Experimental Results for the Special Session on Real-Parameter Optimization at CEC 2005: A Simple, Continuous EDA

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Abstract- A comprehensive set of experiments was conducted with a continuous EDA on 25 test problems provided in the real-parameter optimization special session. It is expected that the results presented here could be used to gain some deeper understanding of the performance of the EDA as well as facilitate the comparison across different algorithms.

1 Introduction

The field of Evolutionary Computation has produced a large number of real-parameter optimization algorithms. Some well-known examples in this class are Evolution Strategies, real-coded Genetic Algorithms, continuous Estimation of Distribution Algorithms, Particle Swarm Optimizer and Differential Evolution.

A common practice in current empirical research is to test a new algorithm with hand-tuned parameter values on a few selected test problems and conclusions on its performance are made based on the corresponding experimental results. However, it has been pointed out that this methodology has some serious issues [2, 5] and it is difficult to effectively compare different algorithms.

This paper presents an empirical study of a continuous Estimation of Distribution Algorithm (EDA). Experiments are conducted following the specifications given for the Special Session on real-parameter optimization at the 2005 Congress on Evolutionary Computation [1]. This session is intended to facilitate comparisons among a variety of real-parameter Evolutionary Algorithms, via a common set of artificially constructed benchmark problems and identical performance criteria.

2 Estimation of Distribution Algorithms

Estimation of Distribution Algorithms (EDAs) [3] refer to a class of novel Evolutionary Algorithms based on probabilistic modelling instead of classical genetic operators such as crossover or mutation. The fundamental mechanism is to conduct searching by sampling new individuals from a probability distribution, which is estimated based on some selected promising individuals in the current population. The major advantage of EDAs is that they can explicitly learn the dependences among variables of the problem and use this structural information to efficiently generate new individuals.

In this paper we utilize a simple, continuous EDA that uses a multivariate Gaussian distribution to model selected individuals and generate new individuals. A framework for such EDAs is $EMNA_{global}$[3] and RECEDA[4] can be seen as one implementation of this framework. The algorithm used here has some modifications (detailed below) and is referred to as EDA_{mvg} (where mvg refers to the MultiVariate Gaussian model involved).

```
Initialize and evaluate the population P
  While stopping criteria not met
    Select some individuals P^sel from P.
    Estimate the mean μ and covariance Σ of P^sel.
    Sample a population P' from G (μ, Σ).
    Evaluate individuals in P'.
    Combine P and P' to create a new P.
  End While
```

Figure 1: The Pseudo Code of EDA_{mvg}

In Figure 1, the general procedure of EDA_{mvg} is to, in each generation, estimate the statistics (i.e., μ and Σ) of selected individuals from the current population and generate new individuals P' (i.e., $|P| = |P'|$) by sampling from the corresponding Gaussian so that new individuals would follow the same distribution as those promising ones. The new population is created by choosing the best individuals from the union of old and new individuals.

The Cholesky decomposition is used to sample individuals from a Gaussian G (μ, Σ):

$$X = \mu + S \cdot Z \qquad \text{Eq. 1}$$

where S is a lower triangular matrix subject to $\Sigma = SS^T$ and Z is an N-by-M matrix with random elements sampled from a standard normal distribution G (0, 1) where N is the dimensionality and M is the number of individuals. It is easy to verify that the covariance of X is equal to Σ:

$$\begin{aligned} Cov(X) &= \frac{1}{M}(X-\mu)(X-\mu)^T \\ &= \frac{1}{M}(S \cdot Z)(S \cdot Z)^T \\ &= S \cdot I \cdot S^T = \Sigma \end{aligned} \qquad \text{Eq. 2}$$

Since the initial population is randomly generated within the whole search space, EDA_{mvg} typically starts with relatively large standard deviations, which makes it capable of searching a wide area and thus less likely to get stuck at local optima compared to local searching methods. On the other hand, once it identifies a promising area possibly containing the global optimum, it can quickly reduce its search scope represented by the eigenvalues of the covariance matrix to achieve fast convergence speed.

An inherent shortcoming of EDA_{mvg} is that it cannot handle multimodal problems efficiently in general, which is mainly due to the single Gaussian distribution in use. However, this does not necessarily mean that it will never work well on such problems. Previous research has shown that as long as the problem presents the "Big-Valley" structure, EDA_{mvg} may still stand a good chance. Furthermore, in some preliminary experiments, a few additional issues have been identified, which may result in significant performance loss.

Firstly, it has been noticed that EDA_{mvg} may even get stuck on unimodal problems such as the Sphere function, especially in high dimensional spaces. Note that an idealized situation for EDA_{mvg} is when the Gaussian is right on top of the global optimum so that it could be found quickly with consistently shrinking search scope. However, when the mean vector is not close to the global optimum, EDA_{mvg} is then required to be able to move towards it in a similar manner as a hill-climbing algorithm. Unfortunately, an investigation into the dynamics of its model parameters reveals that the eigenvalues often quickly dropped to zero while its mean vector was still a bit distant from the global optimum.

The reason is that in each generation those selected individuals are typically distributed in a smaller area compared to the current population. As a result, the new Gaussian model to be built is expected to have smaller eigenvalues compared to the current Gaussian model. If the current mean vector is distant from the global optimum, several generations may be needed for the Gaussian to move close to it but it is possible that the eigenvalues may become close to zero within just a few generations, incapable of making any significant progress.

The key to this issue is to explicitly maintain the population diversity. The first question is when to maintain the diversity because if the Gaussian is very close to the global optimum, maintaining extra diversity may not be helpful and instead it may reduce the convergence speed. A simple heuristic adopted here is the distance between the mean vector and the best individual in the current population. If the best individual is less than a threshold away from the mean vector in each dimension, it may imply that the Gaussian is currently near the global optimum. In this case, no diversity maintenance is to be applied. Otherwise, it may indicate that the current Gaussian model is still on its way to the global optimum and should not shrink too rapidly.

