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► To cite this version:

Ilya Loshchilov. CMA-ES with Restarts for Solving CEC 2013 Benchmark Problems. IEEE Congress on Evolutionary Computation, Jun 2013, Cancun, Mexico. 2013. <hal-00823880>

**HAL Id: hal-00823880**

**<https://hal.inria.fr/hal-00823880>**

Submitted on 18 May 2013

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# CMA-ES with Restarts for Solving CEC 2013 Benchmark Problems

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**Abstract**—This paper investigates the performance of 6 versions of Covariance Matrix Adaptation Evolution Strategy (CMA-ES) with restarts on a set of 28 noiseless optimization problems (including 23 multi-modal ones) designed for the special session on real-parameter optimization of CEC 2013. The experimental validation of the restart strategies shows that: i). the versions of CMA-ES with weighted active covariance matrix update outperform the original versions of CMA-ES, especially on ill-conditioned problems; ii). the original restart strategies with increasing population size (IPOP) are usually outperformed by the bi-population restart strategies where the initial mutation step-size is also varied; iii). the recently proposed alternative restart strategies for CMA-ES demonstrate a competitive performance and are ranked first w.r.t. the proportion of function-target pairs solved after the full run on all 10-, 30- and 50-dimensional problems.

## I. INTRODUCTION

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) proposed by [8], [7] has become a standard for continuous black-box evolutionary optimization. The main advantage of CMA-ES over classical Evolution Strategies comes from the use of correlated mutations instead of axis-parallel ones. The adaptation of the covariance matrix  $\mathbf{C}$  allows to steadily learn appropriate mutation distribution and increase the probability of repeating the successful search steps.

However, there are several properties of black-box optimization problems which may lead to a premature convergence of CMA-ES, among the most common are multi-modality and uncertainty. To increase the probability of finding the global optima, IPOP-CMA-ES [2] and BIPOP-CMA-ES [4] restart strategies for CMA-ES have been proposed. The IPOP-CMA-ES was ranked first on the continuous optimization benchmark at CEC 2005 [3]; and BIPOP-CMA-ES showed the best results together with IPOP-CMA-ES on the black-box optimization benchmark (BBOB) in 2009 and 2010 [1]. Later, alternative restart strategies for CMA-ES proposed in [12] demonstrated an even more competitive performance on some of multi-modal functions during the BBOB 2012. The recently proposed *weighted active* covariance matrix update of CMA-ES [11], [9] is also a competitive alternative to the original update procedure, it allows to substantially improve the performance both on unimodal and multi-modal functions [9]. This paper focuses on analyzing the performance of the restart strategies of CMA-ES with the original and weighted active covariance matrix updates on the CEC 2013 benchmark test [10].

The remainder of this paper is organized as follows. Section

II presents the main principles of the CMA-ES algorithm. Section III describes the restart strategies of CMA-ES. Section IV explains the experimental procedure and comments the experimental results. Section V concludes the paper with a discussion and some perspectives for further research.

## II. THE $(\mu/\mu_w, \lambda)$ -CMA-ES

The CMA-ES algorithm [8], [7] optimizes an objective function  $f : x \in \mathbb{R}^n \rightarrow f(x) \in \mathbb{R}$  by sampling  $\lambda$  candidate solutions from a multivariate normal distribution. It exploits the best  $\mu$  solutions out of the  $\lambda$  ones to adaptively estimate the local covariance matrix of the objective function, in order to increase the probability of successful samples in the next iteration. More formally, at iteration  $t$ ,  $(\mu/\mu_w, \lambda)$ -CMA-ES samples  $\lambda$  individuals ( $k = 1 \dots \lambda$ ) according to

$$\mathbf{x}_k^{(t+1)} = \mathcal{N}(\mathbf{m}^{(t)}, \sigma^{(t)^2} \mathbf{C}^{(t)}) = \mathbf{m}^{(t)} + \sigma^{(t)} \cdot \mathcal{N}(\mathbf{0}, \mathbf{C}^{(t)}), \quad (1)$$

where  $\mathbf{m}^{(t)}$  denotes the mean of a normally distributed random vector,  $\mathbf{C}^{(t)}$  is the covariance matrix and  $\sigma^{(t)}$  is the mutation step-size.

These  $\lambda$  individuals are evaluated and ranked. The mean of the distribution is updated and set to the weighted sum of the best  $\mu$  individuals as  $\mathbf{m}^{(t+1)} = \sum_{i=1}^{\mu} w_i \mathbf{x}_{i:\lambda}^{(t)}$ , with  $w_i > 0$  for  $i = 1 \dots \mu$  and  $\sum_{i=1}^{\mu} w_i = 1$ , where index  $i : \lambda$  denotes the  $i$ -th best individual after the objective function. In the original CMA-ES the information about the remaining (worst  $\lambda - \mu$ ) solutions is used only implicitly during the selection process.

However, it has been shown in [11] that the information from the worst solutions also can be used to reduce the variance of the mutation distribution in unpromising directions. The corresponding *active*  $(\mu/\mu_I, \lambda)$ -CMA-ES algorithm demonstrates a performance gain up to a factor of 2 without loss of performance on any of tested functions in [11]. Later, the *active* update of  $(\mu/\mu_I, \lambda)$ -CMA-ES was extended to the *weighted* case of  $(\mu/\mu_W, \lambda)$ -CMA-ES, where  $w_i > w_{i+1}$  for  $i = 1 \dots \lambda - 1$ . This *weighted active*  $(\mu/\mu_W, \lambda)$ -CMA-ES (also referred to as aCMA-ES) was implemented in the IPOP regime of restarts as IPOP-aCMA-ES and demonstrated improvements up to a factor of 2 on a set of noiseless and noisy functions from the BBOB [9].

