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On the effect of reference point in MOEA/D for multi-objective optimization

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ABSTRACT

Multi-objective evolutionary algorithm based on decomposition (MOEA/D) has continuously proven effective for multi-objective optimization. So far, the effect of weight vectors and scalarizing methods in MOEA/D has been intensively studied. However, the reference point which serves as the starting point of reference lines (determined by weight vectors) is yet to be well studied. This study aims to fill in this research gap. Ideally, the *ideal* point of a multi-objective problem could serve as the reference point, however, since the *ideal* point is often unknown beforehand, the reference point has to be estimated (or specified). In this study, the effect of the reference point specified in three representative manners, i.e., pessimistic, optimistic and dynamic (from optimistic to pessimistic), is examined on three sets of benchmark problems. Each set of the problems has different degrees of difficulty in convergence and spread. Experimental results show that (i) the reference point implicitly impacts the convergence and spread performance of MOEA/D; (ii) the pessimistic specification emphasizes more of exploiting existing regions and the optimistic specification emphasizes more of exploring new regions; (iii) the dynamic specification can strike a good balance between exploitation and exploration, exhibiting good performance for most of the test problems, and thus, is commended to use for new problems.

1. Introduction

Multi-objective optimization problems (MOPs) arise in various disciplines such as engineering, economics and manufacturing [1]. In an MOP, objectives are often in conflict with one another. Thus, the optimal solution of an MOP is not a single one but is a set of trade-off solutions, i.e., Pareto optimal solutions (PS). The image of PS in objective space is termed Pareto optimal front (PF). Multi-objective evolutionary algorithms (MOEAs) are well suited for solving MOPs since their population-based nature can lead to an approximation of the PF in a single run [2, pp. 5–7].

Pareto-dominance based algorithms are some of the earliest MOEAs. They are found to perform well on 2- and 3-objective MOPs, e.g., MOGA [3], NSGA-II [4], but fail to scale up for many-objective problems (usually, with more than three objectives) [5,6]. This is because that the selective pressure induced by the Pareto-dominance relation degrades significantly as the number of objectives increases. Most of solutions become non-dominated in many-objective problems [5]. In addition to Pareto-dominance based algorithms, decomposition-based algorithms [7] which adopt the idea of "divide-and-conquer" become increasingly popular in the last decade. The multi-objective evolutionary algorithm based on decomposition (MOEA/D) [8] is representative of decomposition-based algorithms. The MOEA/D is demonstrated to have high search ability for combinatorial optimization [9,10], high compatibility with local search and also is computationally efficiency [11,12]. In recent years, a large body of MOEAs that employ the MOEA/D algorithmic framework have been proposed e.g., [13–20].

The MOEA/D decomposes an MOP into a set of single-objective problems by means of weighted scalarizing methods, and solves these single-objective problems in a collaborative manner. Usually, in MOEA/D a set of evenly distributed weights is employed. These weight vectors serve as reference lines. A Pareto optimal solution along each reference line is expected to be obtained, see Fig. 1. The effect of MOEA/D components, i.e., the weight vectors and scalarizing methods, has been intensively studied, and is briefly reviewed below.

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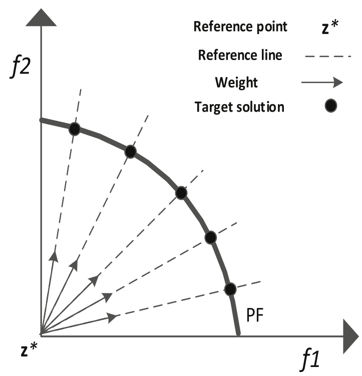


Fig. 1. Illustration of the reference point, reference lines, weights and target solutions.

- Studies with respect to weight vectors: in [21–23] the relation between weight vectors and their obtained solutions is investigated. The main findings are that (i) evenly distributed weights do not always lead to evenly distributed solutions; (ii) under a certain scalarizing method, an optimal distribution of weight vectors exists in order to obtain evenly distributed solutions for a given PF shape. As the PF shape is often unknown in advance, how to adaptively adjust the distribution of weight vectors according to an estimated PF shape are studied, e.g., [13,24,25,14]. In the light of this aspect, there are also methods that adopt the idea of co-evolution to adjust weight vectors, i.e., co-evolving weight vectors with solutions during the search [26,27].
- Studies with respect to scalarizing methods: the weighted sum (WS) method is one of the earliest scalarizing methods. It is reported in [28] that the WS method is ineffective on non-convex PFs. In [29,30] constraints are introduced during the search to handle this shortcoming. Moreover, a localized WS method is proposed in [19] which can effectively address this issue, and offers competitive performance for many-objective optimization. In addition to the WS method, the Chebyshev and penalty boundary intersection (PBI) methods are two other frequently used scalarizing methods. The Chebyshev method can handle non-convex PFs, however, it is inferior to the WS method in terms of convergence property. Moreover, in [9,31] the effect of an adaptive and a simultaneous use of Chebyshev and WS methods is investigated, respectively. Both studies try to utilize advantages of Chebyshev and WS methods, meanwhile to eliminate their disadvantages. In [20,32,33] the effect of different L_p scalarizing methods is investigated. It is found that as the parameter p decreases (from inf to 1), the L_p scalarizing method becomes more efficient in pushing solutions towards the PF while less efficient in handling various PF shapes. As a consequence, the use of a Pareto adaptive L_p scalarizing method is proposed in [20]. With respect to the PBI method, Ishibuchi et al. [10,34] investigate the effect of different penalty values to the PBI performance. It is found that the penalty value plays a crucial role in PBI. Its effect is similar to the p value in the L_p scalarizing method. To improves the performance of PBI method (mitigating the limitation of PBI with small penalty values), the study [35] suggests a localized PBI method, and is shown as effective.

In addition to weights vectors and scalarizing methods, the effect of reference point which serves as the starting point of reference lines is rarely studied. In general, it is a good choice to adopt the *ideal* point which is composed of the best value of each objective function as the reference point. However, since the *ideal* point is usually unknown in advance, one has to approximate the specification of the reference point. It is worth mentioning that there are studies e.g., [36] that employ the *nadir* point as the starting point of reference directions. Such method suits for problems with inverted Pareto front [37]. Furthermore, the simultaneous use both the *ideal* and *nadir* as reference points is also investigated, and is shown to be effective to improve the performance of MOEA/D [38,39]. However, these studies also suffer from the estimation of *ideal* and/or *nadir* points.