Another question is how to maintain the diversity. A simple approach is to amplify the covariance matrix by a factor larger than 1. A more precise way is to treat each dimension separately because the best individual may be close to the mean vector in some dimensions while far away from it in other dimensions. In this paper, the diversity is maintained by enlarging the corresponding eigenvalues so that the distance between the best individual and the mean vector in these dimensions is equal to the threshold (i.e., a maximum amplification value Q is set in advance to avoid too dramatic changes). Note that since the Gaussian model employs a full covariance matrix, the concept of "dimension" in the above analysis is defined with regard to the eigenvectors.

Secondly, it has shown that with the help of the diversity maintenance technique the performance of EDA_{mvg} on a number of test problems could be improved. However, the convergence speed is not always fast enough to enable a very good solution to be found within a limited number of fitness evaluations. This is partially because that all selected individuals are given equal weight in building the model despite of their difference in quality. A straightforward approach is to increase the influence of the best individual by explicitly moving the mean vector towards it in an incremental manner:

$$\mu_{i+1} = (1-\alpha)\cdot\mu_i + \alpha\cdot X^{Best} \qquad \text{Eq. 3}$$

Thirdly, for some problems, there are a huge number of optima around the global optimum with comparable quality and basin sizes. In this case, selected individuals are likely to be distributed in a wide range and the Gaussian model usually has quite large eigenvalues, which may prevent better solutions from being found efficiently. One way to solve this issue is to divide the population into clusters of individuals and build a Gaussian for each of them. However, this would inevitably require an extra clustering algorithm, which should be able to handle a large (unknown) number of clusters. Instead, the solution utilized here is to increase the selection pressure to force EDA_{mvg} to focus on the very best individuals to speedup the convergence.

3 Experimental Configuration

The benchmark suite for the Special Session consisted of 25 artificial test functions most of which are variations of well-known test functions through rotation, shifting and hybridization in the hope of overcoming some known disadvantages of these functions [1].

The major performance criterion was the distance (error) between the best individual found and the global optimum in terms of fitness value, examined after some predefined numbers of fitness evaluations (check points). Additionally, an accuracy level was set for each problem and the success rate was calculated based on the percentage of trials reaching that level and the corresponding number of fitness evaluations required was recorded for comparison. The computational complexity of each algorithm was also measured in order to better reflect its real running time. Please refer to [1] for details of the performance criteria in use.

For EDA_{mvg}, the truncation selection with ratio τ was used as the selection operator. The threshold used in diversity maintenance was set to be half the square root of the eigenvalues (i.e., half the standard deviations with regard to the eigenvectors). The initial population was randomly generated within the search space and individuals during evolution outside the search space were reset to the corresponding boundaries except for problems No. 7&25 where the global optima are not within the initialization region. It was observed that it is also beneficial to assume no boundary for problem No. 5.

Given the fixed amount of FEs, there are four tunable parameters $<P, Q, \alpha, \tau>$ defined as below:

- P: population size
- Q: Maximum amplification value ($1 \le Q$)
- α: Learning rate ($0 \le \alpha \le 1$)
- τ: Selection ratio ($0 < \tau \le 1$)

For the experiments, our intention was to do as little parameter tuning as possible, while achieving reasonable performance across the suite. It was assumed that a population size of 10-50 times the problem dimensionality would be required. Beyond this, only a few selected combinations of coarse values were tested for five trials on the first 5 problems (i.e., P=200, 500 and 1000; Q=1, 1.5 and 2; α=0 and 0.2; τ=0.2 and 0.3). Based on the results from around 150 tuning trials, for all 10D problems, the parameters were chosen as <200, 2.0, 0.2, 0.3> while for all 30D problems the values were <1000, 1.5, 0.2, 0.3>. Additionally, around 50 trials were conducted to try to further improve the performance of EDA_{mvg} on problems No. 6-10, which resulted in the following exceptions:

- α=0 for problem No. 6 (i.e., no incremental learning).
- Q=1 and τ=0.2 for problems No. 9&10 (i.e., strong selection pressure and no diversity maintenance).

4 Results and Discussion

Full results are presented in the Appendix, following the requirements given in the Special Session documentation. In summary, in 10D cases (Tables 1-3), EDA_{mvg} was often able to reach the accuracy level on problems No. 1-6 and also on problems No. 7, 11&12 sometimes (Table 8). In 30D cases (Tables 4-6), EDA_{mvg} was able to solve problems No. 1-4&7 with 100% success rate (Table 9). However, for all composition problems No. 15-25, no satisfactory results could be found. In fact, almost all results (error values) were two orders of magnitude. The complexity of EDA_{mvg} is given in Table 7 showing that it required 50% more time in 30D than in 10D. The convergence graphs for all 30D problems showing the median performance are plotted in Figures 2-6.

There are a number of interesting things and issues that need to be noticed:

- EDA_{mvg} is mainly suitable for optimizing unimodal problems (e.g., No. 1-4) or multimodal problems with the "Big-Valley" structure (e.g., No. 7, 9&10) and is relatively robust against rotation, noise and shifting applied on the test problems.
- EDA_{mvg} is also able to solve the Rosenbrock function by walking along the long path towards the global optimum at the bottom of a deep valley with appropriate diversity maintenance approaches.
- The performance of EDA_{mvg} on problem No. 7 was significantly improved from 10D to 30D (i.e., 4% vs. 100%), which means that the difficulty of this problem, at least for some algorithms, is reduced as the dimensionality goes up. This is because the influence of the local optima created by the Cosine functions becomes trivial in high dimensional spaces.
- The computational time of EDA_{mvg} itself is largely dependent on the number of generations (i.e., the number of times the Gaussian model needs to be built) instead of population size, which means that for a fixed amount of FEs, the running time could vary noticeably with different population sizes. In Table 7, the same population size was used for each dimension as in the main experiments to correctly reflect the real algorithm complexity.
- The values reported in Tables 8 & 9 indicate the number of FEs after half of the final population has been evaluated. This is because the population of solutions is not ordered or sorted and thus the actual number of FEs until termination will contain some randomness depending on when the first individual that satisfies the termination condition is evaluated.
- In Figures 2-6, some curves have not really flattened out, suggesting that more FEs would lead to better results (i.e., the curves of problems No.1&2 are broken at the end as the error values were zero).