More formally, the active CMA-ES only differs from the original CMA-ES in the adaptation of the covariance matrix  $\mathbf{C}^{(t)}$ . Like for CMA-ES, the covariance matrix is computed from the best  $\mu$  solutions,  $\mathbf{C}_\mu^+ = \sum_{i=1}^{\mu} w_i \frac{\mathbf{x}_{i:\lambda} - \mathbf{m}^{(t)}}{\sigma^{(t)}} \times$

$\frac{(\mathbf{x}_{i:\lambda}-\mathbf{m}^t)^T}{\sigma^t}$ . The main novelty is to exploit the worst solutions to compute  $\mathbf{C}_\mu^- = \sum_{i=0}^{\mu-1} w_{i+1} \mathbf{y}_{\lambda-i:\lambda} \mathbf{y}_{\lambda-i:\lambda}^T$ , where  $\mathbf{y}_{\lambda-i:\lambda} = \frac{\|\mathbf{C}^{t-1/2}(\mathbf{x}_{\lambda-\mu+1+i:\lambda}-\mathbf{m}^t)\|}{\|\mathbf{C}^{t-1/2}(\mathbf{x}_{\lambda-i:\lambda}-\mathbf{m}^t)\|} \times \frac{\mathbf{x}_{\lambda-i:\lambda}-\mathbf{m}^t}{\sigma^t}$ . The covariance matrix estimation of these worst solutions is used to decrease the variance of the mutation distribution along these directions:

$$\mathbf{C}^{t+1} = (1 - c_1 - c_\mu + c^- \alpha_{old}^-) \mathbf{C}^t + c_1 \mathbf{p}_c^{t+1} \mathbf{p}_c^{t+1T} + (c_\mu + c^- (1 - \alpha_{old}^-)) \mathbf{C}_\mu^+ - c^- \mathbf{C}_\mu^-, \quad (2)$$

where  $\mathbf{p}_c^{t+1}$  is adapted along the evolution path and coefficients  $c_1$ ,  $c_\mu$ ,  $c^-$  and  $\alpha_{old}^-$  are defined such that  $c_1 + c_\mu - c^- \alpha_{old}^- \leq 1$ . The interested reader is referred to [7], [9] for a more detailed description of these algorithms.

A potential issue of the active update is that the positive definiteness of the covariance matrix cannot be guaranteed anymore, that may result in algorithmic instability. According to [12], this issue is not observed on the BBOB benchmark suite [5]. In our experiments with the CEC 2013 benchmark suite this issue is also never observed.

### III. RESTART STRATEGIES FOR CMA-ES

#### A. Preliminary Analysis

The CMA-ES algorithm is a local search optimizer and its default population size  $\lambda_{default}$  has been tuned for unimodal functions. On multi-modal functions, however, it can get stuck in local optima and the convergence to global optima is not guaranteed. Various approaches to increase the probability of finding global optima have been proposed, many of them belong to i). niching approaches and ii). restart strategies.

A representative approach of the first category is the CMA-ES with the fitness sharing [15], where the niche radius is adapted during the search that allows to keep several running individual CMA-ES instances on a certain distance from each other, and, thus, maintain some diversity. Another example is the NBC-CMA-ES algorithm [14] with the niching via Nearest-Better Clustering (NBC) which is employing a radius-free basin identification method. In this approach, the niches are dynamically identified and the corresponding points are used to form populations for individual CMA-ES instances. According to [14], for very highly multi-modal functions, the effort invested into the coordination of local searches often does not pay off as it becomes almost impossible to identify enough basins of attraction to obtain an advantage over uncoordinated restarts.

The second category of restart strategies is not that different from the first one since restarts also can be viewed as a parallelized search, but rather in the time than in space [14]. A milestone paper [6] investigated the probability of reaching the global optimum (and the overall number of function evaluations needed to do so) w.r.t. the population size of CMA-ES. The analysis of empirical results demonstrated that, indeed, this probability is very sensitive to the population size and that the default population size of CMA-ES is rather too small. The restart strategies described in the following sections are inspired by an idea of exploring CMA-ES hyper-parameters such as the population size and the initial step-size.

#### B. The IPOP-CMA-ES and IPOP-aCMA-ES

As mentioned, [6] demonstrated that increasing the population size improves the performance of CMA-ES on multi-modal functions. The authors of [6] suggested a restart strategy for CMA-ES with successively increasing population size. Such an algorithm was later introduced in [2] as IPOP-CMA-ES. IPOP-CMA-ES only aims at increasing the population size  $\lambda$ . Each time at least one of the stopping criteria is met by the CMA-ES, it launches a new CMA-ES with population size  $\lambda = \rho_{inc}^{i_{restart}} \lambda_{default}$ , where  $i_{restart}$  is the index of the restart and  $\lambda_{default}$  is the default population size. Factor  $\rho_{inc}$  must be not too large to avoid "overjumping" some possibly optimal population size  $\lambda^*$ ; in [2] it is set to  $\rho_{inc} = 2$  that in certain cases allows to keep a potential loss in terms of function evaluations (compared to the "oracle" restart strategy which would directly set the population size to the optimal value  $\lambda^*$ ) by about a factor of 2.

The active version of IPOP-CMA-ES (IPOP-aCMA-ES) has been proposed in [9].

#### C. The BIPOP-CMA-ES and BIPOP-aCMA-ES

In BIPOP-CMA-ES [4] after the first single run with default population size, the algorithm is restarted in one of two possible regimes and account the budget of function evaluations spent in the corresponding regime. Each time the algorithm is restarted, the regime with smallest budget used so far is used.

Under the first regime the population size is doubled as  $\lambda_{large} = 2^{i_{restart}} \lambda_{default}$  in each restart  $i_{restart}$  and use some fixed initial step-size  $\sigma_{large}^0 = \sigma_{default}^0$ . This regime corresponds to the IPOP-CMA-ES.

Under the second regime the CMA-ES is restarted with some small population size  $\lambda_{small}$  and step-size  $\sigma_{small}^0$ , where  $\lambda_{small}$  is set to

$$\lambda_{small} = \left\lfloor \lambda_{default} \left( \frac{1}{2} \frac{\lambda_{large}}{\lambda_{default}} \right)^{U[0,1]^2} \right\rfloor, \quad (3)$$

Here  $U[0,1]$  denotes independent uniformly distributed numbers in  $[0,1]$  and  $\lambda_{small} \in [\lambda_{default}, \lambda/2]$ . The initial step-size is set to  $\sigma_{small}^0 = \sigma_{default}^0 \times 10^{-2U[0,1]}$ .

In each restart, BIPOP-CMA-ES selects the restart regime with less function evaluations used so far. Since the second regime uses a smaller population size, it is therefore launched more often.

The active version of BIPOP-CMA-ES (BIPOP-aCMA-ES) has been proposed in [12].

