Performance Assessment of Generalized Differential Evolution 3 (GDE3) with a Given Set of Problems

Saku Kukkonen, *Student Member, IEEE* and Jouni Lampinen

***Abstract*— This paper presents results for the CEC 2007 Special Session on Performance Assessment of Multi-Objective Optimization Algorithms where Generalized Differential Evolution 3 (GDE3) has been used to solve a given set of test problems. The set consist of 19 problems having two, three, or five objectives. Problems have different properties in the sense of separability, modality, and geometry of the Pareto-front**

According to the results, a near optimal set of solutions was found in the majority of the problems. Rotated problems given caused more difficulty than the other problems. Performance metrics indicate that obtained approximation sets were even better than provided reference sets for many problems.

I. Introduction

In this paper, a general purpose Evolutionary Algorithm (EA) called Generalized Differential Evolution 3 (GDE3) [1], [2] with a diversity maintenance technique suited for many-objective problems [3] has been used to solve multi-objective problems defined for the CEC 2007 Special Session on Performance Assessment of Multi-Objective Optimization Algorithms. The problems have been defined in [4], where also evaluation criteria are given. The problems have two, three, or five objectives, and the number of decision variables varies from 3 to 30. Difficulty of functions vary in means of separability, modality, and geometry of the Pareto-front. Some of the problems pose also difficulty of loosing diversity among decision variables.

This paper continues with the following parts: Multi-objective optimization with constraints is briefly defined in Section II. Section III describes the multi-objective optimization method used to solve the given set of problems. Section IV describes experiments and finally conclusions are given in Section V.

II. Multi-Objective Optimization with Constraints

A multi-objective optimization problem (MOOP) with constraints can be presented in the form [5, p. 37]:

$$\begin{array}{ll} \text{minimize} & \{f_1(\vec{x}), f_2(\vec{x}), \ldots, f_M(\vec{x})\} \\ \text{subject to} & (g_1(\vec{x}), g_2(\vec{x}), \ldots, g_K(\vec{x}))^T \leq \vec{0}. \end{array}$$

Thus, there are M functions to be optimized and K constraint functions.

The objective of Pareto-optimization is to find an approximation of the Pareto-front, *i.e.*, to find a set of solutions that are not dominated by any other solution. Weak dominance relation $\preceq$ between two vectors is defined such a way that $\vec{x}_1$ weakly dominates $\vec{x}_2$, *i.e.*, $\vec{x}_1 \preceq \vec{x}_2$ iff $\forall i : f_i(\vec{x}_1) \leq f_i(\vec{x}_2)$. Dominance relation $\prec$ between two vectors is defined such a way that $\vec{x}_1$ dominates $\vec{x}_2$, *i.e.*, $\vec{x}_1 \prec \vec{x}_2$ iff $\vec{x}_1 \preceq \vec{x}_2 \wedge \exists i : f_i(\vec{x}_1) < f_i(\vec{x}_2)$. The dominance relationship can be extended to take into consideration constraint values and objective values at the same time. A constraint-domination $\prec_c$ is defined in this paper so that $\vec{x}_1$ constraint-dominates $\vec{x}_2$, *i.e.*, $\vec{x}_1 \prec_c \vec{x}_2$ iff any of the following conditions is true [6]:

- $\vec{x}_1$ and $\vec{x}_2$ are infeasible and $\vec{x}_1$ dominates $\vec{x}_2$ in constraint function violation space.
- $\vec{x}_1$ is feasible and $\vec{x}_2$ is not.
- $\vec{x}_1$ and $\vec{x}_2$ are feasible and $\vec{x}_1$ dominates $\vec{x}_2$ in objective function space.

The definition for weak constraint-domination $\preceq_c$ is analogous dominance relation changed to weak dominance in the definition above. This constraint-domination is a special case of more general concept of having goals and priorities that is presented in [7].

III. Optimization Method

A. Differential Evolution

The Differential Evolution (DE) algorithm [8], [9] was introduced by Storn and Price in 1995. The design principles of DE are simplicity, efficiency, and the use of floating-point encoding instead of binary numbers. As a typical EA, DE has a random initial population that is then improved using selection, mutation, and crossover operations. Several ways exist to determine a stopping criterion for EAs but usually a predefined upper limit (G_{max}) for the number of generations to be computed provides an appropriate stopping condition. Other control parameters for DE are the crossover control parameter (CR), the mutation factor (F), and the population size (NP).

In each generation G, DE goes through each D dimensional decision vector $\vec{x}_{i,G}$ of the population and creates the corresponding trial vector $\vec{u}_{i,G}$ as follows [10]:

$$\begin{array}{l} r_1, r_2, r_3 \in \{1, 2, \ldots, NP\}, \text{(randomly selected,} \\ \quad \text{except mutually different and different from } i) \\ j_{rand} = \text{floor}\left(rand_i[0,1) \cdot D\right) + 1 \\ \text{for}(j = 1; j \leq D; j = j+1) \\ \{ \\ \quad \text{if}(rand_j[0,1) < CR \vee j = j_{rand}) \\ \qquad u_{j,i,G} = x_{j,r_3,G} + F \cdot (x_{j,r_1,G} - x_{j,r_2,G}) \\ \quad \text{else} \\ \qquad u_{j,i,G} = x_{j,i,G} \\ \} \end{array}$$

The authors are with the Department of Information Technology, Lappeenranta University of Technology, P.O. Box 20, FIN-53851 Lappeenranta, Finland; email: saku.kukkonen@lut.fi.