The experimental comparison of algorithms is by all means an important yet challenging task. Nevertheless, we believe that it is vital for the future of the field, to continue to conduct large-scale studies of this kind. We hope that this Special Session will provide a seed for further evaluations in the future.

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Appendix

Table 1: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5 for Problems 1-8 (10D)

FES \ Prob		1	2	3	4	5	6	7	8
1e3	1^{st}	7.4429E+02	1.7492E+03	4.3913E+06	2.0791E+03	3.9229E+03	2.2780E+07	2.4657E+02	2.0427E+01
	7^{th}	1.5449E+03	2.8191E+03	7.7070E+06	3.4274E+03	5.6708E+03	5.4174E+07	5.1446E+02	2.0644E+01
	13^{th}	1.8109E+03	2.9987E+03	9.2662E+06	4.5502E+03	6.5111E+03	9.0827E+07	6.7094E+02	2.0748E+01
	19^{th}	2.0962E+03	3.9976E+03	1.4613E+07	5.3307E+03	7.8372E+03	1.7301E+08	9.6598E+02	2.0830E+01
	25^{th}	3.0641E+03	6.3958E+03	2.9237E+07	9.4041E+03	1.0721E+04	5.9184E+08	1.6916E+03	2.0921E+01
	Mean	1.8225E+03	3.4610E+03	1.2649E+07	4.7055E+03	6.8539E+03	1.3452E+08	7.4274E+02	2.0724E+01
	Std	5.9124E+02	1.1530E+03	7.3448E+06	1.6848E+03	1.8869E+03	1.2231E+08	3.2877E+02	1.3330E-01
1e4	1^{st}	1.3633E-07	7.1460E-07	5.0966E-03	2.0959E-06	3.9958E-01	6.4988E+00	4.6597E-01	2.0354E+01
	7^{th}	4.7251E-07	3.0955E-06	9.5634E-03	9.4220E-06	8.8041E-01	8.2194E+00	6.5126E-01	2.0458E+01
	13^{th}	9.0985E-07	3.8031E-06	2.6574E-02	2.0827E-05	1.0970E+00	1.0736E+01	7.1729E-01	2.0545E+01
	19^{th}	1.5927E-06	6.3622E-06	6.1694E-02	7.8753E-05	1.9118E+00	3.5801E+01	7.5294E-01	2.0593E+01
	25^{th}	6.7828E-06	2.1157E-05	3.5398E+03	1.0288E-03	9.6715E+00	1.5888E+02	8.3123E-01	2.0705E+01
	Mean	1.3726E-06	5.7484E-06	2.1530E+02	8.7425E-05	1.8943E+00	2.6413E+01	6.9606E-01	2.0525E+01
	Std	1.4808E-06	4.9475E-06	7.7909E+02	2.0576E-04	2.0081E+00	3.3282E+01	9.2404E-02	9.1877E-02
1e5	1^{st}	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2.0154E+01
	7^{th}	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.8190E-12	0.0000E+00	3.4077E-01	2.0304E+01
	13^{th}	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	3.6380E-12	0.0000E+00	4.7212E-01	2.0341E+01
	19^{th}	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	3.6380E-12	0.0000E+00	5.0593E-01	2.0381E+01
	25^{th}	0.0000E+00	0.0000E+00	3.7548E+02	0.0000E+00	3.6380E-12	6.8255E-01	5.9828E-01	2.0468E+01
	Mean	0.0000E+00	0.0000E+00	2.1207E+01	0.0000E+00	2.8376E-12	4.1817E-02	4.2049E-01	2.0344E+01
	Std	0.0000E+00	0.0000E+00	8.0018E+01	0.0000E+00	1.0606E-12	1.5075E-01	1.3303E-01	6.4467E-02

Table 2: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5 for Problems 9-17 (10D)