Thus, in this study investigate how to appropriately set the reference point so as to mitigate the effect induced by the imprecise estimation of *ideal* and/or *nadir* points. Note that since the use of *ideal* point is still the main stream in MOEA/D [8] and many other decomposition-based algorithms, e.g., [13–15], this study, as the first step along this direction, focuses only on the *ideal* point. Concretely, from the perspective of *ideal* point, the reference point is usually specified by Eq. (1), taking minimization problems as an example.

$$\begin{aligned} z_i^* &= z_i^{\min} - \epsilon_i, \quad \epsilon_i \geq 0 \\ z_i^{\min} &= \min\{f_i(\mathbf{x}), \mathbf{x} \in \mathbb{S}\}, \quad i = 1, 2, \ldots, m \end{aligned} \tag{1}$$

where m is the number of objective functions, z_i^* is the ith component of the reference point $\mathbf{z}^* = (z_1^*, z_2^*, \ldots, z_m^*)$. $z_i^{\min}$ is the minimum objective value of f_i amongst all examined solutions so far. $\mathbb{S}$ consists all examined solutions. ϵ_i is a small but computationally significant non-negative number, e.g., $\epsilon_i = 0$ in [8] and $\epsilon_i > 0$ in [13–15]. Moreover, ϵ_i is often fixed during the entire search process.[1]

We argue that Eq. (1) with a constant and small setting of ϵ_i might be inappropriate as the delivered value of $\mathbf{z}^*$ may be quite different from the *ideal* point, especially in the early stage of the evolution, for some real-world problem. Intuitively, a larger ϵ_i is preferred in the early stage, and the value can gradually decrease as the search progresses. This study is going to systematically examine the effect of ϵ_i, i.e., the effect of reference point adjustment, in MOEA/D. The main contributions of this paper are as follows.

- The effect of the reference point specified in three representative manners, i.e., pessimistic (a small ϵ_i), optimistic (a large ϵ_i) and dynamic (ϵ_i varies from a large value to a small value) in MOEA/D is systematically examined on three sets of benchmark problems. Each set of the problems feature different degrees of difficulty in convergence or diversity. The pessimistic reference point specification strategy which is widely used in literature does not perform well as expected, especially for MOPs with great difficulty in diversity.
- The reference point implicitly impacts the search effort distributed to exploit existing regions and explore new regions. The pessimistic specification emphasizes more of exploiting existing regions. The optimistic specification emphasizes more of exploring new regions.
- The dynamic reference point specification can strike a good balance between exploration and exploitation, and thus, show good and robust performance for most of test problems.

The remainder of this paper is organized as follows. Section 2 provides background knowledge. Section 3 analyzes the effect of three reference point specification strategies in MOEA/D, namely, the pessimistic, optimistic, and dynamic strategies. Sections 4–6 provide experiment description, results and discussions, respectively. Finally, Section 7 concludes this paper and identifies future studies.

[1] Note that in some studies, e.g. [23], $\mathbf{z}^*$ is specified with the form $z_i^* = \alpha_i \times z_i^{\min}$ where α_i is a scalar that is smaller but not much smaller than 1, e.g., 0.99. The two forms essentially have the same effect.

2. Related work

Without loss of generality, a minimization MOP is written as follows.

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})) \\ &\text{subject to } \mathbf{x} \in \boldsymbol{\Omega} \end{aligned} \tag{2}$$

where $\mathbf{x}$ is a decision vector in a feasible region $\boldsymbol{\Omega}$, $\mathbb{R}^m$ refers to the objective space. $\mathbf{F} : \boldsymbol{\Omega} \rightarrow \mathbb{R}^m$ consists of m real-valued objective functions.

Pareto-dominance: $\mathbf{x}$ is said to Pareto dominate $\mathbf{y}$, denoted by $\mathbf{x} \preceq \mathbf{y}$, if and only if $\forall i \in \{1, 2, \ldots, m\}$, $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ and $f_j(\mathbf{x}) < f_j(\mathbf{y})$ for at least one index $j \in \{1, 2, \ldots, m\}$.

Pareto optimal solution: A solution $\mathbf{x}^* \in \boldsymbol{\Omega}$ is said to be Pareto optimal if and only if $\nexists \mathbf{x} \in \boldsymbol{\Omega}$ such that $\mathbf{x} \preceq \mathbf{x}^*$. The set of all Pareto optimal solutions is called the Pareto optimal set (PS). The set of all Pareto optimal vectors, $\text{PF} = \{\mathbf{F}(\mathbf{x}) \in \mathbb{R}^m | \mathbf{x} \in \text{PS}\}$, is called the Pareto optimal front (PF).

Ideal point: An objective vector $\mathbf{z}^{ide} = (z_1^{ide}, \ldots, z_m^{ide})$ where z_i^{ide} is the infimum of $f_i(\mathbf{x})$, $\mathbf{x} \in \boldsymbol{\Omega}$ for every $i \in \{1, 2, \ldots, m\}$.

Nadir point: An objective vector $\mathbf{z}^{nad} = (z_1^{nad}, \ldots, z_m^{nad})$ where z_i^{nad} is the supremum of $f_i(\mathbf{x})$, $\mathbf{x} \in \text{PS}$ for every $i \in \{1, 2, \ldots, m\}$.

2.1. Introduction to MOEA/D

MOEA/D decomposes an MOP into a set of single-objective problems which are defined by means of weighted scalarizing methods. The pseudo-code of MOEA/D is shown in Algorithm 1. These single-objective problems are solved in a collaborative manner. Though a number of scalarizing methods are available for decomposing MOPs, the weighted sum, Chebyshev and PBI methods are three mostly used ones [8]. In this study the Chebyshev method is used for illustration. The Chebyshev method is written as follows.

$$\begin{aligned} &g^{ch}(\mathbf{x}|\mathbf{w}) = \max_{i=1}^{m} \left\{ \lambda_i \frac{f_i - z_i^{ide}}{z_i^{nad} - z_i^{ide}} \right\} \\ &\lambda_i = \left(\frac{1}{w_i} \right) \end{aligned} \tag{3}$$

where $\mathbf{w} = (w_1, w_2, \ldots, w_m)$ is a weight vector and $\sum_{i=1}^{m} w_i = 1, \quad w_i > 0$. The *ideal* point is used as reference point z_i^*. By optimizing $g^{ch}(\mathbf{x}|\mathbf{w})$ with different weight vectors, a set of diversified Pareto optimal solutions (along each reference line) could be obtained.

