# Multimodal Optimization Using Particle Swarm Optimization Algorithms: CEC 2015 Competition on Single Objective Multi-Niche Optimization

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***Abstract*—The aim of multimodal optimization is to locate multiple peaks/optima in a single run and to maintain these found optima until the end of a run. The results of seven variants of particle swarm optimization (PSO) algorithms on IEEE Congress on Evolutionary Computation (CEC) 2015 single objective multi-niche optimization problems are reported in this paper. The PSO algorithms include PSO with star structure, PSO with ring structure, PSO with four clusters structure, PSO with Von Neumann structure, social-only PSO with star structure, social-only PSO with ring structure, and cognition-only PSO. The experimental tests are conducted on fifteen benchmark functions. Based on the experimental results, the conclusions could be made that the PSO with ring structure performs better than the other PSO variants on multimodal optimization. To obtain good performance on the multimodal optimization problems, an algorithm needs to converge the candidate solutions to the global optima while keep the population diversity during whole search process.**

***Index Terms*—Multimodal optimization; particle swarm optimization; topology structure; population diversity**

## I. Introduction

Swarm intelligence is based on a population of individuals [1]. In swarm intelligence, an algorithm maintains and successively improves a collection of potential solutions until some stopping condition is met. The solutions are initialized randomly in the search space. The search information is propagated through the interaction among solutions. Based on the solutions convergence and divergence, solutions are guided toward the better and better areas.

In swarm intelligence algorithms, there are several solutions which exist at the same time. The premature convergence may happen due to the solution getting clustered together too fast. The population diversity is a measure of exploration and exploitation. Based on the population diversity changing measurement, the state of exploration and exploitation can be obtained. The population diversity definition is the first step to give an accurate observation of the search state. Many studies of population diversity in evolutionary computation algorithms and swarm intelligence have been proposed in [2]–[8].

Particle swarm optimization (PSO) is a population-based stochastic algorithm modeled on social behaviors observed in flocking birds [9], [10]. A particle flies through the search space with a velocity that is dynamically adjusted according to its own and its companion's historical behaviors. Each particle's position represents a solution to the problem. Particles tend to fly toward better and better search areas over the course of the search process [11], [12]. Different topology structure can be utilized in PSO, which will have different strategy to share search information for every particle. The PSO algorithm with different topology structure will have different capacity developing, which means that the different structure may suit for different problems or different search stages.

The multimodal optimization is aimed to locate multiple global optima at in a single run and to maintain these found optima until the end of a run [13]–[16]. Two performance criteria can be used to measure the success of search algorithms. One is whether an optimization algorithm could find all desired optima including global and/or local optima, and the other is whether it can maintain multiple candidate solutions stably over a run [15]. The population diversity of swarm intelligence was utilized to measure the average distance among candidate solutions, which could reflect the algorithm's ability of solutions maintenance [6], [17].

There are fifteen benchmark problems in IEEE Congress on Evolutionary Computation (CEC) 2015 Competition on single objective multi-niche optimization [18]. An algorithm needs to test on each problem with three kinds of dimension, and all optima are shifted and rotated in the decision space. In this paper, the effectiveness of different variants of PSO algorithms is verified on these benchmark functions. The PSO variants include seven PSO variants, which are PSO with star structure, PSO with ring structure, PSO with four clusters structure, PSO with Von Neumann structure, social-only PSO with star structure, social-only PSO with ring structure, and cognition-only PSO [17], [19], [20].

The remaining of the paper is organized as follows. Section II reviews the basic concepts of a particle swarm optimization algorithm. Section III introduces the concepts and performance criteria of multimodal optimization. Experiments of seven variants of PSO algorithms on multimodal optimization are conducted in Section IV. Finally, Section V concludes with some remarks and future research directions.

## II. Particle Swarm Optimization

Particle swarm optimization emulates the swarm behavior and the individuals represent points in the $n$-dimensional search space. A particle represents a potential solution. Each particle is associated with two vectors, *i.e.*, the velocity vector and the position vector. For the purpose of generality and clarity, $m$ represents the number of particles and $n$ the number of dimensions. The position of a particle is represented as $x_{ij}$, and the velocity of a particle is represented as $v_{ij}$, $i$ represents the $i$th particle, $i = 1, \cdots, m$, and $j$ is the $j$th dimension, $j = 1, \cdots, n$.

### A. Canonical Particle Swarm

The velocity and position update equations in canonical PSO algorithm are as follow [1], [21], [22]:

$$\mathbf{v}_i \leftarrow w\mathbf{v_i} + c_1\text{rand}()(\mathbf{p}_i - \mathbf{x}_i) + c_2\text{rand}()(\mathbf{p}_n - \mathbf{x}_i) \quad (1)$$

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i \quad (2)$$

where $w$ denotes the inertia weight and usually is less than $1$ [23], $c_1$ and $c_2$ are two positive acceleration constants, rand() is a random function to generate uniformly distributed random numbers in the range $[0, 1)$ and is different for each dimension and each particle, $\mathbf{x}_i$ represents the $i$th particle's position, $\mathbf{v}_i$ represents the $i$th particle's velocity, $\mathbf{p}_i$ is termed as personal best, which refers to the best position found by the $i$th particle, and $\mathbf{p}_n$ is termed as local best, which refers to the position found by the members in the $i$th particle's neighborhood that has the best fitness evaluation value so far.

