEA/D-DE***
	0.186140	0.823351	0.778214	0.157339	0.843885	*MOEA/D-DE+PSO*
UF7	0.007096	0.106932	0.239466	0.244556	0.648772	*MOEA/D-DE*
	0.006726	**0.008776**	**0.245517**	**0.267355**	**0.674953**	***MOEA/D-DE+PSO***
UF8	**0.057705**	**0.076824**	**0.079935**	**0.015975**	**0.138522**	***MOEA/D-DE***
	0.077700	0.087222	0.088455	0.006532	0.101902	*MOEA/D-DE+PSO*
UF9	0.047530	0.151860	0.139659	0.035320	0.160975	*MOEA/D-DE*
	0.035499	**0.038980**	**0.071131**	**0.051008**	**0.149478**	***MOEA/D-DE+PSO***
UF10	0.296028	0.427088	0.442905	0.067564	0.672801	*MOEA/D-DE*
	0.184050	**0.187033**	**0.187158**	**0.001552**	**0.190097**	***MOEA/D-DE+PSO***

for algorithmic improvement. In this paper, we have proposed the hybrid version of MOEA/D [1] known as MOEA/D-DE+PSO by incorporating DE [3] and PSO [4] for multiobjective optimization. Experimental results shows the improvement of the MOEA/D-DE+PSO over MOEA/D-DE on MOEA/D-DE almost all ZDT problems [8] and CEC'09 test instance [9].

References

1. Q. Zhang and H. Li, "MOEA/D," *IEEE Trans on EC*, vol. 11, no. 6, pp. 712–731, 2007.
2. W. Khan and Q. Zhang, " MOEA/D-DRA with Two Crossover Operators," in *Proceeding of the UKCI'10*, 2010, pp. 1–6.
3. R.Storn and K.V.Price, "Differential Evolution," *J.Global Opt*, vol. 11, no. 4, pp. 341–359, 1997.
4. R.Eberhart and J.Kennedy, "A New optimizer using Particle Swarm Theory," in *Proceedings of MHS'95*, pp. 39–43.
5. J. A. Vrugt et al., "AMALGAM," *IEEE Trans On EC*, vol. 13, no. 2, pp. 243–259, 09.
6. J. A. Vrugt et.al, "Improve AMALGAM," *PNAS'07*, vol. 104, no. 3, pp. 708–701.
7. W. Khan, "Integration of NSGA-II and MOEA/D in Multimethod Search Approach," in *GECCO'11 (Companion)*, 2011, pp. 75–76.
8. E. Zitzler et al., "Comparsion of MOEAs," *EC*, vol. 8, no. 2, pp. 173–195, 200.
9. Q. Zhang et al., "Test Instances for the CEC'09," Technical Report CES-487.
10. E. Zitzler et al, "Performance Assessment of Multiobjective Optimizers: An Analysis and Review," *IEEE Trans on EC*, vol. 7, pp. 117–132, 2003.