When the *ideal* and *nadir* points are unavailable, $z_i^{\min}$ and $z_i^{\max}$ which refer to the minimum and maximum objective value amongst the current population, respectively, could be used as approximated *ideal* and *nadir* points. Specifically, Eq. (3) is rewritten as follows:

$$\begin{aligned} &g^{ch}(\mathbf{x}|\mathbf{w}) = \max_{i=1}^{m} \left\{ \lambda_i \frac{f_i - z_i^*}{z_i^{\max} - z_i^*} \right\} \\ &\lambda_i = \left(\frac{1}{w_i} \right), \quad z_i^* = z_i^{\min} - \epsilon_i \end{aligned} \tag{4}$$

Since the approximations are not always accurate especially at the beginning of the search, in this study we are going to study how to set z_i^* (i.e., the ϵ_i) appropriately such that the shortcoming of the inaccurate approximations can be mitigated. Besides, to facilitate the analysis of ϵ_i (as will be shown in Section 3), Eq. (4) is further rewritten as follows.

$$\begin{aligned} &g^{ch}(\mathbf{x}|\mathbf{w}) = \max_{i=1}^{m} \left\{ \lambda_i \frac{f_i - z_i^{\min}}{z_i^{\max} - z_i^{\min}} - \epsilon_i \right\} \\ &\lambda_i = \left(\frac{1}{w_i} \right) \end{aligned} \tag{5}$$

Algorithm 1. MOEA/D

Input: Population size, N, Selection neighbourhood size, ns, replacement neighbourhood size, nr, maximum generations, T
Output: S

Initialize weights, $W \leftarrow \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N\}$, and solutions $S \leftarrow \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$;
Evaluate the objective values of the initial S;
Randomly assign each weight, $\mathbf{w}_i$ with a candidate solution, $\mathbf{x}_i$;
Calculate the Euclidean distance between weights, $\mathbf{w}_i$ and $\mathbf{w}_j$, $i, j \in 1, 2, \ldots, N$;
Find ns neighbouring weights $B(\mathbf{w}_i)$ of $\mathbf{w}_i$ and identify the associated neighbouring solutions $B(\mathbf{x}_i)$ of $\mathbf{x}_i$;
Set the mating pool $Q = \emptyset$, the probability of mating restriction $\delta = 0.8$;
while $t \leq T$ **do**
  Set the reference point $\mathbf{z}^*$ by Eq. (1);
  for $i \leftarrow 1$ **to** N **do**
    if $rand < \delta$ **then**
      $Q \leftarrow B(\mathbf{x}^i)$;
    else
      $Q \leftarrow S$;
    end
    Generate a new solution $\mathbf{x}_{new}$ by applying the simulated binary crossover and polynomial mutation operators to solutions selected from Q;
    Evaluate the objective value of $\mathbf{x}_{new}$;
    Normalize all solutions using Eq. (5) which ensures that $f_i \in [0, 1]$;
    $c \leftarrow 0$;
    while $c \leq nr || Q \neq \emptyset$ **do**
      randomly pick an index j from Q;
      if $g^{ch}(\mathbf{x}_{new}|\mathbf{w}_i, \mathbf{z}^*) < g^{ch}(\mathbf{x}_j|\mathbf{w}_i, \mathbf{z}^*)$ **then**
        $\mathbf{x}_j \leftarrow \mathbf{x}_{new}$;
        $c \leftarrow c + 1$
      end
      remove $\mathbf{x}_j$ from Q;
    end
  end
end

2.2. Existing studies on the reference point specification

The effect of reference point specification has not been actively studied in literature. We are aware of only two studies [23,40] performed by the third author and his colleagues. In [23], the form of $z^i = \alpha_i \times \min\{f_i(\mathbf{x}), \mathbf{x} \in \mathbb{S}\}$ is adopted in MOEA/D. Three settings, i.e., $\alpha_i = \{1.0, 1.01, 1.1\}$ are examined for multi-objective maximization knapsack problems. A higher hypervolume (HV) [41] value is obtained by $\alpha_i = 1.1$. Likewise, $\alpha_i = \{0.9, 0.99, 1.0\}$ are examined for DTLZ2 problem. However, no significant improvement is observed. In [40] reference point specifications in Eq. (6) for minimization are examined.

$$z_i^* = z_i^{\min} - \beta(z_i^{\max} - z_i^{\min}), \quad \beta = \beta_0 \left(\frac{T - t}{T - 1} \right) \tag{6}$$

where β decreases from an initial value β_0 to zero along the search. T is maximum generations and t is the current generation index. The considered settings of β_0 are $\{0, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10\}$. In addition, constant settings of $\beta = \beta_0$ are also examined. Again, good results are found for a changing β for knapsack problems while no clear improvement is obtained for DTLZ problems. Though the above two studies have empirically reported some preliminary results with respect to the effect of reference point specifications in MOEA/D, reasons that lead to the corresponding effects are not analyzed.

3. Analysis of the effect of the reference point in MOEA/D

In general, to facilitate a *posteriori* decision making, an evolutionary multi-objective algorithm tries to drive solutions toward the PF as close as possible (i.e., good convergence), and spread them along the entire PF as much as possible (i.e., good diversity). Unfortunately, MOEAs often struggle to balance convergence and diversity performance.

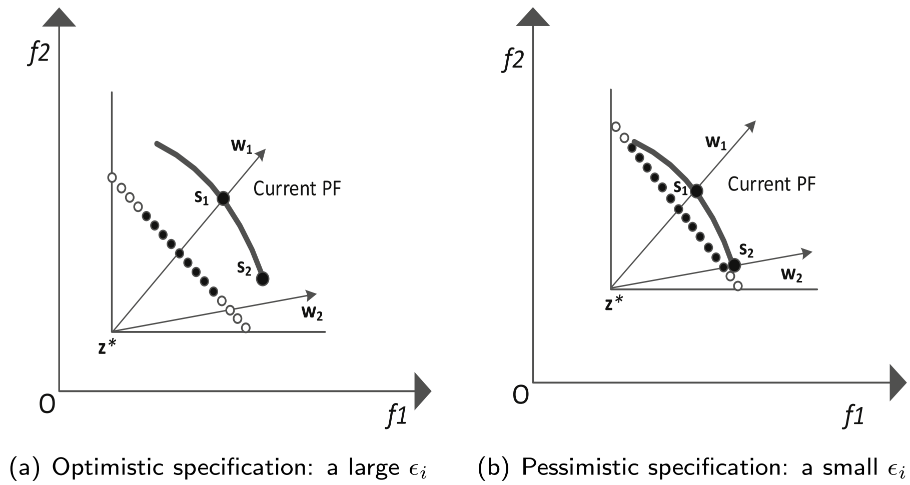


Fig. 2. Optimistic and pessimistic specifications of reference point.

The reference point in MOEA/D, to some extent, influences the algorithm's convergence and spread performance. This can be explained by Fig. 2. The reference point determines the search region of MOEA/D in the objective space. That is, the amount of search effort distributed to explore new regions or to exploit exiting regions is controlled by the reference point.

More specifically, if the reference point is specified in an optimistic manner, i.e., using a large ϵ_i in Eq. (1), many reference lines do not intersect with the approximated PF as shown by open circles in Fig. 2(a). Those reference lines, e.g. $\mathbf{w}_2$, can be categorized as exploration related, being beneficial to improve spread performance. On the contrary, if the reference point is specified in a pessimistic manner, i.e., using a small ϵ_i in Eq. (1), as shown in Fig. 2(b), though the exploitation ability might become more efficient, the exploration ability of MOEA/D is less efficient than the optimistic specification. Actually, most of MOEA/D related algorithms employ a pessimistic reference point specification.