The basic procedure of PSO is shown as Algorithm 1. A particle updates its velocity according to equation (1), and updates its position according to equation (2). The $c_1\text{rand}()(\mathbf{p}_i - \mathbf{x}_i)$ part can be seen as a cognitive behavior, while $c_2\text{rand}()(\mathbf{p}_n - \mathbf{x}_i)$ part can be seen as a social behavior.

### B. Variants of Particle Swarm

The PSO algorithm that updates velocities using Eq. (1) is called full model. In addition, Kennedy [19] introduced two special PSO models: the cognition-only model and the social-only model, which are defined by omitting components of Eq. (1). The cognition-only model is obtained by dropping the social component in the full model. The velocity update equation in particle swarm with cognition-only model is given as follows:

$$\mathbf{v}_i \leftarrow w\mathbf{v_i} + c_1\text{rand}()(\mathbf{p}_i - \mathbf{x}_i) \quad (3)$$

The social-only model is obtained by dropping the cognition component in the full model. The velocity update equation in particle swarm with social-only model is given in (4):

$$\mathbf{v}_i \leftarrow w\mathbf{v_i} + c_2\text{rand}()(\mathbf{p}_n - \mathbf{x}_i) \quad (4)$$

In PSO algorithm, a particle not only learns from its own experience, it also learns from its companions. It indicates that a particle's "moving position" is determined by its own experience and the neighbors' experience [20].

**Algorithm 1:** The basic procedure of particle swarm optimization

**Initialization**: Initialize velocity and position randomly for each particle in every dimension;
**while** *have not found "good enough" solution or not reached the pre-determined maximum number of iterations* **do**
    Calculate each particle's fitness value;
    Compare fitness value between current position and the best position in history (personal best, termed as $\mathbf{p}_i$). For each particle, if the fitness value of the current position is better than $\mathbf{p}_i$, then update $\mathbf{p}_i$ to be the current position;
    Select the particle which has the best fitness value among current particle's neighborhood, this particle is called the neighborhood best (termed as $\mathbf{p}_n$);
    Update each particle's velocity and position, respectively;

### C. Topology Structure

A particle updates its position in the search space at each iteration. The velocity update in Eq. (1) consists of three parts, which are previous velocity, cognition part, and social part. The cognition part means that a particle learns from its own searching experience, and the social part means that a particle can learn from other particles, or learn from the best in its neighbors in particular. Topology defines the neighborhood of a particle.

PSO algorithm has different kinds of topology structures, e.g., star, ring, four clusters, and Von Neumann structure [20], [24]. A particle in a PSO with a different structure has different number of particles in its neighborhood with a different scope. Learning from a different neighbor means that a particle follows different neighborhood (or local) best, in other words, topology structure determines the connections among particles, and the strategy of search information propagation. Although it does not relate to the particle's cognition part directly, topology can affect the algorithm's convergence speed and the ability of avoiding premature convergence, i.e., the PSO algorithm's ability of exploration and exploitation.

Different topology structure can be utilized in PSO, which will have different strategy to share search information for every particle. Topology determines the structure of particles' connections and the transmission of search information in the swarm. Star and ring are the two most commonly used structures. A PSO with a star structure, where all particles are connected to each other, has the smallest average distance in swarm, and on the contrary, a PSO with a local ring structure, where every particle is connected to two near particles, has the biggest average distance in swarm [12], [25].

Four most commonly used topology structures are considered in this paper. They are star, ring, four clusters, and Von Neumann structure.

- **Star** structure: because all particles or nodes are connected, search information is shared in a global scope, this topology is frequently termed as *global* or *all* topology. With this topology, the search information is shared in the whole swarm, and a particle with the best fitness value will be chosen to be the "leader." Other particles will follow the leader to find optimum.
- **Ring** structure: a particle is connected with two neighbors in this topology. A particle compares its fitness value with its left neighbor at first, and then the winner particle compares with the right neighbor. A particle with better fitness value in this small scope is determined by these two comparisons.
- **Four clusters** structure: as the name indicated, the whole swam are divided into four subgroups. Each subgroup is a small star topology, which has a "leader" particle in this subgroup, sharing its own search information. Each subgroup also has three linking particles links to other three subgroups. The linking particles are used to exchange search information with other three subgroups.
- **Von Neumann** structure: in this structure, every particle has four neighbors that are wrapped on four sides, and the swarm is organized as a mesh.