This is the most common DE version, DE/rand/1/bin. Both CR and F remain fixed during the entire execution of the algorithm. Parameter $CR \in [0, 1]$, which controls the crossover operation, represents the probability that an element for the trial vector is chosen from a linear combination of three randomly chosen vectors and not from the old vector $\vec{x}_{i,G}$. The condition "$j = j_{rand}$" ensures that at least one element of the trial vector is different compared to the elements of the old vector. Parameter F is a scaling factor for mutation and its value range is $(0, 1+]$[1]. In practice, CR controls rotational invariance of the search, and its small value (*e.g.*, 0.1) is practicable with separable problems while larger values (*e.g.*, 0.9) are for non-separable problems. Parameter F controls the speed and robustness of the search, *i.e.*, a lower value for F increases the convergence rate but it also increases the risk of getting stuck into a local optimum. Parameters CR and NP have the similar effect on the convergence rate as F has.

After the mutation and crossover operations, the trial vector $\vec{u}_{i,G}$ is compared to the old vector $\vec{x}_{i,G}$. If the trial vector has an equal or better objective value, then it replaces the old vector in the next generation. This can be presented as follows in the case of minimization of an objective [10]:

$$\vec{x}_{i,G+1} = \begin{cases} \vec{u}_{i,G} & \text{if} \quad f(\vec{u}_{i,G}) \leq f(\vec{x}_{i,G}) \\ \vec{x}_{i,G} & \text{otherwise} \end{cases} .$$

DE is an elitist method since the best population member is always preserved and the average objective value of the population will never deteriorate.

B. Generalized Differential Evolution

The first version of Generalized Differential Evolution (GDE) extended DE for constrained multi-objective optimization, and it modified only the selection rule of the basic DE [6]. The basic idea in the selection rule of GDE is that the trial vector is selected to replace the old vector in the next generation if it weakly constraint-dominates the old vector. There was no explicit sorting of non-dominated solutions [11, pp. 33 – 44] during the optimization process or any mechanism for maintaining the distribution and extent of solutions. Also, there was no extra repository for non-dominated solutions.

The second version, GDE2, made the selection based on crowdedness when the trial and old vector were feasible and non-dominating each other in the objective function space [12]. This improved the extent and distribution of the obtained set of solutions but slowed down the convergence of the overall population because it favored isolated solutions far from the Pareto-front until all the solutions were converged near the Pareto-front.

The third and latest version is GDE3 [1], [2]. Besides the selection, another part of the basic DE has also been modified. Now, in the case of feasible and non-dominating solutions, both vectors are saved for the population of next generation. Before continuing to the next generation, the size of the population is reduced using non-dominated sorting and pruning based on diversity preservation. The pruning technique used in the original GDE3 is based on crowding distance, which provides a good crowding estimation in the case of two objectives. However, crowding distance fails to approximate crowdedness of solutions when the number of objectives is more than two [2]. Since, the provided problem set in [4] consists of problems with more than two objectives, a more general diversity maintenance technique proposed in [3] was used. The technique is based on a crowding estimation using the nearest neighbors of solutions in Euclidean sense, and an efficient nearest neighbors search technique.

All the GDE versions handle any number of M objectives and any number of K constraints, including the cases where $M = 0$ (constraint satisfaction problem) and $K = 0$ (unconstrained problem). When $M = 1$ and $K = 0$, the versions are identical to the original DE, and this is why they are referred as *Generalized* DEs.

IV. Experiments

A. Configuration

GDE3 and the given problems were implemented in the ANSI-C programming language and compiled with the GCC compiler. The hardware was an ordinary PC with 1.2 GHz CPU & 1 GB RAM, and the operating system was Linux.

In the case of boundary constraint violations, violating variable values were reflected back from the violated boundary using following rule before the selection operation of GDE3:

$$u_{j,i,G} = \begin{cases} 2x_j^{(lo)} - u_{j,i,G} & \text{if} \quad u_{j,i,G} < x_j^{(lo)} \\ 2x_j^{(up)} - u_{j,i,G} & \text{if} \quad u_{j,i,G} > x_j^{(up)} \\ u_{j,i,G} & \text{otherwise} \end{cases} ,$$

where $x_j^{(lo)}$ and $x_j^{(up)}$ are lower and upper limits respectively for a decision variable x_j.

B. Parameter Setting

Along stopping criterion and size of the population (NP), GDE3 has two control parameters (CR and F) as described in Section III-A, where the effect and ranges of these are also given. As a rule of thumb in single-objective optimization, if nothing is known about the problem in hand then suitable initial control parameter values are $CR = 0.9$ and $F = 0.9$, and $NP = 5 \cdot D \dots 30 \cdot D$, where D is the number of decision variables of the problem [13]. For an easy problem (*e.g.*, moderately multimodal and low dimensional), a small value of NP is sufficient but with difficult problems, a large value of NP is recommended in order to avoid stagnation to a local optimum. In general, increase of control parameter values, will also increase the number of function evaluations (FES) needed. Dependency between NP and FES needed is linear while FES needed increases more rapidly along CR and F [14]. If values of F and/or NP are too low, search is prone to stagnate to a local optimum (with very low control

[1]Notation means that F is larger than 0 and upper limit is in practice around 1 although there is no hard upper limit.

parameter values the search converges without the selection pressure).