#### D. The NIPOP-aCMA-ES

The NIPOP-aCMA-ES [12] is an alternative restart strategy to the IPOP-aCMA-ES, where in addition to increasing of population size in each restart, the initial step-size is also decreased by some factor  $k_{\sigma dec}$ . In [12], this factor is set to  $k_{\sigma dec} = 1.6$  such that  $\sigma$  value after 9 restarts (the default maximum number of restarts in BIPOP-aCMA-ES) roughly corresponds to the minimum possible initial  $\sigma = 10^{-2} \sigma_{default}$ .

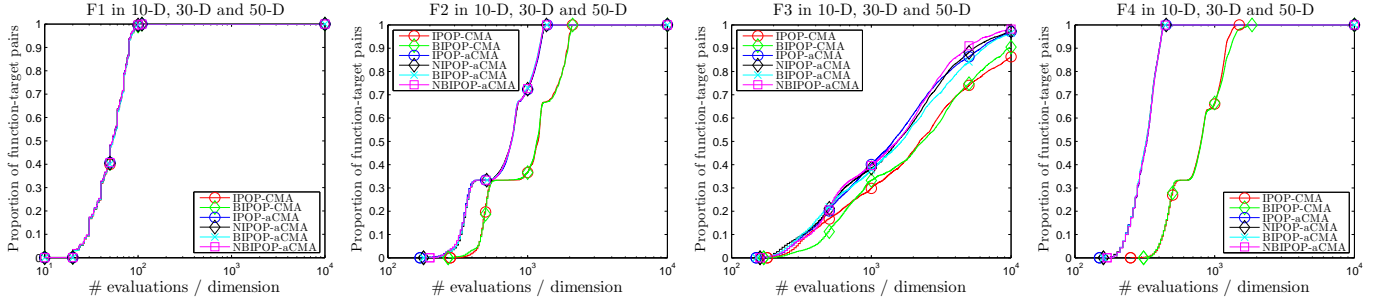


Fig. 2. Empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 300 function-target pairs in  $10^{[-1..4]}$  (100 pairs for each of dimensions 10, 30 and 50) for F1, F2, F3.

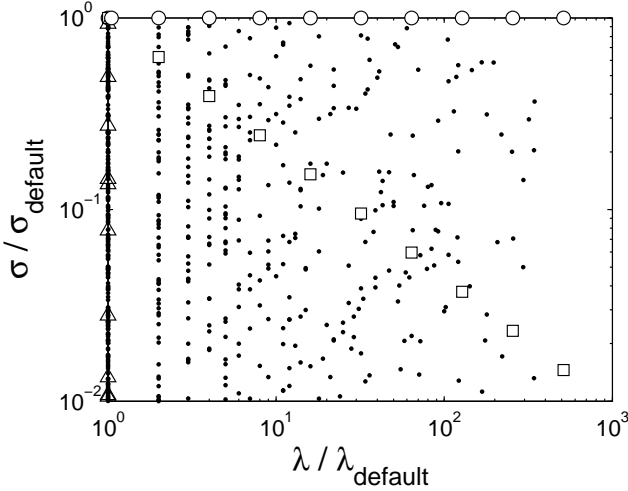


Fig. 1. An illustration of  $\lambda$  and  $\sigma$  hyper-parameters distribution for 9 restarts of IPOP-aCMA-ES ( $\circ$ ), BIPOP-aCMA-ES ( $\circ$  and  $\cdot$  for 10 runs), NIPOP-aCMA-ES ( $\square$ ) and NBIPOP-aCMA-ES ( $\square$  and many  $\triangle$  for  $\lambda/\lambda_{default} = 1$ ,  $\sigma/\sigma_{default} \in [10^{-2}, 10^0]$ ). The first run of all algorithms corresponds to the point with  $\lambda/\lambda_{default} = 1$ ,  $\sigma/\sigma_{default} = 1$ .

used for BIPOP-aCMA-ES. This strategy represents an alternative to the BIPOP-aCMA-ES in the case if the restart strategy is restricted to increasing of population size. It also outperforms IPOP-aCMA-ES and is competitive with BIPOP-aCMA-ES on the BBOB noiseless problems [13].

#### E. The NBIPOP-aCMA-ES

In NBIPOP-aCMA-ES [12] as well as in BIPOP-aCMA-ES there are two restart regimes:

- i). Double the population size and decrease the initial step-size by  $k_{\sigma dec} = 1.6$  (NIPOP-aCMA-ES).
- ii). Launch CMA-ES with default population size  $\lambda_{default}$  and  $\sigma^0 = \sigma_{default}^0 \times 10^{-2U[0,1]}$ .

In contrast with BIPOP-aCMA-ES, where both regimes have the same budget, the budget is adapted here according to the performance of the regime: the best solutions  $x_A^*$  and  $x_B^*$  found by regimes A and B are used as an estimate of the quality of the regimes. Thus,  $k_{budget} = 2$  times larger computation budget is allocated for regime A if it performs better than B (i.e., if  $x_A^*$  is better than  $x_B^*$ ), and vice versa.

The NBIPOP-aCMA-ES typically outperforms IPOP-aCMA-ES, BIPOP-aCMA-ES and NIPOP-aCMA-ES on the BBOB noiseless problems [13], especially in larger dimensions.

All the above described algorithms can be viewed as some search algorithms in the space of hyper-parameters  $\lambda$  and  $\sigma$ . The typical patterns of these search algorithms are shown in Fig. 1.

#### IV. EXPERIMENTAL VALIDATION

The experimental validation investigates the performance of 6 CMA-ES restart strategies: IPOP-CMA-ES, BIPOP-CMA-ES, IPOP-aCMA-ES, BIPOP-aCMA-ES, NBIPOP-aCMA-ES, NBIPOP-aCMA-ES. We use the source code <sup>1</sup> provided by the authors of [12], which is based on the original MATLAB code <sup>2</sup> of CMA-ES provided by N. Hansen. Both for IPOP and BIPOP versions the default parameter settings are used as given in [9], [4], [12]. The initial step-size  $\sigma$  is chosen according to the given search range  $[-100; 100]$  as  $0.6 \cdot 200 = 120$ .

For all functions and dimensions the maximum number of function evaluations was set to  $10000n$ .

##### A. Results

The results individually for each function and problem dimension are given according to [10] in Tables II-XIX after the maximum number of function evaluations.