Next we empirically examine the effect of different reference point specifications in MOEA/D. Specifically, various settings of ϵ_i, i.e., $\epsilon_i = \{10, 5, 1, 0.1, 0.01, 0.005, 0.001, 0.0001, 0\}$ are examined. Unless otherwise specified, $\epsilon_1 = \epsilon_2 =, \ldots, = \epsilon_m$ in this study. Since all objective values are normalized within [0, 1] by Eq. (5), these settings gives that $\mathbf{z}^* = \{-10, -5, -1, -0.1, -0.01, -0.005, -0.001, -0.0001, 0\}$, respectively, which loosely represent different degrees of reference point specifications, i.e., from optimistic to pessimistic. In addition, $\mathbf{z}^* = \mathbf{z}^{ide}$ is considered as a reference. The 2-objective WFG4 (Walking Fish Group) problem [42] under three different parameter settings (k and l) are taken as test instances. Each test instance represents a different type of problem, having different degrees of difficulty in convergence or spread (more details are provided in the next section). For each of the ϵ_i setting, MOEA/D is run for 31 times, and each run with 250 generations. Other settings are the same as [11]. The hypervolume metric [41] is applied to assess the algorithm performance. From Fig. 3, we can clearly observe that different settings of ϵ_i lead to different results.

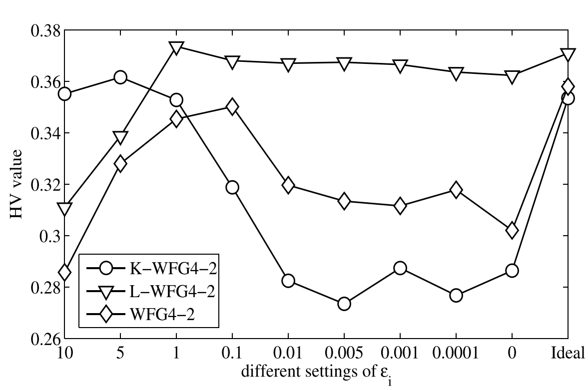


Fig. 3. Performance of MOEA/D (measured by hypervolume (HV) metric, the larger the better) with different reference point specifications for 2-objective WFG4 problems. The WFG problem parameters are set as $k=96$ and $l=4$ in K-WFG, $k=4$ and $l=96$ in L-WFG, and $k=48$ and $l=48$ in WFG.

The pessimistic specification (e.g., $\epsilon_i = 0$ or 0.001) that is used in many studies, e.g., [8,13–15] is not always effective, especially for K-WFG and WFG problems. Whereas the *ideal* point looks the best specification for WFG problems, the best results are obtained from $\epsilon_i = 1$ for L-WFG and $\epsilon_i = 5$ for K-WFG. Therefore, an appropriate setting of ϵ_i, i.e., reference point specification, can be helpful to improve the performance of MOEA/D.

As explained in Fig. 2, intuitively, the optimistic specification helps improve the exploration ability; and the pessimistic specification helps improve the exploitation ability. If the feature of an MOP is difficult to identify, i.e., whether pushing solutions towards the PF or spreading solutions along the PF is more difficult, a dynamic reference point setting from optimistic to pessimistic is a good choice. In other words, in the early stage of the search, the optimistic specification is preferred while in the late stage of the search, the pessimistic specification is preferred. This is because that in the early stage $\mathbf{z}_{min}$ often does not approximate the *ideal* point well, and thus, one can specify the reference point more optimistically, i.e., using a larger ϵ_i.

4. Experiment description

4.1. Test problems and parameter settings

The WFG test problems from 2 to 9 with two, three and four objectives [42] are applied to examine the effect of the reference point in MOEA/D. Properties of these problems include separability or nonseparability, unimodality or multimodality, unbiased or biased parameters, and convex or concave geometries. More precisely, three sets of WFG problems are generated, see Table 1. Each

Table 1
Test problems.

Type	Number of decision variables
K-WFG	$n=100$ ($k=96$, $l=4$)
L-WFG	$n=100$ ($k=6$, $l=94$)
WFG	$n=96$ ($k=48$, $l=48$)

Table 2
Settings of genetic operators and other parameter.

Parameters	Settings
Simulated binary crossover (SBX [44])	$p_c = 1$, $\eta = 30$
Polynomial mutation (PM [4])	$p_m = 1/n$, $\eta = 20$
MOEA/D mating restriction probability	$\delta = 0.8$
MOEA/D selection neighborhood size	$ns = 10$
MOEA/D replacement size	$nr = 2$

set is configured with a different number of WFG position (k) and distance (l) parameters. The first set is with $k = 96$ and $l = 4$, the second set is with $k = 6$ and $l = 94$ and the third set is with $k = 48$ and $l = 48$. A large k creates difficulty in diversity, and a large l creates difficulty in convergence. Thus, the first set of problems, denoted as K-WFG, feature easy convergence while difficult diversity. The second set of problems, denoted as L-WFG, feature easy diversity while difficult convergence. The third set of problems, denoted as WFG, feature comparable difficulty in both convergence and diversity. Hereafter, we use WFGh-m to denote the problem WFGh with m objectives where h refers to the index of WFG problem.

In order to make a statistical analysis, MOEA/D is run for 31 runs on each test problem. In each algorithm run, the maximum generations are $T = 250$. The genetic operators and other parameters are listed in Table 2, and are fixed across all algorithm runs. Evenly distributed weight vectors generated by the simplex-lattice method [43], see Eq. (7), are used in the experiment.

$$w_i \in \left\{0, \frac{1}{H}, \frac{2}{H}, \dots \frac{H}{H}\right\}, \quad i = 1, 2, \dots, m \tag{7}$$
$$w_1 + w_2 + \dots + w_m = 1$$

Given a positive integer H, the number of generated weight vectors is C_{H+m-1}^{m-1} where C stands for the combinatorial formula. H is set to 99, 13 and 9 which lead the number of weights (also the population size, N) of 100, 105 and 220 for 2-, 3- and 4-objective problems, respectively.