FES \ Prob		9	10	11	12	13	14	15	16	17
1e3	1^{st}	3.1118E+01	4.7194E+01	1.0334E+01	1.6150E+04	1.7504E+01	3.7422E+00	5.7889E+02	2.3088E+02	2.3469E+02
	7^{th}	4.6647E+01	5.9538E+01	1.1222E+01	4.4364E+04	2.4316E+01	4.1167E+00	6.6082E+02	2.8378E+02	3.2413E+02
	13^{th}	5.2076E+01	6.3431E+01	1.1844E+01	6.2453E+04	5.2347E+01	4.2461E+00	6.6968E+02	2.9964E+02	3.4355E+02
	19^{th}	5.9478E+01	6.7429E+01	1.2182E+01	7.0122E+04	7.3742E+01	4.3484E+00	6.9037E+02	3.1761E+02	3.6573E+02
	25^{th}	7.3728E+01	8.5372E+01	1.3417E+01	1.2576E+05	3.5896E+02	4.5397E+00	7.3166E+02	3.5488E+02	4.2456E+02
	Mean	5.2430E+01	6.4880E+01	1.1754E+01	5.9272E+04	6.9055E+01	4.2331E+00	6.7187E+02	2.9990E+02	3.3974E+02
	Std	1.0149E+01	9.1246E+00	7.6046E-01	2.3938E+04	7.1379E+01	1.7794E-01	3.4014E+01	2.7915E+01	4.5618E+01
1e4	1^{st}	6.0999E+00	8.4075E+00	2.8515E+00	1.1421E-02	1.6063E+00	3.4144E+00	3.4446E+02	1.4492E+02	1.5685E+02
	7^{th}	2.7632E+01	2.5827E+01	9.6743E+00	1.9561E+00	2.6644E+00	3.7498E+00	4.0384E+02	1.6813E+02	1.8851E+02
	13^{th}	3.1118E+01	3.0808E+01	1.0277E+01	1.0546E+01	3.0282E+00	3.9349E+00	5.1027E+02	1.7171E+02	1.9866E+02
	19^{th}	3.2684E+01	3.4567E+01	1.0783E+01	7.2090E+01	3.8884E+00	3.9991E+00	5.3203E+02	1.8812E+02	2.0746E+02
	25^{th}	3.7142E+01	4.0416E+01	1.1571E+01	5.1846E+03	4.4398E+00	4.0942E+00	5.6325E+02	2.0037E+02	2.1703E+02
	Mean	2.9342E+01	2.9426E+01	9.9626E+00	6.2420E+02	3.2076E+00	3.8730E+00	4.7914E+02	1.7623E+02	1.9664E+02
	Std	6.3410E+00	7.3722E+00	1.6811E+00	1.3720E+03	7.9417E-01	1.8456E-01	6.7786E+01	1.4106E+01	1.3622E+01
1e5	1^{st}	2.2831E+00	1.9899E+00	0.0000E+00	0.0000E+00	8.7299E-01	1.8955E+00	1.3176E+02	1.1950E+02	1.3989E+02
	7^{th}	4.2680E+00	3.0710E+00	9.8396E-01	0.0000E+00	1.6412E+00	2.3809E+00	4.0000E+02	1.3791E+02	1.5072E+02
	13^{th}	5.8297E+00	4.1805E+00	3.5443E+00	1.0003E+01	1.8858E+00	2.6264E+00	4.0000E+02	1.4468E+02	1.5625E+02
	19^{th}	6.1401E+00	7.9776E+00	5.6281E+00	1.7541E+01	2.0400E+00	2.8309E+00	4.0000E+02	1.5317E+02	1.6170E+02
	25^{th}	1.2080E+01	1.0945E+01	9.5647E+00	5.1846E+03	2.4741E+00	3.3198E+00	5.0746E+02	1.6847E+02	1.8020E+02
	Mean	5.4179E+00	5.2891E+00	3.9446E+00	4.4231E+02	1.8412E+00	2.6298E+00	3.6500E+02	1.4392E+02	1.5679E+02
	Std	1.9107E+00	2.7737E+00	3.1202E+00	1.1692E+03	3.4013E-01	3.9422E-01	9.2391E+01	1.3678E+01	1.0050E+01

Table 3: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5 for Problems 18-25 (10D)

FES \ Prob		18	19	20	21	22	23	24	25
1e3	1st	8.4853E+02	8.9952E+02	1.0039E+03	1.0991E+03	8.8535E+02	1.1487E+03	8.3969E+02	1.0257E+03
	7th	1.0093E+03	1.0189E+03	1.0548E+03	1.2861E+03	9.0692E+02	1.2697E+03	1.0081E+03	1.4119E+03
	13th	1.0833E+03	1.0901E+03	1.0772E+03	1.3036E+03	9.3869E+02	1.3058E+03	1.0585E+03	1.4847E+03
	19th	1.1063E+03	1.1173E+03	1.1188E+03	1.3226E+03	9.8322E+02	1.3299E+03	1.1037E+03	1.5967E+03
	25th	1.1454E+03	1.2011E+03	1.1840E+03	1.3527E+03	1.0601E+03	1.3669E+03	1.1911E+03	1.7113E+03
	Mean	1.0522E+03	1.0633E+03	1.0827E+03	1.2870E+03	9.4561E+02	1.2878E+03	1.0406E+03	1.4843E+03
	Std	7.8132E+01	8.6763E+01	4.8733E+01	6.3251E+01	4.7277E+01	5.5108E+01	8.9004E+01	1.4709E+02
1e4	1st	3.0000E+02	3.0000E+02	3.0000E+02	3.0001E+02	7.6696E+02	5.5947E+02	2.0000E+02	3.7738E+02
	7th	3.0000E+02	3.0000E+02	3.0003E+02	5.0000E+02	7.7683E+02	5.5947E+02	2.0000E+02	3.8224E+02
	13th	3.0007E+02	8.0000E+02	8.0000E+02	5.0000E+02	7.7779E+02	5.5947E+02	2.0000E+02	3.8510E+02
	19th	8.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	7.8252E+02	7.2163E+02	2.0000E+02	3.8830E+02
	25th	8.0004E+02	8.0010E+02	9.9557E+02	1.1077E+03	8.4924E+02	1.1645E+03	7.9862E+02	4.0409E+02
	Mean	4.8324E+02	5.6439E+02	6.5858E+02	5.4097E+02	7.8564E+02	6.5166E+02	2.2395E+02	3.8615E+02
	Std	2.4298E+02	2.5072E+02	2.4600E+02	2.1391E+02	2.2050E+01	1.6344E+02	1.1972E+02	6.0176E+00
1e5	1st	3.0000E+02	3.0000E+02	3.0000E+02	3.0000E+02	7.5544E+02	5.5947E+02	2.0000E+02	3.6434E+02
	7th	3.0000E+02	3.0000E+02	3.0000E+02	3.0000E+02	7.6281E+02	5.5947E+02	2.0000E+02	3.7078E+02
	13th	3.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	7.6355E+02	5.5947E+02	2.0000E+02	3.7335E+02
	19th	8.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	7.6708E+02	7.2122E+02	2.0000E+02	3.7538E+02
	25th	8.0000E+02	8.0000E+02	9.0070E+02	8.0000E+02	8.2733E+02	9.7050E+02	2.0000E+02	3.8090E+02
	Mean	4.8319E+02	5.6438E+02	6.5190E+02	4.8400E+02	7.7088E+02	6.4052E+02	2.0000E+02	3.7304E+02
	Std	2.4302E+02	2.5071E+02	2.3809E+02	1.6753E+02	2.0094E+01	1.3858E+02	0.0000E+00	3.7179E+00