To assess the performance of the algorithms we use a procedure similar to one used in BBOB framework: for each objective function we define a set of function-target pairs  $\Delta f_t$  in the range  $[10^{-1}, 10^4]$ . The lower bound of  $10^{-1}$  is chosen because for most of *multi-modal* functions the objective values below  $10^{-1}$  are usually difficult to achieve. Fig. 2 and 3 depict the empirical cumulative distribution of running time of the annotated algorithm individually on all objective functions. Importantly, the results for all 3 problem dimensions and 51 runs are aggregated such that if the proportion of function-target pairs equals to 1 after a given number of function evaluations, then all  $3 \cdot 100 = 300$  function-target pairs have been solved 51 times (i.e., 15300 problems solved) by the corresponding algorithm. For some functions, e.g., F20, the y-axis is scaled to better illustrate the difference in performances.

<sup>1</sup><https://sites.google.com/site/ppsnbipop/>

<sup>2</sup>[https://www.lri.fr/~hansen/cmaes\\_inmatlab.html](https://www.lri.fr/~hansen/cmaes_inmatlab.html)

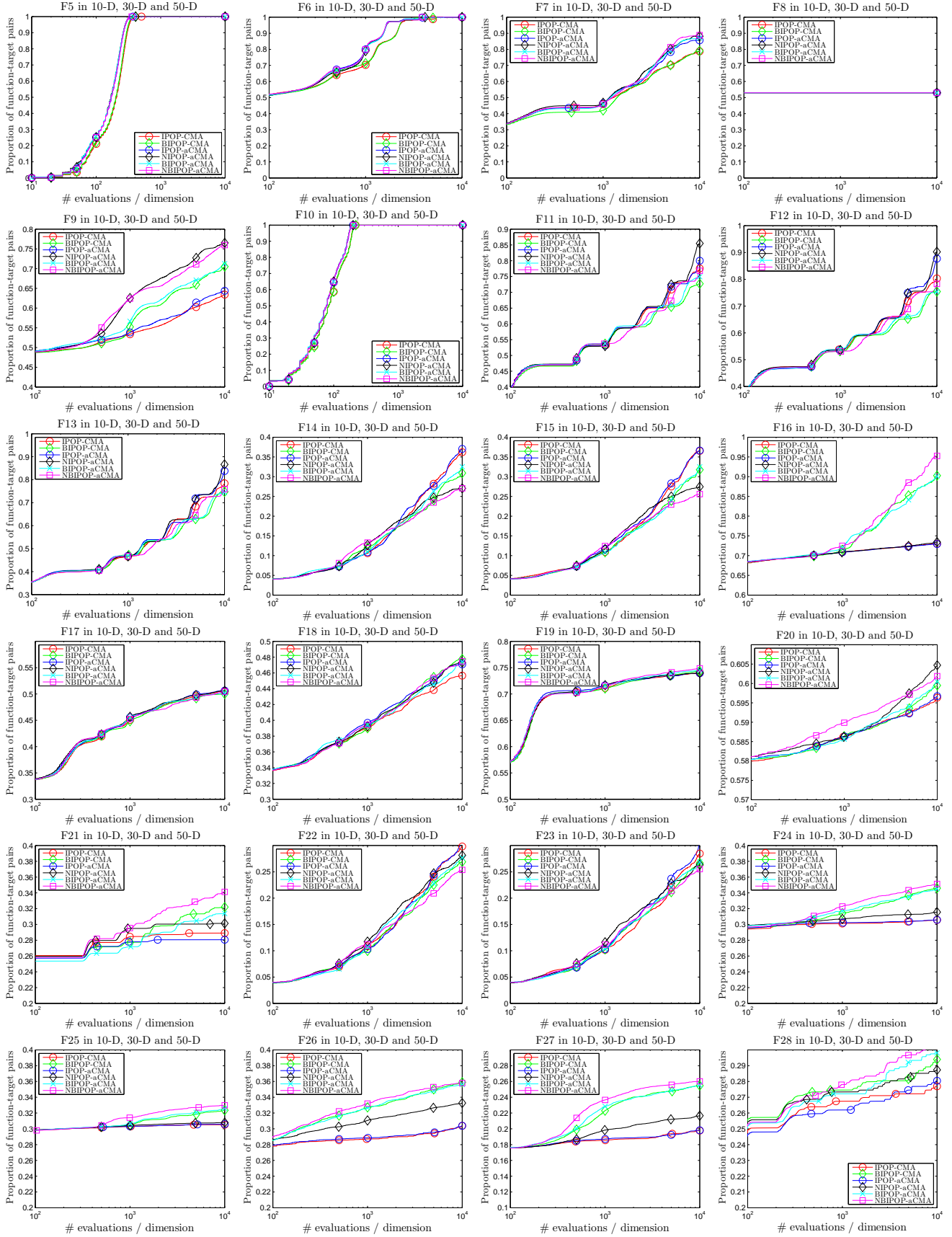


Fig. 3. Continuation of Fig. 2.

TABLE I. COMPUTATIONAL COMPLEXITY OF ALL 6 ALGORITHMS GIVEN FOR 10-, 30- AND 50-DIMENSIONAL SCHWEFEL'S FUNCTION (F14).

	T0	T1	(T2 - T1) / T0 for IPOP-CMA	IPOP-aCMA	BIPOP-CMA	BIPOP-aCMA	NIPOP-aCMA	NBIPOP-aCMA
D=10	0.277	1.778	22.64	25.45	45.97	45.08	59.26	62.39
D=30	0.277	2.929	38.20	45.66	56.20	64.96	63.64	68.11
D=50	0.277	4.159	57.29	69.41	84.06	85.43	102.38	103.79

**Active covariance matrix update.** The active versions of CMA-ES clearly outperform the original ones on unimodal ill-conditioned functions F2, F3, F4. A substantial improvement is also observed on F5, F6, F7. The only function, where the original versions seem to perform better is F21 composition function of functions F1, F3, F4, F5 and F6, i.e., on which the active versions actually perform better. This is an unexpected result and requires further analysis.

**BIPOP vs IPOP.** BIPOP-based algorithms outperform IPOP-based algorithms on F9, F14, F16, F20, F21, F24, F25, F26, F27, F28, and are outperformed by the latter on F11, F12, F13, F14 and F15. While in some cases the difference is minor, in overall, BIPOP-based algorithms perform better on composition functions.