## III. MULTIMODAL OPTIMIZATION

The aim of multimodal optimization is to locate multiple peaks/optima in a single run [16] and to maintain these found optima until the end of a run [14], [15]. An algorithm on solving multimodal optimization problems should have two kinds of abilities: find global/local optima as many as possible and preserve these found solutions until the end of search.

Three performance measures are introduced in [18], [26]. The peak ratio (PR) measures the average percentage of all known global optima found over multiple runs, and the success rate (SR) measures the percentage of successful runs (a successful run is defined as a run where all known global optima are found) out of all runs. The equations of $PR$ and $SR$ calculation are given in (5) and (6), respectively.

$$PR = \frac{\sum_{run=1}^{NR} NPF_i}{NKP \times NR} = \frac{NPF}{NKP \times NR} \quad (5)$$

$$SR = \frac{NSR}{NR} \quad (6)$$

where $NPF_i$ denotes the number of global optima found in the end of the $i$-th run, $NPF$ denotes the number of total global optima found in all runs, $NKP$ the number of known global optima, $NR$ the number of runs, and $NSR$ denotes the number of successful runs [18], [26]. All global optima are known for the CEC 2015 multi-niche benchmark problems. The found optimum is compared with the known global optima to calculate the number of unique optima found. The process for calculating number of the global optima found is given in algorithm 2. A successful run is record when the number of found unique optima equals to the number of the problem's goal optima.

**Algorithm 2:** The algorithm for calculating number of the global optima found.

**Input**: $S_{individuals}$: a set of individuals (candidate solutions) in the population; $S_{optima}$: a set of $q$ global optima solutions; $\epsilon$: accuracy level; $F^{\star}$: the fitness of global optima; $r$: niche radius; $flag$: a set to indicate that $j$th global optimum is found or not

**Output**: $count$: the number of identified global optima found in the end of run

Initialization: $count = 0$, $flag_j = false$, $j = 1, 2, \cdots, q$;
**for** *each individual* $\mathbf{x}_i$ *in the candidate solutions set* $S_{individuals}$ **do**
    **if** $|F^{\star} - fit(\mathbf{x}_i)| \leq \epsilon$ **then**
        **for** *each solution* $\mathbf{o}_j$ *in global optima solutions set* $S_{optima}$ **do**
            **if** *distance*$(\mathbf{x}_i, \mathbf{o}_j) < r$ **then**
                $flag_j = true$;
                break;
$count \leftarrow$ total number of $true$ in $flag$;

The minimum number of function evaluations (FEs) required to locate all known global optima at a specified accuracy level $\epsilon$ is utilized to measure the convergence speed of a search algorithm. The equation (7) gives the calculation the average FEs ($AveFEs$) over multiple runs.

$$AveFEs = \frac{\sum_{run=1}^{NR} FEs_i}{NR} = \frac{FEs}{NR} \quad (7)$$

where $FEs_i$ denotes the number of evaluations used in the $i$-th run, and $FEs$ denotes the number of total evaluations used in all runs. The $MaxFEs$ is used in (7) when the algorithm cannot locate all the global optima until the end of run [26].

## IV. EXPERIMENTAL STUDY

### A. Benchmark Functions and Parameters Setting

The benchmark functions are given in Table I [18]. In all experiments, the parameters of particle swarm optimization algorithms are set as follows: $w = 0.72984$, $c_1 = c_2 = 1.496172$ [17]. The accuracy level $\epsilon$ is 0.1, and the radius $r$ is 0.5. The population size and number of iterations are given in Table II. For each problem with a certain dimension, all algorithms have run 51 times.

### B. Performance

The results of peak ratio are given in Table III, and the results of success rate are given in Table IV. In general, PSO or social-only PSO with star structure will converge to one optimum at the end of run. This is because that there is no solution maintenance strategy in PSO/social-only PSO with star structure; all solutions will be converged to the best found solutions. The PSO/social-only PSO with star structure has no success run on problems with multiple global optima, and it

TABLE I
SUMMARY OF THE CEC'15 MULTI-NICHE TEST FUNCTIONS.