4.2. Reference point specifications

MOEA/D with optimistic, pessimistic and dynamic reference point specifications, denoted as MOEA/D-ORP, MOEA/D-PRP and MOEA/D-DRP, respectively, are examined. Since all solutions are normalized by Eq. (5) during the search, objective values are within [0, 1], that is, $\mathbf{z}^{min} = (0, \dots, 0)$. Thus, $\epsilon_i = 1$ and $\epsilon_i = 0.001$ could represent optimistic $\mathbf{z}^* = (-1, \dots, -1)$ and pessimistic $\mathbf{z}^* = (-0.001, \dots, -0.001)$ specifications, respectively. Also, Eq. (8) is employed to represent a dynamic reference point specification, that is, ϵ_i linearly decreases from $\epsilon_i^{ini} = 1$, an optimistic specification, to $\epsilon_i^{end} = 0.001$, a pessimistic specification, as the search progresses. Note that reference point specifications (ϵ_i) could also change non-linearly, and with different initial and final settings. These will be examined in Section 6.2.

$$z_i^* = z_i^{\min} - \epsilon_i$$
$$\epsilon_i = (\epsilon_i^{ini} - \epsilon_i^{end}) \left(\frac{T-t}{T-1}\right) + \epsilon_i^{end}, \quad \epsilon_i^{ini} = 1, \quad \epsilon_i^{end} = 0.001. \tag{8}$$

4.3. Performance assessment

In order to demonstrate that the DRP strategy can strike a good balance between exploration and exploitation. The hypervolume (HV) metric [41] is used to evaluate the performance of algorithms. The HV metric measures the volume of the space enclosed by the approximated PF and a given reference vector, $\mathbf{z}_{HV}^{ref}$. A favourable HV value (larger, for a minimization problem) implies good convergence with diversity. In this study, the method and software developed by Fonseca et al. [45] is applied to calculate the HV. Prior to the calculation, all solutions are normalized by the true *ideal* and *nadir* points which are (0, 0, . . ., 0) and (2, 4, . . ., 2m), respectively (which assumes equal relative importance of normalized objectives across the search domain). Therefore, the PF is included in a m-dimensional unit hypercube $[0, 1]^m$. Then, the reference vector $\mathbf{z}_{HV}^{ref}$ is set to (1.1, 1.1, . . ., 1.1). Readers are referred to [46,47] for details of the dependency of HV value and the chosen reference vector.[2] For each run, the HV is calculated using all non-dominated solutions found during the search, however, for reasons of computational feasibility, the set of non-dominated solutions is pruned to a maximum size of N using the SPEA2 truncation procedure [48].

5. Experimental results

5.1. Attainment surfaces

Qualitatively, median attainment surfaces [49] are plotted to visualize the performance of MOEA/D-PRP, MOEA/D-ORP and MOEA/D-DRP on 2-objective instances, see Fig. 4. These allow visual inspection of the effect of different reference point specifications.

From Fig. 4 it is observed that for the K-WFG4-2 problem which features difficulty for algorithms in spreading well along the PF, the ORP leads to the best results, followed by the DRP strategy, then the PRP strategy. The three strategies show comparable convergence performance; For the L-WFG4-2 problem which features difficulty for algorithms in converging to the PF, the PRP strategy offers better convergence than the OPR and DRP strategies. The ORP strategy is the worst in terms of convergence. The three strategies have comparable spread performance; For the WFG4-2 problem which features comparable difficulty in convergence and spread. The PRP strategy offers the best convergence performance while it is worse than PRP and DRP in terms of diversity. The DRP strategy offers better convergence performance than the ORP strategy, and the two strategies lead to comparable diversity performance.

Therefore, it can be tentatively concluded that the DRP strategy strikes a good balance between convergence and diversity, and tends to be robust for different types of problems. The ORP strategy is good in terms of diversity, however, its convergence is the worst amongst the three strategies. On the contrast, the PRP strategy (which is frequently used in literature) shows good convergence but poor diversity for some problems, e.g., K-WFG type problems. In addition, for comparison the same computational experiments where the reference point $\mathbf{z}^* = 0$ is performed. Almost the same results are obtained from $\mathbf{z}^* = 0$ as those from $\mathbf{z}^* = -0.001$ (i.e., the PRP strategy).

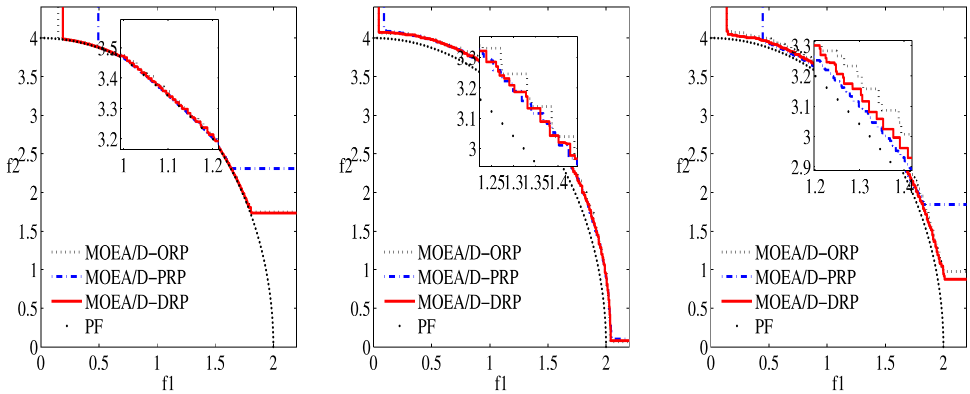
Though only attainment surfaces for 2-objective WFG4 and its variant problems are shown, similar results are observed for other problems, and they are provided as supplementary material.

5.2. HV results

Next we quantitatively discuss the effect of ORP, PRP and DRP strategies based on the HV results. The mean HV values of MOEA/D-ORP, MOEA/D-PRP and MOEA/D-DRP are shown in Table 3 for all test problems. The non-parametric Wilcoxon–Ranksum two-sided method at the 95% confidence level is applied to test whether the results are statistically different.

From Table 3 the following results are observed.

[2] We have also provided the HV results with different reference vectors as supplementary material.



(a) K-WFG4-2 (b) L-WFG4-2 (c) WFG4-2

Fig. 4. Attainment surfaces of MOEA/D-ORP, MOEA/D-PRP and MOEA/D-DRP for K-WFG4-2, L-WFG4-2 and WFG4-2.

- For K-WFG test problems, MOEA/D-DRP is worse than MOEA/D-ORP for six 2-objective instances, but it tends to beat MOEA/D-ORP for 3- and 4-objective instances. Specifically, MOEA/D-DRP is better than MOEA/D-ORP for four 3-objective and eight 4-objective problems. Regarding MOEA/D-PRP, MOEA/D-DRP completely dominates it.
- For L-WFG test problems, MOEA/D-DRP performs better than MOEA/D-ORP for three 2-objective, two 3-objective and six 4-objective problems. It performs better than MOEA/D-PRP for five 2-objective, five 3-objective and seven 4-objective problems.
- For WFG test problems, MOEA/D-DRP is only worse than MOEA/D-ORP for WFG2-3 and WFG6-3. For all the other problems, MOEA/D-DRP is superior or comparable to MOEA/D-ORP. With respect to MOEA/D-PRP, MOEA/D-DRP beats it for all problems except for WFG7-3 and WFG8-3 where the two algorithms perform comparably.

Overall, the PRP strategy which is widely used in existing studies has not shown good performance as expected. Moreover, this strategy evidently is not a good choice for K-WFG type problems. The ORP strategy is acceptable for 2-objective problems. The DRP strategy is the most robust which performs well on most of problems, in particular for 4-objective problems. This indicates that the DRP strategy, by employing a linear dynamic process, strikes a good balance between the exploration and exploitation.