**NBIPOP and NIPOP vs BIPOP and IPOP.** The alternative restart strategies outperform the original ones on F9, F12, F16, F20, F24, F25, F26, F27, F28, and demonstrate a comparable performance on other functions.

**Computational Complexity.** The results of experimental runs on F14 Schwefel's function are given in Table I according to [10]. The restart strategies where smaller population sizes are used (e.g., NBIPOP-aCMA-ES) spend more time on internal computations per function evaluation, and are typically up to 2 times slower in terms of time than IPOP-CMA-ES.

## V. CONCLUSION AND PERSPECTIVES

In this paper, we have compared the original and recently proposed restart strategies for CMA-ES on the CEC 2013 test suite. The aggregated results depicted in Fig. 4 demonstrate a slightly better performance of the NBIPOP-aCMA-ES and NIPOP-aCMA-ES. A possible reason is that a smaller initial step-size is especially useful on composition functions where the basins of attractions are relatively small. The results also confirm some superiority of the active covariance matrix update.

The main limitation of all tested approaches is that the search in the hyper-parameter space of the population size and initial step-size seems to be inefficient and some potentially useful information from the restarts (e.g., the location of the best found solution) is not used. Another important limitation inherited from the CMA-ES is a lack of functionality which would allow to detect and exploit the separability of the objective function. Thus, the algorithms which specifically focus on separable and partially-separable functions will very likely outperform the CMA-ES and its restarts strategies. The above-described issues need to be addressed in future work.

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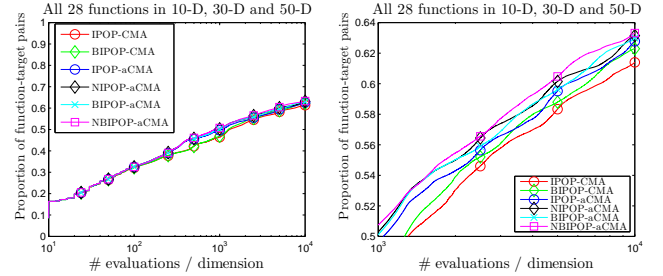


Fig. 4. Empirical cumulative distribution of all function-target pairs solved on all functions, dimensions and runs (in overall, 428400 pairs).

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TABLE II. IPOP-CMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	20.187	20.471	20.352	20.342	0.070
9	0.000	3.121	0.199	0.582	0.721
10	0.000	0.000	0.000	0.000	0.000
11	0.000	1.990	0.000	0.332	0.551
12	0.000	0.995	0.000	0.098	0.299
13	0.000	1.990	0.000	0.313	0.508
14	3.602	167.830	18.535	26.681	26.673
15	0.312	58.398	18.535	21.955	15.243
16	0.905	1.542	1.124	1.152	0.136
17	10.382	12.430	10.984	11.068	0.430
18	10.258	11.953	10.951	10.974	0.414
19	0.440	0.919	0.646	0.646	0.115
20	1.547	4.019	3.019	2.763	0.592
21	100.000	400.190	400.190	374.677	71.735
22	9.902	313.190	56.512	73.252	49.446
23	14.224	318.930	59.212	86.163	66.062
24	200.000	225.200	208.450	209.465	7.022
25	200.000	224.120	203.610	205.517	6.709
26	106.960	218.160	205.810	204.201	15.094
27	319.700	560.530	446.630	454.562	79.234
28	300.000	300.000	300.000	300.000	0.000

TABLE III. BIPOP-CMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.002	0.000	0.000	0.000
8	20.185	20.517	20.359	20.339	0.082
9	0.000	2.638	0.104	0.554	0.684
10	0.000	0.000	0.000	0.000	0.000
11	0.000	2.985	0.995	0.936	0.806
12	0.000	1.990	0.995	0.585	0.603
13	0.000	3.651	0.995	0.944	0.838
14	3.665	359.080	33.529	54.762	69.842
15	3.665	258.190	39.989	52.837	57.800
16	0.000	1.593	0.090	0.305	0.457
17	10.550	13.109	11.500	11.551	0.559
18	6.299	14.018	11.706	11.565	1.159
19	0.378	0.951	0.591	0.604	0.120
20	0.828	3.559	2.621	2.582	0.552
21	100.000	400.190	300.000	284.407	122.356
22	38.725	339.800	78.204	99.325	65.612
23	34.232	302.550	110.520	117.411	65.091
24	100.000	207.740	110.700	130.304	38.730
25	109.600	207.740	202.260	192.516	28.442
26	100.000	200.020	107.960	118.570	24.895
27	186.920	447.100	354.120	346.434	44.001
28	100.000	300.000	300.000	280.392	60.065

TABLE IV. NIPOP-ACMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	20.189	20.470	20.357	20.353	0.064
9	0.000	1.782	0.000	0.254	0.457
10	0.000	0.000	0.000	0.000	0.000
11	0.000	0.995	0.000	0.267	0.441
12	0.000	0.995	0.000	0.078	0.270
13	0.000	1.026	0.000	0.254	0.439
14	3.602	336.200	142.300	140.243	97.669
15	3.727	332.870	109.320	129.231	96.480
16	0.000	1.529	1.121	1.055	0.314
17	10.310	11.968	10.980	11.006	0.382
18	10.346	11.439	10.824	10.849	0.257
19	0.059	0.953	0.679	0.658	0.159
20	1.563	3.606	2.479	2.417	0.455
21	100.000	400.190	400.190	350.538	87.857
22	21.790	375.240	98.857	146.533	110.690
23	18.237	506.830	180.540	196.642	117.469
24	100.000	206.400	108.070	149.495	49.703
25	100.000	207.430	200.000	196.967	19.858
26	49.144	200.020	100.990	123.776	43.711
27	300.000	547.980	325.980	350.654	67.183
28	109.340	300.000	300.000	292.859	35.738