| | No. | **Function Name** | Dimension | Goal optima No. global/local | $F_i^\star = F_i(x^\star)$ |
|---|---|---|---|---|---|
| Expanded Scalable Function | $f_1$ | Shifted and Rotated Expanded Two-Peak Trap | 5 | 1 / 15 | 100 |
| | | | 10 | 1 / 55 | |
| | | | 20 | 1 / 210 | |
| | $f_2$ | Shifted and Rotated Expanded Five-Uneven-Peak Trap | 2 | 4 / 21 | 200 |
| | | | 5 | 32 / 0 | |
| | | | 8 | 256 / 0 | |
| | $f_3$ | Shifted and Rotated Expanded Equal Minima | 2 | 25 / 0 | 300 |
| | | | 3 | 125 / 0 | |
| | | | 4 | 625 / 0 | |
| | $f_4$ | Shifted and Rotated Expanded Decreasing Minima | 5 | 1 / 15 | 400 |
| | | | 10 | 1 / 55 | |
| | | | 20 | 1 / 210 | |
| | $f_5$ | Shifted and Rotated Expanded Uneven Minima | 2 | 25 / 0 | 500 |
| | | | 3 | 125 / 0 | |
| | | | 4 | 625 / 0 | |
| | $f_6$ | Shifted and Rotated Expanded Himmelblau's Function | 4 | 16 / 0 | 600 |
| | | | 6 | 64 / 0 | |
| | | | 8 | 256 / 0 | |
| | $f_7$ | Shifted and Rotated Expanded Six-Hump Camel Back | 6 | 8 / 0 | 700 |
| | | | 10 | 32 / 0 | |
| | | | 16 | 256 / 0 | |
| | $f_8$ | Shifted and Rotated Modified Vincent Function | 2 | 36 / 0 | 800 |
| | | | 3 | 216 / 0 | |
| | | | 4 | 1296 / 0 | |
| Composition Func. | $f_9$ | Composition Function 1 | 10, 20, 30 | 10 / 0 | 900 |
| | $f_{10}$ | Composition Function 2 | 10, 20, 30 | 1 / 9 | 1000 |
| | $f_{11}$ | Composition Function 3 | 10, 20, 30 | 10 / 0 | 1100 |
| | $f_{12}$ | Composition Function 4 | 10, 20, 30 | 10 / 0 | 1200 |
| | $f_{13}$ | Composition Function 5 | 10, 20, 30 | 10 / 0 | 1300 |
| | $f_{14}$ | Composition Function 6 | 10, 20, 30 | 1 / 19 | 1400 |
| | $f_{15}$ | Composition Function 7 | 10, 20, 30 | 1 / 19 | 1500 |

only has some success runs on function $f_1$ and $f_4$. This is because that function $f_1$ and $f_4$ only have one global optimum, respectively. PSO with ring structure outperforms the other PSO variants, which is due to that particles follow different local best positions. The existing of different local best could maintain several solutions until the end of run.

The Table III gives the number of the global optima found by variants of PSO algorithms. For some problems, the global optimum could not be found in the predefined number of iteration by all variants of PSO algorithms. For these problems, the statistical results of 51 runs are given in Table V. The values of "best", "median", "Worst", and "Mean" are measured on the best fitness values for each run.

The computational complexity is also important in multimodal optimization. For the success runs, the average fitness evaluations numbers are given in Table VI. The time costs of variants of PSO with star structure, ring structure, and Von Neumann structure are given in Table VII. There is no significant difference among different PSO variants.

## V. CONCLUSIONS

In this paper, seven variants of PSO algorithms are utilized to solve fifteen problems in CEC 2015 multi-niche benchmark suite [18]. The experimental results show that the PSO with star structure is not suit for the multimodal optimization. The PSO with star structure cannot maintain multiple solutions; and all solution will converge to one optimum until end of run. The PSO with ring structure outperforms the other PSO variants, which is due to that particles follow different local best positions.

From the experimental results, we can conclude that to obtain good performance on the multimodal optimization problems, an algorithm needs to converge the candidate solutions to the global optima while keep the population diversity during whole search process. The search effectiveness and population diversity maintenance should be balanced in the PSO algorithm. For PSO with star structure, an extra archive to store the found global optima or an additional strategy to maintain the population diversity may be a way to improve its performance in multimodal optimization. For PSO with ring structure, the search effectiveness should be improved in multimodal optimization.

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TABLE II
THE PARAMETERS SET IN VARIANTS OF PSO ALGORITHMS.