Table 4: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5, FES=3e5 for Problems 1-8 (30D)

FES \ Prob		1	2	3	4	5	6	7	8
1e3	1st	6.4766E+04	6.4464E+04	8.7083E+08	7.4316E+04	3.2596E+04	1.2940E+10	1.0214E+04	2.1062E+01
	7th	7.9338E+04	1.0243E+05	1.3872E+09	1.1350E+05	3.5919E+04	4.0659E+10	1.0666E+04	2.1174E+01
	13th	8.1474E+04	1.0988E+05	1.4972E+09	1.2930E+05	3.8150E+04	6.1536E+10	1.1037E+04	2.1211E+01
	19th	8.7593E+04	1.1928E+05	1.7421E+09	1.4862E+05	3.9168E+04	6.7127E+10	1.1209E+04	2.1234E+01
	25th	1.0330E+05	1.3817E+05	2.5262E+09	1.7114E+05	4.3070E+04	8.3975E+10	1.1713E+04	2.1284E+01
	Mean	8.3569E+04	1.0883E+05	1.5591E+09	1.2729E+05	3.7872E+04	5.6316E+10	1.0978E+04	2.1199E+01
	Std	8.8251E+03	1.6422E+04	3.7759E+08	2.7827E+04	2.6937E+03	1.6743E+10	4.0520E+02	5.7618E-02
1e4	1st	9.6670E+03	1.7309E+04	5.9897E+07	1.8732E+04	1.5866E+04	2.0813E+09	3.6010E+03	2.0916E+01
	7th	1.0946E+04	2.1458E+04	8.5518E+07	2.5122E+04	1.7319E+04	3.2404E+09	4.3054E+03	2.1031E+01
	13th	1.2085E+04	2.2351E+04	1.0220E+08	2.7478E+04	1.8545E+04	3.6156E+09	4.5595E+03	2.1080E+01
	19th	1.3816E+04	2.4235E+04	1.1284E+08	3.0061E+04	2.0087E+04	4.5719E+09	5.0857E+03	2.1134E+01
	25th	2.7035E+04	2.8186E+04	1.8176E+08	3.8106E+04	2.2583E+04	6.5199E+09	5.9215E+03	2.1168E+01
	Mean	1.3394E+04	2.2781E+04	1.0575E+08	2.7868E+04	1.8880E+04	3.9473E+09	4.6831E+03	2.1078E+01
	Std	3.8168E+03	2.6586E+03	3.0794E+07	4.8535E+03	1.9123E+03	1.0857E+09	5.9336E+02	6.1353E-02
1e5	1st	8.8117E-04	5.7950E-03	3.0707E+01	1.3907E-01	1.6556E+02	4.2679E+02	3.3931E-01	2.0814E+01
	7th	2.3338E-03	1.1152E-02	5.0916E+01	2.2696E-01	2.4458E+02	8.3183E+02	6.7917E-01	2.0973E+01
	13th	3.8545E-03	1.3118E-02	6.8676E+01	2.5974E-01	2.8990E+02	1.0108E+03	7.5221E-01	2.1003E+01
	19th	5.1188E-03	2.1318E-02	1.0528E+02	4.3105E-01	3.5157E+02	1.1703E+03	8.1013E-01	2.1021E+01
	25th	1.0657E-02	4.8730E-02	1.7465E+02	8.5895E-01	5.0557E+02	2.4757E+03	9.1287E-01	2.1074E+01
	Mean	4.1528E-03	1.6243E-02	8.3579E+01	3.3773E-01	3.0630E+02	1.0672E+03	7.3268E-01	2.0990E+01
	Std	2.3862E-03	9.5842E-03	4.1951E+01	1.7282E-01	9.0395E+01	4.7259E+02	1.3773E-01	5.6130E-02
3e5	1st	0.0000E+00	0.0000E+00	1.1369E-13	3.4106E-13	1.3231E-02	1.9084E+01	0.0000E+00	2.0814E+01
	7th	0.0000E+00	0.0000E+00	6.2528E-13	8.5265E-13	4.0436E-02	2.0694E+01	2.8422E-14	2.0925E+01
	13th	0.0000E+00	0.0000E+00	1.1937E-12	1.3642E-12	4.8325E-02	2.0936E+01	2.8422E-14	2.0969E+01
	19th	0.0000E+00	0.0000E+00	2.2737E-12	3.3538E-12	7.1404E-02	2.1621E+01	2.8422E-14	2.0980E+01
	25th	5.6843E-14	5.6843E-14	7.2760E-12	7.8444E-12	1.1709E-01	2.2739E+01	1.9895E-13	2.1018E+01
	Mean	9.0949E-15	9.0949E-15	1.7758E-12	2.2601E-12	5.5133E-02	2.1103E+01	3.8654E-14	2.0945E+01
	Std	2.1269E-14	2.1269E-14	1.7241E-12	1.9799E-12	2.5375E-02	8.2756E-01	3.6582E-14	4.9832E-02

Table 5: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5, FES=3e5 for Problems 9-17 (30D)