In the case of multi-objective optimization and conflicting objectives, lower control parameter values (*e.g.*, 0.2) for CR and F can be used than in single-objective optimization because conflicting objectives already maintain diversity and restrain the search speed. This has been noted in [15], [16], where the effect of the control parameters has been studied empirically. Also, if the complexity of objectives differ (*e.g.*, as in the case of the ZDT problems [11, pp. 356–360]), then a high value for CR might lead to premature convergence with respect to one objective compared to another. The value of NP can be selected in same way as in single-objective optimization or it can be selected according to a desired approximation set size of the Pareto-optimal front.

To keep the setup as simple as possible, same set of control parameter values were used for all the problems. It would had been also possible to apply some kind of dependency on the number of objectives and/or the number of decision variables. As well, it would had been possible to use some control parameter adaptation strategy as in [17]–[19]. However, these were not applied, because then it would had been unclear, how parameter adjustment vs. the optimization algorithm itself contributes to the results.

The control parameter values used were $CR = 0.1$, $F = 0.5$, $NP = 100$, and $G_{max} = 4999$. First two control parameter values were obtained by a couple of preliminary tests with the problems. Value of CR was relatively low but suitable because most of the problems were separable and because the complexity of objectives differ for the problems modified from the ZDT problems [4]. Value of F is a compromise between speed and robustness. Value $F = 0.5$ has been noted to be especially suitable for the ZDT4 problem because of equally spaced local optima [15], [16]. The size of the population was set according to the smallest desired approximation set size given in [4]. With chosen NP and G_{max} values, FES is exactly 500 000, which was an upper limit given in [4].

In [4], approximation sets of different sizes were demanded for different problems. In GDE3, the size of the approximation set is usually same as NP. Now, NP was kept fixed for all the problems, and solutions for the approximation set were collected during generations. Populations of 200 last generations (49 last generations in the case of 5 000 function evaluations) were merged together and non-dominated solutions were selected from this merged set of solutions. If the size of non-dominated set was larger than the desired approximation set size, then the set was reduced to desired size using the pruning technique described in [3].

C. Results of Experiments

The problems given in [4] were solved 25 times and achieved results are presented in Tables I–VII and Figs. 1–3.

Tables II–VII show the best, the worst, median, mean, and standard deviation values of two performance indicators after different FES. Binary indicators used were R [20] and Hypervolume indicator ($I_{\bar{H}}$) [21]. When these are used to compare against a reference set, then a smaller indicator value express better performance (value 0 indicates equal performance respect to the reference set). According to the R indicator values in Tables II–IV, a better approximation set than the given reference set (*i.e.*, negative indicator value) was found for problems OKA2, S_ZDT4, S_ZDT6, WFG1 (M=3), WFG8 (M=3,5), and WFG9 (M=3). According to the Hypervolume indicator values in Tables V–VII, a better approximation set than the given reference set was found for problems OKA2, S_ZDT6, S_DTLZ2 (M=3), WFG1 (M=3), WFG8 (M=3,5), WFG9 (M=3,5), and R_DTLZ3 (M=5).

Covered sets (CS) metric [22] measures number of covered Pareto-subsets in the decision variable space. The SYMPART problem has been designed to have several Pareto-subsets in the the decision variable space mapping into a single Pareto-set in the objective space. CS values for the SYMPART problem are shown in Table I and these indicate that solutions converged into a single Pareto-subset in the decision variable space. This is not surprising since GDE3 does not explicitly maintain diversity in the decision variable space but in the objective space according to common goals of multi-objective optimization [11].

TABLE I
THE RESULTS FOR COVERED SETS CS FOR TEST FUNCTION SYMPART

FES	5e+3	5e+4	5e+5
Best	2.0000e+00	1.0000e+00	1.0000e+00
Median	1.0000e+00	1.0000e+00	1.0000e+00
Worst	0.0000e+00	1.0000e+00	1.0000e+00
Mean	1.0000e+00	1.0000e+00	1.0000e+00
Std	0.5000e+00	0.0000e+00	0.0000e+00

Attainment surfaces [23] in the case of two and three objectives are shown in Figs. 1 and 2. Results in Fig. 1 indicate that the Pareto-optimal front is found reliably in all the two-objective cases except with the ZDT4 problem modifications and S_ZDT2. According to Fig. 1, the rotated version of ZDT4 (R_ZDT4) is harder to solve than the unrotated version (S_ZDT4). 100 % attainment surface for S_ZDT4 is a single point (1, 2), thus sometimes the result can be a single solution for this problem. Different control parameter values would give different results. Attainment surfaces for the three-objective problems in Fig. 2 indicate good performance in the most cases. The rotated DTLZ2 problem (R_DTLZ2) seems to be the most difficult one, and also S_DTLZ3 causes some difficulty.

Figure 3 illustrates pairwise interaction of objective functions in the case of the five-objective WFG8 and WFG9 problems. The results indicate good coverage of the Pareto-optimal front since the approximation sets appear to be close to the true Pareto-front and also the area of the Pareto-front is well covered.