| **Function** | $D$ | size | Iteration | $MaxFEs$ | $2000 \times D \times \sqrt{q}$ |
|---|---|---|---|---|---|
| $f_1$ | 5 | 100 | 400 | 40000 | 40000 |
| | 10 | 200 | 750 | 150000 | 149666.29 |
| | 20 | 480 | 1211 | 581280 | 581033.56 |
| $f_2$ | 2 | 100 | 200 | 20000 | 20000 |
| | 5 | 200 | 283 | 56600 | 56568.54 |
| | 8 | 400 | 640 | 256000 | 256000 |
| $f_3$, $f_5$ | 2 | 100 | 200 | 20000 | 20000 |
| | 3 | 200 | 336 | 67200 | 67082.03 |
| | 4 | 800 | 250 | 200000 | 200000 |
| $f_4$ | 5 | 64 | 625 | 40000 | 40000 |
| | 10 | 100 | 1497 | 149700 | 149666.29 |
| | 20 | 200 | 2906 | 581200 | 581033.56 |
| $f_6$ | 4 | 100 | 320 | 32000 | 32000 |
| | 6 | 200 | 480 | 96000 | 96000 |
| | 8 | 400 | 640 | 256000 | 256000 |
| $f_7$ | 6 | 100 | 340 | 34000 | 33941.12 |
| | 10 | 160 | 708 | 113280 | 113137.08 |
| | 16 | 400 | 1280 | 512000 | 512000 |
| $f_8$ | 2 | 100 | 240 | 24000 | 24000 |
| | 3 | 240 | 368 | 88320 | 88181.63 |
| | 4 | 1200 | 240 | 288000 | 288000 |
| $f_{10}$ | 10 | 100 | 633 | 633000 | 63245.55 |
| | 20 | 200 | 633 | 126600 | 126491.10 |
| | 30 | 200 | 949 | 189800 | 189736.65 |
| $f_9$, $f_{11}$, $f_{12}$, $f_{13}$ | 10 | 80 | 791 | 63280 | 63245.55 |
| | 20 | 100 | 1265 | 126500 | 126491.10 |
| | 30 | 100 | 1898 | 189800 | 189736.65 |
| $f_{14}$, $f_{15}$ | 10 | 100 | 895 | 89500 | 89442.71 |
| | 20 | 160 | 1118 | 178880 | 178885.43 |
| | 30 | 200 | 1342 | 268400 | 268328.15 |

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TABLE III
THE RESULT OF PEAK RATIO (PR).

| Function | $D$ | | Star | Ring | Four Clusters | Von Neumann | Social Star | Social Ring | Cognition |
|---|---|---|---|---|---|---|---|---|---|
| $f_1$ | 5 | $NPF$ | 18 | **42** | 25 | 36 | 11 | 32 | 0 |
| | | $PR$ | 0.35294 | **0.82352** | 0.49019 | 0.70588 | 0.21568 | 0.62745 | 0 |
| | 10 | $NPF$ | 0 | **3** | **3** | 2 | 0 | 1 | 0 |
| | | $PR$ | 0 | **0.05882** | **0.05882** | 0.03921 | 0 | 0.01960 | 0 |
| $f_2$ | 2 | $NPF$ | 51 | 202 | 139 | 185 | 51 | **203** | 32 |
| | | $PR$ | 0.25 | 0.99019 | 0.68137 | 0.90686 | 0.25 | **0.99509** | 0.15686 |
| | 5 | $NPF$ | 51 | **672** | 185 | 469 | 49 | 647 | 0 |
| | | $PR$ | 0.03125 | **0.41176** | 0.11335 | 0.28737 | 0.03002 | 0.39644 | 0 |
| | 8 | $NPF$ | 49 | **820** | 165 | 630 | 48 | 604 | 0 |
| | | $PR$ | 0.00375 | **0.06280** | 0.01263 | 0.04825 | 0.00367 | 0.04626 | 0 |
| $f_3$ | 2 | $NPF$ | 70 | **825** | 200 | 538 | 51 | 723 | 175 |
| | | $PR$ | 0.05490 | **0.64705** | 0.15686 | 0.42196 | 0.04 | 0.56705 | 0.13725 |
| | 3 | $NPF$ | 55 | **1987** | 203 | 1010 | 51 | 1338 | 5 |
| | | $PR$ | 0.00862 | **0.31168** | 0.03184 | 0.15843 | 0.008 | 0.20988 | 0.00078 |
| | 4 | $NPF$ | 52 | **5975** | 202 | 3838 | 51 | 4621 | 0 |
| | | $PR$ | 0.00163 | **0.18745** | 0.00633 | 0.12040 | 0.0016 | 0.14497 | 0 |
| $f_4$ | 5 | $NPF$ | 4 | **19** | 7 | 12 | 1 | 7 | 0 |
| | | $PR$ | 0.07843 | **0.37254** | 0.13725 | 0.235294 | 0.01960 | 0.13725 | 0 |
| $f_5$ | 2 | $NPF$ | 71 | **815** | 201 | 513 | 51 | 686 | 100 |
| | | $PR$ | 0.05568 | **0.63921** | 0.15764 | 0.40235 | 0.04 | 0.53803 | 0.07843 |
| | 3 | $NPF$ | 52 | **1975** | 203 | 1023 | 51 | 1351 | 6 |
| | | $PR$ | 0.00815 | **0.30980** | 0.03184 | 0.16047 | 0.008 | 0.21192 | 0.00094 |
| | 4 | $NPF$ | 53 | **6008** | 202 | 3811 | 51 | 4524 | 1 |
| | | $PR$ | 0.00166 | **0.18848** | 0.00633 | 0.11956 | 0.0016 | 0.14192 | 0.00003 |
| $f_6$ | 4 | $NPF$ | 51 | **459** | 171 | 249 | 51 | 414 | 0 |
| | | $PR$ | 0.0625 | **0.5625** | 0.20955 | 0.30514 | 0.0625 | 0.50735 | 0 |
| | 6 | $NPF$ | 51 | **976** | 192 | 513 | 50 | 767 | 0 |
| | | $PR$ | 0.01562 | **0.29901** | 0.05882 | 0.15716 | 0.01531 | 0.23498 | 0 |
| | 8 | $NPF$ | 51 | **1928** | 204 | 1044 | 51 | 1335 | 0 |
| | | $PR$ | 0.00390 | **0.14767** | 0.01562 | 0.07996 | 0.00390 | 0.10225 | 0 |
| $f_7$ | 6 | $NPF$ | 45 | **284** | 119 | 213 | 43 | 256 | 0 |
| | | $PR$ | 0.11029 | **0.69607** | 0.29166 | 0.52205 | 0.10539 | 0.62745 | 0 |
| | 10 | $NPF$ | 33 | **323** | 99 | 262 | 26 | 227 | 0 |
| | | $PR$ | 0.02022 | **0.19791** | 0.06066 | 0.16053 | 0.01593 | 0.13909 | 0 |
| | 16 | $NPF$ | 15 | **282** | 36 | 287 | 2 | 197 | 0 |
| | | $PR$ | 0.00114 | **0.02159** | 0.00275 | 0.02198 | 0.00015 | 0.01508 | 0 |
| $f_8$ | 2 | $NPF$ | 60 | **513** | 168 | 277 | 51 | 423 | 164 |
| | | $PR$ | 0.03267 | **0.27941** | 0.09150 | 0.15087 | 0.02777 | 0.23039 | 0.08932 |
| | 3 | $NPF$ | 52 | **1504** | 197 | 656 | 51 | 1035 | 15 |
| | | $PR$ | 0.00472 | **0.13652** | 0.01788 | 0.05954 | 0.00462 | 0.09395 | 0.00136 |
| | 4 | $NPF$ | 52 | **4439** | 194 | 2413 | 51 | 3895 | 5 |
| | | $PR$ | 0.00078 | **0.06715** | 0.00293 | 0.03650 | 0.00077 | 0.05892 | 0.00007 |
| $f_9$ | 10 | $NPF$ | 2 | **9** | 2 | 0 | 0 | 0 | 0 |
| | | $PR$ | 0.00392 | **0.01764** | 0.00392 | 0 | 0 | 0 | 0 |
| $f_{11}$ | 10 | $NPF$ | 0 | 12 | 6 | 8 | 0 | **44** | 0 |
| | | $PR$ | 0 | 0.02352 | 0.01176 | 0.01568 | 0 | **0.08627** | 0 |
| $f_{12}$ | 10 | $NPF$ | 0 | 7 | 2 | 0 | 0 | 0 | 0 |
| | | $PR$ | 0 | **0.01372** | 0.00392 | 0 | 0 | 0 | 0 |
| $f_{13}$ | 10 | $NPF$ | 2 | 2 | 6 | 10 | 3 | 7 | 0 |
| | | $PR$ | 0.00392 | 0.00392 | 0.01176 | **0.01960** | 0.00588 | 0.01372 | 0 |
| | 20 | $NPF$ | **2** | 0 | 0 | 0 | 1 | 0 | 0 |
| | | $PR$ | **0.00392** | 0 | 0 | 0 | 0.00196 | 0 | 0 |
| | 30 | $NPF$ | 0 | 0 | 0 | **3** | 0 | 0 | 0 |
| | | $PR$ | 0 | 0 | 0 | **0.00588** | 0 | 0 | 0 |