FES	Prob	9	10	11	12	13	14	15	16	17
1e3	1st	4.3267E+02	6.0253E+02	4.0776E+01	1.3967E+06	1.9539E+05	1.3860E+01	9.5589E+02	6.9913E+02	7.4932E+02
	7th	4.6218E+02	7.1045E+02	4.4829E+01	1.6740E+06	4.5304E+05	1.4113E+01	1.1372E+03	8.2778E+02	9.8379E+02
	13th	4.8279E+02	7.3974E+02	4.5598E+01	1.7640E+06	5.5122E+05	1.4247E+01	1.1650E+03	9.7815E+02	1.0433E+03
	19th	4.9762E+02	7.8264E+02	4.6641E+01	1.8759E+06	6.7365E+05	1.4309E+01	1.1892E+03	1.0170E+03	1.1203E+03
	25th	5.2759E+02	8.7458E+02	4.7758E+01	2.1114E+06	9.1741E+05	1.4422E+01	1.2306E+03	1.0955E+03	1.1894E+03
	Mean	4.8069E+02	7.4799E+02	4.5612E+01	1.7633E+06	5.4610E+05	1.4211E+01	1.1472E+03	9.3427E+02	1.0336E+03
	Std	2.3251E+01	5.9182E+01	1.5074E+00	1.8674E+05	1.9682E+05	1.4239E-01	6.8119E+01	1.1896E+02	1.0988E+02
1e4	1st	2.1796E+02	2.6602E+02	4.0776E+01	6.6886E+05	6.8473E+02	1.3563E+01	5.3282E+02	2.4419E+02	3.1513E+02
	7th	2.4092E+02	2.9026E+02	4.2281E+01	8.9468E+05	3.9579E+03	1.3814E+01	5.3867E+02	3.0468E+02	3.6598E+02
	13th	2.5779E+02	3.0117E+02	4.3226E+01	9.8146E+05	5.4160E+03	1.3917E+01	5.5386E+02	3.3590E+02	3.8099E+02
	19th	2.6686E+02	3.0559E+02	4.3766E+01	1.1274E+06	8.0248E+03	1.3995E+01	5.7049E+02	3.5537E+02	4.0299E+02
	25th	2.8964E+02	3.4612E+02	4.4573E+01	1.2351E+06	1.7558E+04	1.4181E+01	7.3842E+02	4.1708E+02	5.3539E+02
	Mean	2.5405E+02	3.0088E+02	4.3037E+01	9.9405E+05	6.2895E+03	1.3895E+01	5.6912E+02	3.3281E+02	3.8926E+02
	Std	1.9430E+01	1.9196E+01	1.0942E+00	1.5475E+05	3.6730E+03	1.5393E-01	5.3611E+01	3.7350E+01	4.3495E+01
1e5	1st	1.5976E+02	1.6636E+02	3.7561E+01	1.1726E+03	1.4808E+01	1.3303E+01	2.3021E+02	2.0682E+02	2.0761E+02
	7th	1.8533E+02	1.9566E+02	4.0027E+01	5.8597E+03	1.6399E+01	1.3499E+01	4.0000E+02	2.1657E+02	2.3931E+02
	13th	1.8872E+02	2.0723E+02	4.0599E+01	1.3917E+04	1.7182E+01	1.3579E+01	4.0001E+02	2.1867E+02	2.4605E+02
	19th	1.9661E+02	2.1849E+02	4.1359E+01	2.5676E+04	1.7770E+01	1.3645E+01	4.0001E+02	2.3680E+02	2.6051E+02
	25th	2.1360E+02	2.4118E+02	4.1871E+01	6.5465E+04	1.9365E+01	1.3735E+01	5.0001E+02	3.1260E+02	4.4007E+02
	Mean	1.8950E+02	2.0536E+02	4.0491E+01	1.8684E+04	1.7201E+01	1.3554E+01	3.9722E+02	2.2865E+02	2.5850E+02
	Std	1.2597E+01	1.8600E+01	1.0321E+00	1.7230E+04	1.2072E+00	1.2494E-01	4.0124E+01	2.4775E+01	4.6959E+01
3e5	1st	1.5685E+02	1.4763E+02	3.5960E+01	3.6695E+00	1.3243E+01	1.2763E+01	2.0003E+02	1.7733E+02	1.8536E+02
	7th	1.7291E+02	1.7779E+02	3.9224E+01	5.0595E+02	1.5131E+01	1.3219E+01	4.0000E+02	1.9339E+02	2.0915E+02
	13th	1.7997E+02	1.8734E+02	3.9924E+01	3.6196E+03	1.5509E+01	1.3321E+01	4.0000E+02	2.0644E+02	2.2651E+02
	19th	1.8293E+02	2.0024E+02	4.0194E+01	7.0942E+03	1.6009E+01	1.3453E+01	4.0000E+02	2.1377E+02	2.3471E+02
	25th	2.0275E+02	2.2489E+02	4.1147E+01	2.7886E+04	1.6676E+01	1.3565E+01	5.0000E+02	2.8982E+02	4.3153E+02
	Mean	1.7868E+02	1.8852E+02	3.9453E+01	6.0542E+03	1.5310E+01	1.3321E+01	3.9600E+02	2.0789E+02	2.3759E+02
	Std	1.0242E+01	1.9372E+01	1.2844E+00	7.7717E+03	9.5194E-01	1.7390E-01	4.5455E+01	2.6580E+01	5.0793E+01

Table 7: Computational Complexity (Seconds)*

	T0	T1	T2	(T2-T1)/T0
D=10	7.029	0.810	3.646	0.4035
D=30		1.371	5.648	0.6085

* **System**: Windows XP (SP2) **CPU**: Xeon 2.4G Hz **RAM**: 1G **Language**: Matlab 6.5

For D=10: Population Size= 200 (applied to T1&T2)
For D=30: Population Size= 1000 (applied to T1&T2)

Table 6: Error Values Achieved When FES=1e3, FES=1e4, FES=1e5, FES=3e5 for Problems 18-25 (30D)

