Noname manuscript No.
(will be inserted by the editor)

Differential Evolution – A diversity approach

Joel Chacón Castillo · Carlos Segura

Received: date / Accepted: date

Abstract Diferential evolution is a popular evolutionary algorithm widely used in optimization. However, it is well known that the performance of differential evolution is affected by the premature convergence. An alternative for dealing with this problem is to explicitly try to maintain proper diversity. In this paper is proposed a replacement strategy that preserves useful diversity. The novelty of our method is that it induces a balance between exploration and exploitation to various optimization stages. Specifically, in the initial phases, larger amounts of diversity are accepted. The diveristy measure is based on calculating distances to the closest surviving individual. The experimental validation is carry out with several benchmark tests and the top-rank algorithms of the competitions organized in the Congress on Evolutionary Computation. The restults illustrate the usefulness of the proposal. The new method significantly improves on the best results of the state-of-the-art.

Keywords Diversity · Differential Evolution · Evolutionary

1 Introduction

Evolutionary Algorithms (EAs) are built to deal with optimization problems, which are designed from many scientific and application fields, such as science, economic and engineering [1,2]. Principally, EAs can be classified into following categories, such as Genetic Algorithms (GAs) [3, 4] , Evolutionary Strategies (ESs) [5], Genetic Programming (GP) [6], Evolutionary Programing (EP) [7], Differential Evolution (DE) [8] and other natural-inspired algorithms [9]. DE

Joel Chacón
Guanajuato, Gto.
E-mail: joel.chacon@cima.mt

Carlos Segura
Guanajuato Gto.
E-mail: carlos.segura@cimat.mx

was introduced by Storn and Price [8], also is cosidered as one of the most effective EAs used to deal with real-world optimization problems, mainly for its convergency properties. Similarly than with other EAs, DE follows the natural evolution process which involves mutation, recombination and selection to evolve a population throuth an iterative progress until the criteria stop is reached. However, the peculiarity of DE resides in considering difference of vectors parameters to explore the search space, being very similar than its precursor algorithms namely the Nelder-Mead [10] and the Controled Random Search (CRS) [11]. In spite of the popularity and effectiveness of DE, there exists several weakness that had been partially solved througth learning techniques. One of the first weakness and possibly the most important, is the performance of DE which is very sensitive to choice of the strategy parameters depending in the objective function [12]. Several strategies as adaptive and self-adaptive have been proposed to alliviate this drawback [13, 14]. However, none of them has shown superior results than the rest.

A second weakness of DE algorithms resides in the reproduction phase. In DE this phase involves the vector differences, therefore it depends on the content of the population affecting the search process, as result a limited number of solutions are produced. In fact, this issue can lead to converge into a local optima or lost of diverse solutions better known as premature convergence [15]. On the other hand, there exist situations where the search process could not progress and the population remains diverse, this phenomena is known as stagnation [16]. Its well known that stagnation ocurrs with small populations size. Although that large populations are not prone to stagnate, it involves more evaluation functions and in certain situations is not available a large population e.g. expensive optimization problems ref CEC.

The last one drawback is highly related with the diversity of the population. Generally speaking, the search process of all the EAs involves two process: exploration and intensification. A desirable behavior of an algorithm is to produce a proper balance between these two process. So that first it induces an exploration in the search space and after that an exploitation of the knowledge gathered during the search process [17]. Both exploration as exploitation are equally important, since that with a excessive exploitation, the population loses its diversity and the populations members can be located in a reduced sub-optimal region of the search space. On the other hand, if the exploration is dominant, the algorithm waste resources on uninteresting regions, resulting in too slow convergence and in poor quality-solutions. Principally, DE algorithms are very likely to prematurely converge, since that introduce a high selection pressure [15]. Several strategies have been proposed in DE to deal with premature convergence, as parameter adaptation based on the idea of controlling the population diversity [17], autho-enhanced population diversity mechanism [18], alternative selection strategy [15].

A recient and novel approach to deal with these diversity issues, is through a sopisticated replacement strategy that explictly preserves the diversity [19]. This method trnasforms a single-objective problem into a multi-objective one, by considereing diversity as an explicit objective, with the idea of adapting the

balance induced between exploration and exploitation to various optimization stages. Thus, the ideal balance is reached considering the criteria stop of the algorithm.