TABLE IV
THE RESULTS OF SUCCESS RATE (SR).

| Function | $D$ | | Star | Ring | Four Clusters | Von Neumann | Social Star | Social Ring | Cognition |
|---|---|---|---|---|---|---|---|---|---|
| $f_1$ | 5 | $NSR$ | 19 | 31 | 20 | **33** | 8 | 30 | 0 |
| | | $SR$ | 0.37254 | 0.60784 | 0.39215 | **0.64705** | 0.15686 | 0.588235 | 0 |
| | 10 | $NSR$ | 0 | 4 | 1 | **5** | 0 | 2 | 0 |
| | | $SR$ | 0 | 0.07843 | 0.01960 | **0.09803** | 0 | 0.03921 | 0 |
| $f_2$ | 2 | $NSR$ | 0 | **51** | 5 | 33 | 0 | 50 | 0 |
| | | $SR$ | 0 | **1.0** | 0.09803 | 0.64705 | 0 | 0.98039 | 0 |
| $f_4$ | 5 | $NPF$ | 3 | **15** | 7 | 13 | 2 | 10 | 0 |
| | | $PR$ | 0.05882 | **0.29411** | 0.13725 | 0.25490 | 0.03921 | 0.19607 | 0 |
| $f_7$ | 6 | $NSR$ | 0 | **2** | 0 | 0 | 0 | **2** | 0 |
| | | $SR$ | 0 | 0.03921 | 0 | 0 | 0 | 0.03921 | 0 |