6. Discussions

Two issues are further studied, as part of a wider discussion on the effect of the reference point specification: (i) convergence and diversity performance led by the three strategies, and (ii) variants of dynamic specifications.

Table 3
The mean HV values of MOEA/D-ORP, MOEA/D-PRP and MOEA/D-DRP for all test problems. The symbol '–', '=' or '+' means the considered algorithm is statistically worse than, comparable to or better than MOEA/D-DRP.

		$m=2$			$m=3$			$m=4$		
		ORP	PRP	DRP	ORP	PRP	DRP	ORP	PRP	DRP
K-WFG	2	0.6401^{-}	0.6255^{-}	0.6459	$0.8106^{=}$	0.8011^{-}	0.8122	0.9079^{-}	0.9081^{-}	0.9165
	3	0.6451^{+}	0.5433^{-}	0.6344	$0.7771^{=}$	0.7463^{-}	0.7750	0.9148^{-}	0.9017^{-}	0.9222
	4	0.3576^{+}	0.3006^{-}	0.3541	0.4924^{-}	0.5080^{-}	0.5320	0.6082^{-}	0.6143^{-}	0.6424
	5	0.3341^{+}	0.2730^{-}	0.3288	$0.5394^{=}$	0.4922^{-}	0.5326	0.6655^{-}	0.6522^{-}	0.6983
	6	0.3728^{+}	0.2949^{-}	0.3633	0.5866^{-}	0.5408^{-}	0.5907	0.7395^{-}	0.6901^{-}	0.7898
	7	$0.3474^{=}$	0.3072^{-}	0.3468	0.4959^{-}	0.4985^{-}	0.5036	0.6457^{-}	0.6405^{-}	0.6720
	8	0.2664^{+}	0.2193^{-}	0.2483	$0.4186^{=}$	0.3869^{-}	0.4128	0.5658^{-}	0.5558^{-}	0.5832
	9	0.3978^{+}	0.3711^{-}	0.3957	0.6086^{-}	0.5966^{-}	0.6113	0.7123^{-}	0.7290^{-}	0.7453
$^{+}/^{=}/^{-}$		6/1/1	0/0/8		0/4/4	0/0/8		0/0/8	0/0/8	
L-WFG	2	0.6469^{+}	0.6202^{-}	0.6328	0.9438^{+}	1.0281^{+}	0.8972	$1.2864^{=}$	1.2579^{-}	1.3003
	3	0.5872^{-}	0.5842^{-}	0.6039	$0.7075^{=}$	0.7305^{+}	0.6923	0.8159^{-}	0.8319^{+}	0.8222
	4	$0.3728^{=}$	$0.3725^{=}$	0.3725	0.6537^{+}	0.6095^{-}	0.6405	0.9540^{+}	0.8575^{-}	0.9442
	5	0.3099^{-}	0.3104^{-}	0.3184	$0.5239^{=}$	0.5337^{+}	0.5252	0.8279^{-}	0.7658^{-}	0.8363
	6	0.3716^{+}	0.3617^{-}	0.3677	0.6599^{+}	0.5874^{-}	0.6475	0.8768^{-}	0.9048^{-}	0.9731
	7	0.3776^{-}	$0.3727^{=}$	0.3766	0.6862^{+}	0.6415^{-}	0.6533	0.9953^{-}	0.9544^{-}	1.0180
	8	0.2834^{+}	$0.2868^{=}$	0.2919	0.4870^{-}	0.5038^{-}	0.5186	0.7255^{-}	0.7399^{-}	0.8086
	9	$0.3800^{=}$	0.3761^{-}	0.3824	0.6342^{-}	0.6096^{-}	0.6550	0.8827^{-}	0.8671^{-}	0.9261
$^{+}/^{=}/^{-}$		3/2/3	0/3/5		4/2/2	3/0/5		1/1/6	1/0/7	
WFG	2	0.5573^{-}	0.5510^{-}	0.5645	0.7131^{+}	0.6977^{-}	0.7012	$0.8246^{=}$	0.8172^{-}	0.8288
	3	$0.5910^{=}$	0.5130^{-}	0.5971	0.6870^{-}	0.6865^{-}	0.6964	$0.8463^{=}$	0.8051^{-}	0.8499
	4	0.3490^{-}	0.3105^{-}	0.3621	0.4822^{-}	0.4932^{-}	0.5077	0.6262^{-}	0.5984^{-}	0.6592
	5	0.3185^{-}	0.2849^{-}	0.3233	0.4566^{-}	0.4528^{-}	0.4632	0.6746^{-}	0.6037^{-}	0.6933
	6	0.3491^{-}	0.3173^{-}	0.3585	0.5433^{+}	0.5136^{-}	0.5387	0.7461^{-}	0.6771^{-}	0.7821
	7	0.2813^{-}	0.2747^{-}	0.2910	$0.4423^{=}$	$0.4430^{=}$	0.4466	0.6840^{-}	0.6759^{-}	0.7074
	8	0.2090^{-}	0.2057^{-}	0.2135	$0.3728^{=}$	$0.3692^{=}$	0.3700	$0.5959^{=}$	0.5598^{-}	0.6008
	9	$0.3601^{=}$	0.3485^{-}	0.3613	0.5282^{-}	0.5347^{-}	0.5541	0.6916^{-}	0.7095^{-}	0.7443
$^{+}/^{=}/^{-}$		0/2/6	0/0/8		2/2/4	0/2/6		0/3/5	0/0/8	

Table 4
The mean GD and MS values of MOEA/D-ORP, MOEA/D-PRP and MOEA/D-DRP for WFG4 and its variant test problems. The symbol '–', '=' or '+' means the considered strategy is statistically worse than, comparable to or better than the DRP strategy under the Wilcoxon–Ranksum statistical test at the 95% confidence level.

		K-WFG			L-WFG			WFG		
		4-2	4-3	4-4	4-2	4-3	4-4	4-2	4-3	4-4
GD	ORP	0.0008⁻	0.0037⁻	0.0036⁻	0.0065⁻	0.0139⁻	0.0066⁻	0.0055⁻	0.0195⁻	0.0081⁻
	PRP	0.0002⁺	0.0030⁺	0.0032⁺	0.0059⁺	0.0105⁻	0.0049⁻	0.0029⁺	0.0146⁺	0.0074⁻
	DRP	0.0004	0.0034	0.0033	0.0061	0.0095	0.0044	0.0039	0.0150	0.0066
MS	ORP	0.7089⁺	0.7388⁻	0.7596⁻	1.1200⁺	1.0031⁼	1.0153⁺	0.8785⁺	0.9192⁻	0.9242⁻
	PRP	0.4509⁻	0.7264⁻	0.7747⁼	1.0060⁼	1.0071⁺	1.0125⁼	0.5746⁻	0.8987⁻	0.9240⁻
	DRP	0.7011	0.7946	0.7757	1.0195	1.0029	1.0066	0.8656	0.9297	0.9362

Table 5
The mean HV values of MOEA/D-DRP and MOEA/D-IDRP for WFG4 and its variant test problems. The symbol '–', '=' or '+' means MOEA/D-IDRP is statistically worse than, comparable to or better than MOEA/D-DRP under the Wilcoxon–Ranksum statistical test at the 95% confidence level.

