Dear Editor:

Attached is the revised version of our paper "Differential Evolution with Enhanced Diversity Maintenance" (manuscript number: #OPT-D-18-00396).

We have studied the review report very carefully. The comments and suggestions provided are very interesting and, in our opinion, they have allowed us to improve on the general quality of the manuscript.

The rest of this document presents our detailed responses to the reviewers' comments.

We would like to thank you and the reviewers for the time invested in evaluating our work. We really appreciate all valuable reviewer's comments.

Sincerely,
The authors

Reviewer 1

This paper concerns a variant of differential evolution with enhanced diversity maintenance. The method has been well described, in proper context of the current literature. Results seem to be quite extensive as it's done for both CEC2016 and CEC2017. Mean distance measure has been used to check diversity. The paper can be accepted after some minor revision.

It's not quite clear to what extent the idea is new, how the proposed approach is from other methods of enhancing diversity in population?

Response 1

Even though several variants related with diversity and Differential Evolution have been proposed, few works relate the level of diversity promoted with the stopping criteria, which is the main novelty of our proposal. The main motivation behind this principle is to provide a mechanism that is automatically adapted depending on the stopping criterion set by the user. At initial stages exploration is promoted, whereas as the execution progresses the search behavior is gradually moved towards exploitation. There are some somewhat popular algorithms that incorporates this principle. They are the ones proposed by Montgomery et al. [18, 19, 20]. However the main difference is that in our work, the diversity is promoted in the replacement phase, whereas in the Montgomery et al. case, modifications are done in the mutation operator. Another difference, is that in their work, premature convergence might appear in some cases. However, in our proposal, the premature convergence is explicitly avoided. Regarding other methods based on modifying the replacement strategy, some related methods are the "Restricted Tournament Selection (RTS)" and the "Hybrid Genetic Search with Adaptive Diversity Control (HGSADC)". To better clarify the effectiveness of our proposal, the corrected manuscript incorporates two new diversity-based methods in the Differential Evolution scheme (see Section 5.1). The benefits against these methods are quite clear. In addition, the "Literature review" section have been extended to better clarify the main difference between our proposal and other state-of-the-art methods.

Notation in Eq.(4) is a bit confusing. Is d in the the (b^d-a^d) a power or what? If not, use a proper notation. If it's a power, what is the mathematical reason for this?

Response 2

The d has been converted to a subindex with the aim of avoiding misunderstanding. This comment has been incorporated in the manuscript.

Fig. 2, there seems to have some significant differences between CEC2016 and CEC17 rates. What is the reason?

Response 3

The problems proposed in the Congress on Evolutionary Computation are designed independently each year. As it is analyzed by Daniel Molina et al [17], differences among years are large and proper methods for a specific benchmark are not always promising for other benchmarks. In order to better deal with this issue and with the aim of better analyzing the generality of optimizers, it is recommended to use several benchmarks simultaneously. For the specific problems that has been incorporated in this paper, there are several functions of the CEC2016 that have not been solved by any of the state-of-the-art schemes. This is not an issue specific of our method as it can be analyzed from Table 2 and Table 3, so solving the CEC2016 benchmark is more difficult than solving the CEC2017 benchmark, therefore the results attained in Figure 2 are as expected.

Reviewer 2

The authors propose a novel replacement strategy that combines the use of an elite population and a mechanism to preserve diversity in the differential evolution (DE) algorithm. Experimental testing on benchmarks from IEEE Congress on Evolutionary Computation (CEC) indicates that even the top-rank algorithms of CEC competitions 2017 and 2016 are super-ceded by the proposed modification of the DE. The paper is well written and well structured. The survey seems to be adequate.

My only concern is that the paper does not present any theoretical or experimental analysis to answer the question: Why the proposed modification of the DE is so good? I guess that this question might require a separate paper. It is recommended, however, that the authors provide as much information as possible for the further research in this direction. In particular, it would be helpful to give more information in the introduction about the theoretical results on DE algorithms, which may be related to this modification. Besides that, it would be helpful to see more detailed presentation of the experimental results with the analysis (on some instances) of the positive input given by the new mechanism in comparison to the previously known version of the DE.