Our proposal follows a similar guideline, where it aims an ideal balance between exploration and exploitation considering the criteria stop. However, we keepkeep the single-objective context and focus in DE algorithms.

The rest of the paper is organized as follows. In section .. is described a the classic DE. A brief revision of the last EAs is showed in section ... Our proposal based in diversity is described in the section ... In the section .. are showed the experimental results including some of the most popular EAs. Finally, our conclusions and some lines of future work are given in section ...

**Techniques to deal with this hybridization with anneling procedures to reduce the selection pressure ref37 **Generational replacement ref3. **Incrementing the population size ref19. **Organization of the paper.ss

2 Literature Review

2.1 Differential Evolution: Basic Concepts

In the literature are present several variants of DE ref1 ref8. For simplicity, in this work is used the classic DE scheme references... "survey-state-art" Originally DE was proposed as direct search method for single-objective continuous optimization problems ref44(improving). Usually, the parameters governing the system performance are presented in a vector like $\mathbf{X} = [x_1, x_2, ..., x_D]^T$, which is identified as an individual. Particularly, for real parameter optimization each parameter x_i is a real number.

In single-objective optimization, the aim is to obtain the vector $\mathbf{X}^*$ wich minimizes (or maximize) a defined objective function, mathecally denoted by $f(\mathbf{X})(f : \Omega \subseteq \Re^D \rightarrow \Re)$, i.e., $f(\mathbf{X}^*) < f(\mathbf{X})$ for all $\mathbf{X} \in \Omega$, where Ω is a non-empty large finite set identified as the domain of the search.

The basic scheme of DE consists that given the target parameter vectors (each vector of the population), a new mutant (or donant) vector is created using a vector generation strategy. After that, the mutant vector is combined with the target vector to generate the trial vector. In the same vein, each one of the trial vectors is compared with the correspond target vector, and the vector with the best fitness is select to survive as trial vector of the next generation. In case of tie, the new generated trial vector survives.

2.1.1 Initialization

The DE algorithms as is usual begins with a randomly initiated population of NP parameter vectors. Subsequent generations in DE are denoted by $G = 0, 1, ..., G_{max}$. The ith vector of the population at the current generation is denoted as:

$$\mathbf{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, ..., X_{D,i,G}]. \quad (1)$$

The initial population should cover a bounded range and is reached by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum bounds. Hence, each jth component of the ith vector is initialized as follow:

$$X_{j,i,0} = x_{j,min} + rand_{i,j}[0,1](x_{j,max} - x_{j_min}) \tag{2}$$

where $rand_{i,j}[0,1]$ is a uniformly distribuited random number lying between 0 and 1.

2.1.2 Mutation

The mutation can be seen as a change or perturbation with a random element. Particularly, in DE a parent vector called *target* vector is combined through a defined strategy to form the *donor* vector. In one simple form, a mutant vector $V_{i,G}$ is created from the ith target vector and is stablished as follows:

$$\mathbf{V}_{i,G} = \mathbf{X}_{r1,G} + F(\mathbf{X}_{r2,G} - \mathbf{X}_{r3,G}) \quad r1 \neq r2 \neq r3 \tag{3}$$

The indices $r1, r2, r3 \in [1, NP]$ are mutually exclusive integers randomly chosen from the range $[1, NP]$. It is important take into account that the difference of any two vectors is scaled by a scalar number F and usually is defined in the interval $[0.4, 1]$, also the scale difference is added to the third one.

2.1.3 Crossover

In order to increase the diversity of the perturbed parameter vectors, a crossover operation is aplied to the generated donor vector. Accordingly this, the target vector is mixed with the mutated vector to form the trial vector $\mathbf{U_{i,G}} = [u_{1,i,G}, u_{2,i,G}, ..., u_{D,i,G}]$. In the DE-context are present two kinds of crossover methods –*exponential* and *binomial*(or uniform), however in this paper only is considered the binomial crossover. In the binomial crossover strategy, the trial vector $\mathbf{U}_{i,G}$ is generated as follows:

$$\mathbf{U}_{j,i,G} = \begin{cases} \mathbf{V}_{j,i,G}, & \text{if}(rand_{i,j}[0,1] \leq CR \quad or \quad j = j_{rand}) \\ \mathbf{X}_{j,i,G}, & \text{otherwise} \end{cases} \tag{4}$$

where $rand_{i,j}[0,1]$ is a uniformly distribuited random number, which is generated for each jth component of the ith vector parameter. j_{rand} is a randomly chosen index, which ensures that $\mathbf{U}_{i,G}$ has at least one component from $\mathbf{V}_{i,G}$. CR is the crossover constant $\in [0,1]$, which has to be determined by the user.