TABLE V
EXPERIMENTAL RESULTS OF THE FUNCTIONS THAT THE GLOBAL OPTIMUM IS NOT FOUND.

| Fun. | $F_i^\star$ | $D$ | Best | Median | Worst | Mean | Std. | Best | Median | Worst | Mean | Std. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | PSO with Star Structure | | | | | PSO with Ring Structure | | | | |
| $f_1$ | 100.0 | 20 | 260.000 | 420.000 | 700.000 | 412.915 | 92.682 | 140.222 | 222.151 | 300.093 | 231.399 | 35.876 |
| $f_4$ | 400.0 | 10 | **400.248** | 401.874 | 403.482 | 401.883 | 0.7061 | 400.374 | 401.289 | 402.448 | 401.265 | 0.4596 |
| | | 20 | 402.497 | 405.434 | 408.672 | 405.429 | 1.1991 | 402.497 | 404.990 | 406.490 | 404.970 | 0.84838 |
| $f_9$ | 900.0 | 20 | **906.979** | 920.289 | 946.712 | 921.158 | 10.919 | 909.979 | 916.212 | 931.116 | **919.625** | 7.413 |
| | | 30 | **913.485** | 915.0407 | 945.538 | **922.877** | 10.251 | 916.220 | 930.319 | 930.862 | 926.527 | 4.891 |
| $f_{10}$ | 1000.0 | 10 | **1070** | 1070 | 1080 | 1070.58 | 2.352 | 1070.00 | 1070.00 | 1070.00 | **1070.00** | 0.00053 |
| | | 20 | **1080.00** | 1080.00 | 1080.00 | **1080.00** | 8.6E-06 | 1082.92 | 1100.00 | 14163.2 | 1434.66 | 1819.31 |
| | | 30 | **1080.00** | 1080.00 | 1080.14 | **1080.00** | 0.01979 | 1167.00 | 32793.8 | 33467.0 | 23056.0 | 13333.7 |
| $f_{11}$ | 1100.0 | 20 | **1100.41** | 1102.80 | 1105.19 | 1102.78 | 0.93541 | 1100.81 | 1102.14 | 1103.32 | 1102.12 | 0.50924 |
| | | 30 | 1102.34 | 1104.00 | 1108.58 | 1103.97 | 1.18417 | **1101.50** | 1102.51 | 1103.56 | 1102.45 | 0.43287 |
| $f_{12}$ | 1200.0 | 20 | 1203.65 | 2654.14 | 3261.17 | 2566.20 | 400.29 | 1511.40 | 2288.10 | 2808.51 | 2281.25 | 298.65 |
| | | 30 | 2577.95 | 3217.34 | 4097.15 | 3275.41 | 317.16 | 1678.28 | 3003.50 | 3609.10 | 3008.48 | 334.08 |
| $f_{14}$ | 1400.0 | 10 | **1440** | 2222.20 | 2943.70 | 2235.22 | 352.73 | 1440.00 | 2127.53 | 2523.49 | 2044.92 | 286.21 |
| | | 20 | 2610.61 | 3567.01 | 4190.12 | 3502.56 | 443.39 | 2923.79 | 3319.04 | 3871.90 | 3372.66 | 249.97 |
| | | 30 | 3246.86 | 4959.30 | 5887.91 | 4752.40 | 687.177 | 3552.79 | 4691.64 | 5572.76 | 4625.48 | 522.90 |
| $f_{15}$ | 1500.0 | 10 | **1640** | 1640 | 1864.92 | 1653.19 | 43.003 | 1640.00 | 1640.00 | 1640.00 | 1640.00 | 1.2E-09 |
| | | 20 | **1640** | 2101.97 | 2285.56 | 1996.46 | 228.87 | 1640.00 | 1640.00 | 2097.73 | 1740.82 | 168.143 |
| | | 30 | **1640** | 2092.89 | 2748.20 | 2113.88 | 226.82 | 1640.16 | 1964.92 | 2210.75 | 1909.44 | 166.86 |
| | | | PSO with Four Clusters Structure | | | | | PSO with Von Neumann Structure | | | | |
| $f_1$ | 100.0 | 20 | 220.00 | 340.00 | 460.000 | 338.866 | 57.253 | **140.00** | 220.00 | 300.00 | **230.99** | 36.379 |
| $f_4$ | 400.0 | 10 | 400.41 | 401.37 | 402.46 | 401.37 | 0.55489 | 400.33 | 400.83 | 401.999 | **400.90** | 0.42035 |
| | | 20 | 402.73 | 404.70 | 406.52 | 404.60 | 0.97076 | **401.99** | 403.25 | 405.01 | **403.36** | 0.75458 |
| $f_9$ | 900.0 | 20 | 907.20 | 931.11 | 936.14 | 928.80 | 5.8583 | 907.92 | 931.11 | 932.56 | 926.07 | 7.442 |
| | | 30 | 913.63 | 930.45 | 934.41 | 929.13 | 5.12314 | 915.28 | 930.31 | 930.81 | 928.63 | 4.2447 |
| $f_{10}$ | 1000.0 | 10 | 1070.00 | 1070.00 | 1080.00 | 1071.76 | 3.81220 | 1070.00 | 1070.00 | 1070.00 | **1070.00** | 2.3E-06 |
| | | 20 | **1080.00** | 1080.00 | 1080.00 | **1080.00** | 0.00014 | 1080.07 | 1080.64 | 1114.49 | 1082.56 | 5.34876 |
| | | 30 | **1080.00** | 1080.04 | 1083.03 | 1080.50 | 0.8561 | 1083.35 | 10059.39 | 33422.26 | 15956.61 | 14048.6 |
| $f_{11}$ | 1100.0 | 20 | 1101.21 | 1102.38 | 1104.59 | 1102.47 | 0.76258 | 1100.98 | 1101.74 | 1102.60 | **1101.77** | 0.38803 |
| | | 30 | 1101.17 | 1103.15 | 1105.29 | 1103.24 | 0.79828 | 1101.51 | 1102.22 | 1103.50 | **1102.27** | 0.40843 |
| $f_{12}$ | 1200.0 | 20 | 1530.44 | 2074.00 | 2843.49 | 2109.93 | 373.22 | **1200.89** | 1781.37 | 2712.28 | **1849.65** | 402.16 |
| | | 30 | 2086.93 | 2927.89 | 3635.05 | 2905.79 | 340.15 | **1203.42** | 2552.25 | 3178.09 | **2514.37** | 335.21 |
| $f_{14}$ | 1400.0 | 10 | **1440** | 2012.85 | 2721.57 | 1976.61 | 325.73 | **1440** | 1574.89 | 2344.90 | **1749.02** | 276.93 |
| | | 20 | 2448.33 | 2931.06 | 4058.28 | 3005.33 | 349.31 | **1520.01** | 2754.62 | 3406.36 | **2782.69** | 320.71 |
| | | 30 | 3211.84 | 4198.42 | 5594.12 | 4292.06 | 631.79 | **3083.80** | 3641.49 | 5137.65 | **3770.07** | 574.12 |
| $f_{15}$ | 1500.0 | 10 | **1640** | 1640 | 1640 | **1640** | 1.8E-13 | **1640** | 1640 | 1640 | **1640** | 2.5E-13 |
| | | 20 | **1640** | 1640 | 2194.13 | 1825.02 | 216.04 | 1640.00 | 1640.00 | 2059.47 | **1653.52** | 68.56 |
| | | 30 | 1640.00 | 1963.45 | 2320.31 | 1918.77 | 211.81 | 1640.00 | 1640.00 | 1948.40 | **1658.91** | 65.77 |