TABLE V. IPOP-ACMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	20.156	20.474	20.359	20.353	0.075
9	0.000	3.000	0.000	0.504	0.720
10	0.000	0.000	0.000	0.000	0.000
11	0.000	1.990	0.000	0.351	0.520
12	0.000	0.995	0.000	0.078	0.270
13	0.000	1.990	0.000	0.254	0.482
14	0.187	65.170	21.825	23.576	14.898
15	0.250	125.390	18.472	24.250	22.438
16	0.526	1.598	1.222	1.169	0.228
17	10.262	12.014	10.784	10.846	0.317
18	10.227	13.002	11.021	11.076	0.548
19	0.458	0.873	0.658	0.655	0.097
20	1.512	4.035	2.604	2.719	0.609
21	200.000	400.190	400.190	380.564	60.122
22	16.683	259.830	58.989	70.667	42.842
23	16.404	243.670	59.730	80.393	49.941
24	200.000	225.180	205.850	209.341	8.700
25	200.000	222.810	203.370	203.944	4.543
26	108.950	220.010	202.730	199.885	20.951
27	304.010	559.070	462.610	450.354	89.518
28	300.000	300.000	300.000	300.000	0.000

TABLE VI. BIPOP-ACMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	20.000	20.468	20.355	20.345	0.082
9	0.000	2.635	0.263	0.522	0.644
10	0.000	0.000	0.000	0.000	0.000
11	0.000	2.985	0.000	0.587	0.721
12	0.000	2.985	0.013	0.644	0.766
13	0.000	2.396	0.995	0.733	0.688
14	0.312	271.630	27.012	40.118	49.940
15	0.125	317.210	28.596	47.743	65.849
16	0.000	1.386	0.051	0.174	0.331
17	4.490	13.014	11.466	11.409	1.101
18	7.703	13.686	11.340	11.336	0.974
19	0.339	0.900	0.575	0.589	0.116
20	1.001	3.547	2.551	2.538	0.583
21	100.000	400.190	400.190	315.805	113.884
22	29.353	233.900	66.714	82.324	45.521
23	25.760	367.530	85.528	96.398	55.902
24	102.720	212.520	108.570	127.376	37.218
25	103.460	206.820	201.300	184.849	36.093
26	73.031	200.020	107.420	121.129	32.253
27	300.000	400.000	337.710	340.883	29.434
28	100.000	300.000	300.000	260.784	80.196

TABLE VII. NBIPOP-ACMA-ES IN 10-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	20.000	20.520	20.353	20.339	0.090
9	0.000	1.503	0.000	0.232	0.440
10	0.000	0.000	0.000	0.000	0.000
11	0.000	1.990	0.000	0.364	0.506
12	0.000	2.985	0.000	0.238	0.542
13	0.000	2.836	0.001	0.484	0.676
14	6.892	356.120	76.816	114.997	92.377
15	18.597	659.330	151.310	158.161	117.317
16	0.011	1.369	0.054	0.120	0.263
17	10.333	12.390	11.369	11.334	0.545
18	7.956	16.995	11.071	11.288	1.276
19	0.010	0.876	0.518	0.525	0.139
20	1.198	3.795	2.761	2.726	0.650
21	100.000	200.000	200.000	152.941	50.410
22	36.355	451.250	141.820	175.131	114.655
23	24.453	512.870	129.180	174.230	122.831
24	100.000	202.240	107.870	119.885	32.220
25	100.000	205.770	200.060	176.972	39.918
26	100.000	246.650	105.970	111.035	24.986
27	172.980	360.600	311.620	316.684	29.556
28	100.000	300.000	300.000	249.020	88.029



TABLE VIII. IPOP-CMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	64.878	0.000	1.732	9.296
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	55.451	2.843	16.835	19.624
8	20.765	21.006	20.956	20.931	0.059
9	1.213	41.165	37.269	24.463	16.090
10	0.000	0.000	0.000	0.000	0.000
11	0.000	6.965	2.096	2.290	1.452
12	0.071	5.970	1.990	1.853	1.164
13	0.000	12.135	1.990	2.414	2.266
14	60.072	1277.400	185.840	287.008	272.130
15	29.083	1055.100	344.610	537.708	241.796
16	1.914	3.191	2.539	2.528	0.273
17	32.431	39.212	33.577	34.073	1.355
18	32.044	181.730	40.312	81.650	61.282
19	1.177	3.203	2.527	2.484	0.402
20	13.737	15.000	14.585	14.603	0.349
21	200.000	300.000	300.000	254.902	50.254
22	120.550	1483.100	420.510	502.379	309.407
23	91.710	1869.600	517.520	576.071	350.245
24	219.630	306.160	300.270	285.725	30.214
25	205.270	302.720	298.280	286.874	28.505
26	200.000	403.450	323.380	314.510	81.420
27	483.550	1326.300	1281.600	1141.729	290.392
28	300.000	300.000	300.000	300.000	0.000

TABLE IX. BIPOP-CMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	3.638	0.000	0.082	0.509
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	46.223	1.156	9.426	13.302
8	20.799	21.017	20.936	20.935	0.051
9	0.231	12.008	6.419	6.489	2.380
10	0.000	0.000	0.000	0.000	0.000
11	0.000	6.965	2.985	3.082	1.519
12	0.000	5.970	1.990	2.410	1.430
13	0.000	6.853	1.990	2.391	1.474
14	51.251	4146.200	514.060	669.207	697.842
15	56.319	2228.500	492.590	609.778	450.755
16	0.002	2.826	0.042	0.775	1.143
17	33.612	40.999	36.002	36.343	1.770
18	32.257	172.370	41.362	54.364	33.956
19	1.265	3.309	2.497	2.395	0.418
20	12.392	15.000	14.344	14.237	0.636
21	100.000	300.000	200.000	200.000	28.284
22	113.670	2906.700	705.840	838.581	577.102
23	189.780	2776.000	664.470	716.942	452.235
24	117.230	300.540	161.900	180.398	50.160
25	214.860	302.750	224.960	231.191	21.156
26	111.940	205.460	148.750	163.826	32.299
27	378.550	660.750	513.360	503.581	71.345
28	100.000	300.000	300.000	292.157	39.208