Response 1

Some theoretical analyses of the convergence properties of DE have been performed by Zaharie et al.[34] and other authors. Nevertheless, in order to attain solid statements, in these previous works, authors usually make several strong assumptions about the functions to optimize, so such analyses are only applicable for a narrow set of functions. Many of the problems proposed in the Congress Evolutionary Computation incorporate features that are difficult to be analyzed when used in stochastic algorithms, such as, dependencies, multi-modality, combination of functions, etc. Therefore, performing such kinds of analyses might require a separate paper (and a lot of work). Additionally, there are some empirical studies performed by Motgomery et al. [18], which have a quite solid theoretical background, that analyze the kinds of movements that appear in Differential Evolution, and shed some light on aspects related to diversity and convergence on Differential Evolution. The Literature Review Section have been extended to better show the current status regarding the relations among Differential Evolution, Diversity and Stopping Criteria, and has also included some diversity-based methods for population-based strategies different to Differential Evolution. Additionally, some new experiments related to these kinds of methods have been incorporated in Section 5.1.
Regarding the experimental analysis, we also consider that Fig. 1 is quite illustrative. Note how the diversity is reduced almost linearly depending on the percentage of evaluations, which is not a so typical behaviour. This means that the selection of the region where the search is focused on is delayed and thus, it might be better selected, which is the basic principle behind our proposal.

Can the population of DE contain identical vectors? If so, the population should be referred to as a multi-set rather than a set (page 3, line 19).

Response 2
Effectively, a population of DE could contain identical vectors. Therefore term “set” has been changed to “multi-set” in the manuscript. In addition, we have changed the term set to multi-set in some other parts of the manuscript. Thanks for this comment.

Notation "NP" for the population size is hard to justify (page 3, line 32 and below). It looks like the class of Nondeterministic Polynomial time decision problems or as a multiple of N and P. I recommend using a single letter for the population size.

Response 3

This comment has been incorporated in the manuscript.

In expression (1), the symbols r1, r2 and r3 should be converted to r_1, r_2 and r_3 (with numbers in subscripts).

Response 4

This observation has been corrected in the manuscript.

Which norm is meant in line 47 of page 4?

Response 5
It is the normalized Euclidean Distance. This has been clarified in the paper.

It seems that the terminology from the area of DEs and from the area of genetic algorithms is mixed in this paper, which makes it difficult to read. In particular, at the bottom of page 7, the term "individuals" seems to replace the "trial vectors" and the term "parent population" is undefined at all. I personally would prefer the terminology from the field of evolutionary algorithms (which is rather well-known). Anyway, any term in use should have a clear definition in the paper.

Response 6

Since the main topic addressed in this work is Differential Evolution, we decided to adopt the DE terminology, thus the parent population and offspring population are referred to as target and trial vectors. This is clarified in the corrected manuscript with the aim of improving the readability of the paper.

The set Elite is not properly commented in Algorithm 2 and in the text describing it.

Response 7

The Elite vectors are managed similarly than the target vectors in standard DE, i.e., they contain a copy of the best vector that has been located in a given position of the population. This has been clarified in the paper.

The proposed method has many parameters besides the population size NP and D_I: parameter 0.9 involved in the update rule (line 2 of Algorithm 2), the parameters 0.2, 0.1, 0.9, 0.1 of crossover (5) and the parameter 0.5 of the mutation in (6). Is the algorithm sensitive to all these parameters? Is it possible to get rid of some of them? (E.g. it seems to be redundant to have four parameters for crossover.) To keep the same style of presentation of parameters with the other algorithms on page 11, it is recommended that all parameters of the new algorithm are named and listed together with D_I=0.3, NP=250 for the DE_EDM in the list of parameters on page 11.

Response 8
We have performed this change in the new version of the manuscript. Regarding the option of reducing the set of parameters, we believe that by incorporating some adaptive rules, as it is done in SHADE for instance, this might be achieved. However, this increases the complexity of our proposal. For this reason, we decided to use a more simple approach and we selected some reproduction schemes designed by other authors, that have attained promising results.

More information should be provided for each testing instance from CEC 2016 and 2017. What is the dimension of the search space? What are the features of these problems? How are they built? Obviously, one can find this information in the literature, but it is quite relevant in this paper to make it self-contained and to discuss the strong and weak points of the new method.

Response 9
The problems of each benchmark are divided in the following groups: uni-modal, simple multi-modal, hybrid and composition functions. We have incorporated Table 1 to offer this important information more quickly. We have also attached some comments about some of the most important features of these groups, as well as the dimension and bounds of the variables.

The phrase "In fact it attained the third and second places in CEC 2016 and CEC 2017 respectively." (top of page 13) seems to contradict the fact that in Tables 1 and 2 the proposed modification of DE is the best method.

Response 10

This sentence applies to the standard version of DE and not to the modification proposed. This has been clarified and the paragraph is now as follows:

Note that in base of this definition, the best attainable score is 100. This happens when a given approach obtains both SR_{min} and SE_{min}. DE-EDM attained the best attainable score in both years, which confirms its clear superiority when compared both with state-of-the-art and standard DE. In these long-term executions, standard DE attained the third and second places in the problems of the CEC 2016 and CEC 2017, respectively. This means that the performance of the state-of-the-art algorithms is not so impressive in long-term executions.