TABLE VI

THE RESULTS OF CONVERGENCE SPEED. THE "-" INDICATES THAT ALL GLOBAL OPTIMA IS NOT LOCATED IN EACH RUN.

| Function | $D$ | Star | Ring | Four Clusters | Von Neumann | Social Star | Social Ring | Cognition |
|---|---|---|---|---|---|---|---|---|
| $f_1$ | 5 | 31562.74 | 23298.03 | 24556.86 | **22241.17** | 33801.96 | 23558.82 | - |
| | 10 | - | 143639.21 | 147317.64 | **137807.84** | - | 146043.13 | - |
| $f_2$ | 2 | - | 5623.52 | 17666.66 | 7321.56 | - | **4960.78** | - |
| $f_4$ | 5 | 38644.70 | 33549.80 | 33295.05 | **31713.88** | 35641.72 | 33625.09 | - |
| $f_7$ | 6 | - | **33007.84** | - | - | - | 33490.19 | - |

TABLE VII

COMPUTATIONAL COMPLEXITY: THE RESULTS OF TIME COSTS. THE UNIT IS THE SECOND, AND $T0 = 0.061s$.

| | PSO with Star Structure | | | PSO with Ring Structure | | | PSO with Von Neumann structure | | |
|---|---|---|---|---|---|---|---|---|---|
| | $T1$ | $\hat{T}2$ | $(\hat{T}2 - T1)/T0$ | $T1$ | $\hat{T}2$ | $(\hat{T}2 - T1)/T0$ | $T1$ | $\hat{T}2$ | $(\hat{T}2 - T1)/T0$ |
| $D = 10$ | 20.5 | 20.7 | 3.278 | 20.5 | 20.7 | 3.278 | 20.5 | 20.8 | 4.918 |
| $D = 20$ | 45.0 | 45.3 | 4.918 | 45.0 | 45.7 | 11.47 | 45.0 | 45.1 | 1.639 |
| $D = 30$ | 73.0 | 74.0 | 16.39 | 73.0 | 74.3 | 21.31 | 73.5 | 74.5 | 16.39 |


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