TABLE X. NIPOP-ACMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	40.445	0.044	4.055	9.172
8	20.755	21.057	20.928	20.921	0.062
9	0.000	5.486	2.927	2.823	1.228
10	0.000	0.000	0.000	0.000	0.000
11	0.000	3.980	0.995	1.032	1.040
12	0.000	2.985	0.031	0.656	0.825
13	0.000	3.049	0.995	0.931	0.928
14	201.210	1328.000	712.620	716.645	244.372
15	90.572	1334.200	668.950	670.256	280.430
16	1.508	3.092	2.549	2.484	0.314
17	31.754	39.261	34.100	34.248	1.716
18	31.905	171.940	35.104	53.961	44.520
19	1.130	3.324	2.482	2.408	0.465
20	10.012	15.000	13.529	13.365	1.260
21	200.000	300.000	200.000	241.176	49.705
22	116.060	2326.200	530.650	572.759	341.016
23	82.274	1546.600	632.300	667.436	326.261
24	220.740	306.150	298.520	290.623	22.816
25	207.770	303.430	298.610	278.962	35.199
26	125.110	361.420	249.560	251.038	57.982
27	320.160	1329.300	639.970	870.294	422.247
28	300.000	300.000	300.000	300.000	0.000

TABLE XI. IPOP-ACMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	120.400	0.118	8.854	22.129
8	20.834	21.023	20.950	20.944	0.045
9	1.500	41.265	38.879	27.216	16.452
10	0.000	0.000	0.000	0.000	0.000
11	0.000	4.396	0.997	1.174	1.092
12	0.000	2.985	0.327	0.704	0.833
13	0.000	4.120	0.995	1.117	1.225
14	54.523	1195.800	225.710	271.876	220.390
15	60.736	1203.100	298.050	336.269	257.532
16	1.968	2.987	2.550	2.529	0.252
17	31.726	39.384	33.396	33.764	1.442
18	31.678	176.400	36.888	70.653	56.460
19	1.207	3.398	2.532	2.466	0.449
20	13.716	15.000	14.537	14.596	0.329
21	200.000	300.000	300.000	254.902	50.254
22	99.474	1249.100	401.690	477.206	293.492
23	119.140	1195.500	444.300	492.216	292.454
24	218.430	304.830	297.660	276.327	34.673
25	211.300	303.330	299.040	289.376	26.732
26	200.000	406.060	350.090	329.264	77.967
27	380.180	1334.000	1272.900	1079.047	344.867
28	300.000	300.000	300.000	300.000	0.000

TABLE XII. BIPOP-ACMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	67.819	0.027	2.727	10.937
8	20.797	21.040	20.950	20.939	0.055
9	1.305	9.197	5.168	5.214	1.949
10	0.000	0.000	0.000	0.000	0.000
11	0.000	5.970	2.985	3.142	1.426
12	0.000	5.970	2.985	2.810	1.575
13	0.995	6.495	1.990	2.646	1.437
14	113.210	1461.700	429.630	495.496	277.647
15	46.739	2217.100	465.140	544.527	416.219
16	0.000	2.914	0.060	0.940	1.203
17	32.490	38.849	35.802	35.702	1.712
18	32.241	178.520	37.687	58.053	43.411
19	1.350	3.275	2.291	2.285	0.323
20	11.551	15.000	14.155	14.015	0.770
21	200.000	300.000	200.000	213.725	34.754
22	118.070	1347.500	636.750	662.775	301.895
23	190.510	1403.400	649.110	702.919	312.690
24	120.200	238.170	213.840	186.292	41.916
25	210.660	301.460	225.560	226.475	13.732
26	118.910	203.450	154.730	168.368	30.175
27	400.000	909.700	513.390	516.864	93.743
28	100.000	300.000	300.000	284.314	54.305

TABLE XIII. NBIPOP-ACMA-ES IN 30-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	0.003	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.000	33.998	0.057	2.313	6.049
8	20.797	21.013	20.946	20.942	0.048
9	0.401	7.630	2.768	3.300	1.383
10	0.000	0.000	0.000	0.000	0.000
11	0.000	6.965	2.985	3.043	1.413
12	0.000	5.970	2.985	2.907	1.376
13	0.000	7.963	2.985	2.778	1.453
14	278.650	2221.100	739.970	810.125	360.294
15	282.650	1674.500	744.850	765.493	294.867
16	0.014	2.784	0.041	0.440	0.926
17	32.451	40.384	33.593	34.419	1.869
18	32.191	186.960	39.560	62.289	45.591
19	1.103	2.866	2.233	2.228	0.341
20	11.117	13.636	13.131	12.940	0.598
21	100.000	200.000	200.000	192.157	27.152
22	129.850	2390.800	734.260	838.392	459.988
23	188.220	1835.500	666.730	667.086	289.554
24	122.800	230.390	155.520	161.757	30.045
25	154.010	229.260	221.920	219.984	11.094
26	128.850	291.790	146.760	158.223	29.999
27	350.550	606.710	471.890	468.925	73.770
28	100.000	300.000	300.000	268.627	73.458



TABLE XIV. IPOP-CMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	173190.000	1.988	6506.587	27617.764
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.048	195.500	11.322	22.928	39.616
8	20.841	21.178	21.134	21.123	0.052
9	3.167	75.313	71.931	59.601	25.270
10	0.000	0.000	0.000	0.000	0.000
11	0.001	21.889	6.965	8.506	5.594
12	0.000	20.894	5.970	6.117	4.359
13	0.000	93.992	5.573	10.804	16.777
14	150.500	13329.000	780.840	1625.565	2921.834
15	102.580	12866.000	801.890	1357.597	2387.453
16	2.717	3.776	3.336	3.315	0.277
17	53.291	79.900	57.218	58.214	4.370
18	54.106	360.530	328.620	228.534	135.806
19	2.022	5.935	4.518	4.413	0.789
20	25.000	25.000	25.000	25.000	0.000
21	200.000	1122.200	200.000	516.822	408.086
22	152.770	13113.000	1042.700	1825.791	2860.190
23	164.350	13349.000	1133.400	2986.475	4190.174
24	244.560	391.680	385.340	375.023	33.360
25	239.930	388.480	383.120	373.787	33.452
26	200.000	491.740	481.770	382.372	129.421
27	699.690	2200.700	2130.500	1936.220	454.537
28	400.000	3400.600	400.000	1034.771	1222.556