FES \ Prob		18	19	20	21	22	23	24	25
1e3	1st	1.2662E+03	1.2696E+03	1.2082E+03	1.3992E+03	1.5345E+03	1.3915E+03	1.4021E+03	1.8770E+03
	7th	1.3107E+03	1.3068E+03	1.2933E+03	1.4597E+03	1.6464E+03	1.4566E+03	1.4905E+03	1.9057E+03
	13th	1.3300E+03	1.3310E+03	1.3341E+03	1.4784E+03	1.7022E+03	1.4777E+03	1.5102E+03	1.9226E+03
	19th	1.3576E+03	1.3549E+03	1.3683E+03	1.4929E+03	1.7413E+03	1.5091E+03	1.5213E+03	1.9323E+03
	25th	1.3995E+03	1.3898E+03	1.4107E+03	1.5156E+03	1.8136E+03	1.5686E+03	1.5557E+03	1.9590E+03
	Mean	1.3311E+03	1.3300E+03	1.3290E+03	1.4711E+03	1.6951E+03	1.4805E+03	1.5024E+03	1.9205E+03
	Std	3.5310E+01	3.1844E+01	4.6848E+01	2.9806E+01	7.4641E+01	4.0611E+01	3.4449E+01	1.8909E+01
1e4	1st	1.0330E+03	1.0235E+03	1.0284E+03	1.0950E+03	1.0449E+03	1.1079E+03	1.0641E+03	1.3035E+03
	7th	1.0492E+03	1.0476E+03	1.0474E+03	1.1652E+03	1.0850E+03	1.1766E+03	1.1489E+03	1.5510E+03
	13th	1.0577E+03	1.0614E+03	1.0537E+03	1.1946E+03	1.1073E+03	1.1956E+03	1.1701E+03	1.5758E+03
	19th	1.0671E+03	1.0695E+03	1.0617E+03	1.2050E+03	1.1254E+03	1.2058E+03	1.1871E+03	1.5973E+03
	25th	1.1226E+03	1.1005E+03	1.0944E+03	1.2575E+03	1.1608E+03	1.2360E+03	1.2288E+03	1.7143E+03
	Mean	1.0630E+03	1.0592E+03	1.0547E+03	1.1892E+03	1.1049E+03	1.1885E+03	1.1673E+03	1.5703E+03
	Std	2.3791E+01	1.7925E+01	1.4730E+01	3.5545E+01	3.2993E+01	2.9857E+01	3.7693E+01	6.9217E+01
1e5	1st	8.0001E+02	8.0000E+02	8.0000E+02	5.0000E+02	8.6184E+02	5.3416E+02	2.0000E+02	2.0882E+02
	7th	8.0002E+02	8.0001E+02	8.0003E+02	5.0000E+02	8.8283E+02	5.3417E+02	2.0000E+02	2.0966E+02
	13th	8.0006E+02	8.0004E+02	8.0006E+02	5.0000E+02	8.8657E+02	5.3417E+02	2.0000E+02	2.1017E+02
	19th	9.1149E+02	9.1007E+02	9.1024E+02	5.0000E+02	8.9473E+02	5.3417E+02	2.0000E+02	2.1121E+02
	25th	9.1942E+02	9.1482E+02	9.1522E+02	5.0000E+02	9.2280E+02	5.3417E+02	2.0000E+02	2.1459E+02
	Mean	8.4968E+02	8.4462E+02	8.5339E+02	5.0000E+02	8.9073E+02	5.3417E+02	2.0000E+02	2.1067E+02
	Std	5.7205E+01	5.5779E+01	5.6704E+01	5.7234E-04	1.5028E+01	3.0520E-04	1.0350E-03	1.4716E+00
3e5	1st	8.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	8.5405E+02	5.3416E+02	2.0000E+02	2.0841E+02
	7th	8.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	8.6553E+02	5.3416E+02	2.0000E+02	2.0852E+02
	13th	8.0000E+02	8.0000E+02	8.0000E+02	5.0000E+02	8.7156E+02	5.3417E+02	2.0000E+02	2.0855E+02
	19th	9.0570E+02	9.0545E+02	9.0587E+02	5.0000E+02	8.7646E+02	5.3417E+02	2.0000E+02	2.0859E+02
	25th	9.0696E+02	9.0650E+02	9.0717E+02	5.0000E+02	9.0699E+02	5.3417E+02	2.0000E+02	2.0864E+02
	Mean	8.4665E+02	8.4223E+02	8.5085E+02	5.0000E+02	8.7193E+02	5.3416E+02	2.0000E+02	2.0855E+02
	Std	5.3712E+01	5.2794E+01	5.4021E+01	3.3968E-13	1.1648E+01	2.7077E-04	2.9008E-14	5.2943E-02

Table 8: Number of FES to Achieve the Fixed Accuracy Levels (10D)*

Prob	1st	7th	13th	19th	25th	Mean	Std	Success Rate	Success Performance
1	9.30E+03	9.70E+03	9.90E+03	1.03E+04	1.09E+04	9.96E+03	4.35E+02	100	9.96E+03
2	9.90E+03	1.05E+04	1.07E+04	1.07E+04	1.15E+04	1.06E+04	3.65E+02	100	1.06E+04
3	1.33E+04	1.41E+04	1.45E+04	1.49E+04	☹	1.46E+04	1.10E+03	92	1.59E+04
4	1.03E+04	1.11E+04	1.13E+04	1.19E+04	1.29E+04	1.15E+04	6.43E+02	100	1.15E+04
5	2.33E+04	2.45E+04	2.51E+04	2.59E+04	2.71E+04	2.51E+04	9.76E+02	100	2.51E+04
6	3.05E+04	5.45E+04	5.89E+04	7.37E+04	☹	6.00E+04	1.40E+04	88	6.82E+04
7	7.55E+04	☹	☹	☹	☹	7.55E+04	0.00E+00	4	1.89E+06
11	5.03E+04	☹	☹	☹	☹	6.60E+04	1.43E+04	12	5.50E+05
12	1.01E+04	1.55E+04	☹	☹	☹	1.42E+04	3.40E+03	40	3.54E+04