		K-WFG			L-WFG			WFG		
		4-2	4-3	4-4	4-2	4-3	4-4	4-2	4-3	4-4
HV	IDRP	0.3439⁻	0.4829⁻	0.6162 ⁻	0.3716⁻	0.6416	0.9437	0.3429⁻	0.4959⁻	0.6259⁻
	DRP	0.3541	0.5320	0.6424	0.3725	0.6405⁻	0.9442	0.3621	0.5077	0.6592

6.1. Convergence and diversity

The convergence and diversity performance measured by the generational distance (GD) [50] and maximum spread metric (MS) [51], respectively are shown in Table 4. It is observed that,

- in terms of the GD metric (the smaller the better), the PRP strategy shows the best performance (as expected) for a majority of problems. The DRP also offers good convergence for some problems. The ORP shows the least well convergence for all problems. In addition, between the PRP and ORP strategies, the PRP is better for all problems.
- in terms of the MS metric (the larger the better), the ORP strategy is the best for all 2-objective problems. However, for 3- and 4-objective problems, the DRP shows better performance. Comparing the ORP with the PRP, the ORP strategy is better for most of problems.

Though only the results for K-WFG4, L-WFG4 and WFG4 problems are shown, similar results are obtained for most of other problems. Thus, we can further confirm that the PRP, being good in exploitation, helps improve the convergence performance while sacrifice the spread performance. The ORP, on the contrary, being good in exploration, helps improve the spread performance while degrade the convergence performance. An interesting observation is that the OPR strategy shows inferior spread performance than the DRP strategy for the K-WFG and WFG problems with over two objectives. The reason might be that the ORP strategy emphasizes too much on exploration such that solution near the centre of the PF are lost, resulting in a poor spread performance. The DRP strategy relatively makes a good balance between convergence and spread performance, and thus, is recommended to use for new problems.

6.2. Different dynamic specifications

6.2.1. Different initial ϵ_i^{ini} and final ϵ_i^{end} settings

Previous experimental results have demonstrated the superiority of the DRP strategy when $\epsilon_i^{ini} = 1$ and $\epsilon_i^{end} = 0.001$. In this section, other settings of ϵ_i^{ini} and ϵ_i^{end} are examined.

First, we consider the case that settings of ϵ_i^{ini} and ϵ_i^{end} are reversed, i.e., $\epsilon_i^{ini} = 0.001$ and $\epsilon_i^{end} = 1$. This returns an inverse dynamic process (denoted as IDRP). That is, ϵ_i linearly increases from 0.001 (pessimistic specification) to 1 (optimistic specification) as the search progresses.

Comparison results between MOEA/D-DRP and MOEA/D-IDRP in terms of the HV metric for K-WFG4, L-WFG4 and WFG4 problems are shown in Table 5. It is observed from Table 5 that MOEA/D-IDRP is worse than MOEA/D-DRP for all problems except for L-WFG4-3. This clearly shows that the IDRP strategy is not as efficient as the DRP strategy.

Second, a wide variety of ϵ_i^{ini} and ϵ_i^{end} settings are examined, i.e., $\epsilon_i^{ini} \in \{10, 5, 1, 0.5, 0.1\}$ and $\epsilon_i^{end} \in \{0.01, 0.005, 0.001, 0.0005, 0.0001\}$. This returns 25 different settings. Note that ϵ_i^{ini} is set to be larger than ϵ_i^{end}. This is to ensure a decreased dynamic process, i.e., from optimistic to pessimistic. Experimental results in terms of the HV metric are shown in Fig. 5 for the 3-objective K-WFG4, L-WFG4 and WFG4 problems.

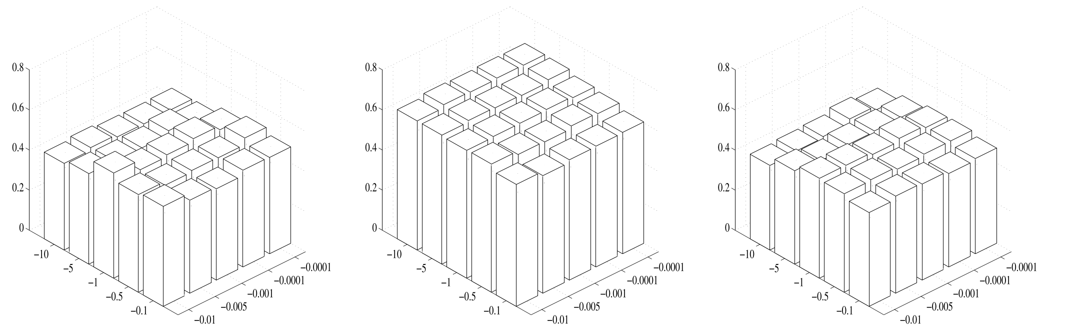
It is observed that MOEA/D-DRP is affected by ϵ_i^{ini} and ϵ_i^{end}. $\epsilon_i^{ini} = 1$ and $\epsilon_i^{end} = 0.001$ relatively exhibit good performance, though such settings are not the best for all problems. As for ϵ_i^{ini}, a large value, e.g., 10 or 5, is not appropriate for K-WFG and WFG problems. As for ϵ_i^{end}, there is no clear-cut observation amongst the considered settings. In addition, different settings of ϵ_i^{ini} (e.g., 10 and 1) lead to a large performance difference, however, different settings of ϵ_i^{end} (e.g., 0.001 and 0.0001) do not lead to a large performance difference. For example, the algorithm performance is comparable between the setting of $\epsilon_i^{ini} = 1, \epsilon_i^{end} = 0.001$ and $\epsilon_i^{ini} = 1, \quad \epsilon_i^{end} = 0.0001$, however, the algorithm performance is clearly different between the settings $\epsilon_i^{ini} = 10, \epsilon_i^{end} = 0.001$ and $\epsilon_i^{ini} = 1, \epsilon_i^{end} = 0.001$. Overall, from the experimental results the settings that $\epsilon_i^{ini} \in (0.1, 1)$ and $\epsilon_i^{end} \in (0.0001, 0.01)$ are recommended for new problems.