2.1.4 Selection

Once generated the trial vectors, is performed a greedy selection scheme. This selection determine wheter the target or the trial vector survives to the next generation, and is described as follows:

$$\mathbf{X}_{j,i,G+1} = \begin{cases} \mathbf{U}_{i,G}, & \text{if} \quad f(\mathbf{U}_{i,G}) \leq f(\mathbf{X}_{i,G}) \\ \mathbf{X}_{i,G}, & \text{otherwise} \end{cases} \tag{5}$$

where $f(\mathbf{X})$ is the objective function to be minimized. Hence, the population eigher gets better or remains the same fitness status, but never deteriorates.

The mutation scheme decribed with the crossover proposed is refered as DE/rand/1/bin. The general convention is DE/x/y/x, where DE indicates "differential evolution", x denotes the base vector to be perturbed, y is the number of difference vectors considered for perturbation and z is the type of crossover to use.

——Diversity Revision *Explain the influence of the paramters. *Show the implication of these operators with the population diversity. *Talk about hybrid and adaptive strategies.

2.2 Diversity Work with Differential Evolution

3 Differential Evolution Through Years

In the last decade, DE has been recognized as one of the most eficient and simple algorithm. Particularly, the algorithms derivated from DE have obtained one of the top rank in the Congress on Evolutionary Computation (CEC). In this work we are interested in the bias of DE algorithms and CEC competitions problems througth the last years. It is important to take into account that althougth the kind of problems have changed, the DE are in the top rank. In 2005 CEC competition on real parameter optimization, on 10-D problems classical DE secured 2nd rank and a self-adaptive DE variant called SaDE secured third rank although they performed poorly over 30-D problems. Although a powerful variant of ES, known as restart covariance matrix adaptation ES (CMA-ES) yielded better results than classical and self-adaptiveDE, later on many improver DE variants were proposed in the period 2006-2009. Hence, another rigorous comparison is needed to determine how well these variants might compete against the restart CMA-ES and many othe real parameter optimizers over the standard numerical benchmarks. It is also interesting to note that the variants of DE continued to secure front ranks in the subsequent CEC competitions like CEC 2006 competition on constrained real parameter optimization (first rank), CEC 2007 competition on multi-objective optimization (second rank), CEC 2008 competition on large scale global optimization (third rank), CEC 2009 competition on multi-objective optimization (first rank was