D. Algorithm Complexity

GDE3 is well scalable algorithm with the simple genetic operations of DE. Therefore, also problems with a large

TABLE II

THE RESULTS FOR R INDICATOR ON TEST FUNCTIONS 1–7

FES		1. OKA2	2. SYMPART	3. S_ZDT1	4. S_ZDT2	5. S_ZDT4	6. R_ZDT4	7. S_ZDT6
5e+3	Best	-9.1667e-04	1.9591e-02	3.1691e-02	8.5101e-02	5.9457e-02	9.4119e-03	1.3016e-01
	Median	-3.2720e-04	2.7242e-02	4.4954e-02	9.8426e-02	7.5723e-02	1.7792e-02	1.3353e-01
	Worst	8.7530e-03	3.8250e-02	5.2071e-02	1.0832e-01	8.4380e-02	2.5286e-02	1.3880e-01
	Mean	1.6276e-04	2.8114e-02	4.4531e-02	9.8161e-02	7.5424e-02	1.7979e-02	1.3425e-01
	Std	1.9278e-03	5.1089e-03	5.2328e-03	6.0500e-03	5.1217e-03	3.5124e-03	2.7636e-03
5e+4	Best	-1.0604e-03	1.5354e-05	6.1755e-05	1.1911e-04	5.6863e-03	3.0167e-04	1.7994e-02
	Median	-9.8426e-04	2.2455e-05	1.0604e-04	1.8268e-04	6.8634e-03	7.7423e-04	1.9935e-02
	Worst	-8.6300e-04	4.8377e-05	1.9624e-04	4.0119e-02	9.1519e-03	2.2171e-03	2.1445e-02
	Mean	-9.8540e-04	2.4138e-05	1.1611e-04	3.3835e-03	6.9527e-03	8.6134e-04	1.9906e-02
	Std	5.2305e-05	7.0431e-06	3.5321e-05	1.1056e-02	8.8275e-04	4.6544e-04	7.2638e-04
5e+5	Best	**-1.0646e-03**	1.1824e-06	2.9681e-08	4.2118e-07	**-5.6401e-09**	1.7335e-04	**-1.0788e-06**
	Median	-1.0593e-03	1.6037e-06	3.7213e-06	7.6896e-06	-3.6526e-09	5.7908e-04	-1.0788e-06
	Worst	-1.0401e-03	1.9349e-06	1.5023e-05	4.0053e-02	7.1672e-05	1.9619e-03	-1.0788e-06
	Mean	-1.0583e-03	1.5629e-06	4.5328e-06	3.2126e-03	1.1464e-05	6.7173e-04	-1.0788e-06
	Std	5.4274e-06	2.0334e-07	3.6209e-06	1.1088e-02	2.6818e-05	4.2043e-04	6.4837e-22

TABLE III

THE RESULTS FOR R INDICATOR ON TEST FUNCTIONS 8–13 WHEN M = 3

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
5e+3	Best	1.2102e-04	3.3374e-04	3.5014e-04	6.5228e-02	-1.2425e-02	-6.4830e-03
	Median	1.9031e-04	4.3908e-04	4.8860e-04	8.0660e-02	-1.0336e-02	-3.1983e-03
	Worst	3.5465e-04	4.9278e-04	7.4403e-04	8.2120e-02	-7.6820e-03	2.5751e-04
	Mean	2.0985e-04	4.3003e-04	4.8803e-04	8.0179e-02	-9.9919e-03	-3.3098e-03
	Std	6.1425e-05	4.5257e-05	1.0472e-04	3.2198e-03	1.4138e-03	2.2460e-03
5e+4	Best	2.3955e-05	9.9901e-05	1.0047e-05	4.5607e-02	-2.7319e-02	-1.1960e-02
	Median	4.7156e-05	1.2175e-04	1.7569e-05	5.8939e-02	-2.5931e-02	-8.9520e-03
	Worst	9.4718e-05	1.7084e-04	2.4134e-05	6.2100e-02	-2.4959e-02	-5.6245e-03
	Mean	5.0877e-05	1.2273e-04	1.7951e-05	5.5117e-02	-2.5956e-02	-8.9123e-03
	Std	2.0136e-05	1.6097e-05	2.7603e-06	6.0483e-03	6.2452e-04	1.2524e-03
5e+5	Best	1.7526e-06	1.2872e-05	2.8851e-10	**-2.9818e-04**	**-2.8890e-02**	**-1.5883e-02**
	Median	5.1877e-06	2.0366e-05	1.7172e-07	5.2467e-04	-2.8532e-02	-8.9031e-03
	Worst	1.9259e-05	3.8971e-05	1.4113e-06	6.9913e-03	-2.7996e-02	-5.4468e-03
	Mean	6.0540e-06	2.1897e-05	3.4950e-07	1.6775e-03	-2.8504e-02	-9.2222e-03
	Std	3.9601e-06	7.1629e-06	3.5416e-07	2.2177e-03	1.6545e-04	1.9794e-03