Table 9: Number of FES to Achieve the Fixed Accuracy Levels (30D)*

Prob	1st	7th	13th	19th	25th	Mean	Std	Success Rate	Success Performance
1	1.44E+05	1.49E+05	1.50E+05	1.53E+05	1.57E+05	1.50E+05	3.30E+03	100	1.50E+05
2	1.55E+05	1.57E+05	1.59E+05	1.63E+05	1.68E+05	1.60E+05	3.99E+03	100	1.60E+05
3	2.04E+05	2.12E+05	2.14E+05	2.19E+05	2.28E+05	2.15E+05	5.13E+03	100	2.15E+05
4	1.90E+05	1.96E+05	2.00E+05	2.02E+05	2.08E+05	1.99E+05	4.43E+03	100	1.99E+05
7	1.21E+05	1.29E+05	1.32E+05	1.34E+05	1.47E+05	1.31E+05	5.10E+03	100	1.31E+05

* For population-based algorithms, the sequence of individuals in the population to be evaluated is random and the above numbers are based on the middle position of the population in the last generation.

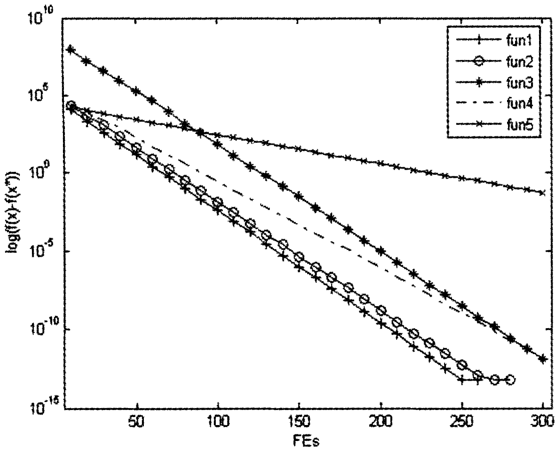

Figure 2: Convergence Graph for Problems 1-5 (30D)

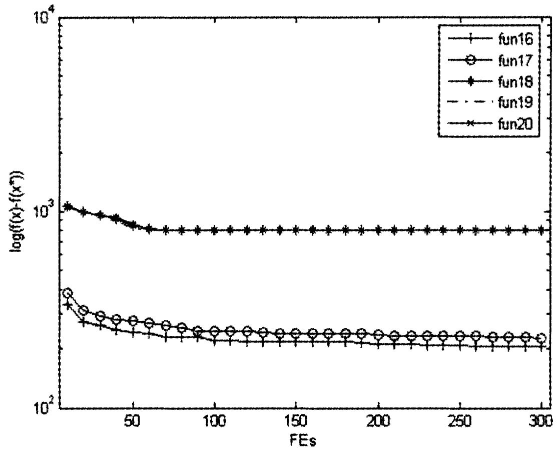

Figure 5: Convergence Graph for Problems 16-20 (30D)

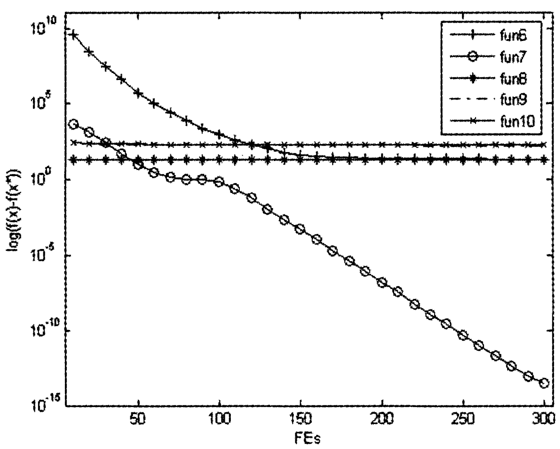

Figure 3: Convergence Graph for Problems 6-10 (30D)

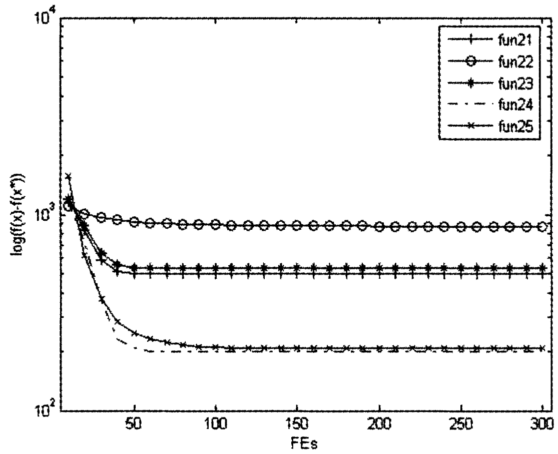

Figure 6: Convergence Graph for Problems 21-25 (30D)

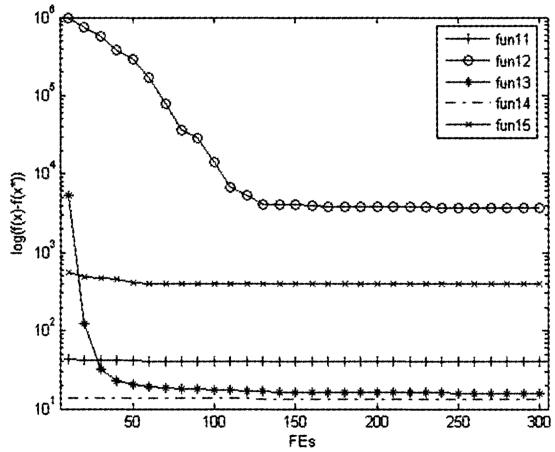

Figure 4: Convergence Graph for Problems 11-15 (30D)