Table 1 My caption

Year	Topic	Places
2005	Single Objective Real-Parameter Optimization	1th Restart Covariance Matrix Adaptation (IPOP-CMA-ES)
		2th Estimation of Distribution Algorithm (EDA)
		3th Real Coded Memetic Algorithm (RCMA)
2006	Constrained Real-Parameter Optimization	1th ϵ Constrained Based DE (ϵ RDE)
		2th Dynamic Multi-swarm Optimizer (DMS-PSO)
		3th Self-adaptive Differential Evolution (SaDE)
2008	Large Scale Global Optimization	1th Multiple Trayectory Search (MTS)
		2th Dynamic Multi-swarm Optimizer (DMS-PSO)
		3th Self-adaptive DE (jDEdynNP-F)
2009	Dynamic Optimization	1th Self-adaptive DE (jDE)
2010	Constrained Real-Parameter Optimization	1th ϵ Constrained DE with gradient based mutation (ϵ Deg)
		2th Ensemble of Constraint Handling Techniques (ECHT)
		3th Self-adaptive DE (jDEsoco)
	Large Scale Global Optimization	1th Memetic Algorithm Based on Local Search Chains (MA-SW-Chains)
		2th Two Stage Based Ensemble Optimization Evolutionary Algorithm (EOEA)
		3th Dynamic Multi-Swarm Particle Swarm Optimizer (DMS-PSO)
2011	Real World Optimization Problems	1th Genetic Algorithm with Multi-parent Crossover (GA-MPC)
		2th Hybrid DE (DE-Λ_{CR})
		3th Self-adaptive Multi-Operator DE (SAMODE)
2013	Single Objective Real-Parameter Optimization	1th Restart Covariance Matrix Adaptation (NBIPOPaCMA)
		2th Hybrind CMA-ES and Iterated Local Search (iCMAES-ILS)
		3th Dynamically updated Region-based Memetic Algorithm with Local Search Chaining and CMA-ES (DRMA-LSCh-CMA)
2014	Single Objective Real-Parameter Optimization	1th ϵ Constrained Based DE (ϵ RDE)
	Computationally Expensive Optimization	1th Mean Variance Mapping Optimization (MVMO)
		2th Surrogate-assited Optimization with Memetic Optimization Dimensionally-controled Search (SO-MODS)
		3th Simple Algorithm Withouth Approximation (WA)
2015	Learned Based Single Objective Real-Parameter Optimization	1th Succesful Parent Selecting L-SHADE with Eigenvector-Based Crossover (SPS-L-SHADE-EIG)
		2th DE with success Parameter Adaptation (DesPA)
		3th Mean Variance Mapping Optimization (MVMO) and Neurodynamic L-SHADE (LSHADE-ND)
	Multi-Niche Single Objective Real-Parameter Optimization	1th Algorithm Based on Decomposition
		2th Particle Swarm Optimization (PSO)
		3th Neighborhood Based Speciation Differential Evolution (NSDE)
	Computationally Expensive Optimization	1th Mean Variance Mapping Optimization (MVMO)
		2th Parameter Tuned CMA-ES
2016	Learned Based Single Objective Real-Parameter Optimization	1th Mean-Variance Mapping Optimization (MVMO-PHM)
		2th Cooperative Co-evolution L-SHADE with Restarts (CCL-SHADE)
		3th L-SHADE with four strategies (L-SHADE44)
	Single Objective Real-Parameter Optimization	1th United Multi-Operator Evolutionary Algorithms (UMOEAs-II)
		2th Ensemble Sinusoidal Parameter Adaptation with L-SHADE (L-SHADE-EpSin)
		3th Improved L-SHADE (iL-SHADE)
	Computationally Expensive Optimization	1th Mean Variance Mapping Optimization (MVMO)
		2th Artificial super-Bee enhanced Colony (AsBeC)
		3th Reduced Yin-Yang-Pair Optimization (RYYPO)
2017	Single Objective Real-Parameter Optimization	1th Effective Butterfly Optimizer with Covariance Matrix Adapted Retreat Phase (EBOwithCMAR)
		3th Ensemble Sinusoidal DE Covariance Matrix Adaptation with Euclidean Neighborhood (L-SHADE-cnEpSin)

taken by a DE-based algorithm MOEA/D for unconstrained problems), and CEC 2009 competition on evolutionary computation in dynamic and uncertain enviroments (first rank).

Some of the CEC competitions are described as follows:

4 Proposal

Our proposal has a similar direction that the one described by Carlos Segura et. al., where is proposed a multi-objective replacement strategy. The main advantage of the new proposal is that the balance between exploration and exploitation is automatically adjusted based on the given stopping criterion. Thus, the stopping criterion, as well as the elapsed time or the evaluations already executed, are used as inputs to the replacement strategy, which is one of the novelties of the new design. In this way, for shorter stoping criteria the method induces a faster reduction in diversity than for longer stopping criteria.

One of the basic principles behind the development of the replacement strategy devised in this paper is that ndividuals that contribute too little to diversity –the contribution is measured with the DCN value– should not be accepted regardless of their original objective value. In out approach, individuals that contribute too little are penalized. The value D represent the minimum DCN required to avoid being penalized. Any individual whose DCN value is lower than this threshold value is penalized. The key principle residesin how to evaluate wheter an individual contributes enough or not, i.e., how to set the value D. The value of D should depend on the optimization stage. Specifically, this value should be rediced as the stopping criterion is approached.