TABLE XV. BIPOP-CMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	226180.000	0.044	8858.325	37582.750
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.095	358.290	11.802	45.355	79.408
8	20.996	21.190	21.139	21.128	0.039
9	4.838	25.721	13.283	13.711	4.507
10	0.000	0.000	0.000	0.000	0.000
11	1.054	21.889	7.960	9.275	5.319
12	0.995	17.909	6.965	8.116	3.704
13	0.000	33.058	5.970	7.636	6.106
14	217.950	5743.500	1354.700	1697.035	1445.075
15	159.450	4883.900	930.790	1305.709	1101.514
16	0.003	3.801	0.069	1.562	1.662
17	54.361	94.885	61.199	61.619	5.778
18	54.870	359.620	74.722	138.883	117.742
19	3.099	5.892	4.264	4.347	0.560
20	22.118	25.000	23.573	23.527	0.642
21	200.000	836.440	200.000	224.958	124.767
22	269.680	12181.000	1262.300	1765.215	1731.078
23	202.850	6981.000	1329.100	1922.464	1655.305
24	169.060	385.270	248.760	246.799	32.156
25	229.770	386.390	253.630	259.932	32.909
26	128.880	205.240	178.600	177.402	23.863
27	400.060	929.000	743.580	736.518	101.757
28	400.000	400.000	400.000	400.000	0.000

TABLE XVI. NIPOP-ACMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	832.750	0.000	18.773	116.751
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.035	207.120	1.910	11.714	37.728
8	20.963	21.183	21.117	21.111	0.044
9	2.058	11.924	7.054	6.882	1.819
10	0.000	0.000	0.000	0.000	0.000
11	0.000	5.970	1.990	1.991	1.305
12	0.000	5.970	0.995	1.366	1.289
13	0.000	3.982	0.995	1.481	1.009
14	450.280	2700.500	1204.200	1257.493	442.403
15	365.230	2537.000	1331.800	1352.059	504.095
16	2.580	4.025	3.386	3.370	0.296
17	53.824	67.491	57.287	57.737	2.373
18	55.423	356.310	106.450	193.660	134.807
19	2.571	5.272	4.544	4.467	0.521
20	19.790	25.000	22.969	22.985	1.370
21	200.000	1122.200	200.000	365.287	325.247
22	218.400	2165.900	895.340	1017.509	466.984
23	253.900	3175.900	938.890	1186.484	690.262
24	237.470	392.150	382.040	370.404	37.743
25	215.640	387.580	382.570	365.032	48.659
26	200.000	493.560	311.420	288.263	98.170
27	520.440	2183.300	2119.100	1898.799	520.000
28	400.000	3332.600	400.000	571.590	693.198

TABLE XVII. IPOP-ACMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	151.320	0.002	5.446	22.085
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.074	334.040	3.701	14.791	47.056
8	21.012	21.184	21.124	21.122	0.031
9	4.115	75.639	72.368	51.913	29.071
10	0.000	0.000	0.000	0.000	0.000
11	0.000	21.889	7.960	8.545	4.595
12	0.000	11.940	1.991	3.535	3.597
13	0.000	66.642	3.980	6.066	10.345
14	155.470	11724.000	632.460	1138.034	2040.146
15	115.590	11956.000	639.630	1067.475	1732.580
16	2.593	3.883	3.392	3.356	0.271
17	55.082	76.011	57.460	58.697	4.181
18	54.672	360.110	73.931	164.545	129.500
19	2.439	6.125	4.507	4.472	0.553
20	25.000	25.000	25.000	25.000	0.000
21	200.000	1122.200	836.440	645.944	407.072
22	238.960	12547.000	832.130	1406.937	1989.910
23	225.600	12190.000	895.690	1201.597	1669.776
24	240.740	389.950	385.920	380.249	27.161
25	218.540	387.800	382.090	366.253	45.626
26	200.000	492.920	360.670	370.441	125.046
27	664.700	2181.700	2128.900	2048.754	315.653
28	400.000	3345.400	400.000	856.214	1067.395

TABLE XVIII. BIPOP-ACMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	1030.200	0.012	28.663	150.674
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.084	212.400	8.221	20.688	36.151
8	20.996	21.185	21.127	21.127	0.034
9	5.075	24.628	12.349	12.505	3.778
10	0.000	0.000	0.000	0.000	0.000
11	0.000	14.924	6.965	7.323	3.542
12	2.985	17.909	6.965	6.773	2.896
13	1.026	26.939	6.064	7.131	4.187
14	220.000	4419.200	1129.500	1253.627	857.811
15	195.040	3854.700	1218.600	1399.566	923.669
16	0.005	3.916	0.418	1.673	1.696
17	54.955	68.581	58.751	60.136	3.608
18	54.805	360.920	81.181	158.085	123.762
19	2.587	5.346	4.294	4.240	0.572
20	22.080	25.000	23.663	23.752	0.626
21	200.000	1122.200	200.000	267.999	211.417
22	283.460	4940.200	1402.500	1628.679	1025.878
23	102.160	4956.800	1330.600	1752.374	1054.910
24	142.300	387.080	248.860	245.757	32.071
25	223.190	383.050	251.220	256.735	33.322
26	117.910	204.170	200.000	184.264	24.067
27	400.000	988.050	735.360	716.665	141.348
28	400.000	400.000	400.000	400.000	0.000

TABLE XIX. NBIPOP-ACMA-ES IN 50-D

Func.	Best	Worst	Median	Mean	Std
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000
3	0.000	866.850	0.000	18.166	121.348
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000
7	0.039	19.629	3.646	4.971	5.724
8	20.969	21.186	21.131	21.119	0.045
9	2.022	12.466	7.058	7.220	2.286
10	0.000	0.000	0.000	0.000	0.000
11	0.998	11.940	5.970	5.505	2.959
12	0.007	9.950	4.975	5.371	2.540
13	0.995	29.051	6.965	7.595	5.468
14	381.240	3312.300	1335.000	1375.403	566.544
15	333.750	3413.000	1495.500	1553.688	548.191
16	0.018	3.864	0.044	0.878	1.441
17	54.741	66.344	56.737	57.369	2.726
18	55.241	352.130	104.030	133.647	100.310
19	3.040	5.370	4.422	4.458	0.593
20	18.746	24.587	22.738	22.547	1.175
21	100.000	200.000	200.000	198.039	14.003
22	188.710	3858.600	1336.300	1448.353	601.295
23	477.380	4233.400	1492.400	1712.552	809.352
24	194.580	265.320	244.990	239.643	20.380
25	233.430	257.410	248.680	247.570	5.059
26	113.930	223.360	200.000	196.091	14.340
27	390.610	878.060	777.220	727.829	144.098
28	400.000	400.000	400.000	400.000	0.000