6.2.2. Non-linear change

This section examines the effect of a non-linear DRP controlled by Eq. (9). $\gamma \in \{0.2, 0.5, 2, 5\}$ are considered. $\gamma > 1$ indicates that the reference point increases fast in the early stage of evolution (the larger the faster) and slowly at late stage of the evolution. $\gamma < 1$ indicates the opposite.

$$\begin{aligned} z_i^* &= z_i^{\min} - \epsilon_i \\ \epsilon_i &= (\epsilon_i^{ini} - \epsilon_i^{end})\left(\frac{T-t}{T-1}\right)^{\gamma} + \epsilon_i^{end}, \quad \epsilon_i^{ini} = 1, \quad \epsilon_i^{end} = 0.001. \end{aligned} \tag{9}$$

MOEA/D with the four dynamic strategies are examined on the three sets of problems. For instance, the HV results for K-WFG4, L-WFG4 and WFG4 problems are shown in Table 6. It is observed that



(a) K-WFG4-3 (b) L-WFG4-3 (c) WFG4-3

Fig. 5. Performance of MOEA/D-DRP with different initial and final settings measured by the HV metric for K-WFG4-3, L-WFG4-3 and WFG4-3 problems.

for K-WFG problems MOEA/D-DRP (i.e., $\gamma = 1$) provides better performance than the other settings for most of problems, especially for the 3- and 4-objective problems. For the L-WFG problems, $\gamma = 1$ is better than $\gamma = 2$ and 5 but is inferior to $\gamma = 0.2$ and 0.5. For WFG problems, $\gamma = 1$ clearly beats $\gamma = 5$, and is better than or comparable to the other three settings for most of problems. Overall, the above results indicate that $\gamma \leq 1$ is in general superior to $\gamma > 1$. However, there is no clear- cut amongst $\gamma = 1$, 0.2, 0.5.

6.3. *Effect of* ϵ *with the true* ideal *point*

Under the assumption that the *ideal* point is not known in advance, we have demonstrated the superiority of the DRP strategy over the other two strategies. In this section we assume that the *ideal* point is known, and examine the effect of ϵ_i. Specifically, the reference point is specified as follows.

$$z_i^* = z_i^{ide} - \epsilon_i, \quad \epsilon_i \geq 0 \tag{10}$$

Three settings $\epsilon_i = 1, 0.001, 0$ are considered. Likewise, $\epsilon_i = 1$ represents the ORP strategy; $\epsilon_i = 0.001$ represents the PRP strategy; and $\epsilon_i = 0$ indicates that the *ideal* point is directly applied as reference point. In addition, the DRP strategy (ϵ_i varying from 1 to 0.001) is considered. Experimental results, i.e., the number of problems that an algorithm performs in terms of the HV metric, *nbest*, are summarized in Table 7.

From the results it is observed that $\epsilon_i = 1$ is obviously inferior to the other settings. $\epsilon_i = 0.001$ and 0 show almost comparable performance, however, they are slightly worse than the DRP strategy. Thus, even when the *ideal* point is known in advance, the DRP strategy still compares favourably against the other settings.

Table 6
The mean HV values of obtained by the four strategies. The symbol '–', '=' or '+' means the considered algorithm is statistically worse than, comparable to or better than MOEA/D-DRP ($\gamma = 1$) under the Wilcoxon–Ranksum statistical test at the 95% confidence level.

		$\gamma = 0.2$	$\gamma = 0.5$	$\gamma = 2$	$\gamma = 5$
K-WFG	4-2	0.3572^{+}	$0.3557^{=}$	0.3419^{-}	0.3350^{-}
	4-3	0.4845^{-}	0.4868^{-}	0.4883^{-}	0.4864^{-}
	4-4	0.6304^{-}	0.6360^{-}	0.6349^{-}	0.6076^{-}
L-WFG	4-2	0.3749^{+}	0.3731^{+}	$0.3718^{=}$	0.3714^{-}
	4-3	0.6540^{+}	0.6487^{+}	0.6325^{-}	0.6131^{-}
	4-4	0.9568^{+}	0.9595^{+}	0.9148^{-}	0.8816^{-}
WFG	4-2	$0.3618^{=}$	0.3595^{-}	0.3606^{-}	0.3597^{-}
	4-3	$0.5022^{=}$	$0.4997^{=}$	$0.5054^{=}$	0.4735^{-}
	4-4	0.6262^{-}	0.6475^{-}	$0.6563^{=}$	0.6101^{-}

Table 7
Comparison results for $\epsilon_i = 1$, 0.001, 0 and the DRP strategy when the *ideal* point is known. Note that the sum of *nbest* in each row might not be identical to 24 (the number of m-objective problems). The is because that for some problems more than one setting is statistically the best.

nbest	$\epsilon_i = 1$	$\epsilon_i = 0.001$	$\epsilon_i = 0$	DRP
$M = 2$	6	7	14	7
$M = 3$	2	9	3	15
$M = 4$	3	10	10	14

7. Conclusion

Multi-objective evolutionary algorithm based on decomposition (MOEA/D) has proven effective for a variety of MOPs. Due to its efficiency and simplicity, a number of studies with respect to MOEA/D have been proposed so far, for example, proposal of MOEA/D variants, investigation of MOEA/D's components, i.e., weight vectors, scalarizing methods. However, the reference point, as an important component of the MOEA/D, is far from being well studied. This study systematically investigates the effect of three representative strategies for the specification of reference point, i.e., pessimistic, optimistic and dynamic. MOEA/D with the three strategies are examined for three sets of problems generated using the WFG test suite with different k and l parameters. That is, K-WFG problems ($k = 96$, $l = 4$) which present easy convergence while difficult spread; L-WFG problems ($k = 6$, $l = 94$) which present easy spread while difficult convergence, and WFG problems ($k = 48$, $l = 48$) which present comparable difficulty in convergence and spread. Experimental results show that different reference point specifications lead to different performance of exploitation and exploration. The PRP strategy is inclined to exploitation while the ORP strategy is inclined to exploration. Experimental results show that neither of the two strategies offers good performance for various types of problems. The DRP strategy strikes a good balance between exploitation and exploration, and thus, can provide good performance for a wide variety of problems. Therefore, the DPR is recommend to use for unknown problems.

With respect to future studies, first we would like to assess the findings on other state-of-the-art decomposition-based algorithms (e.g., NSGA-III [16], θ-DEA [52], RVEA [53], MOEA/D-LWS [19]), and for other problem types e.g. multi-objective combinatorial problems, and some real-world problems, e.g., the size of renewable energy systems [54,55], scheduling problems [56,57]. Second, though the dynamic strategy shows a good performance in general, more effective methods could be developed, e.g., adaptively adjusting the reference point based on the improvement of PF, or based on the number of convergence and diversity related decision variables [58]. Third, the reference point impacts the search resources distributed in the search space. It therefore can be considered for devising strategies for search resources assignment [59] in MOEA/D. Lastly, the reference point might also be used to search for preferred solutions [60].

All experimental results and source codes are available at http://ruiwangnudt.gotoip3.com/optimization.html.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.asoc.2017.04.002.

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