In our scheme, an initial D_I value must be set. Then, a linear reduction of D is done. The reduction is calculated in such way that by the end of the xecution, the resulting value is 0. In this work, the stopping criterion is set by function evaluations (nfes). Thus, if max_nfes is the maximum number of evaluations and $nfes$ the elapsed number of evaluations, D can be calculated as $D = D_I - D_I * (nfes/max\_nfes)$. According to REFERENCE updating D is more approiate through a linear reduction. This process is formulated as

$$arg_{x_j \in C_i} \quad min \quad dist(x_j, o_i) \sqrt{\sum_{d=1}^{D} (x_j^d - o_i^d)} \tag{6}$$

$$S_i = \left\{ \sqrt{\sum_{d=1}^{D} (x_{seed}^d - x_i^d)} \leq r \right\} \tag{7}$$

Algorithm 1 DE scheme

1: Randomly initialize the population of NP individuals, where each one is uniformly distirbuited.
2: **while** stopping criterion is nor satisfied **do**
3: **for** $i = 1$ to NP **do**
4: Mutation: Generate the Donor vector according Eq. (3)
5: Crossover: Recombine the mutate vector according Eq. (4)
6: Selection: Update the parent vector according Eq. (5)
7: Replacement:

Algorithm 2 Replacement Phase

1: $Survivors = Penalized = \emptyset$.
2: $Current = Population \cup Offspring \cup Elite$.
3: Sort $Current$ according to fitness.
4: **while** $Survivos < pop\_size$ **do**
5: Select the best individual $Current_best$ in $Current$ as a new seed.
6: Find the other individuals nearest according to Eq. DISTANCE and move to $Penalized$.
7: Move the best individual $Current_best$ to $Survivors$.
8: **while** $Survivos < pop\_size$ **do**
9: Select the individual $Penalized$ with maximum distance to closest $Survivor$.
10: Move individual $Penalized$ to $Survivors$.

5 Experimental Study

6 Conclusion

References

1. N. Noman, H. Iba, Differential evolution for economic load dispatch problems, Electric Power Systems Research **78**(8), 1322 (2008)
2. U.K. Chakraborty, *Advances in differential evolution*, vol. 143 (Springer, 2008)
3. M. Srinivas, L.M. Patnaik, Genetic algorithms: A survey, computer **27**(6), 17 (1994)

4. H.P. Schwefel, Numerische optimierung von computer-modellen (phd thesis), Reprinted by Birkhuser (1977)
5. H. John. Holland, adaptation in natural and artificial systems: An introductory analysis with applications to biology, control and artificial intelligence (1992)
6. J.R. Koza, *Genetic Programming II, Automatic Discovery of Reusable Subprograms* (MIT Press, Cambridge, MA, 1992)
7. D.B. Fogel, L.J. Fogel, J.W. Atmar, in *Signals, systems and computers, 1991. 1991 Conference record of the twenty-fifth asilomar conference on* (IEEE, 1991), pp. 540–545
8. R. Storn, K. Price, Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces, Journal of global optimization **11**(4), 341 (1997)
9. S. Das, P.N. Suganthan, Differential evolution: A survey of the state-of-the-art, IEEE transactions on evolutionary computation **15**(1), 4 (2011)
10. J.A. Nelder, R. Mead, A simplex method for function minimization, The computer journal **7**(4), 308 (1965)
11. W. Price, Global optimization by controlled random search, Journal of Optimization Theory and Applications **40**(3), 333 (1983)
12. R. Gämperle, S.D. Müller, P. Koumoutsakos, A parameter study for differential evolution, Advances in intelligent systems, fuzzy systems, evolutionary computation **10**(10), 293 (2002)
13. J. Brest, S. Greiner, B. Boskovic, M. Mernik, V. Zumer, Self-adapting control parameters in differential evolution: A comparative study on numerical benchmark problems, IEEE transactions on evolutionary computation **10**(6), 646 (2006)
14. J. Zhang, A.C. Sanderson, Jade: adaptive differential evolution with optional external archive, IEEE Transactions on evolutionary computation **13**(5), 945 (2009)
15. Â.A. Sá, A.O. Andrade, A.B. Soares, S.J. Nasuto, in *AISB 2008 Convention Communication, Interaction and Social Intelligence*, vol. 1 (2008), vol. 1, p. 57
16. J. Lampinen, I. Zelinka, et al., in *Proceedings of MENDEL* (2000), pp. 76–83
17. D. Zaharie, in *Proc. of MENDEL*, vol. 9 (2003), vol. 9, pp. 41–46
18. M. Yang, C. Li, Z. Cai, J. Guan, Differential evolution with auto-enhanced population diversity, IEEE transactions on cybernetics **45**(2), 302 (2015)
19. C. Segura, C.A.C. Coello, E. Segredo, A.H. Aguirre, A novel diversity-based replacement strategy for evolutionary algorithms, IEEE transactions on cybernetics **46**(12), 3233 (2016)