TABLE IV

THE RESULTS FOR R INDICATOR ON TEST FUNCTIONS 8–13 WHEN M = 5

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
5e+3	Best	1.1874e-04	1.4489e-04	1.8060e-04	5.5501e-02	2.5957e-03	3.4959e-03
	Median	2.0113e-04	1.9262e-04	2.0902e-04	5.6547e-02	6.3762e-03	6.5687e-03
	Worst	3.0906e-04	2.4053e-04	3.7126e-04	5.7383e-02	8.4641e-03	1.2315e-02
	Mean	2.0537e-04	1.9155e-04	2.2656e-04	5.6468e-02	5.8783e-03	7.0251e-03
	Std	5.4517e-05	2.3081e-05	4.6965e-05	4.5748e-04	1.6508e-03	2.4993e-03
5e+4	Best	2.5119e-05	3.8708e-05	8.2593e-06	4.3338e-02	-8.7779e-03	1.2384e-03
	Median	3.2789e-05	5.4383e-05	1.3803e-05	4.4686e-02	-7.5197e-03	2.5167e-03
	Worst	5.6378e-05	7.1649e-05	2.2322e-05	4.8814e-02	-5.6231e-03	3.9287e-03
	Mean	3.5491e-05	5.4718e-05	1.3649e-05	4.5536e-02	-7.5371e-03	2.4667e-03
	Std	7.2419e-06	8.8530e-06	3.3250e-06	1.7500e-03	8.5325e-04	7.4804e-04
5e+5	Best	1.1620e-05	2.1338e-05	8.6601e-09	2.5696e-03	**-1.1963e-02**	5.4102e-04
	Median	2.2048e-05	3.2824e-05	1.7526e-07	4.0640e-03	-1.1466e-02	2.2168e-03
	Worst	2.9654e-05	4.7707e-05	1.1207e-06	8.3991e-03	-1.0254e-02	3.2987e-03
	Mean	2.1511e-05	3.3115e-05	2.5426e-07	4.6089e-03	-1.1345e-02	1.9398e-03
	Std	4.5056e-06	5.1085e-06	2.7395e-07	1.5691e-03	3.9922e-04	7.0266e-04

TABLE V

THE RESULTS FOR HYPERVOLUME INDICATOR $I_{\bar{H}}$ ON TEST FUNCTIONS 1–7

FES		1. OKA2	2. SYMPART	3. S_ZDT1	4. S_ZDT2	5. S_ZDT4	6. R_ZDT4	7. S_ZDT6
5e+3	Best	-8.1822e-04	5.6037e-02	1.4412e-01	2.0390e-01	1.7806e-01	2.9857e-02	3.3013e-01
	Median	-1.2067e-04	7.7552e-02	1.5541e-01	2.3654e-01	2.2822e-01	5.5198e-02	3.3673e-01
	Worst	1.1253e-02	1.0814e-01	1.7080e-01	2.6279e-01	2.5523e-01	7.8043e-02	3.5134e-01
	Mean	5.7619e-04	7.9939e-02	1.5566e-01	2.3674e-01	2.2904e-01	5.5075e-02	3.3957e-01
	Std	2.3879e-03	1.4267e-02	6.4482e-03	1.5411e-02	1.5735e-02	1.0553e-02	6.3348e-03
5e+4	Best	-1.2271e-03	4.5698e-05	4.5187e-04	5.3297e-04	1.7081e-02	1.3164e-03	4.1598e-02
	Median	-1.1387e-03	6.7513e-05	5.4824e-04	6.3449e-04	2.0508e-02	2.9080e-03	4.4862e-02
	Worst	-9.9987e-04	1.4421e-04	7.9815e-04	4.8012e-02	2.7175e-02	6.8484e-03	4.8148e-02
	Mean	-1.1430e-03	7.2177e-05	5.7459e-04	4.4286e-03	2.0748e-02	3.0337e-03	4.4900e-02
	Std	5.7123e-05	2.1111e-05	9.1660e-05	1.3115e-02	2.5913e-03	1.3294e-03	1.5190e-03
5e+5	Best	**-1.2359e-03**	3.6903e-06	1.5672e-04	1.9244e-04	8.5011e-07	7.0434e-04	**-2.3166e-04**
	Median	-1.2280e-03	4.6079e-06	1.7207e-04	2.1011e-04	8.8127e-07	2.1165e-03	-2.3057e-04
	Worst	-1.2108e-03	5.9303e-06	2.0140e-04	4.7812e-02	1.6596e-04	6.0925e-03	-2.2998e-04
	Mean	-1.2271e-03	4.6477e-06	1.7484e-04	4.0254e-03	2.7290e-05	2.2690e-03	-2.3067e-04
	Std	5.6516e-06	6.1853e-07	1.0255e-05	1.3178e-02	6.1760e-05	1.2492e-03	4.6401e-07

TABLE VI

THE RESULTS FOR HYPERVOLUME INDICATOR $I_{\bar{H}}$ ON TEST FUNCTIONS 8–13 WHEN M = 3

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
5e+3	Best	1.1903e-03	7.0009e-03	5.8468e-03	3.3514e-01	-8.4283e-02	-3.9985e-02
	Median	1.7331e-03	1.0490e-02	7.3463e-03	4.0795e-01	-7.1249e-02	-1.9643e-02
	Worst	2.7208e-03	1.6287e-02	1.0130e-02	4.1498e-01	-5.7687e-02	4.6558e-03
	Mean	1.8318e-03	1.0864e-02	7.4275e-03	4.0540e-01	-7.1250e-02	-1.9533e-02
	Std	3.9030e-04	2.4615e-03	1.0603e-03	1.5231e-02	7.1969e-03	1.4889e-02
5e+4	Best	1.9213e-05	3.6020e-04	1.0362e-06	2.3857e-01	-1.6707e-01	-7.0592e-02
	Median	1.0317e-04	5.4497e-04	2.4363e-06	3.0322e-01	-1.6076e-01	-5.6592e-02
	Worst	3.0643e-04	8.6963e-04	7.7858e-06	3.1826e-01	-1.5562e-01	-3.8630e-02
	Mean	1.1763e-04	5.5892e-04	2.9691e-06	2.8495e-01	-1.6072e-01	-5.6463e-02
	Std	8.1453e-05	1.4571e-04	1.6691e-06	2.9185e-02	3.0068e-03	6.0416e-03
5e+5	Best	**-1.1926e-05**	1.8820e-06	2.9421e-13	**-7.7648e-03**	**-1.7596e-01**	**-9.1893e-02**
	Median	-7.2509e-06	7.1121e-06	7.7444e-11	-7.3850e-04	-1.7490e-01	-5.7558e-02
	Worst	5.9468e-06	2.7589e-05	2.0865e-08	3.4607e-02	-1.7279e-01	-3.8588e-02
	Mean	-6.8285e-06	8.5274e-06	2.2984e-09	5.5177e-03	-1.7461e-01	-5.9748e-02
	Std	4.1187e-06	6.5709e-06	5.0220e-09	1.2774e-02	9.6867e-04	9.8129e-03

TABLE VII

THE RESULTS FOR HYPERVOLUME INDICATOR $I_{\bar{H}}$ ON TEST FUNCTIONS 8–13 WHEN M = 5

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
5e+3	Best	8.2136e-04	2.1075e-03	1.8676e-03	6.2449e-01	-7.0962e-02	7.1520e-03
	Median	1.6175e-03	3.0403e-03	2.4715e-03	6.3553e-01	-4.2623e-02	5.9194e-02
	Worst	3.4713e-03	4.5852e-03	4.5449e-03	6.4442e-01	-1.6277e-02	1.2018e-01
	Mean	1.7376e-03	3.1397e-03	2.6169e-03	6.3496e-01	-4.2629e-02	5.6182e-02
	Std	6.6919e-04	8.1164e-04	6.3012e-04	4.8418e-03	1.6789e-02	2.7955e-02
5e+4	Best	1.4208e-05	-8.1147e-05	1.9484e-06	4.9639e-01	-2.8735e-01	-1.1565e-01
	Median	3.0415e-05	1.0186e-05	7.4486e-06	5.1423e-01	-2.6913e-01	-1.0207e-01
	Worst	1.0524e-04	8.0324e-05	1.7884e-05	5.5708e-01	-2.4381e-01	-7.7629e-02
	Mean	3.7278e-05	-1.7555e-06	7.6879e-06	5.2167e-01	-2.6760e-01	-1.0011e-01
	Std	2.1466e-05	4.0644e-05	3.8216e-06	1.9071e-02	1.2959e-02	9.4746e-03
5e+5	Best	2.3565e-06	**-1.5004e-04**	4.8850e-15	2.6997e-02	**-3.5647e-01**	**-1.2640e-01**
	Median	9.5935e-06	-1.3870e-04	1.9213e-11	4.5399e-02	-3.5044e-01	-1.1320e-01
	Worst	2.0628e-05	-9.8051e-05	3.8028e-09	1.0187e-01	-3.1668e-01	-1.0204e-01
	Mean	1.0020e-05	-1.3436e-04	4.9525e-10	5.2961e-02	-3.4706e-01	-1.1315e-01
	Std	4.8347e-06	1.3372e-05	9.0507e-10	2.0532e-02	9.3962e-03	6.2539e-03

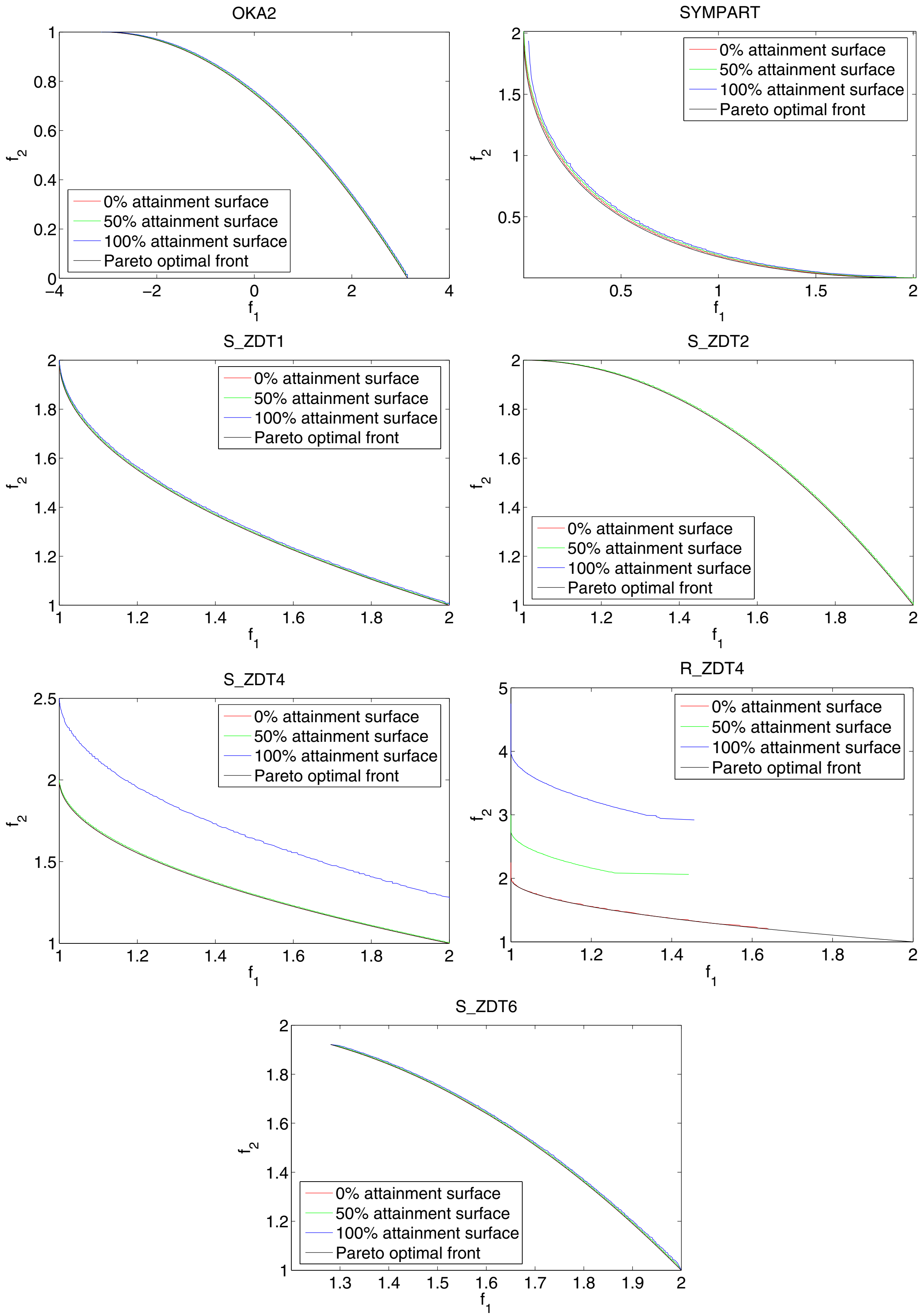

Fig. 1. Pareto-optimal front and 0%, 50%, 100% attainment surfaces after 5e+5 FES on test functions 1–7.

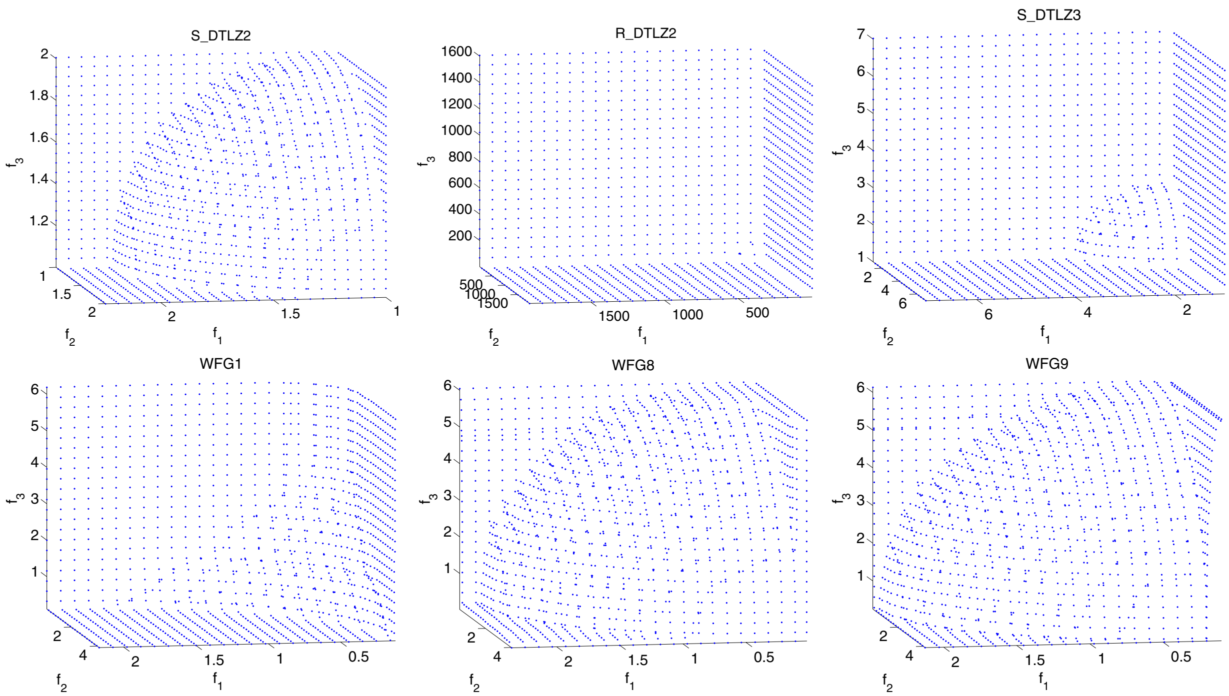

Fig. 2. 50% attainment surfaces after 5e+5 FES on test functions 8–13 (M = 3).

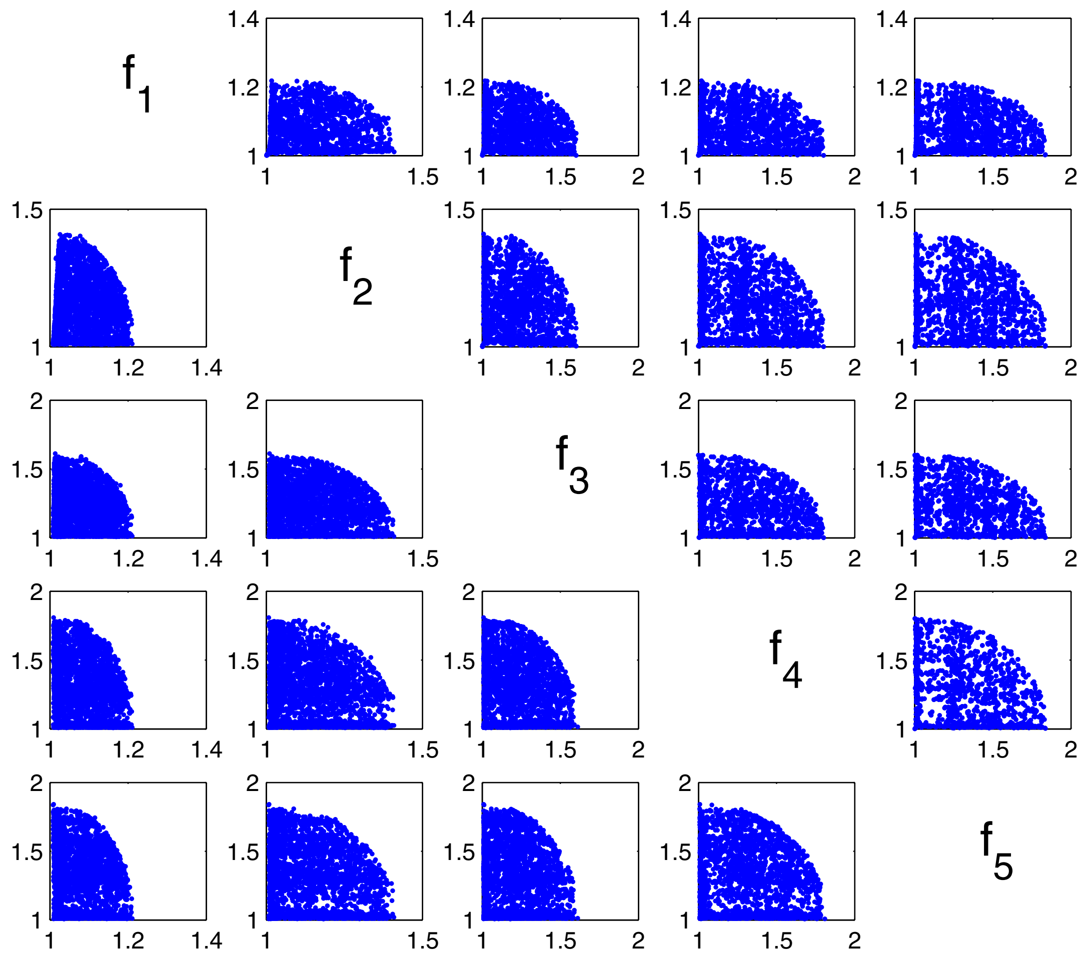

Fig. 3. Upper diagonal plots are for WFG8 (M = 5) and lower diagonal plots are for WFG9 (M = 5).

number of decision variables and/or a large population size as in [24] are solvable in reasonable time. The most complex operation in GDE3 is non-dominated sorting, which time complexity is $O\left(N \log^{M-1} N\right)$ [25][2].

The time complexity of GDE3 was measured according to instructions given in [4], and observed CPU times are reported in Table VIII.

TABLE VIII

COMPUTATIONAL COMPLEXITY: T1 IS MEAN TIME FOR EVALUATING ALL THE PROBLEMS 10000 TIMES AND T2 IS MEAN TIME FOR SOLVING ALL THE PROBLEMS USING GDE3 AND 10000 FUNCTION EVALUATIONS

$T1$	$T2$	$(T2-T1)/T1$
1.4563 s	1.7284 s	0.1868

V. Conclusions

Results of Generalized Differential Evolution 3 (GDE3) for the CEC 2007 Special Session on Performance Assessment of Multi-Objective Optimization Algorithms have been reported. The problems given were solved with the same fixed control parameter settings, *i.e.*, there was no parameter adaption based on problem characteristic or other criteria.

According to the results, GDE3 performed well especially with the problems without artificial rotation according to the attainment surfaces obtained. Worse performance with rotated problems was probably due to selected control parameter values. In several cases, the performance metrics indicate that the obtained approximation sets were even better than the given reference sets.

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